# Determination of the Conductor Sag According to the Period of Own Harmonic Oscillations, Taking into Account the Difference in Heights of the Suspension Points

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## 1 Introduction

Electricity is delivered to consumers via overhead power transmission lines (OTL). In this regard, there is an acute problem of monitoring the state of OTL in order to ensure a reliable power supply to consumers [1]. Recently, work has been actively carried out on the creation of intelligent OTL, which will be more efficient and safe [2]. However, any system for monitoring the state of OTL elements is based on methods and approaches to receiving and interpreting input data.

There are many approaches to assessing the technical condition of OTL: the study of the conductor heating temperature in order to increase its carrying capacity for the most effective use of existing OTL instead of building new ones [3–5]; use of unmanned aerial vehicles in order to estimate the geometric parameters of OTLs (use of video cameras or laser scanners), as well as control of insulators and conductor connections (video recording in the ultraviolet and infrared ranges) [6]; control of ice-rime deposits on the OTL conductors by the method of echolocation (inputting a signal into the conductor and analyzing its reflected value, which determines the wall thickness of the ice-rime deposits) [7]; determination of the conductor sag to assess the mechanical loads acting on it [8, 9].

In this article, we propose the method for determining the sag of the conductor, since it allows one to describe its geometry, and, therefore, determine the mechanical loads it experiences. This may be relevant when the OTL is exposed to external factors, for example, ice-rime deposits [10, 11]. To control the sag of the conductor, the following methods can be used: optical (a source and receiver of optical radiation are used, the shift of one of which determines the change in the conductor sag value)

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[12–14]; inclinometric (the angle of inclination of the conductor is determined, which is then used to calculate the mechanical loads experienced by the conductor) [9, 15].

Previously, our team developed methods where the mechanical loads experienced by OTL were determined using such parameters as the angle of inclination of the OTL conductor [8, 16] and the angle of its rotation [8, 17, 18]. At present for the first time, a method has been developed for determining the conductor sag by the period of its oscillations [19, 20]. This technique is also applicable for the case when the points of the conductor suspension are at different heights [19].

This article describes a method for determining the sag of a conductor by the period of its oscillations, and theoretical estimations are compared with the data of a model experiment. Comparison of the calculated values of the oscillation period of the conductor with the experimental ones should be carried out at different sags and heights of one of the points of the conductor suspension.

## 2 Brief Theory of the Method for Determining the Conductor Sag at Different Heights of Its Suspension Points

A conductor in the span of an OTL is considered as an absolutely rigid isotropic construction. This construction has one rotational degree of freedom relative to the axis passing through the suspension points of the OTL conductors, which are at different heights (Fig. 1).

The geometry of the conductor in the OTL span can be determined using the equations of the parabolic sag of a flexible thread [21]:

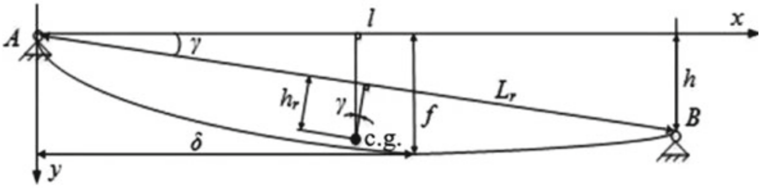

**Fig. 1** Model of OTL conductor, fixed at the points of its suspension at different heights and representing a physical pendulum, with the designation of the main geometric parameters: $L_r$—the minimum distance between the suspension points, m; $l$—the OTL span length, m; $f$ is the vertical distance between the highest point of the suspension and the lowest point of the conductor in the OTL span, m; $h$ is the difference in the heights of the points of the OTL conductor suspension, m; $h_r$ is the distance from the center of gravity (c.g.) of the OTL conductor to the segment with the ends at the points of suspension of the conductor $A$ and $B$, m; $\delta$ is the distance from the suspension point $A$ to the projection of the lower point of the conductor on the $x$-axis, m; $\gamma$ is the angle between the segment AB and the $x$-axis

$$y = \frac{1}{a}\left(\delta x - \frac{x^2}{2}\right), \quad y' = \frac{\delta - x}{a}, \quad \delta = \frac{l}{2} + \frac{h}{l}a, \tag{1}$$

where $a$ is the coefficient of the parabola, $x$ is the coordinate of the conductor section and the origin is at point $A$.

The minimum distance from a straight line passing through the suspension points of the conductor to the center of swinging of the OTL conductor is determined as follows:

$$L_k = \frac{l^2\left(l^4 + 56a^2l^2\right) + 28a^2h^2}{14a\left(l^4 + 40a^2l^2 + 20a^2h^2\right)}. \tag{2}$$

The distance from the axis of rotation to the center of oscillation can be found from the oscillation period $T$:

$$L_k = \frac{gT^2}{4\pi^2}. \tag{3}$$

Joint consideration of Eqs. (2) and (3) allows finding the coefficient of the parabola $a$.

We can calculate the parameter $f$ from the equations of the parabolic sag of a flexible thread (1), substituting the value $\delta$ instead of $x$ and using the coefficient of the parabola $a$:

$$f = \frac{\delta^2}{2a}. \tag{4}$$

The vertical distance from the lowest point of the curve to the straight line passing through the points of suspension of the OTL conductor, according to [22], is the sag of the conductor. This distance can be found using the following expression:

$$f_p = f - \delta tg\gamma = f - \delta\frac{h}{l}. \tag{5}$$

Thus, to determine the sag of a conductor for OTL spans with suspension points located at different heights, additional data is required, such as the difference in heights of the points of suspension of the OTL conductor, the length of the OTL span and an acute angle between the horizontal straight line and the straight line passing through the points of the OTL conductor suspension (Fig. 1).

This data is difficult to obtain experimentally, which complicates the calculation of the conductor sag arrow.

In this regard, this work considers the possibility of applying the formula for determining the sag of a conductor, where only the period of own harmonic oscillations of the OTL conductor is used as input data [20, 23]:

$$f_p = \frac{5gT^2}{16\pi^2} \approx 0.31T^2. \tag{6}$$

## 3 Technique for Measuring Own Harmonic Vibration Period of a Conductor when Changing its Sag and the Height of One of the Points of the Conductor Suspension

The stand for measuring the own harmonic vibrations of a conductor when changing its sag and the height of one of the conductor suspension points includes a span of 5.368 m (horizontal distance between the conductor suspension points) with a conductor; flexible bearings at the conductor attachment points (in this case, the conductor at point *B* is attached so that the parameter *h* is 0, 0.25, 0.503, 0.753 and 1.003 m); laser level for easy measurement of the heights of the suspension points and the lower point of the conductor sag; a ruler for measuring vertical distances (the difference in the heights of the points of the conductor suspension and its sag).

The period *T* is determined as follows: (1) the conductor is retracted to the side and released, 40 full oscillations of the conductor are counted (the conductor returns to its original extreme position in which it was at the initial moment); (2) the time is measured during which these 40 complete oscillations of the conductor occurred; (3) the period of the conductor oscillation is determined as the arithmetic mean. Oscillations were observed visually.

To measure distances, for example, a sag, a ruler 1000 mm long is used (the minimum length measurement step is 0.5 mm). To increase the accuracy of measuring the sag, a BOSCH PCL 20 SET laser level is used, using which the horizontal and vertical laser beams are projected onto the wall. The use of a laser level allows one to project the position of the suspension point *B* and the lower point of the conductor onto a ruler. This allows the parameters $h, f$ and $\delta$ to be determined. The conductor sag arrow in the experiment is determined in accordance with expression (5).

The description of the experimental setup at $h = 250$ mm is shown in Fig. 2.

## 4 Comparison of the Results of Calculating the Conductor Own Oscillations Period with the Available Experimental Data

A series of experiments is carried out at $h = 0 \div 1.003$ m. The oscillation period is determined in accordance with Formula (6) based on the data on the conductor sag:

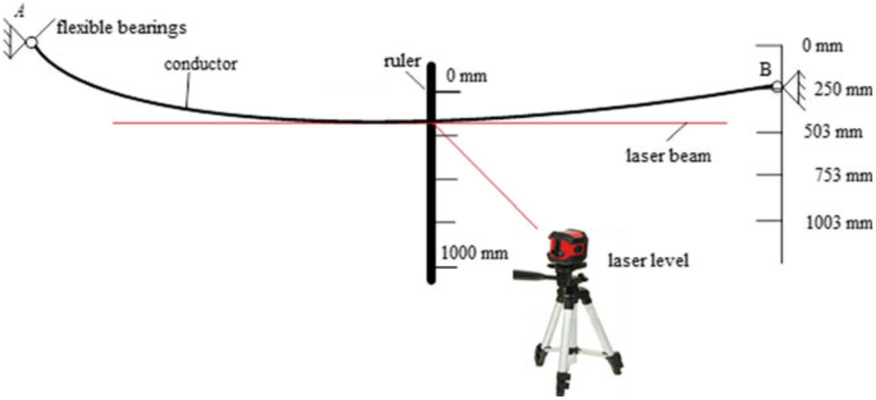

**Fig. 2** Experimental setup with $h = 250$ mm

$$T = \sqrt{\frac{16\pi^2 f_p}{5g}}. \tag{7}$$

The error in calculating the period of own harmonic oscillations of a conductor at various sags is determined by the following formula:

$$\delta_T = \left[\left(T_{\text{exp}} - T_{\text{calc}}\right)/T_{\text{exp}}\right] \cdot 100\%, \tag{8}$$

where $T_{\text{exp}}$ is the oscillation period of the conductor, determined experimentally at a certain sag $f_{\text{exp}}$, estimated experimentally; $T_{\text{calc}}$ is the oscillation period of the conductor, calculated by Formula (7) with the current sag $f_{\text{exp}}$, estimated experimentally.

Conductor's own harmonic oscillation period on its sag at the difference in height between the suspension points $h = 0$ m dependence is considered in this article [23].

The dependences of the period of own harmonic oscillations of the conductor on its sag at a difference in heights between the suspension points $h = 0.25$ m and $h = 0.503$ m are shown in Fig. 3. The experimental values of the conductor sag ($f_{\text{exp}}$, m) are plotted along the abscissa axis, and the periods of the conductor's own harmonic oscillations ($T_{\text{calc}}$ are the periods calculated by Formula (7) for the values of $f_{\text{exp}}$; $T_{0.25\text{exp}}$ and $T_{0.503\text{exp}}$ are the periods measured experimentally at $h = 0.25$ m and $h = 0.503$ m, respectively).

The sag of the conductor at $h = 0.25$ m varies from 0.1 to 0.553 m. The period of own harmonic oscillations of the conductor in this case changes from 0.578 to 1.331 s. The error in calculating the period of own harmonic oscillations of a conductor at different sags at $h = 0.25$ m lies in the range from 0.01 to 1.34%. At $h = 0.503$ m, the sag of the conductor changes from 0.106 to 0.667 m. The period of own harmonic oscillations of the conductor in this case changes from 0.592 to 1.456 s; the error in

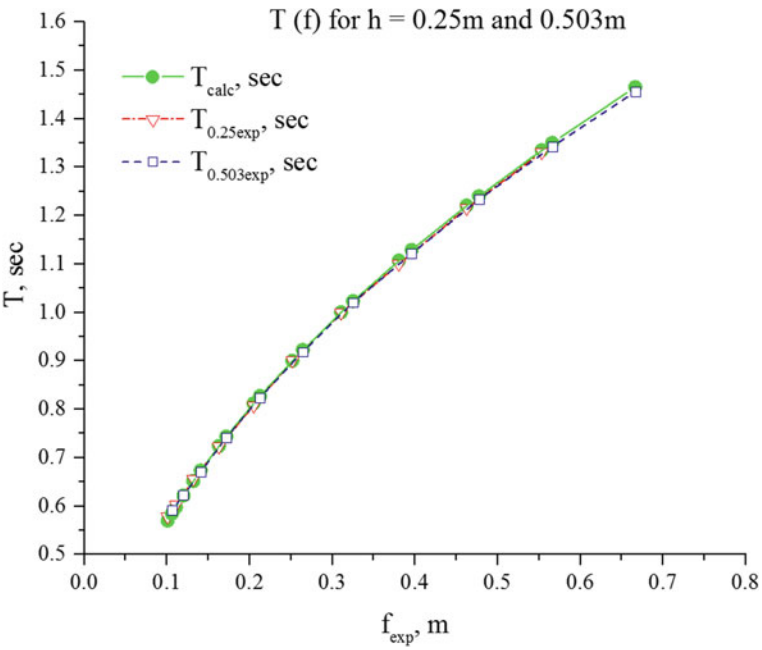

**Fig. 3** Dependences of the period of own harmonic oscillations of the conductor on its sag at $h =$ 0.25 m and $h = 0.503$ m

calculating the period of oscillations of a conductor with different sags is from 0.17 to 1.26%.

The dependences of the period of own harmonic oscillations of the conductor on its sag at $T_{1.003\mathrm{exp}}$, and the difference in heights between the suspension points $h = 0.753$ m and $h = 1.003$ m are shown in Fig. 4. The experimental values of the conductor sag ($f_{\mathrm{exp}}$, m) are plotted along the abscissa axis, and the conductor's own harmonic oscillation period values are plotted along the ordinate axis ($T_{\mathrm{calc}}$—periods calculated by Formula (7) for $f_{\mathrm{exp}}$ values; $T_{0.753\mathrm{exp}}$—periods obtained experimentally at $h = 0.753$ m; $T_{1.003\mathrm{exp}}$—periods obtained experimentally at $h = 1.003$ m).

The sag of the conductor at $h = 0.753$ m during the experiment changes from 0.102 to 0.757 m. The period of own harmonic oscillations of the conductor in this case changes from 0.579 to 1.546 s. The error in calculating the period of the conductor oscillations at various sags at $h = 0.753$ m is from 0.04 to 1.16%. At $h = 1.003$ m, the sag of the conductor during the experiment changes from 0.080 to 0.705 m. The period of own harmonic oscillations of the conductor in this case changes from 0.495 to 1.503 s. The error in calculating the period of oscillations of a conductor at different sags at $h = 1.003$ m lies in the range from 0.06 to 2.67%.

The dependences of the period of own harmonic oscillations of the conductor on its sag, obtained by the proposed theoretical model, indicate that expression (6) for determining the sag of the conductor by the period of its own harmonic oscillations

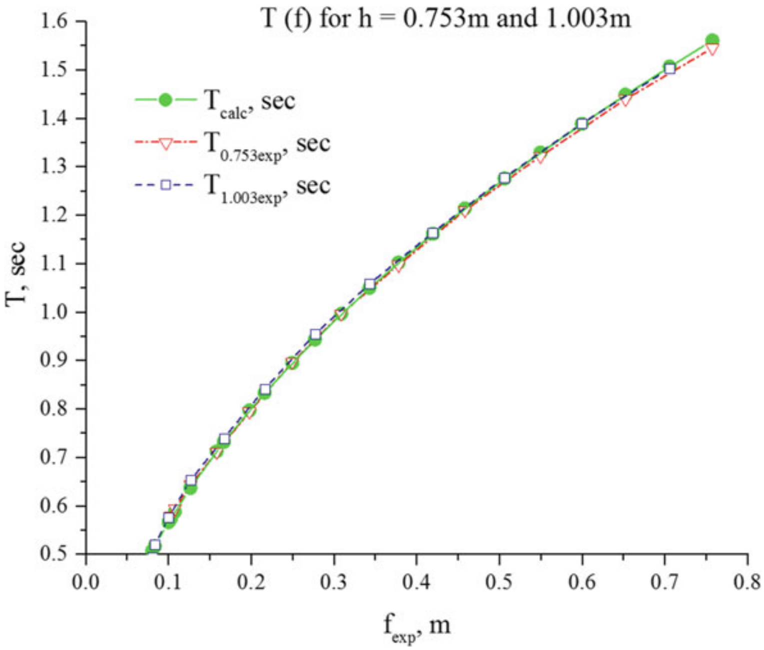

**Fig. 4** Dependences of the period of own harmonic oscillations of the conductor on its sag at $h = 0.753$ m and $h = 1.003$ m

makes it possible to reliably estimate the boom of the conductor sag even if there is a difference in heights between the points of the conductor suspension.

## 5 Conclusion

In this work, the possibility of using the method for determining the sag of a conductor by the period of its own harmonic oscillations at the nonzero difference in heights between the points of the conductor suspension is experimentally confirmed. A mathematical model is proposed that describes well the experimental dependences of the period of oscillations of the conductor on its sag. The error of the model for determining the oscillation period of the conductor, depending on its sag, does not exceed 2.735% when a span length does not exceed 5.368 m, a difference in the heights of the suspension points ranges from 0 to 1.003 m and a change in the sag varies from 0.080 to 0.757 m.

In the future, the method for determining the sag of a conductor by the period of its own harmonic oscillations can be implemented on the basis of existing and newly developed systems for monitoring the state of OTL [24].

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