An Approach to Determining the Integrated Reliability of Technical Systems at the Development Stage

Ruslan Litvinenko, Aver Auhadeev, Azat Khusnutdinov, Irina Antipanova, and Leylya Kisneeva

1 Introduction

When setting the requirements for the reliability of an engineering object at the stage of development of basic specifications and terms of reference, the nomenclature and values of reliability measures (RM) are determined and agreed upon between the customer and the designer. The components of the targeted RMs are selected from among the measures regulated by international standards IEC 62347:2006 and in accordance with IEC 60300-3-15:2009. The number of targeted RMs should be minimal in order to reduce the cost of verification, confirmation and evaluation of RMs available during manufacture and operation. At the same time, these RMs should be sufficient and fully describe the reliability of the engineering object at all stages of the life cycle. For comprehensive engineering systems, complex RMs or a specific single characteristic of reliability and maintainability, as well as their combinations, are used. If the engineering object during operation can be stored or shipped, then it is also necessary to set the preservation factors and the durability factors, if the criteria for reaching the limit state are determined for the engineering object. The numerical values of RMs are found by calculation, trial or computational and experimental methods using reference statistical data on the reliability of equivalents (prototypes) of the engineering object being designed, as well as operation and test data obtained from component suppliers. The engineering object being designed meets the requirements for reliability, subject to compliance with the applicable requirements of all RMs [1, 2].

At the design stage, the reliability requirements are possible to update with an appropriate feasibility study in the course of considering possible options for the engineering object followed by calculation of their reliability; selection of a schematic and design version of the structure that meets the customer's requirements in terms

R. Litvinenko (✉) · A. Auhadeev · A. Khusnutdinov · I. Antipanova · L. Kisneeva
Kazan State Power Engineering University, Kazan, Russia

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of the set of RMs and costs; clarification of the values of RMs of the engineering object.

The analysis of international standard IEC 60300-3-15:2009 makes it possible to conclude that for comprehensive engineering systems, if they are recoverable, provided that requirements for durability and preservation are defined for them, the total number of single and complex RMs is within 5–7 units. In the situation where the engineering system is unique, consisting of promising, unparalleled components, it is sufficient that all the RMs set comply with the requirements described in the specification. In practice, the designed or upgraded engineering system includes, however, in part or in whole, existing components for which statistical data on reliability are known. Therefore, at the design stage, there may be several possible schematic and design options for building such systems that meet the specified reliability requirements. A balanced approach to the issue of choosing a particular system design comes into importance, which is a solution to the multicriteria challenge of comparative assessment of reliability levels, as one of the technical requirements for the design engineering of options. Similar to the requirements for reliability, the specification indicates the requirements, characteristics, norms, indicators and other parameters that determine the purpose, performance of the products being designed. Therefore, the designer shall choose such a version of the engineering system that would satisfy all technical requirements (requirements for operation, stability, electromagnetic compatibility, etc.), including the reliability requirements described in the specification. Various schematic and design versions of the engineering system have different quantitative RMs, which do not allow unambiguously giving preference to one or another engineering solution. At the same time, the difference between RMs can vary from insignificant to significant, and the superiority of one option of the engineering solution over another is possible only in certain measures from those specified in the reliability requirements.

Thus, the relevance of the study is due to the objective need for developing such a reliability measure R that would characterize the entire set of basic reliability properties of the engineering system, if all the individual measures described in the specification meet the reliability requirements [3]. As IEC 62347:2006 provides exact definitions and the list of complex measures, the authors suggest calling this measure as the Integrated Reliability Measure (IRM).

Note that the authors do not set out to develop an approach for assessing the IRM of the engineering system; the purpose of the proposed methodology is to determine generalized indicators for several schematic and design options that synthesize individual RM into a single one. At the next stage of assessing the technical level of alternatives for the system being designed, taking into account all the requirements of the specification, the numerical value of reliability will be represented by one single indicator—IRM, and not a set of single and complex RMs.

2 Materials and Methods. Methodology for Determining the IRM Based on the Analytic Hierarchy Process

Generally, the Analytic Hierarchy Process (AHP) is a mathematical apparatus of the systematic approach to decision-making technology based on calculations and the use of the method of pairwise comparisons, which makes it possible to find such an option (alternative) that best suits the essence of the challenge and the requirements to its solution. This process was developed by Thomas L. Saaty in the 1970s and is actively used in addressing various multicriteria problems (analysis of possible scenarios, ranking, resource allocation, risk management, etc.). In addition, AHP has become widespread in the practical solution of problems of comparative analysis of the technical level of alternatives for engineering objects being designed [4–6].

The solution to the problem by means of AHP consists of the following steps:

1. Setting the goal, main criteria (performance indicators) and alternatives.
2. Decomposition of the problem into a hierarchical structure: a tree from the goal through criteria to alternatives.
3. Building matrices of pairwise comparisons of criteria by purpose and alternatives by criteria.
4. Checking the consistency of experts' judgments (criterion of the quality of the experts' performance).
5. Application of the mathematical apparatus for the analysis of the resulting matrices.
6. Determination of local priorities of criteria and global priorities of alternatives.
7. Selection of the dominant alternative.

A more detailed description of the content of these AHP steps is widely presented in various sources [7–9].

The AHP has the following advantages:

- takes various factors and multiple goals into account;
- regards the possible effect of the interaction of factors;
- simplicity of mathematical calculations;
- for pairwise comparisons, a scale of relative importance is used, which streamlines and simplifies the procedure for setting expert assessments;
- is able to assign ranks to alternative options (numerical indicators reflecting the significance or importance of an object).

The latter provision is recommended to be used for building the quantitative value of the IRM for alternative options of the engineering system, where the corresponding RMs will act as criteria [10]. It is suggested using the calculated values of global (composite) priorities as the IRM. Note that the IRM is determined for specific structural and schematic options for constructing the system being designed and the adequate quantitative values of the RM. Modification of any of these provisions will result in a re-determination of the IRM.

As a disadvantage of the AHP, it can be noted that.

- the number of measures should not exceed 7 ± 2, since human consciousness is not capable of simultaneous perception and processing of more than the specified number of information units [11, 12];
- the AHP allows for finding the ranks of the selected alternatives, but does not have internal means of interpreting these ranks;
- despite checking the consistency of the judgments of experts, expert assessments are subjective.

The performance and adequacy of the proposed approach to the AHP-based determination of IRM for the subsequent assessment of the technical level of design and schematic versions of the engineering system being designed will be then specifically exemplified.

3 Results. AHP-Based Calculation of the IRM for a Multi-motor Drive

For the multi-motor drive (MMD) used in electric transport, the customer described the following reliability requirements in the specification:

- gamma-percentile time to failure (*GPTF*)—minimum 7500 h;
- mean time between failures (*MTBF*)—minimum 12,000 h;
- average overhaul life (*AOL*)—minimum 5 years;
- availability factor (*AF*)—minimum 0.99;
- mean lifetime till discarding (*MLTD*)—minimum 20 years.

The designer proposed three possible options for MMD with pre-calculated values of the measures shown in Table 1.

Using the AHP, it is required to calculate the IRM R_1, R_2 and R_3 for alternative options of the MMD for the subsequent comprehensive analysis of the compliance of all technical requirements with the system being designed.

Decomposition of the problem into a hierarchy. The decomposition of the IRM calculation problem is shown in Fig. 1. In the most elementary form, the hierarchy consists of a top (conditionally, there is a common goal—generating the IRM), from which there are intermediate levels, consisting of 5 criteria (reliability measures) that

Table 1 Reliability measures for MMD versions

	Reliability measures				
	GPTF, kilohours	*MTBF*, kilohours	*AOL*, years	*AF*	*MLTD*, years
Alternative 1	7.6	12.5	7	0.998	22
Alternative 2	8	13.5	6.5	0.999	24
Alternative 3	5.2	14	5	0.994	25

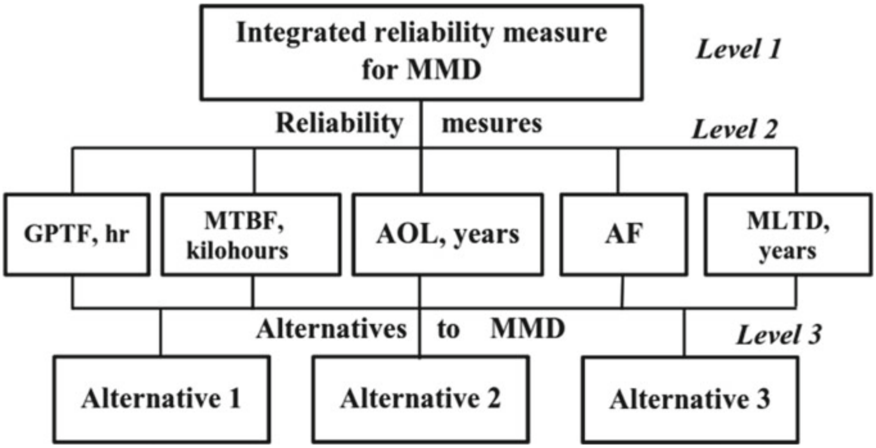

Fig. 1 Decomposition of the IRM calculation problem for the MMD options

clarify the goal, to the lower level, consisting of 3 alternative options of the MMD. This hierarchy is called dominant.

Building matrices of pairwise judgments. When using AI, the problem to be solved was presented hierarchically; therefore, the matrix of comparing the importance of the second-level RM is made available relative to the overall goal (level 1). Similar matrices are also built for pairwise comparisons of each alternative MMD in relation to the elements of level 2. For subjective paired comparisons, Thomas L. Saaty developed a numerical scale of relative importance [8, 12], according to which experts determine the weight of functions (measures) that describe the system being designed.

To build a pairwise judgment matrix for level 2 A_1 with the dimension $k \times k$, where $k = 5$ is the number of specified RMs, a table is drawn up in k rows and columns, in which the headers of the columns and rows refer to the measures used in generating the IRM and described in the specification. Actions begin with the indicator located in the heading of row 1 (gamma-percentile time to failure *GPTF*), while a question arises how much this measure is more important than the measures referred to in the column heading, respectively, the mean time between failures *MTBF* and then the rest of the measures. When comparing the measure with itself, the ratio is equal to one. If the compared measure is more important than the RM from the column heading, then an integer from the relative importance scale [13] is used, otherwise, the reciprocal. Thus, in turn, the importance of all RMs is compared with each other and all elements of the matrix A_1 of pairwise comparisons for level 2 (RM level) are determined (Table 2).

The compilation of matrices A_{2j}, $\left(j = \overline{1, k}\right)$ for level 3 (the level of alternatives) is greatly simplified, since the RM values are expressed quantitatively, not qualitatively. Elements of matrices A_{2j}, $\left(j = \overline{1, k}\right)$ are generated by dividing the RM values of the corresponding alternative options of the MMD for each measure j (matrices

Table 2 Matrix A_1 of pairwise comparisons for level 2

Measures	*GPTF*, $j = 1$	*MTBF*, $j = 2$	*AOL*, $j = 3$	*AF*, $j = 4$	*MLTD*, $j = 5$
GPTF	1	3	4	1	5
MTBF	1/3	1	4	2	3
AOL	1/4	1/4	1	1/5	3
AF	1	1/2	5	1	6
MLTD	1/5	1/3	1/3	1/6	1

Note $j = \overline{1, k}$ is the serial number of the reliability measure

A_{21}, A_{22}, A_{23}, A_{24} and A_{25}). Table 3 lists matrices A_{2j} of pairwise comparisons for level 3.

Thus, actions of this stage resulted in a matrix A_1 of pairwise comparisons for the RM level and a matrix $A_{21}...A_{25}$ for the level of alternatives to the MMD.

Determination of local priorities and consistency of expert opinions for level 2. For the matrix A_1 of pairwise comparisons of the measure level, a set of local priorities is determined, which describe the relative influence of the set of top elements on the bottom element of the hierarchy. This identifies the weight of each individual component of the matrix. In view of this, it is necessary to calculate the eigenvector $\overline{A} = [x_1 \ldots x_k]^{\mathrm{T}}$ of the matrix A_1, the components of which are the characterization of the priority vector by the rows of the matrix x_i, determined by the formula

$$x_i = \frac{\overline{a}_i}{\sum_{i=1}^{k} \overline{a}_i},$$

where $\overline{a}_i = \sqrt[k]{\prod_{i=1}^{k} a_i}$ is the geometric mean of the elements of the *i*th row of the matrix A_1, $i = \overline{1, k}$.

For solving the given problem, the eigenvector $\overline{A}$ of the matrix A_1 received the following values:

$$\overline{A} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0.3573 \\ 0.2388 \\ 0.0817 \\ 0.2708 \\ 0.0514 \end{bmatrix},$$

where x_1 is the characteristic of priority of the *GPTF*; x_2—*MTBF*; x_3—*AOL*; x_4—*AF*; x_5—*MLTD*.

The vector of local priorities (eigenvectors of the matrix A_1) $\lambda_{\max} = [\lambda_{\max 1}...\lambda_{\max k}]$ is calculated by reducing the value x_i to a normalized form formula:

Table 3 Matrices of pairwise comparisons for the level of alternatives to the MMD

Matrix A_{21} of pairwise comparisons for the measure GPTF, $j = 1$			
	Alternative 1	Alternative 2	Alternative 3
Alternative 1	**1**	7.6/8 = **0.95**	7.6/8.2 = **0.9268**
Alternative 2	8/7.6 = **1.0526**	**1**	8/8.2 = **0.9756**
Alternative 3	8.2/7.6 = = **1.0789**	8.2/8 = **1.025**	**1**
Matrix A_{22} of pairwise comparisons for the measure MTBF, $j = 2$			
	Alternative 1	Alternative 2	Alternative 3
Alternative 1	**1**	12.5/13.5 = = **0.9259**	12.5/14 = **0.8929**
Alternative 2	13.5/12.5 = = **1.08**	**1**	13.5/14 = **0.9643**
Alternative 3	14/12.5 = **1.12**	14/13.5 = **1.037**	**1**
Matrix A_{23} of pairwise comparisons for the measure AOL, $j = 3$			
	Alternative 1	Alternative 2	Alternative 3
Alternative 1	**1**	7/6.5 = **1.0769**	7/5 = **1.4**
Alternative 2	6.5/7 = **0.9586**	**1**	6.5/5 = **1.3**
Alternative 3	5/7 = **0.7143**	5/6.5 = **0.7692**	**1**
Matrix A_{24} of pairwise comparisons for the measure AF, $j = 4$			
	Alternative 1	Alternative 2	Alternative 3
Alternative 1	**1**	0.998/0.999 = = **0.999**	0.998/0.994 = = **1.004**
Alternative 2	0.999/0.998 = = **1.001**	**1**	0.999/0.994 = = **1.005**
Alternative 3	0.994/0.998 = = **0.996**	0.994/0.999 = = **0.995**	**1**
Matrix A_{25} of pairwise comparisons for the measure MLTD, $j = 5$			
	Alternative 1	Alternative 2	Alternative 3
Alternative 1	**1**	22/24 = **0.999**	22/25 = **1.004**
Alternative 2	24/22 = **1.001**	**1**	24/25 = **1.005**
Alternative 3	25/22 = **0.996**	25/24 = **0.995**	**1**

$$\lambda_{\max i} = x_i \cdot \sum_{j=1}^{k} a_{ij},$$

where $\lambda_{\max i}$ are the eigenvalues of the Perron vector [12, p. 15]; a_{ij} is the value of the element of matrix A_1 in the ith row of the jth column, $j = \overline{1, k}$.

Using the described procedure, the local priorities $\lambda_{\max i}$ will be determined for level 2 (level of measures). For solving the problem of generating the IRM for the alternative MMDs, the vector of local priorities is represented by the following values:

$$\lambda_{\max} = \begin{bmatrix} 0.9945 \\ 1.2139 \\ 1.171 \\ 1.1825 \\ 0.9252 \end{bmatrix}.$$

The sum of all the elements of the resulting vector of local priorities $\lambda_{\max}$ is 5.4871, called Perron's eigenvalue, and it is denoted by the letter Λ [12, p. 16]. If the matrix is absolutely consistent, the condition $\Lambda = k$ is met.

After determining local priorities $\lambda_{\max i}$ and the value Λ, the procedure for assessing the consistency of expert opinions will be applied when generating the matrix of pairwise comparisons A_1 for level 2, since the generalized opinion of the group of experts is not devoid of subjectivity, because of using a qualitative rating scale. For this, the AHP provides for the use of the consistency index (μ), which gives information on how much the numerical and ordinal consistency is disturbed. If the consistency is significantly disturbed, then it is recommended to search for additional information and revise the judgments of experts in the second round of the examination.

The consistency index will be calculated by the formula

$$\mu = \frac{\Lambda - k}{k - 1} = \frac{5.4871 - 5}{4} = 0.1218.$$

The value μ will be compared with the value of random consistency (μ_{rand}), which would be obtained with a random set of quantitative judgments from the scale $\frac{1}{9}, \frac{1}{8}, \frac{1}{7}, \ldots, 1, 2, \ldots, 9$, but provided that an inversely symmetric matrix was created. The source [6] provides a table that allows determining the average consistency μ_{rand} for random matrices of various orders. For solving the given problem for $k = 5$, the value $\mu_{\text{rand}} = 1.12$. To compare the values μ and μ_{rand}, the consistency ratio (CR) will be found by the formula

$$\text{CR} = \frac{\mu}{\mu_{\text{rand}}} \cdot 100\% = 10.9\%.$$

The CR must be 10% or less to be acceptable. In some cases, 20% is possible, but not more. If CR exceeds 20%, it is necessary to conduct a second round of examination and clarify the elements of the matrix of pairwise comparisons A_1 for level 2 [12, 14].

Since CR = 10.9%, it is possible to conclude about the admissible consistency of the matrix of pairwise comparisons A_1 for level 2 of the problem of generating the IRM for alternative MMDs.

Determination of local priorities for level 3. Further, in a similar way, local priorities are determined for level 3 (the level of alternative MMDs); the calculation results are listed in Table 3. It should be noted that if the measures of alternatives are given in the form of numerical values, then it is not necessary to determine μ, since $\mu = 0$, and therefore $\mu_{\text{rand}} = 0$. Hence, no determination of μ_{rand} was made for level 3.

Determination of global priorities. To generate the IRM of alternatives, priorities are synthesized starting from the level of measures. Local priorities are multiplied by the priority of the corresponding RM at a higher level and are summed up for each element in accordance with the criterion driven by this element (each element of level 2 is multiplied by 1, i.e. by the weight of the single goal of the highest level). This gives the composite or global priority of that element, which is then used as a criterion for weighing the local priorities of the elements below it. As mentioned above, it is proposed to accept the value of the global priority as the IRM for alternatives.

The results summarized in Table 4 will be presented in the form of a 5 × 3 matrix A_3, where the columns will correspond to the values of the vector of level 2 RM priorities, and the rows will correspond to the alternative MMDs.

Thus, for each measure the priority matrix A_3 will take the following form:

$$A_3 = \begin{bmatrix} x_1^{(GPTF)} & x_1^{(MTBF)} & x_1^{(AOL)} & x_1^{(AF)} & x_1^{(MLTD)} \\ x_2^{(GPTF)} & x_2^{(MTBF)} & x_2^{(AOL)} & x_2^{(AF)} & x_2^{(MLTD)} \\ x_3^{(GPTF)} & x_3^{(MTBF)} & x_3^{(AOL)} & x_3^{(AF)} & x_3^{(MLTD)} \end{bmatrix}$$
$$= \begin{bmatrix} 0.3193 & 0.3337 & 0.3784 & 0.3125 & 0.3099 \\ 0.3361 & 0.334 & 0.3514 & 0.3375 & 0.338 \\ 0.3445 & 0.3323 & 0.2703 & 0.35 & 0.3521 \end{bmatrix}.$$

The vector of global priorities will be determined by the formula

$$\overline{W} = A_3 \cdot \overline{A} = \begin{bmatrix} x_1^{(GPTF)} & x_1^{(MTBF)} & x_1^{(AOL)} & x_1^{(AF)} & x_1^{(MLTD)} \\ x_2^{(GPTF)} & x_2^{(MTBF)} & x_2^{(AOL)} & x_2^{(AF)} & x_2^{(MLTD)} \\ x_3^{(GPTF)} & x_3^{(MTBF)} & x_3^{(AOL)} & x_3^{(AF)} & x_3^{(MLTD)} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix},$$

where W_1, W_2, W_3 are the global priorities of the corresponding alternative MMDs.

Substituting the numerical values obtained for A_3 and $\overline{A}$ will result in the following values of global (eigen) priorities:

Table 4 Determination of local priorities for level 3 (the level of alternatives)

Alternatives to MMD	Alternative 1	Alternative 2	Alternative 3	Values of vector of priorities x_i	Vector of local priorities $\lambda_{\max i}$
For specified failure-free service hours GPTF					
Alternative 1	1	0.95	0.9268	0.3193	0.9999
Alternative 2	1.0526	1	0.9756	0.3361	0.9999
Alternative 3	1.0789	1,025	1	0.3445	0.9999
Definition Λ_1					3
For mean time between failures MTBF					
Alternative 1	1	0.9259	0.8929	0.3337	1.001
Alternative 2	1.08	1	0.9643	0.334	1
Alternative 3	1.12	1.037	1	0.3323	0.9999
Definition Λ_2					3
For specified overhaul life AOL					
Alternative 1	1	1.0769	1.4	0.3784	1.0001
Alternative 2	0.9586	1	1.3	0.3514	1.0001
Alternative 3	0.7143	0.7692	1	0.2703	1.0001
Definition Λ_3					3.0003
For availability factor AF					
Alternative 1	1	0.999	1.004	0.3125	1
Alternative 2	1.001	1	1.005	0.3375	1
Alternative 3	0.996	0.995	1	0.35	1
Definition Λ_4					3
For specified life MLTD					
Alternative 1	1	0.999	1.004	0.3099	1.0001
Alternative 2	1.001	1	1.005	0.338	0.9999
Alternative 3	0.996	0.995	1	0.3521	1
Definition Λ_5					3

$$\overline{W} = \begin{bmatrix} 0.3193 & 0.3337 & 0.3784 & 0.3125 & 0.3099 \\ 0.3361 & 0.334 & 0.3514 & 0.3375 & 0.338 \\ 0.3445 & 0.3323 & 0.2703 & 0.35 & 0.3521 \end{bmatrix} \times \begin{bmatrix} 0.3573 \\ 0.2388 \\ 0.0817 \\ 0.2708 \\ 0.0514 \end{bmatrix} = \begin{bmatrix} 0.3268 \\ 0.3372 \\ 0.3363 \end{bmatrix}.$$

The sum of global priorities must be equal to one.

Equating the calculated values of global priorities W_1, W_2, W_3 to the measures R_1, R_2 and R_3 will yield the numerical values of the IRM of the corresponding alternatives to MMD, characterizing the set of single and complex RMs. Because of solving the given problem, the highest IRM corresponds to the alternative 2 to MMD, as $R_2 = 0.3372 > 0.3363 > 0.3268$. These measures, which are a generalized (unified) RM, can be then used along with other technical and economic requirements to the MMD for the subsequent selection of the most preferable schematic and the design version of the multi-motor drive being designed.

4 Discussion

The analysis of the calculation results has shown the adequacy and correctness of the proposed approach, and its suitability for use for further analysis of the IRM of alternatives of the sophisticated engineering systems being designed. The practical effect of determining the IRM by the analytic hierarchy process can be further developed because of the integration of the expert system used at the stage of constructing the matrix of pairwise judgments for the level of reliability measures, and artificial neural network technologies. The ability of neural networks to learn on multiple qualitative and quantitative examples with unknown regularities between input and output data will further allow the developed neuro-expert system to automate the process of determining the weight of specified reliability measures [15, 16]. Also, the use of neural network technologies in determining the IRM will make it possible to increase the number of individual analyzed RMs in connection with the ability to process a large amount of information.

5 Conclusion

Thus, the approach to determining the IRM of engineering systems at the design stage, synthesizing up to 5–7 individual measures that describe various properties of reliability, makes it possible to give a comprehensive assessment and, based on this, to rank options of schematic and design solutions. The proposed mechanism for determining the IRM for the alternatives of the engineering system being designed is a convenient tool for scientific research at the stage of basic specifications and terms of reference. The use of the AHP for this purpose facilitates a deep analysis of a large volume of expert and statistical information about the specified reliability criteria, taking into account the weight characteristics of the analyzed measures. The integral measure to determine the level of reliability is useful to get a formalized result, which is expressed through the corresponding value of the vector of global priorities and enables to quantitatively assess the superiority of one alternative over another.

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