Key-Block Theorem Application on Discontinuous Rock Slope Instabilities and Rock Mass Description

Mohammad Azarafza[1(✉)] and Hong-Hu Zhu[2]

[1] Department of Civil Engineering, University of Tabriz, Tabriz, Iran
m.azarafza.geotech@gmail.com

[2] School of Earth Sciences and Engineering, Nanjing University, Nanjing, China
zhh@nju.edu.cn

Abstract. Key-block theorem, also known as block theory or Goodman's approach, is considered the most accurate and flexible method for understanding structural instability conditions of discontinuous rock slopes. This procedure provides useful information on slope failure mechanism, rock blocks geometry, discontinuity networks and slope stability conditions. In this article, the application of the key-block analysis method was considered, which attempted to present a practical review of Goodman's theory in recent advancements.

Keywords: Slope stability · Discontinuous rock slope · Key-block method · Limit equilibrium analysis

1 Introduction

Slope stability is one of the essential issues in geoengineering for more than 300 years. Slope stability is involved in numerous geotechnical projects and covers different aspects of infrastructural developments [1–3]. The slope stability analysis can be categorized in various instabilities of earth or rock slopes, which are classified as primary and secondary groups. The primary group consists of wedge failure, toppling failure (rock slopes), planar failure, rotational failure (rock and earth slopes), and the secondary group consists of composite slips, exceptional cases, glacial slips, etc. [4]. The wedge failure is the most common instability type in discontinuous rock slopes with slip geometry in wedge form, leading to the unstable mass during the functional interface of two or more discontinuities in rock slope. The failure that contains several metres to hundreds of metres of geometric volume movable mass above the main slip surface can be triggered progressively [5]. The planar failure is considered a particular case of wedge failure due to its geometrical status of the discontinuity network of slope mass. This failure occurs along a planar surface, such as bedrock, main discontinuity surface, resistant layer, and dense clay lenses [6, 7]. The toppling failure is considered as anti-dip instability occurred in a discontinuous rock slope. The main discontinuity set and slope surface orientations are opposite, and rock columns are overturned at the geometric centre of the sliding mass [8]. Goodman and Bray (1976) categorized these types of instabilities, which called toppling into the main

H.-H. Zhu et al. (Eds.): BRWSG 2021, LNCE 230, pp. 200–208, 2022.
https://doi.org/10.1007/978-981-16-9963-4_16

and secondary groups [9]. The main group contains systematic toppling named flexural failure, block failure and block-flexural failure. The secondary group includes the non-systematic and complex form of toppling [10, 11]. Finally, the rotational or massive failure mainly occurs in homogeneous masses (i.e., soil, heavily jointed rock, weak geomaterials) where slip surfaces pass through the body of the mass in the most fragile state [7, 12, 13]. Generally, various approaches are used for slope stability assessments to investigate the instability conditions and quantify the failure mechanisms, classified as simple evaluations, planar failures, limit state criteria for limit equilibrium analysis, numerical methods, hybrid, and high-order approaches [2, 3, 14]. In the meantime, limit equilibrium methods (LEMs) are considered the most flexible approach due to their simplicity, continuous access, rapid implementation, accessible assumptions, closed-form analysis, and providing multiple answers, coupled with different methods [15]. This flexibility helps to develop different capable formulations like Goodman's block theory [16].

2 Key-Block Theorem

Block theory, also known as Key-block theory or Goodman's theorem, was introduced by Goodman and Shi in 1985 [16]. It is based on stability assessment of rock blocks using LEM and two primary mechanisms of failure called 'structural failure' because of discontinuities and 'stress-based failure' because of the presence of high stresses. The key-block theory uses the geometrical status of rock blocks and the discontinuity network on rock mass to claim the logical relationship between the block geometry and the critical failure surface of excavated or natural rock slopes [15]. Goodman's theorem aims to search and identify the finite key blocks involving local or progressive instability on discontinuous rock slopes [17]. Thus, the rock blocks' geometric position, discontinuity network, and emplacement on rock mass play essential roles instability assessments and are considered evaluation variables. This theory classified the blocks into finite and infinite groups, which the finite blocks are capable of movement, which causes structural failure. These blocks are divided into different blocks, as well as a presentation in Fig. 1.

Key-block theory uses "finiteness theorem" and "removability theorem" to identify, recognize, classify and analyze the discontinuity network based on spatial equations/inequalities of discontinuities which lead to recorded block information in polar coordinates (i.e., discontinuities' dip/dip direction) as rock mass which can easily convert to Cartesian coordinates [18]. Based on the removability theorem, the polyhedral geometry of blocks was convex convergence form to block's mobility. The non-convergent concave blocks are not movable. According to the finiteness theorem, the joint pyramid (the shared spaces between half-spaces of discontinuity planes that form part of the block pyramid), the excavation pyramid a group of extraction half-spaces (excavation half-spaces) that are displaced to create a block pyramid), space pyramid (complementary half-space of excavation pyramid), and block pyramid (the factor for determining the convexity and concavity of blocks) have to define and categorized [19]. These continuous equations give a mathematical definition for block geometry and their stability analysis leading to [16]:

Infinite blocks

$$\begin{aligned} &\text{Block pyramid} \neq \varnothing \\ &\text{Block pyramid} = \text{Joint pyramid} \cap \text{Excavation pyramid} \\ &\text{Joint pyramid} \cap \text{Excavation pyramid} \neq \varnothing \\ &\text{Joint pyramid} \not\subset \text{Space pyramid} \end{aligned}$$

Finite blocks

$$\begin{aligned} &\text{Block pyramid} = \varnothing \\ &\text{Block pyramid} = \text{Joint pyramid} \cap \text{Excavation pyramid} \\ &\text{Joint pyramid} \cap \text{Excavation pyramid} = \varnothing \\ &\text{Joint pyramid} \subset \text{Space Pyramid} \end{aligned}$$

Conical/trapped blocks

$$\begin{aligned} &\text{Block pyramid} = \varnothing \\ &\text{Block pyramid} = \text{Joint pyramid} \cap \text{Excavation pyramid} \\ &\text{Joint pyramid} \cap \text{Excavation pyramid} = \varnothing \\ &\text{Joint pyramid} = \varnothing \end{aligned}$$

Key blocks

$$\begin{aligned} &\text{Block pyramid} = \varnothing \\ &\text{Joint pyramid} \neq \varnothing \\ &\text{Joint pyramid} \cap \text{Excavation pyramid} = \varnothing \\ &\text{Joint pyramid} \subset \text{Space pyramid} \end{aligned}$$

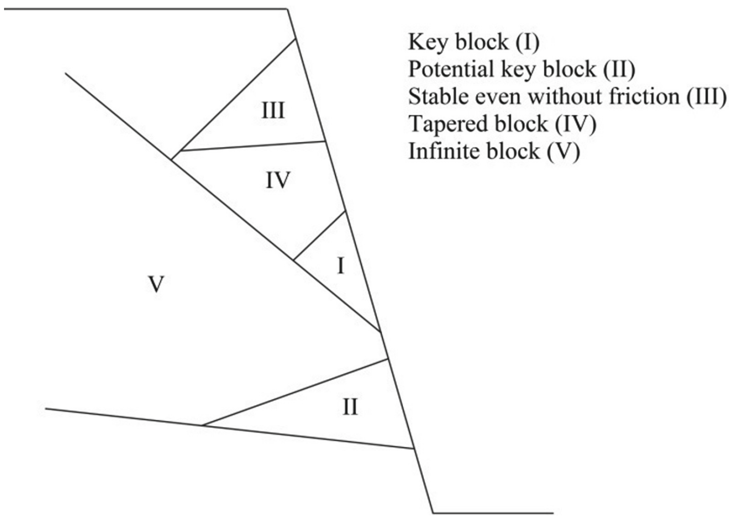

Fig. 1. Rock block classification based on block theory [13].

It is easy to restrict the trending evaluation loops to identify the active key-blocks based on the equations. This advantage has focused on key-block theory in stability analysis [20, 21]. Also, Goodman and Shi (1985) applied the removability theorem to express the tendency trend and removable blocks. They can displace in a particular direction without encountering the neighbour's actions. This is the definition of key-block as it is a key of instability [16, 22–24]. For the removable block [16]:

Block pyramid $= \emptyset$
Block pyramid $=$ Joint pyramid $\cap$ Excavation pyramid
Joint pyramid $\cap$ Excavation pyramid $= \emptyset$
Joint pyramid $\subset$ Space pyramid
Joint pyramid $\neq \emptyset$

They also defined non-removable blocks that blocks cannot move in any direction. The removable and non-removable status is presented in Fig. 2. So, the slope instability in discontinuous rock masses is limited to finite and removable blocks. This finiteness in the LEM-base stability analysis provides the evaluation much faster, more accurately, and less time than traditional methods like manual methods or commercial software.

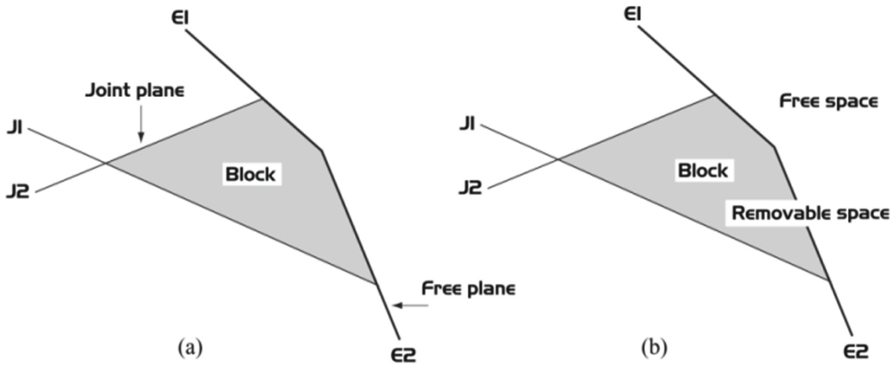

Fig. 2. Rock block in an excavation: (a) intersection of two joint surfaces and two free spaces, (b) removable space of the block [16].

3 Review of Literature on Key-Block Method

Um and Kulatilake (2001) provide a kinematic and LEM analysis based on block theory for stability evaluations on rock slope in Gorges dam site in China [25]. Huang et al. (2000) used key-block basics to describe discontinuity networks in rock slopes [26]. Yarahmadi-Bafghi and Verdel (2003) developed a key-group method based on Goodman's theory [27]. Jimenez-Rodriguez et al. (2006) utilized the systematic quantitative procedure for rock slope stability assessments based on key-block theory [28]. Haswanto and Abd-Ghani (2008) performed a rock slope stability analysis based on key-block theory in Fraser's Hill Pahang in Malaysia [29]. Kulatilake et al. (2011) applied the key-block method to estimate the stability condition of rock slopes at the

Yujian River dam site [15]. Greif and Vlčko (2013) used key-block analyses to evaluate the LEM-based stability status for 45 rock slopes in Slovakia [30]. Azarafza et al. (2013) applied key-block theory to stability analysis of discontinuous rock slopes in the southwest of Iran [31]. Sun et al. (2014) utilized key-block procedure for LEM-based stability analysis of rock slopes in the Jinping-I hydropower station in China [32]. Li et al. (2016) introduced a method based on block-group, and Sarma approaches stability assessment for rock slope mass instabilities [33]. Azarafza et al. (2017) provided the algorithm for three-dimensional simulation of discontinuities emplacements in jointed rock slopes named 3DDGM (three-dimensional discontinuity geometrical modelling) [18]. Also, the key-block based stability analysis was performed on a discontinuous rock slope in South Pars Gas Complex, southwest of Iran [17]. Liu et al. (2017) present the novel semi-deterministic method based on block theory named NSDBT and successfully applied it to Changhe dam in Sichuan, China. They mentioned that NSDBT is capable of identifying block geometry based on key-block instructions [23]. Wang et al. (2018) utilized a multi-level framework based on block theory and analytic hierarchy process, named GeoSMA-3D, to characterize rock block dimension in rock mass [21]. Azarafza et al. (2020) provided the key-block-based fuzzy logical decision-making procedure on estimating the wedge and planar instability status in jointed rock slopes [34]. Later on, Azarafza et al. (2020) introduce the novel approach to investigate the main toppling failures based on the key-block theorem [35].

4 Key-Block Principles

Key-block is built on principles related to the rock blocks geometry, key-block location, discontinuity networks, discontinuity friction, and applied loading direction [36]. The analysis process is mainly covered by LEM and computed safety factors representing the movable body's stability [16]. As Goodman's statements, the key-blocks are the key to rock blocks instabilities that can be single-handed triggered the movements or motivated as key-group [27], leading to the progressive instability in slope mass. In this regard, the following basics have to be satisfied [37]:

- Discontinuity network was considered planar and extended to slope boundaries,
- Key-block have to identify by applying vectorial method,
- Active finite key-block considered as unstable single key when LEM-based safety factor less than 1.0,
- Recognition of finite neighbour blocks as potential triggering key-blocks,
- Building all possible groups related to active finite key-blocks,
- Stability analysis based on LEM as evaluation loops,
- Iteration of new geometry for final stable state slope condition.

5 Real Case Implementation

Several case studies were used to apply the concept of the key-block method to investigate the stability analysis of discontinuous rock slopes. Figure 3 picked the view of the studied slope from Tabriz -Tehran highway path near Tabriz city, located in East-Azerbaijan province, in the northwest of Iran. The studied slope is composed of jointed limestone units in which three joint sets have been identified by field survey. According to the recent excavation, the slope is geometrically modified, leading to local instability in slope mass. The mechanism of failure is obtained as wedge failure and block sliding. So, conducting the key-block method can provide a fast and accurate way to locate the key-block and perform suitable stabilization procedures. Figure 4 illustrates the key-block-based geometrical identification of studied rock slope. The results of experiments and field studies have been used to estimate the geomechanical properties of rock slopes. The Mohr-Coulomb criterion has been used for obtaining the strength properties and LEM analyses. Also, Table 1 shows the results of slope mass mechanical properties. Table 2 and Fig. 5 present the stability analysis of slope based on LEM, which can be stated that the domain is generally stable and the estimated safety factor is up than 1.0. But it is detected some local failures which are required to conducting stabilizations.

Fig. 3. Photograph of the selected slope.

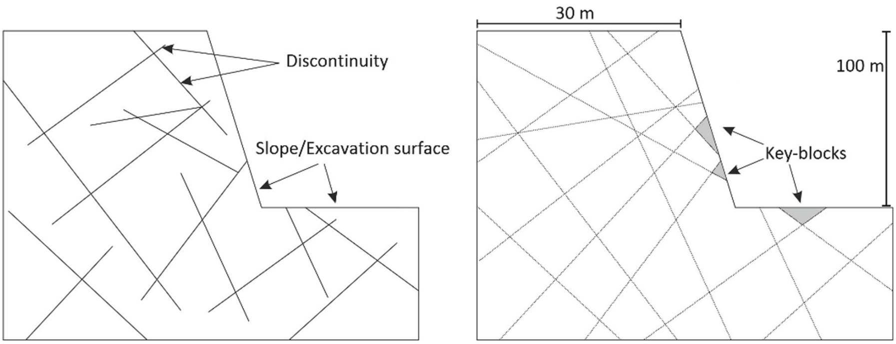

Fig. 4. Key-block-based geometrical identification of studied rock slope.

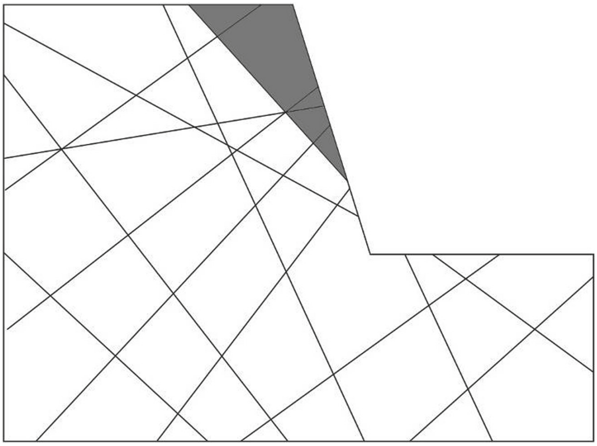

Fig. 5. LEM-based stability status of the rock slope under investigation.

Table 1. Mechanical properties of the slope mass

Parameter	c (kPa)	ϕ (°)	γ (kN/m^3)	E (GPa)	K_n (GPa)	K_s (GPa)	ν
Value	12	32	18.75	75	0.5	0.5	0.33

c: cohesion, ϕ: angle of internal friction, γ: unit weight, E: Young's modulus, ν: Poisson's ratio, K_n & K_s: Normal and shear joint stiffness coefficients

Table 2. Stability analysis by using the key-block method.

Stability	Weight of unstable stone (T_n)	Factor of safety	Analysis method
Stable	1.02	1.17	LEM

6 Conclusions

This paper tried to presents an illustration of the key-block methodology to provide a proper understanding of the outlook of this theory during the past decades. In this regard, the application of key-block theory on jointed rock slope stability is discussed. The principles and basics of the jointed rock slopes that lead to preparing the instability conditions are considered, and the roles of discontinuity network and rock block geometry in failure occurrence were stressed. Different sliding mechanisms of discontinuous rock slopes were mentioned.

Acknowledgments. This research was funded by the National Natural Science Foundation of China (Grant No. 42077235).

References

1. Frasheri, A.: Outlook on the possibility for slope stability evaluation according to petrophysical and integrated geophysical data. In: Proceedings of the 6th International Conference and Exposition on Petroleum Geophysics, Kolkata (2006)
2. Chen, X.-P., Zhu, H., Huang, J., Liu, D.: Stability analysis of an ancient landslide considering shear strength reduction behavior of slip zone soil. Landslides **13**(1), 173–181 (2015). https://doi.org/10.1007/s10346-015-0629-7
3. Zhu, H.H., Wang, Z.Y., Shi, B., Wong, J.K.W.: Feasibility study of strain based stability evaluation of locally loaded slopes: insights from physical and numerical modeling. Eng. Geol. **208**, 39–50 (2016)
4. Bromhead, E.N.: The Stability of Slopes. Spon Press, Milton Park (1998)
5. Zhou, J., Wang, J.: Lower bound limit analysis of wedge stability using block element method. Comput. Geotech. **86**, 120–128 (2017)
6. Wyllie, D.C., Mah, C.: Rock Slope Engineering, 4th edn. Spon Press, Milton Park (2004)
7. Huang, Y.H.: Slope Stability Analysis by the Limit Equilibrium Method. ASCE Publications, Virginia (2014)
8. Nikoobakht, S., Azarafza, M.: Stability analysis and numerical modelling of toppling failure of discontinuous rock slope (A case study). J. Geotech. Geol. **12**(2), 169–178 (2016)
9. Goodman, R.E., Bray, J.W.: Toppling of rock slopes. In: Proceedings of the ASCE Speciality Conference on Rock Engineering for Foundations and Slopes, Boulder, Colorado (1976)
10. Alejano, L.R., Gómez-Márquez, I., Martínez-Alegría, R.: Analysis of a complex toppling-circular slope failure. Eng. Geol. **114**(1–2), 93–104 (2010)
11. Amini, M., Ardestani, A., Khosravi, M.H.: Stability analysis of slide-toe-toppling failure. Eng. Geol. **228**, 82–96 (2017)
12. Abramson, L.W., Lee, T.S., Sharma, S., Boyce, G.M.: Slope Stability Concepts: Slope Stabilization and Stabilization Methods, 2nd edn. Wiley, New York (2001)
13. Albataineh, N.: Slope stability analysis using 2D and 3D methods. Master's Thesis, University of Akron, Ohio (2006)
14. Kliche, C.A.: Rock Slope Stability. 2nd edn. Society for Mining, Metallurgy, and Exploration Press, Englewood (2018)
15. Kulatilake, P.H.S.W., Wang, L., Tang, H., Liang, Y.: Evaluation of rock slope stability for Yujian river dam site by kinematic and block theory analyses. Comput. Geotech. **38**, 846–860 (2011)
16. Goodman, R.E., Shi, G.: Block Theory and its Application to Rock Engineering. Prentice-Hall, New York (1985)
17. Azarafza, M., Asghari-Kaljahi, E., Akgün, H.: Assessment of discontinuous rock slope stability with block theory and numerical modeling: a case study for the South Pars Gas Complex, Assalouyeh, Iran. Environ. Earth Sci. **76**(11), 397 (2017)
18. Azarafza, M., Asghari-Kaljahi, E., Akgün, H.: Numerical modeling of discontinuous rock slopes utilizing the 3DDGM (three-dimensional discontinuity geometrical modeling) method. Bull. Eng. Geol. Env. **76**(3), 989–1007 (2016). https://doi.org/10.1007/s10064-016-0879-1
19. Azarafza, M., Nikoobakht, S., Asghari-Kaljahi, E., Moshrefy-far, M.R.: Stability analysis of jointed rock slopes using block theory (case study: gas flare site in phase 7 of South Pars Gas Complex). In: Proceedings of the 32th National and the 1st International Geosciences Congress, Sari (2014). (in Persian)
20. Zhang, Y., Chen, G., Wu, J., Zheng, L., Zhuang, X.: Numerical simulation of seismic slope stability analysis based on tension-shear failure mechanism. SEAGS-AGSSEA J. **43**(2), 18–28 (2012)

21. Wang, F., Wang, S., Hashmi, M.Z., Xiu, Z.: The characterization of rock slope stability using key blocks within the framework of GeoSMA-3D. Bull. Eng. Geol. Env. **77**(4), 1405–1420 (2018). https://doi.org/10.1007/s10064-018-1291-9
22. Ling, H.I.: Recent applications of sliding block theory to geotechnical design. Soil Dyn. Earthq. Eng. **21**, 189–197 (2001)
23. Liu, T., Deng, J., Zheng, J., Zheng, L., Zhang, Z., Zheng, H.: A new semi-deterministic block theory method with digital photogrammetry for stability analysis of a high rock slope in China. Eng. Geol. **216**, 76–89 (2017)
24. Ma, Q., Li, G., Hong, Y.: The evaluation of stability of rock mass slope based on the block theory and DDA numerical. Appl. Mech. Mater. **353–356**, 1051–1056 (2013)
25. Um, J.G., Kulatilake, P.H.S.W.: Kinematic and Block theory analyses for shiplock slopes of the three gorges dam site in China. Geotech. Geol. Eng. **19**, 21–42 (2001)
26. Huang, C.C., Tsai, C.C., Chen, Y.H.: Generalized method for three-dimensional slope stability analysis. J. Geotech. Geoenviron. Eng. **128**, 836–848 (2000)
27. Yarahmadi-Bafghi, A.R., Verdel, T.: The key-group method. Int. J. Numer. Anal. Meth. Geomech. **27**, 495–511 (2003)
28. Jimenez-Rodriguez, R., Sitar, N., Chacón, J.: System reliability approach to rock slope stability. Int. J. Rock Mech. Min. Sci. **43**, 847–859 (2006)
29. Haswanto, W.A., Abd-Ghani, R.: Stability analysis of rock slopes using Block theory. In: Proceedings of the American Rock Mechanics Association Annual Conference (ARMA), Salt Lake (2008)
30. Greif, V., Vlčko, J.: Key block theory application for rock slope stability analysis in the foundations of medieval castles in Slovakia. J. Cult. Herit. **14**, 359–364 (2013)
31. Azarafza, M., Yarahmadi, A.R., Asghari-Kaljahi, E., Bahmannia, G.R., Moshrefy-far, M.R.: Stability analysis of jointed rock slopes using key block method (case study: gas flare site in 6, 7 and 8 phases of south pars gas complex). J. Geotech. Geol. **9**(3), 169–185 (2013). (in Persian)
32. Sun, G., Zheng, H., Huang, Y.: Stability analysis of statically indeterminate blocks in key block theory and application to rock slope in Jinping-I hydropower station. Eng. Geol. **186**, 57–67 (2014)
33. Li, M., Zhou, S., Wang, G.: 3D identification and stability analysis of key surface blocks of rock slope. Trans. Tianjin Univ. **22**(4), 317–323 (2016). https://doi.org/10.1007/s12209-016-2596-z
34. Azarafza, M., Akgün, H., Feizi-Derakhshi, M.R., Azarafza, M., Rahnamarad, J., Derakhshani, R.: Discontinuous rock slope stability analysis under blocky structural sliding by fuzzy key-block analysis method. Heliyon **6**(5), e03907 (2020)
35. Azarafza, M., Akgün, H., Ghazifard, A., Asghari-Kaljahi, E.: Key-block based analytical stability method for discontinuous rock slope subjected to toppling failure. Comput. Geotech. **124**, 103620 (2020)
36. Shi, G.: Discontinuous Deformation Analysis: A New Numerical Method for the Statics and Dynamics of Block Systems. Thesis, University of California Berkeley (1988)
37. Noroozi, M., Jalali, S.E., Yarahmadi-Bafghi, A.R.: 3D key-group method for slope stability analysis. Int. J. Numer. Anal. Meth. Geomech. **36**(16), 1780–1792 (2011)