

# Mathematical Simulation of Electrodynamic and Thermal Processes in Electrical Process Plants



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## 1 Introduction

Electrical process plants are widely used in various industries. The electrical energy consumed by these plants is about 25% of the total consumption [1]. Induction and microwave plants, which can increase the productivity and quality of products, are used in various technological processes. The use of electromagnetic waves of various frequencies to heat materials is common in these processes.

Induction heating is widely used in various technological processes [2–7] for heat treatment and melting of conductive materials. The advantage of induction heating is the ability to achieve high temperatures and the required uniformity in the heated material.

Along with induction heating, electrical technology uses microwave heating, which is used to intensify the heat treatment of various dielectric materials [8–12]. In comparison with conventional methods for heating dielectric materials, microwave heat treatment provides fast volumetric heating of dielectrics, high efficiency of conversion of the electromagnetic field into thermal energy, and improved product quality.

In addition, a non-thermal effect on the physical and mechanical properties of polymeric materials was identified during microwave exposure, which contributes to an increase in operational parameters.

An urgent task is to develop energy-efficient induction and microwave plants that meet the increased requirements for the quality of products. In addition, electrical process plants such as high-tech power consumers can be classified as objects, the control of which should be organized on the basis of digital twins along with the distributed generation system in the power industry. Therefore, the development of

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mathematical models of electrical process plants and systems is an urgent problem [13].

The purpose of the work is a mathematical simulation of electrodynamic and thermal processes during copper melting in an induction crucible furnace and in a microwave plant for thermal modification of composite materials.

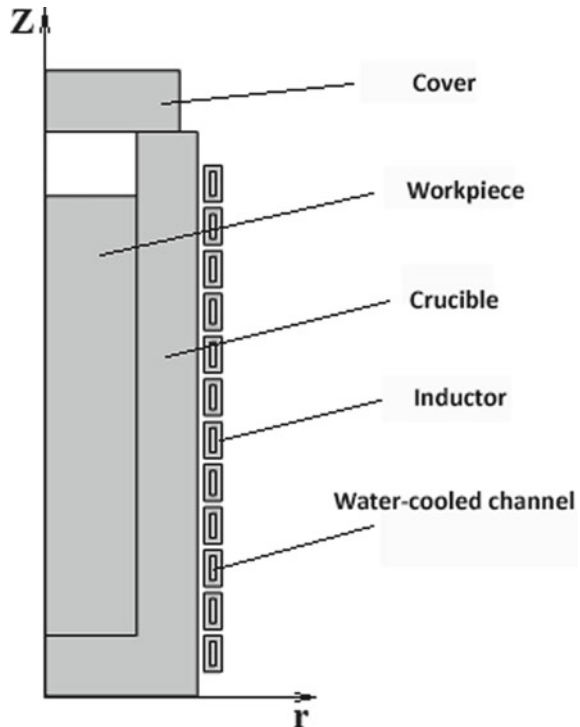
## 2 Description of the Mathematical Model

### 2.1 Mathematical Model of Induction Heating

This paper considers the mathematical simulation of technological processes in an induction plant for melting copper. The induction plant contains the following main components: a water-cooled inductor made of a rectangular copper tube; a graphite crucible with a heated workpiece, which is placed inside the inductor (Fig. 1).

For mathematical simulation of the processes in the crucible furnace during induction melting, the equations of electrodynamics and thermal conductivity are used [2]. The electrodynamic process in the induction plant is described by differential

**Fig. 1** Simplified geometric model of the induction crucible furnace



equations:

$$(j\omega\sigma - \omega^2\varepsilon_0\varepsilon_r)A + \nabla \times (\mu_0^{-1}\mu_r^{-1}B) = J_e, \quad (1)$$

$$B = \nabla \times A, \quad (2)$$

where  $A$ —vector of magnetic potential;  $B$ —vector of magnetic induction;  $\omega$ —circular frequency;  $\sigma$ —electrical conductivity;  $\varepsilon_0, \varepsilon_r$ —vacuum dielectric constant and relative dielectric constant of the medium;  $\mu_0, \mu_r$ —magnetic permeability of vacuum and relative magnetic permeability of the medium;  $J_e$ —current density vector [7, 8].

The heat conduction equation has the following form [14]:

$$\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) + q_v, \quad (3)$$

where  $\rho, C_p$ —density, the specific heat capacity of the material of the corresponding medium;  $q_v$ —specific power of the heat sources due to the induction heating of the workpiece by the induced currents;  $\lambda$ —coefficient of thermal conductivity.

The water flow cooling the inductor is turbulent. In this case, the process of cooling the inductor can be approximately described by the following Eq. (2):

$$Q_{\text{ohl}} = \frac{v\rho C(T_b - T_{\text{ust}})}{V}, \quad (4)$$

where  $v$ —speed of water flow inside the inductor channel;  $\rho$ —density of water;  $T_b$ —temperature at the initial moment of time;  $T_{\text{ust}}$ —steady-state temperature at a certain point in time;  $C$ —specific heat capacity of water.

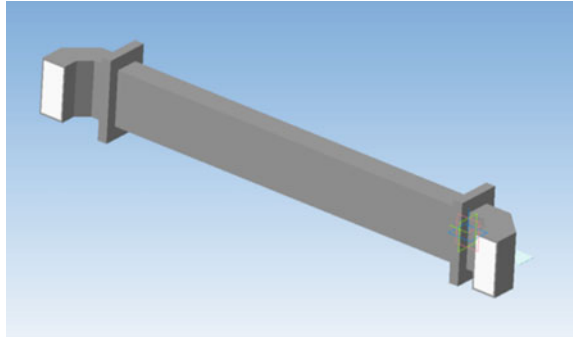
## 2.2 Mathematical Model of Dielectric Microwave Heating

Mathematical simulation of the conveyor-type microwave electrical process plant based on a traveling wave chamber for the modification of polymer composite materials was carried out. The microwave plant is designed to improve the physical and technical properties of the processed materials (Fig. 2).

The waveguide with a cross section of 45 mm × 90 mm, which contains a transporting belt, was selected. A 3 kW water-cooled magnetron operating at a frequency of 2450 MHz was used as a microwave power source.

For mathematical simulation of microwave thermal modification of polymer composite materials (PCM), a system of interrelated equations of thermal conductivity (3) and electrodynamics [15] was used:

**Fig. 2** The geometry of the microwave working chamber based on the waveguide



$$\nabla \times (\mu_r^{-1} \nabla \times \mathbf{E}) - k_0^2 \left( \varepsilon_r' - \frac{j\sigma}{\omega\varepsilon_0} \right) \mathbf{E} = 0$$

$$q_v = 0,5\omega\varepsilon_0\varepsilon_r'' \cdot |\mathbf{E}|^2 \quad (5)$$

where  $\mathbf{E}$ —vector of the electric field strength,  $q_v$ —specific power of heat sources due to dielectric losses,  $\mu_r$ —relative magnetic permeability,  $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ —wave number,  $\varepsilon_r'$ —relative dielectric constant (real part),  $\sigma = \omega\varepsilon_0\varepsilon_r''$ —electrical conductivity,  $\omega = 2\pi f$ —circular frequency,  $\varepsilon_0$ —electric constant,  $\varepsilon_r''$ —effective loss factor (imaginary part of the dielectric constant), and  $f$ —frequency of the electromagnetic field.

The mathematical simulation was carried out using the COMSOL Multiphysics software product. This software product allows solving interrelated differential equations of electrodynamics and thermal conductivity, which characterize technological processes in both induction and microwave heating of materials [15].

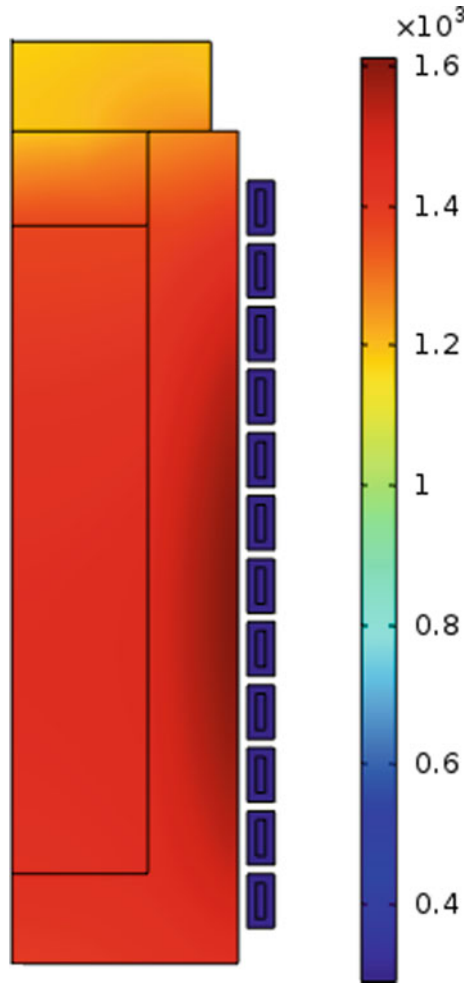
### 3 Results

#### 3.1 The Results of Modeling Copper Melting Processes in the Crucible Induction Plant

The results of mathematical simulation of the processes of heating and melting copper in the crucible induction plant are considered. Figures 3, 4, and 5 show, respectively, the temperature field in the longitudinal section of the induction plant, in the volume of the plant, and the boundary of the phase transition during melting of copper at the time 3600 s.

Figure 6 shows a graph of temperature changes in time in the center of the copper workpiece until 7200 s. The results of studying the influence of various graphite grades, from which the crucible is made, on the heating rate of the copper workpiece are shown in Fig. 7.

**Fig. 3** Temperature distribution in the copper workpiece during 3600 s

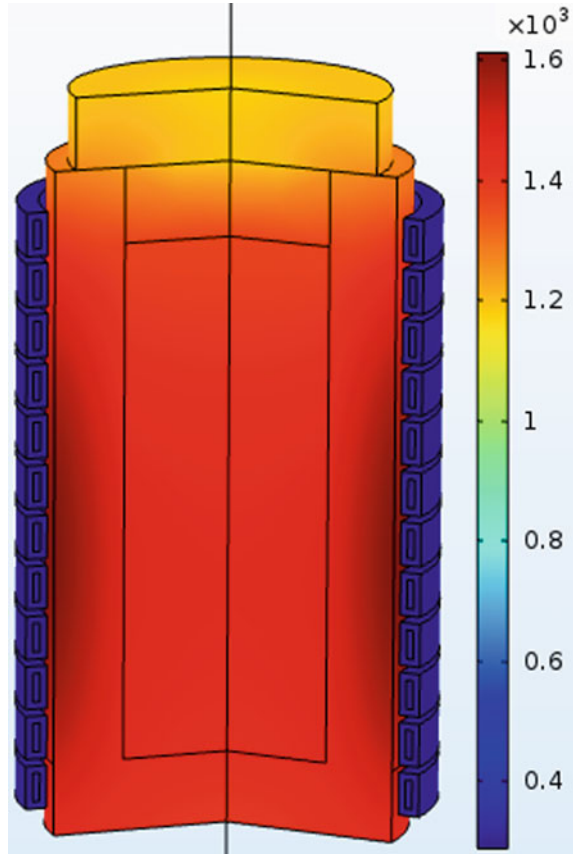


By analyzing the above dependences (Fig. 7), it is possible to select a material for the manufacture of the crucible with higher electrical conductivity and faster heating in order to increase the energy efficiency of the copper melting process.

It was found that the phase transformation of copper from the solid phase to the melt occurs in the time range from 3600 to 4000 s. In this case, the copper workpiece reaches a temperature of 1300–1400 K, which corresponds to the real experiment.

Application of the methodology for simulating melting processes in crucible induction furnaces using the COMSOL Multiphysics software package allows developing energy-efficient plants of this type.

**Fig. 4** Temperature distribution in the copper workpiece during 3600 s



### ***3.2 Results of Simulation of PCM Microwave Heat Treatment***

As a result of mathematical simulation in COMSOL Multiphysics, the distribution of temperature fields in the processed polymer composite material was obtained (Fig. 8).

The distribution of the temperature field in the cross section of the material for different times is shown in Fig. 9.

As was previously found in the works of the authors [10], the modifying effect on the PCM is due to the magnitude of the intensity of the microwave electromagnetic field. In this regard, when simulating the technological process, it is necessary to determine the distribution of the microwave electric field strength. The result of this simulation is the distribution of the electric field strength in the microwave working chamber (Fig. 9) and in the processed material (Fig. 10).

Figures 10 and 11 show that when passing through the dielectric, the distribution pattern of the electric field strength changes. In particular, the distance between the

### 1- Boundary of the phase transition

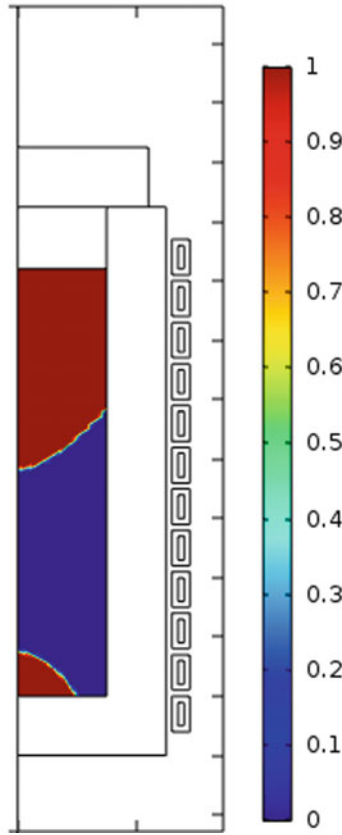


Fig. 5 Phase transition at 3600 s

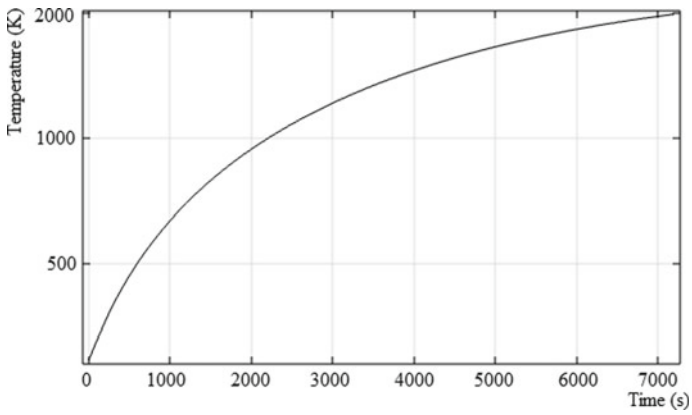
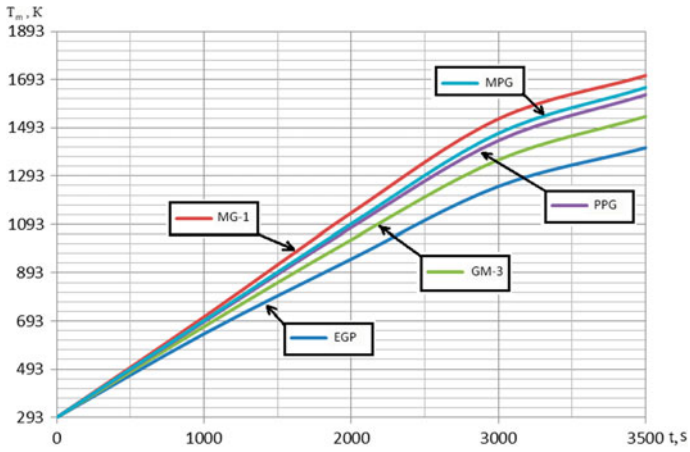
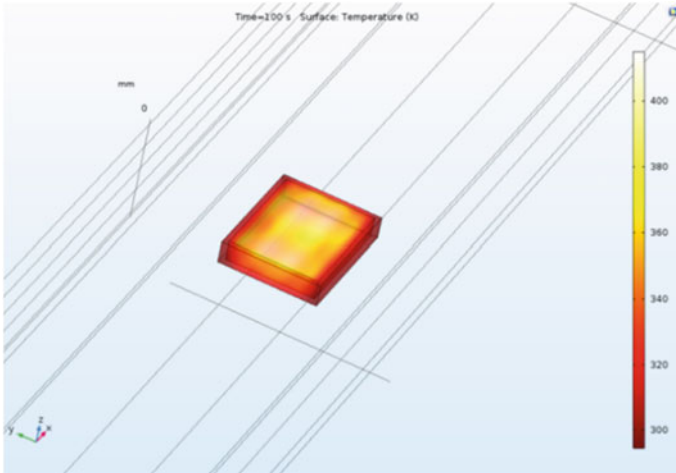


Fig. 6 A graph of temperature changes in time in the center of the copper workpiece during 7200 s



**Fig. 7** Influence of graphite grade on induction heating



**Fig. 8** Distribution of the temperature field in the material

maxima and minima of the electric field strength changes. These features must be taken into account when carrying out a non-thermal modification of PCM.

## 4 Conclusion

Based on the mathematical simulation, it has been established that the most energy-efficient graphite grade for the induction crucible furnace for melting copper is MG-1.



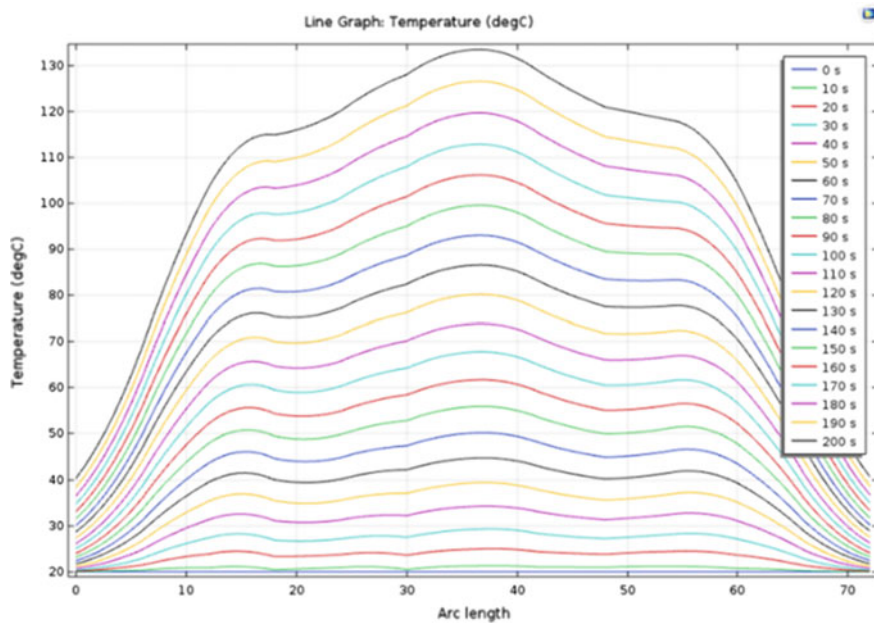


Fig. 9 Distribution of the temperature field in the material

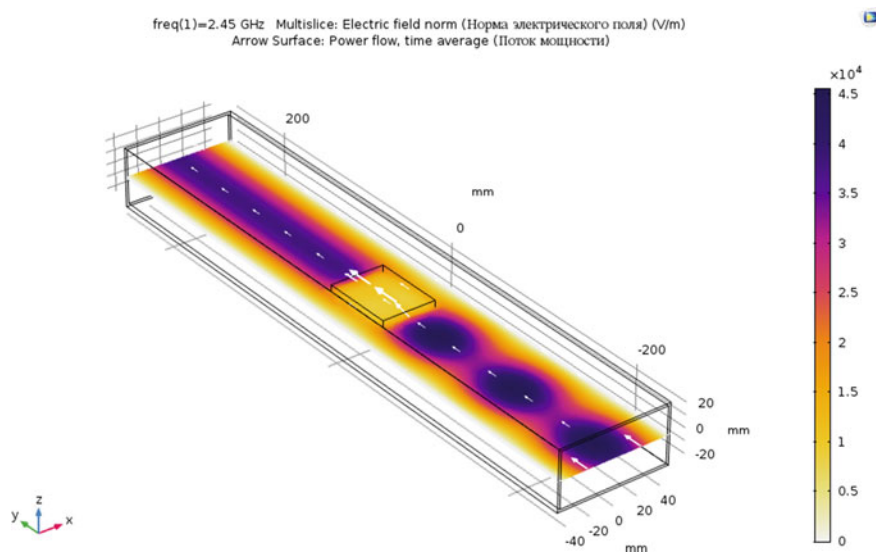
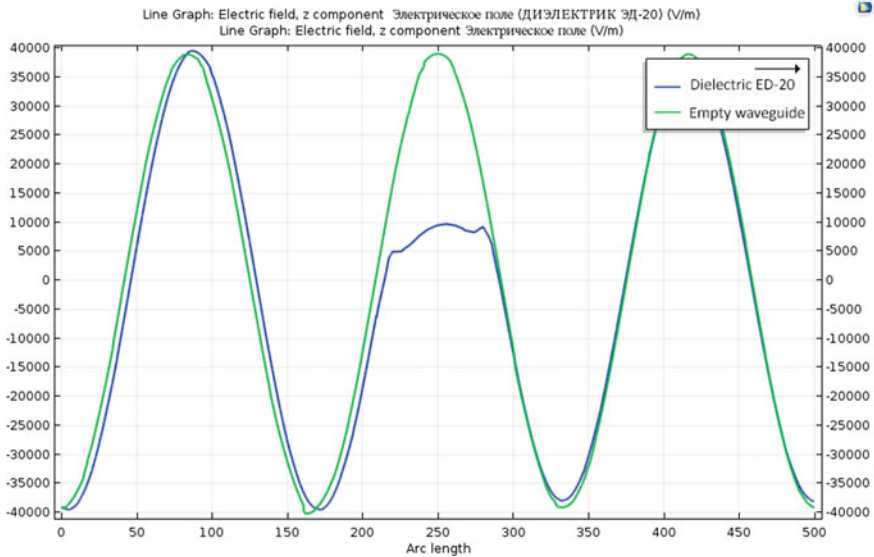


Fig. 10 Electric field distribution in the microwave chamber with dielectric



**Fig. 11** Electric field strength distribution in the dielectric along the  $z$  coordinate

In this case, the rate of heating and melting of copper increases by 7.5% in comparison with similar graphite crucibles of other brands.

When designing process microwave chambers of a rectangular type with a traveling wave for the modification of various materials, it is advisable to carry out numerical simulation of thermal and electric fields in the material. This allows predicting changes in the distribution of temperature and electric fields depending on the properties of polymer composite materials and to select the best parameters of microwave exposure.

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