Mathematical Simulation of the Flux of the Solar Radiation Coming to the Collector

Valentina Khoreva, Sergey Elistratov, Nadezhda Vorogushina, and Ivan Sadkin

1 Introduction

Solar energy resource maps are often used to determine the potential for solar power generation. These maps are created from satellite imagery and interpolation of data from ground-based weather stations, which are often very far from each other, and their data are not always accurate. The data on the maps are not always of high quality and of sufficient scale to be a reliable basis for making decisions on installing a solar collector. A mistake in choosing the location or angle of inclination of the solar collector can lead to a significant decrease in the energy of solar radiation incident on the collector.

In the course of work, several methods of calculating the power of solar radiation coming to the collector were studied and analyzed [1–12]; as a result, five methods were found to calculate the power of solar radiation falling on the surface of the solar collector. These calculation methods are not universal, since the final value of the solar radiation power in some methods does not depend on the installation height of the solar collector above sea level; the final height of the atmosphere is not taken into account; the calculation can be carried out only for zenith angles up to 75°; the dependence of the power of solar radiation falling on the Earth's surface on the collector installation height is linear, which has no physical justification. The task of this work is to create a universal scientifically based methodology for assessing the efficiency of using solar radiation.

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2 Materials and Methods

At the moment, a program has been created that calculates the power of solar radiation arriving at the collector as a dependence on the air density (not linearly changing depending on the collector installation altitude above sea level), the path of the Sun's rays through the atmosphere (not linearly changing at every moment of time) and the angle between the normal to the collector plane and the Sun. This calculation method is fundamentally new and scientifically grounded, since it is based on geometry, astronomy and physical laws.

The calculation program shows the maximum potential (exergy) of solar radiation in a given area (at a given geographic latitude and altitude) [13–15], calculates the value of the solar radiation power coming to the collector for any angle of inclination and azimuth of the solar collector and allows you to choose the optimal angle of inclination of the solar collector, taking into account the latitude, altitude and possible azimuths of the installation. At the moment, the program is written in the SMath Studio.

The absorption of light in the atmosphere depends on its density. The density of the atmosphere is calculated using the Mendeleev-Clapeyron Eq. (1):

$$\rho = \frac{P \cdot M}{R \cdot T} \tag{1}$$

At altitudes of 40 km and above, the density of the atmosphere can be neglected. For the accuracy of calculations, we take the significant height of the atmosphere $h_{\max} = 40,000$ m. We introduce the concept of the density coefficient. The density coefficient is calculated by the formula (2), where $\rho_0 = 1.225$ kg/m^3 is the density of dry air at a temperature of 15 °C and a pressure of 101,330 Pa. For the equator with a collector installation height above sea level $h = 0$ m, $k_\rho = 1$. When $h = 40,000$ m, $k_\rho = 0$.

$$k_\rho = \frac{P \cdot M}{\rho_0 \cdot R \cdot T} \tag{2}$$

For the equator, when the Sun is at its zenith, the power of the solar radiation flux arriving at the plane perpendicular to the flow depends on the density coefficient k_ρ:

$$\frac{dI}{I} = -dk_\rho$$

Integrating the right and left sides of the equation and substituting the integration limits for the density factor from 0 to k_ρ and for the radiation power from I to I_0, where $I_0 = 1362$ W/m^2 [16], we get

$$I = I_0 \cdot e^{-k_\rho} \tag{3}$$

The maximum solar radiation flux at sea level at the equator is $I = 1050$ W/m^2. Therefore, we add the correction factor to (3) $k_e = 0.26016$:

$$I = I_0 \cdot e^{-k_\rho \cdot k_e} \tag{4}$$

For the general case, it is necessary to take into account the path that the Sun's rays travel through the atmosphere.

$$L_{\text{rays}} = \sqrt{h_{\max}^2 + 2R_{\text{Earth}} h_{\max} + R_{\text{Earth}}^2 \sin^2 a} - R_{\text{Earth}} \cdot \sin a, \tag{5}$$

where $R_{\text{Earth}} = 6371$ km is the radius of the Earth;

a is the angle of the Sun's rise above the horizon.

Thus, taking into account (4) and (5), the intensity of direct solar radiation on a perpendicular surface is calculated by the formula (6):

$$I = I_0 \cdot e^{-k_\rho \cdot k_e \cdot \frac{L_{\text{rays}}}{h_{\max}}} \tag{6}$$

The angle of the rise of the Sun above the horizon is a function of time and the width of the collector installation (7):

$$a = f(\varphi, t) \tag{7}$$

Instantaneous power is a function of the collector's height above sea level and the angle of the Sun's elevation above the horizon:

$$N = f[h, a(\varphi, t)] \tag{8}$$

Integrating the instantaneous power function (8) over a time interval from 0 to 24 h, we obtain the daily production of electrical energy:

$$Q_{\text{day}} = \int_0^{24} N[h, a(\varphi, t)]dt \tag{9}$$

Integrating the instantaneous power function (9) over a time interval from 0 to 365 days, we obtain the annual electric power generation:

$$Q_{\text{year}} = \int_0^{365} N[h, a(\varphi, t)]dt = \sum_{\text{day}=1}^{365} Q_{\text{day}} \tag{10}$$

The written program allows at any time to calculate the height of the Sun above the horizon and azimuth at any latitude, which can be used to control the system of collectors that track the Sun (Fig. 1).

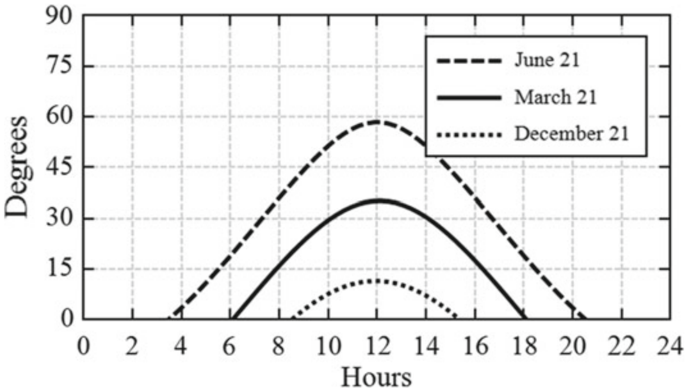

Fig. 1 Relationship between the height of the Sun above the horizon and time. For 55° north latitude

The written program allows us to calculate the relationship between the power of solar radiation and the angle of the Sun's rise above the horizon at any height and for any angle of inclination of the collector (Figs. 2, 3).

Using the written program, you can calculate the power of solar radiation arriving at receivers with different tilt angles and compare them (Figs. 4, 5). The graphs are calculated for Kazan (55° north latitude, 60 m above sea level).

Using the written program, you can calculate the energy of solar radiation arriving at receivers with different tilt angles during the year and compare them (Figs. 6, 7).

Annual production of solar energy for Kazan, calculated according to (10): $Q_{\text{year}} = 2680$ kW h/year m^2 (for the surface perpendicular to the solar radiation flux).

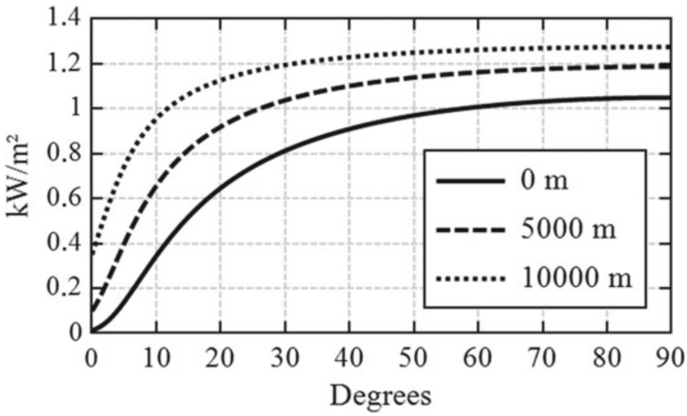

Fig. 2 Relationship between the specific power of radiation entering the surface perpendicular to the flow and the Sun's height above the horizon

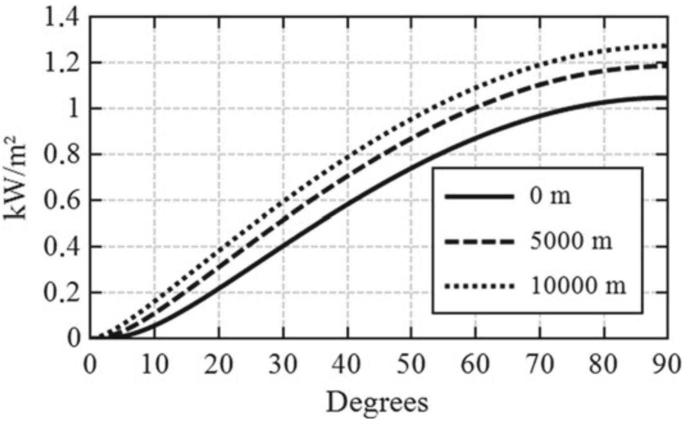

Fig. 3 Relationship between the specific power of radiation arriving at the horizontal surface and the height of the Sun above the horizon

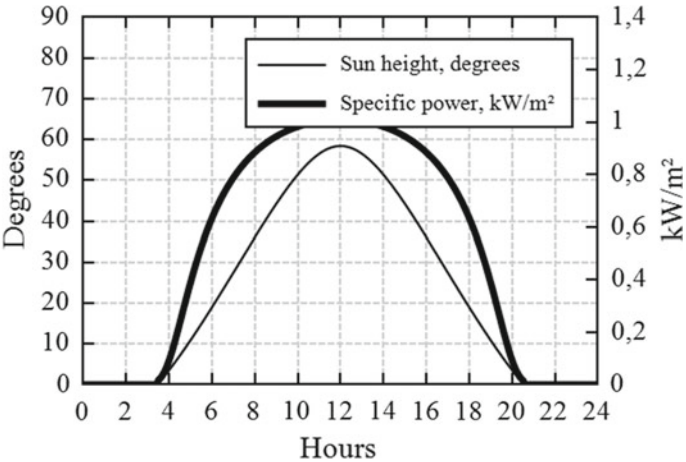

Fig. 4 Relationship between the height of the Sun above the horizon, specific power and time for the surface perpendicular to the flow (55° north latitude, 60 m above sea level, June 21)

Annual production of solar energy for Kazan, calculated according to (10): $Q_{\text{year}} = 1341$ kW h/year m^2 (for a horizontal surface). The obtained calculations were compared with the data obtained by long-term measurements of solar radiation in a clear sky [17]. According to the results of long-term observations for Kazan:

$Q_{\text{year}} = 2676$ kW h/year m^2 (for the surface perpendicular to the solar radiation flux);

$Q_{year} = 1253$ kW h/year m^2 (for a horizontal surface).

In the laboratory of the International Training Center at the Novosibirsk State Technical University, a vacuum solar collector and a heat-insulated storage tank

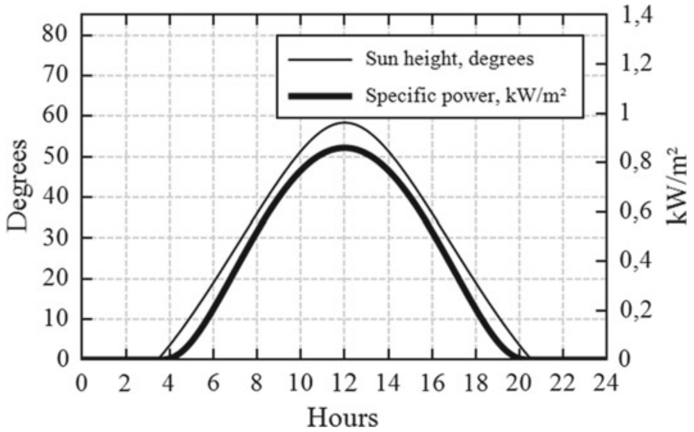

Fig. 5 Relationship between the height of the rise of the Sun above the horizon and specific power from time for horizontal collector surface (55° north latitude, 60 m above sea level, June 21)

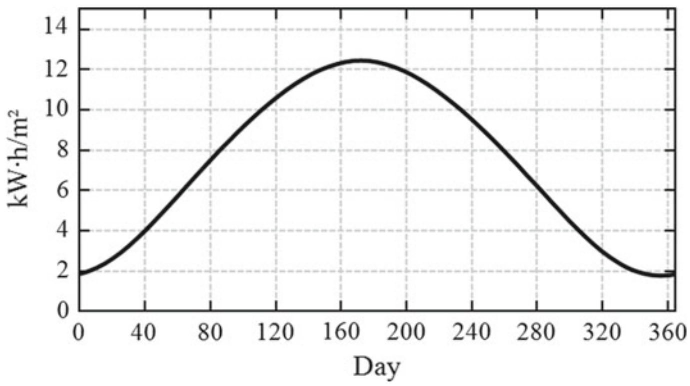

Fig. 6 Relationship between the energy of solar radiation arriving at the surface perpendicular to the flow and the day of the year (55° north latitude, 60 m above sea level)

with a volume of 950 L are installed. The collector and the tank supply hot water to the underfloor heating system and heating convectors. The vacuum manifold is installed on the southwest wall of the building and is tilted at an angle of 15 degrees, and azimuth of the collector is 135 degrees. The graph for this collector is shown in Fig. 8 Using the written program, you can calculate these graphs for any period of time.

Annual production of solar energy for the collector installed at the NSTU, calculated according to (10): $Q_{\text{year}} = 1390$ kW h/year m^2.

With the help of the written program, you can calculate the most optimal installation angle of the solar collector at a known latitude, altitude and azimuth. Figure 9

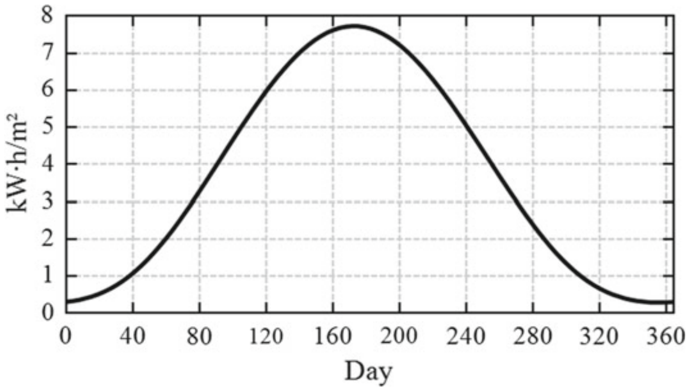

Fig. 7 Relationship between the energy of solar radiation arriving at the horizontal surface and the day of the year (55° north latitude, 60 m above sea level)

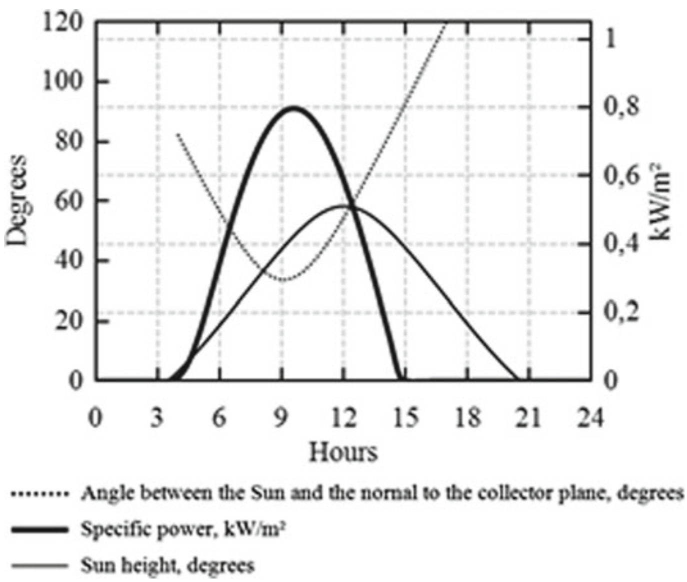

Fig. 8 Relationship between the specific power of solar radiation arriving at the collector installed in NSTU; the height of the rise of the Sun above the horizon; the angle between the Sun and the collector and hours (55° north latitude, 150 m above sea level, June 21)

shows a graph for the collector installed at the Novosibirsk State Technical University. The angle of inclination of the solar collector installed in NSTU is 15 degrees. Figure 9 shows that the power of solar radiation coming to the collector would be maximum if the collector was installed at an angle of 40–60 degrees.

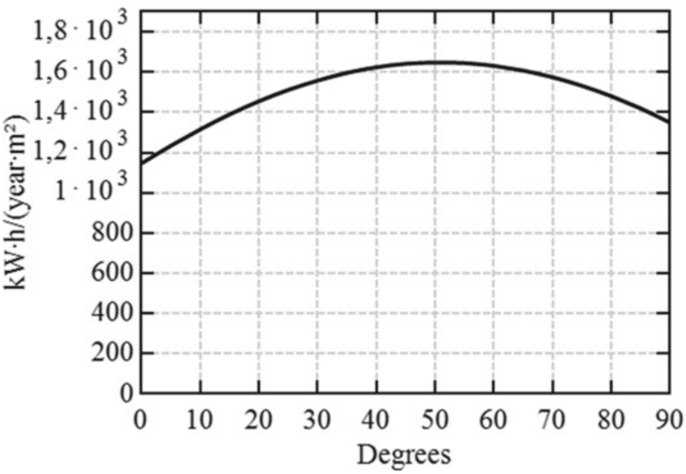

Fig. 9 Relationship between the energy of solar radiation supplied to 1 m^2 of the collector during the year and the collector tilt angle (55° north latitude, 150 m above sea level, 135 degrees azimuth)

3 Conclusion

The written program allows you to calculate the amount of solar radiation entering the solar collector installed at any angle to the solar radiation flux; evaluate the effectiveness of its installation in any region; calculate the power of the solar collector for any day or for a certain period of time and for any height above sea level; choose the optimal angle of installation of the collector. The obtained calculations coincide with the data of long-term observations.

Further work on the program is to ensure that the program calculates the optimal installation angle of the solar collector for a given region, taking into account the limitations of the installation angles when mounting on a specific object; calculate the most optimal period of use of the collector using meteorological data; based on the installation price of 1 m^2 of collector, choose the most optimal manufacturer.

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