Constrained Control for Systems on Lie Groups with Uncertainties via Tube-Based Model Predictive Control on Euclidean Spaces

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Abstract. In this paper, the constrained control of systems evolving on matrix Lie groups with uncertainties is considered. The proposed methodology is composed of a nominal Model Predictive Control (MPC), and a feedback controller. The previous work on the control of systems on manifolds is applied to design the nominal MPC, which generates the nominal trajectory. In the nominal MPC, the state and input constraints on the Lie group are transformed into the constraints on the Euclidean space. While to deal with uncertainties, the feedback control used to track the nominal trajectory is designed directly on the Lie group. The tracking error in the feedback control is proved to be bounded in invariant sets. Such invariant sets are further used to revise the constraints in nominal MPC. We prove that by using this methodology, the stability and safety of the system can be guaranteed simultaneously. The proposed methodology is applied to the constrained attitude control of rigid bodies. In the application example, the detailed mathematical proof and the numerical simulation are presented, illustrating the feasibility of the proposed methodology.

Keywords: Matrix lie group · Model predictive control · Robust control · Attitude control

1 Introduction

1.1 Motivation and Background

Many systems subject to constraints such as state and input constraints, and uncertainties. The state and input constraints are critical for safety. The uncertainties somehow may let the system violet the state and input constraints, hurting the safety of the system. How to address the state and input constraints under uncertainties in terms of safety is, therefore, a meaningful and challenging problem.

The tube-based MPC is a useful tool that can deal with the state and input constraints of a dynamic system with consideration of the disturbance. It has

F. Sun et al. (Eds.): ICCSIP 2021, CCIS 1515, pp. 156–173, 2022.
https://doi.org/10.1007/978-981-16-9247-5_12

been applied to varieties of dynamic systems. Dimarogonas et al. investigated the decentralized control of uncertain nonlinear multi-agent systems using tube-based MPC [10,16]. They also studied the constrained control problem of underwater vehicles by tube-based MPC [12]. Chen et al. addressed the trajectory-tracking control problem for mobile robots by combining tube-based MPC [8]. Kobilarov et al. proposed a tube-based MPC whose tube is expressed by ellipsoids [11]. Yue et al. proposed a robust tube-based model predictive control for lane change maneuver of tractor-trailer vehicles [22]. Some researchers also applied tube-based MPC to the control problem of networks, e.g. [17].

On the other side, the state space of many systems is non-Euclidean manifold, e.g., the configuration of many mechanical systems. The motion control on non-Euclidean Lie group configuration space arouses great interests, which can mainly be categorized into two groups, the coordinate-based control and the geometric control [2,20]. The former method usually uses a local homomorphic map to obtain the local coordinates of the non-Euclidean Lie groups. Taking $SO(3)$ as an example, the local coordinates include Euler angle [24], exponential coordinate [18,20], quaternion [15], etc. While the latter normally builds the error function directly on the Lie group [13,21]. Because of the topological properties of the frequent Lie groups in mechanical systems, the global map between the Lie group and the Euclidean space usually does not exist [3,7,9]. To investigate the global control problem, some researchers adopt various methodologies, e.g., the hybrid system tools [14,19]. Also, the geometric control developed directly on the non-Euclidean Lie group can achieve almost global stability. However, nonlinear control for complex systems with non-Euclidean Lie group configuration space is still a challenging problem.

Recent attempts to control of systems on manifolds include the method by embedding the manifold into ambient Euclidean space [5,6], where the design procedure is usually divided into two steps. First, the given manifold is embedded into the Euclidean space and the system dynamics is stably extended into the Euclidean space. Then the controller is designed on the ambient Euclidean space. As the system dynamics on the manifold is stably extended, the stability of the controlled system on the manifold can therefore be obtained. Such a methodology does not need a local coordinate chart on the manifold, thus can avoid frequent problems induced by the local coordinate chart. In the authors' previous work, the MPC on manifold via embedding is also considered [4]. By stably extending the system dynamics from manifold to ambient Euclidean space, the MPC on Euclidean space can be applied directly. However, the previous work does not consider the uncertainties of the system, which may make the actual trajectory differs from the nominal trajectory. In this way, the safety of the systems may be hurt. The constrained control problem of the system on the manifold is therefore a meaningful problem. It is noted that there are some significant challenges to address this problem for systems in non-Euclidean space. In order to guarantee the safety of the system, one may need to express the volume of the tube, i.e., the invariant set of the tracking error. However, it is difficult to express the tube if the manifold is embedded into the ambient Euclidean space, as the invariant set is not preserved anymore after the extension of the dynamics.

1.2 Contributions

In this paper, we aim to solve the constrained control problem for the systems evolving on the matrix Lie group. We will extend the previous methodology which embeds the matrix Lie group into ambient Euclidean space. Inspired by the methodology of tube-based MPC, we design the nominal trajectory of the tracking error dynamics on Euclidean space. And considering the disturbance, the tube on Euclidean space is defined. We will design the feedback controller directly on the Lie group by transferring the tube from the Euclidean space to the Lie group. We will show that in such a framework, the safety of the system can be ensured.

In summary, the contribution of this paper can be summarized as,

- We propose a framework to address the constrained control problem for systems on matrix Lie groups. The proposed methodology does not rely on any local coordinates of the Lie group and can apply the existing MPC technique on Euclidean spaces.
- The mathematical proof of the proposed methodology in terms of stability and safety is presented.
- The proposed methodology is applied to the attitude control of rigid bodies, demonstrating its feasibility.

This paper is organized into five sections. In Sect. 2, some background and the problem definition are presented. In Sect. 3, the framework of the tube-based MPC on the manifold is designed and analyzed. Section 4, the proposed methodology is applied to the constrained attitude control of rigid bodies.

1.3 Notation

Given a matrix Lie group G and sets $S_1, S_2 \subset G$, we define the following set operations

$$S_1 \odot S_2 = \{s_1 s_2 : s_1 \in S_1, s_2 \in S_2\}$$
$$S_1 \oslash S_2 = \{s_1 : s_2 s_1 \in S_1, \forall s_2 \in S_2\}$$

Also we define the following set operations for sets in Euclidean spaces,

$$\begin{aligned} S_1 \oplus S_2 &= \{s_1 + s_2 : s_1 \in S_1, s_2 \in S_2\} \\ S_1 \ominus S_2 &= \{s_1 : s_2 + s_1 \in S_1, \forall s_2 \in S_2\} \end{aligned} \tag{1}$$

Furthermore, let us define $\mathrm{Ad}_A B = ABA^{-1}$ for all $A \in \mathrm{GL}(n)$, $B \in \mathbb{R}^{n\times n}$. Then, $\mathrm{Ad}_g \xi \in \mathfrak{g}$ for all $g \in G$ and $\xi \in \mathfrak{g}$, where $\mathfrak{g}$ is the Lie algebra of G.

2 Background and Problem Formulation

2.1 System Dynamics and Preliminaries

Systems evolving on an m-dimensional matrix Lie group can be expressed by the following equation of motion (EOM),

$$\begin{aligned} \dot{g} &= g\xi \\ \dot{\xi} &= f(\xi, u) + d \end{aligned} \tag{2}$$

where $g \in G$, $\xi \in \mathfrak{g}$, and $u \in \mathbb{R}^m$, $f(\cdot, \cdot)$ is the left-invariant vector field, $d \in \mathbb{R}^m$ is the disturbance induced by modeling uncertainties, and external disturbances. We suppose d satisfies

$$\|d\| \le b_1 \tag{3}$$

where b_1 is a positive constant.

Denote the reference trajectory of the system by

$$\mathbb{R} \ni t \mapsto (g_0(t), \xi_0(t)) \in G \times \mathfrak{g} \tag{4}$$

and the corresponding reference input of the system by,

$$\mathbb{R} \ni t \mapsto u_0(t) \in \mathbb{R}^m \tag{5}$$

We embed the matrix Lie group G into the Euclidean space $\mathbb{R}^{n\times n}$. The tracking error trajectory can be defined on the Euclidean space as,

$$\mathbb{R} \ni t \mapsto (E(t), \varXi(t)) := (xg_0^{-1} - I, \xi - \xi_0) \in \mathbb{R}^{n\times n} \times \mathfrak{g} \tag{6}$$

If we let the trajectory of the system (2) track the reference trajectory, the tracking error dynamics of (2) can therefore be expressed as,

$$\begin{aligned} \dot{E} &= (g_0 + Eg_0)\varXi g_0^{-1} \\ \dot{\varXi} &= f(\varXi + \xi_0, u) - f(\xi_0, u_0) + d \end{aligned} \tag{7}$$

It is noticed that the system (7) also evolves on the Lie group, not on the Euclidean space. By applying the technique of embedding the matrix Lie group G into Euclidean space $\mathbb{R}^{n\times n}$, we can obtain the following equation evolving on $\mathbb{R}^{n\times n} \times \mathbb{R}^m$,

$$\begin{aligned} \dot{E} &= (g_0 + Eg_0)\varXi g_0^{-1} - \alpha \nabla V(g_0 + Eg_0)g_0^{-1} \\ \dot{\varXi} &= f(\varXi + \xi_0, u) - f(\xi_0, u_0) + d \end{aligned} \tag{8}$$

where $E = xg_0^{-1} - I \in \mathbb{R}^{n\times n}$, and V is a function $\mathbb{R}^{n\times n} \mapsto V(x) > 0$ satisfying

$$\begin{aligned} &V^{-1}(0) = G \\ &V(xg) = V(x), \forall x \in \mathbb{R}^{n\times n}, g \in G \\ &\nabla^2 V(I)(y, y) > 0, \forall y \in \mathfrak{g} \end{aligned} \tag{9}$$

In this way, we say that the system dynamics (2) is embedded into the Euclidean space stably.

2.2 Problem Formulation

In this paper we will consider the control problem of dynamic systems evolving on matrix Lie groups, under state constraints, input boundedness, and uncertainties. The control problem can therefore be expressed as follows.

Problem 1. *Consider the system evolving on matrix Lie groups governed by the EOM (2). Given specific configuration constraint $g \in \mathcal{X}$, and specific velocity constraint $V_0 \in \mathcal{V}$, input constraints $u \in U$, for reference state and input $(g_0, \xi_0) \in G \times \mathfrak{g}$, deign control input $u : t \mapsto (\tau, T)$ which forces $\|E(t)\| \leq \epsilon_1$ and $\|\Xi(t)\| \leq \epsilon_2$ as $t \to \infty$ with small positive constant ϵ_1 and ϵ_2 while fulfilling all the above constraints for all disturbance satisfying (3).*

The configuration error E can further be divided into the parallel direction error and the transversal direction error as,

$$E \in \mathbb{R}^{n\times n} \mapsto E^{\perp} \in g^{\perp}, E \in \mathbb{R}^{n\times n} \mapsto E^{\parallel} \in g \tag{10}$$

where $g^{\perp}$ is the orthogonal component of g in Euclidean space $\mathbb{R}^{n\times n}$, under the Euclidean metric defined by $\langle A, B\rangle = trace(A^T B)$ for all $A, B \in \mathbb{R}^{n\times n}$.

Given the reference trajectory $g_0(t)$ satisfying $\alpha_1 I \leq g_0(t)g_0(t)^T \leq \alpha_2 I$ for all t, we linearize (8) along the reference trajectory, the tracking error dynamics can be expressed as,

$$\begin{aligned}
\dot{E}^{\perp} &= -\alpha((\nabla^2 V(I)\cdot E^{\perp})(g_0 g_0^{\perp})^{-1})^{\perp} \\
\dot{E}^{\parallel} &= g_0 \Xi g_0^{-1} - \alpha(\nabla^2 V(I)\cdot E^{\perp})(g_0 g_0^{T})^{-1})^{\parallel} \\
\dot{\Xi} &= \frac{\partial f}{\partial \xi}(\xi_0, u_0)\Xi + \frac{\partial f}{\partial u}\delta u + d
\end{aligned} \tag{11}$$

where $\delta u = u - u_0$. As stated in [5], the first equation in (11) is exponentially stable at the origin. It is also possible to design control based on the linearized system (11). However, in order to solve Problem 1, we need to carefully consider the set of tracking errors, which may influence the admissible input and state set. As it is difficult to estimate the boundedness of the tracking error for the linearized system, we will therefore develop a methodology which generates the nominal trajectory based on (11), and tracks the nominal trajectory based on (2) directly.

3 Tube-Based MPC Design

3.1 Nominal MPC

By excluding the disturbance from the actual system, the nominal EOM of the system is given by,

$$\begin{aligned}
\dot{\bar{g}} &= \bar{g}\bar{\xi} \\
\dot{\bar{\xi}} &= f(\bar{\xi}, \bar{u})
\end{aligned} \tag{12}$$

where $\bar{*}$ represents the nominal value.

We will solve Problem 1 inspired by the idea of tube-based MPC. The tube-based MPC is composed of a nominal MPC and a feedback controller. The nominal MPC is designed from the nominal tracking error dynamics. By embedding the nominal EOM into Euclidean space, we design the nominal tracking error as $\bar{E} = \bar{X}g_0^{-1} - I \in \mathbb{R}^{n\times n}, \bar{\Xi} = \bar{\xi} - \xi_0$. Then excluding the disturbance from (11), the nominal tracking error dynamics embedded into the Euclidean space is obtained as,

$$\begin{aligned} \dot{\bar{E}}^{\perp} &= -\alpha((\nabla^2 V(I)\cdot \bar{E}^{\perp})(g_0 g_0^{\perp})^{-1})^{\perp} \\ \dot{\bar{E}}^{\parallel} &= g_0\bar{\Xi}g_0^{-1} - \alpha(\nabla^2 V(I)\cdot \bar{E}^{\perp})(g_0 g_0^{T})^{-1})^{\parallel} \\ \dot{\bar{\Xi}} &= \frac{\partial f}{\partial \xi}(\xi_0, u_0)\bar{\Xi} + \frac{\partial f}{\partial u}\bar{\delta}u \end{aligned} \tag{13}$$

In the nominal MPC design, we define the initial tracking error $\bar{E} = E$ and $\bar{\Xi} = \Xi$, i.e., we let $\bar{g} = g$ and $\bar{\xi} = \xi$ at the initial time. The purpose of the nominal MPC is to let $\bar{E}^{\parallel}$ converge to the origin while satisfying the nominal input and state constraints.

To deal with the state and input constraints, we express the admissible set of the configuration and velocity error as $\bar{\mathcal{X}}$ and $\bar{\mathcal{V}}$ and the admissible control input as $\bar{U}$. Therefore, the nominal MPC is expressed as,

$$\begin{aligned} &\min_{\bar{\delta}u(s)} J(\bar{\zeta}, \bar{u}_0) = \quad V_r(\bar{\zeta}(t_k + \Gamma)) + \\ &\int_{t_k}^{t_k+\Gamma} N_r(\bar{\zeta}(s), \bar{\delta}_u(s))ds \\ &s.t.\ \dot{\bar{E}}^{\parallel} = \bar{g}_0\bar{\Xi}\bar{g}_0^{-1} - \alpha(\nabla^2 V(I)\cdot \bar{E}^{\perp})(\bar{g}_0\bar{g}_0^{T})^{-1})^{\parallel} \\ &\dot{\bar{\Xi}} = \frac{\partial f}{\partial \xi}(\xi_0, u_0)\bar{\Xi} + \frac{\partial f}{\partial u}\bar{\delta}_u \\ &(\bar{E}^{\parallel}, \bar{\Xi}) \in \bar{\mathcal{X}} \times \bar{\mathcal{V}}, \bar{\delta}u(s) \in \bar{U} \end{aligned} \tag{14}$$

where $\zeta = (E^{\parallel}, \Xi)$ is the state, $V_r(\cdot)$ and $N_r(\cdot)$ are positive definite functions used to ensure the stability of the MPC. Notice that $\bar{\mathcal{X}}$, $\bar{\mathcal{V}}$, and $\bar{U}$ will be given later, according to the actual admissible state and input set and the feedback controller.

3.2 Feedback Control for the Disturbed System on Matrix Lie Group

The nominal MPC can generate the nominal trajectory of the system on the matrix Lie group. Suppose the nominal error trajectory is given by $\mathbb{R} \ni t \mapsto (\bar{E}, \bar{\Xi})$, and the nominal input error trajectory is denoted by $\mathbb{R} \ni t \mapsto (\bar{\delta_u})$. Then the nominal state trajectory is obtained as $\bar{g} = (\bar{E} + I)g_0, \bar{\xi} = \xi + \bar{\Xi}$, and the nominal input trajectory is obtained as $\bar{\tau} = \tau_0 + \delta_\tau$.

It is noted that using the nominal MPC, the generated nominal state trajectory is already restricted on the matrix Lie group. Therefore, we design the feedback control for the actual systems on the matrix Lie group directly.

For the actual system with uncertainties, it is necessary to design the tracking error carefully. We first define the tracking error between the nominal state and the actual state as $\tilde{E} = g\bar{g}^{-1} - I, \tilde{\Xi} = \xi - \bar{\xi}$. The feedback controller should ensure the boundedness of the tracking error $(\tilde{E}, \tilde{\Xi})$ and the input error $\tilde{\tau} = \tau - \bar{\tau}$ so that the constraints in the nominal MPC can be derived from the actual admissible input and state sets.

As the nominal trajectory always evolves on the matrix Lie group, the feedback controller can be designed in a cascaded format. Given the nominal trajectory generated by the NMPC, design the velocity ξ_r which is the output of the outer loop controller such that

$$\mathrm{Ad}_{\bar{g}}(\xi_r - \bar{\xi}) = -k_g(\tilde{E}^T\tilde{E} + \tilde{E}^T)^{\parallel} \tag{15}$$

where k_g is a positive constant.

Then design the following control law of the inner loop to let ξ track ξ_r,

$$u = u_r - k_{\xi}(\xi - \xi_r) \tag{16}$$

where $u_r = f^{-1}(\dot{\xi})$.

Lemma 1. *[23] Given two vectors $x, y \in \mathbb{R}^n$ their convex hull is defined by $Co(x,y) := \{\xi : \xi = \theta x + (1-\theta)y, 0 < \theta < 1\}$. Consider a vector-valued function $f : \mathbb{R}^n \mapsto \mathbb{R}^m$. Assume that f is differentiable on an open set $S \subseteq \mathbb{R}^n$. Let x, y two points of S such that $Co(x,y) \subseteq S$. Then, there exist constant vectors $c_1, \cdots, c_m \in Co(x,y)$ such that,*

$$f(x) - f(y) = \left[\sum_{k=1}^{m}\sum_{j=1}^{n} l_m(k) l_n(j)^T \frac{\partial f_k(c_k)}{\partial x_j}\right](x-y)$$

We define an intermediate tracking error $\xi_e := \xi - \xi_r$. From the second element of (2) and Lemma 1, there are $c_1, c_2, ..., c_m \in Co(u, u_r)$ such that,

$$\begin{aligned}\dot{\xi}_e =& f(\xi,u) - f(\xi_r,u) + f(\xi_r,u) - f(\xi_r,u_r) + d\\ =& f(\xi,u) - f(\xi_r,u) + \left[\sum_{k=1}^{m}\sum_{j=1}^{n} l_m(k) l_n(j)^T \frac{\partial f_k(\xi_r, c_k)}{\partial x_j}\right](u-u_r) + d\end{aligned} \tag{17}$$

where L_1 is the Lipschitz constant of the function $f(\cdot, u)$.

Defining $\varphi_1 = \frac{1}{2}\xi_e^T\xi_e$, we arrive at,

$$\begin{aligned}\dot{\varphi}_1 = \xi_e^T\dot{\xi}_e \le& L_1\|\xi_e\|^2 - k_{\xi}\frac{J(\xi_r) + J^T(\xi_r)}{2}\|\xi_e\|^2 + \xi_e^T d\\ \le& -(k_{\xi}J_{min}(\xi_e) - L_1)\|\xi_e\|^2 + \frac{1}{4\rho_g}\|\xi_e\|^2 + \rho_g b_1^2\end{aligned} \tag{18}$$

where ρ_g is a positive constant. Then it is concluded that $\dot{\varphi}_1 < 0$ if $\|\xi_e\| > \frac{\rho_g}{k_\xi J_{min} - L_1 - \frac{1}{4\rho_g}} d_m$. As we let $\|\xi_e\| = 0$ at the initial instant, the velocity tracking error ξ_e is bounded by

$$\|\xi_e\| \leq b_v := \frac{\rho}{k_\xi J_{min} - L_1 - \frac{1}{4\rho_g}} b_1 \tag{19}$$

Then we have the following Lemma.

Lemma 2. *Consider system (2). Suppose the nominal state and input trajectory are generated by solving (14), the control law (15) and (16) are used to track the nominal state. Then the tracking error $\tilde{E}$ and ξ_e converge to the invariant set $\tilde{\Omega}_E := \{\tilde{E} : \|(\tilde{E}^T\tilde{E} + \tilde{E}^T)^{\|}\| \leq \frac{\sqrt{2\rho_\xi} b_v}{\sqrt{2k_g - \frac{1}{2\rho_\xi}}}\}$, $\tilde{\Omega}_\xi := \{\xi_e : \|\xi_e\| \leq b_v\}$.*

Proof. We define the candidate Lyapunov function as,

$$\varphi_2 = \|g\bar{g}^{-1} - I\|^2 = \langle g\bar{g}^{-1} - I, g\bar{g}^{-1} - I\rangle. \tag{20}$$

Then, taking the time derivative of φ_2 yields,

$$\begin{aligned}
\dot{\varphi}_2 &= 2\langle \tilde{E}, g(\xi - \bar{\xi})\bar{g}^{-1}\rangle \\
&= 2\langle \tilde{E}, g(\xi - \xi_r)\bar{g}^{-1}\rangle + 2\langle \tilde{E}, g(\xi_r - \bar{\xi})\bar{g}^{-1}\rangle \\
&= 2\langle \tilde{E}, g\bar{g}^{-1}\,\mathrm{Ad}_{\bar{g}}\,\xi_e\rangle + 2\langle \tilde{E}, g\bar{g}^{-1}\,\mathrm{Ad}_{\bar{g}}(\xi_r - \bar{\xi})\rangle \\
&= 2\langle \tilde{E}, (\tilde{E} + I)\,\mathrm{Ad}_{\bar{g}}\,\xi_e\rangle + 2\langle \tilde{E}, (\tilde{E} + I)\,\mathrm{Ad}_{\bar{g}}(\xi_r - \bar{\xi})\rangle \\
&= 2\langle \tilde{E}^T\tilde{E} + \tilde{E}, \mathrm{Ad}_{\bar{g}}\,\xi_e\rangle + 2\langle \tilde{E}^T\tilde{E} + \tilde{E}, \mathrm{Ad}_{\bar{g}}(\xi_r - \bar{\xi})\rangle \\
&= 2\langle (\tilde{E}^T\tilde{E} + \tilde{E})^{\|}, \mathrm{Ad}_{\bar{g}}\,\xi_e\rangle + 2\langle (\tilde{E}^T\tilde{E} + \tilde{E})^{\|}, \mathrm{Ad}_{\bar{g}}(\xi_r - \bar{\xi})\rangle
\end{aligned} \tag{21}$$

Substituting (15) into (21) and applying Young's inequality we have,

$$\begin{aligned}
\dot{\varphi}_2 &= -2k_g\|(\tilde{E}^T\tilde{E} + \tilde{E})^{\|}\|^2 + 2\langle (\tilde{E}^T\tilde{E} + \tilde{E})^{\|}, \mathrm{Ad}_{\bar{g}}(\xi_r - \bar{\xi})\rangle \\
&\leq -2k_g\|(\tilde{E}^T\tilde{E} + \tilde{E})^{\|}\|^2 + \frac{1}{2\rho_\xi}\|(\tilde{E}^T\tilde{E} + \tilde{E})^{\|}\|^2 + 2\rho_\xi\|b_v\|^2 \\
&\leq -(2k_g - \frac{1}{2\rho_\xi})\|(\tilde{E}^T\tilde{E} + \tilde{E})^{\|}\|^2 + 2\rho_\xi b_v^2
\end{aligned}$$

where ρ_ξ is a positive constant. It is seen that $\dot{\varphi}_2 \leq 0$ if $\|(\tilde{E}^T\tilde{E} + \tilde{E}^T)^{\|}\| \geq \frac{\sqrt{2\rho_\xi} b_v}{\sqrt{2k_g - \frac{1}{2\rho_\xi}}}$. Aslo we let $\tilde{E} = 0$ at the initial instant, it is then concluded that $\tilde{\Omega}_E := \{\|(\tilde{E}^T\tilde{E} + \tilde{E}^T)^{\|}\| \leq \frac{\sqrt{2\rho_\xi} b_v}{\sqrt{2k_g - \frac{1}{2\rho_\xi}}}\}$ is an invariant set for the closed-loop system under the control law (15) and (16).

3.3 Constraints Revision from Tube

The MPC synthesis should consider the revision of the admissible sets of state and control. As we have shown, the feedback control law is designed such that the tracking error and the input fall into the invariant set, the state and input constraints for the nominal system can be revised accordingly. In this way, the safety of the actual system is guaranteed in the presence of tracking error induced by the uncertainties.

From the configuration tracking error invariant set $\tilde{\Omega}_E$, the invariant set of $\tilde{g} = g\bar{g}^{-1}$ can be obtained as $\tilde{\Omega}_g = \tilde{\Omega}_E \oplus \{I\}$. Then the admissible set of $\bar{g}$ can be derived as $\bar{\mathcal{X}} = \mathcal{X} \oslash \tilde{\Omega}_g$, from which we can further derive the admissible set of the nominal parallel tracking error $\bar{\mathcal{X}}^{\parallel}$. And combining the results of the previous subsections, the constraints in the nominal MPC can therefore be revised as,

$$\bar{\mathcal{V}} = \mathcal{V} \ominus (\tilde{\Omega}_\xi \oplus k_g \tilde{\Omega}_E), \bar{U} = U \ominus k_\xi \tilde{\Omega}_\xi \tag{22}$$

4 Application Example

In this section, we will take the rotational motion of the rigid body as an application example to illustrate the theoretical results of this paper.

4.1 Rotational Dynamics of Rigid Body

The attitude control of the rigid body is started from the rotational motion of the rigid body, which is given by,

$$\begin{aligned} \dot{R} &= R\hat{\omega} \\ \dot{\omega} &= M^{-1}(\tau - \hat{\omega} M \omega) + d_r \end{aligned} \tag{23}$$

where $R \in SO(3)$ is the rotation matrix of the rigid body, $\omega \in \mathbb{R}^3$ is the angular velocity, $M \in \mathbb{R}^{3\times 3}$ is the inertia tensor, $\tau \in \mathbb{R}^3$ is the torque, and $d_r \in \mathbb{R}^3$ is the disturbance bounded by $\|d_r\| \leq b_r$ with positive constant b_r.

Given a reference trajectory,

$$\mathbb{R} \ni t \mapsto (R_0(t), \omega_0(t)) \tag{24}$$

It is natural to derive that the error dynamics follows the following format,

$$\begin{aligned} &R \ni t \mapsto (E(t), e(t)) := (X(t)R_0^{-1} - I, \omega(t) - \omega_0(t)) \\ &\in \mathbb{R}^{3\times 3} \times \mathbb{R}^3 \end{aligned} \tag{25}$$

By embedding the manifold into the Euclidean space, and splitting the tracking error E into parallel error $E^{\parallel}$ and transversal error $E^{\perp}$, we have the following linearized tracking error dynamics,

$$\begin{aligned} \dot{E}^{\perp} &= -2\alpha E^{\perp} \\ \dot{E}^{\parallel} &= R_0 \hat{e} R_0^{-1} \\ \dot{e} &= M^{-1}(Me \times \omega_0 + M\omega_0 \times e) + M^{-1}\delta_\tau \end{aligned} \tag{26}$$

It has been proved that the first error dynamics of (26) is stable, while the second and third dynamics can be stabilized to zeros. Therefore, we can design the MPC controller for the rotational dynamics (26).

The constraints on the attitude, angular velocity, and input are expressed as,

$$RR_0^T \in \mathcal{X}, \omega \in \mathcal{V}, \tau \in U \tag{27}$$

4.2 Feedback Control and Invariant Set of Tracking Error

For the system constraints (27), we can design the nominal error trajectory using MPC. Suppose the nominal error trajectory is given by $\mathbb{R} \ni t \mapsto (\bar{E}, \bar{\Xi})$, and the nominal input error trajectory is denoted by $\mathbb{R} \ni t \mapsto (\bar{\delta_\tau})$. Then the nominal state and input trajectory is obtained as $\bar{R} = (\bar{E} + I)R_0, \bar{\omega} = \omega + \bar{\Xi}, \bar{\tau} = \tau_0 + \bar{\delta}_\tau$.

As shown in the previous section, we need to design a feedback control law to force the actual trajectory to track the nominal trajectory $(\bar{R}, \bar{\omega}, \bar{\tau})$, and the tracking error should be bounded in a robust invariant set, which is called the tube of the tracking error. A cascaded structure feedback controller will also be considered for this purpose.

First, we design the following reference angular velocity for the feedback attitude control. As the system dynamics always evolves on $SO(3)$, we design the angular velocity ω_r such that,

$$\mathrm{Ad}_{\bar{R}}(\hat{\omega}_r - \hat{\bar{\omega}}) = -k_1 \tilde{E}^{\parallel} \tag{28}$$

where $\tilde{E}^{\parallel} = R\bar{R}^T - I$ is the parallel error between $\bar{R}$ and R, k_1 is a positive constant. Note that $\bar{R}$ always evolves on $SO(3)$ for system (23).

Then design the body torque as,

$$\tau = \tau_r - k_2 e_\omega \tag{29}$$

where $e_\omega = \omega - \omega_r$, $\tau_r = M\dot{\omega}_r + \hat{\omega}_r M \omega_r$, and k_2 is a positive constant.

Let us consider the tracking error of the angular velocity of the rigid body. Substituting (28) into (23) yields,

$$\begin{aligned} \dot{e}_\omega =& M^{-1}\tau - M^{-1}\hat{\omega}M\omega - M^{-1}\tau_r + M^{-1}\hat{\omega}_r M\omega_r \\ =& M^{-1}(\tau - \tau_r) + M^{-1}\hat{\omega}_r M\omega_r - M^{-1}\hat{\omega}M\omega \end{aligned} \tag{30}$$

Define a function $\eta_1(\omega) : \{\omega \in \mathbb{R}^3 : \|\omega\| \leq \omega_m\} \ni \omega \mapsto M^{-1}\hat{\omega}M\omega \in \mathbb{R}^3$ with positive constant ω_m, then we have,

$$\|\eta_1(\omega) - \eta_1(\omega_r)\| \leq L_2 \|e_\omega\| \tag{31}$$

where L_2 is the Lipchitz constant of the function $\eta_1(\cdot)$.

In order to derive the results, we further define $\eta_2(\omega) : \{\omega \in \mathbb{R}^3 : \|\omega\| \leq \omega_m\} \ni \omega \mapsto \hat{\omega}M\omega \in \mathbb{R}^3$, hence we have

$$\|\eta_2(\omega_{0,r}) - \eta_2(\bar{\omega}_0)\| \leq L_3 \|\omega_{0,r} - \bar{\omega}_0\| \leq L_3 k_1 \|e_{\tilde{R},0}\| \tag{32}$$

where L_3 is the Lipchitz constant of $\eta_2(\cdot)$.

Proposition 1. *Consider the system dynamics (23). The nominal state and input trajectory are represented by $\bar{R}(t), \bar{\omega}(t), \bar{\tau}(t)$. Suppose the control torque is determined by the feedback control law (29). If the positive constants k_1 and k_2 satisfy*

$$k_1 - \frac{1}{4\rho_1} > 0,$$
$$k_2 \lambda(M)^{-1} - \rho_1 - \frac{1}{4\rho_2} - L_2 > 0$$

then the state tracking error and the input of the closed-loop system falls into the following sets,

$$\begin{aligned}
&E^{\|} \in \tilde{\Omega}_{E^{\|}} = \{E^{\|} : \|E^{\|}\| \leq \sqrt{\frac{\rho_2}{\min(\beta_1, \beta_2)}} b_r := L_R\} \\
&\tilde{\omega} \in \tilde{\Omega}_\omega = \{\tilde{\omega} : \|\tilde{\omega}\| \leq (k_1 + 1)\sqrt{\frac{\rho_2}{\min(\beta_1, \beta_2)}} b_r\} \\
&\tilde{\tau} \in \tilde{\Omega}_\tau = \{\tilde{\tau} : \|\tilde{\tau}\| \leq (\|M\| k_1(k_1 + 1) + L_3 k_1 + k_2) \\
&b_r \sqrt{\frac{\rho_2}{\min(\beta_1, \beta_2)}} := \Delta_\tau \Big\}
\end{aligned} \tag{33}$$

where $k_5 = \frac{k_3}{k_4}$, $L_R = \sqrt{\frac{\rho_2}{\min(\beta_1,\beta_2)}} b_r$, $\beta_1 = k_1 - \frac{1}{4\rho_1}$, $\beta_2 = k_2\lambda(M)^{-1} - \rho_1 - \frac{1}{4\rho_2} - L_2$ *with positive constants* ρ_1, ρ_2, $\lambda(M)$ *is the minimum eigenvalue of* M.

Proof. We define the following Lyapunov candidate as,

$$V = \operatorname{tr}(I - R\bar{R}^T) + \frac{1}{2} e_\omega^T e_\omega \tag{34}$$

which is positive definite.

From (30), we can obtain the time derivative of V,

$$\begin{aligned}
\dot{V} =& \operatorname{tr}\left[(\bar{R}R^T)^T \left(\mathrm{Ad}_{\bar{R}}(\hat{\omega} - \hat{\bar{\omega}})\right)\right] + e_\omega^T \dot{e}_\omega \\
=& \operatorname{tr}\left[(\bar{R}R^T)^T \left(\mathrm{Ad}_{\bar{R}}\, \hat{e}_\omega\right)\right] \\
&+ \operatorname{tr}\left[(\bar{R}R^T)^T \left(\mathrm{Ad}_{\bar{R}}(\hat{\omega}_r - \hat{\bar{\omega}})\right)\right] \\
&- k_2 e_\omega^T M^{-1} e_\omega + e_\omega^T [\eta_1(\omega_r) - \eta_1(\omega)] + e_\omega^T d_r \\
=& \left(\bar{R}\hat{e}_\omega \bar{R}^T\right)\left(R\bar{R}^T - \bar{R}R^T\right)^\vee \\
&+ \left(\bar{R}(\hat{\omega}_r - \hat{\bar{\omega}})\bar{R}^T\right)\left(R\bar{R}^T - \bar{R}R^T\right)^\vee \\
&+ -k_2 e_\omega^T M^{-1} e_\omega + e_\omega^T [\eta_1(\omega_r) - \eta_1(\omega)] + e_\omega^T d_r \\
\leq& -k_1 \|\tilde{E}^{\|}\|^2 + \|\tilde{E}^{\|}\| \|e_\omega\| - k_2 \lambda(M)^{-1} \|e_\omega\|^2 \\
&+ L_2 \|e_\omega\|^2 + e_\omega^T d_r \\
\leq& -(k_1 - \frac{1}{4\rho_1})\|\tilde{E}^{\|}\|^2 - (k_2\lambda(M)^{-1} - \rho_1 \\
&- \frac{1}{4\rho_2} - L_2)\|e_\omega\|^2 + \rho_2 b_r^2
\end{aligned} \tag{35}$$

Taking $\beta_1 = k_1 - \frac{1}{4\rho_1}$ and $\beta_2 = k_2\lambda(M)^{-1} - \rho_1 - \frac{1}{4\rho_2} - L_2$, if the parameters are selected such that $\beta_1 > 0$ and $\beta_2 > 0$, then

$$\dot{V} \leq -\min(\beta_1, \beta_2)\|(e_{\tilde{R}}^T, e_\omega^T)^T\|^2 + \rho_2 b_r^2 \tag{36}$$

It is seen that $\dot{V} < 0$ if $\|\tilde{E}^T, e_\omega^T)^T\| > \sqrt{\frac{\rho_2}{\min(\beta_1,\beta_2)}} b_r$. The bound of $\tilde{E}$ and e_ω can be expressed as,

$$\begin{aligned} \|\tilde{E}\| &\leq \|(e_{\tilde{R}}^T, e_\omega^T)^T\| \leq L_R \\ \|e_\omega\| &\leq \|(e_{\tilde{R}}^T, e_\omega^T)^T\| \leq L_R \end{aligned} \tag{37}$$

Recalling the definition of e_ω we arrive at,

$$\|\tilde{\omega}\| \leq k_1\|e_{\tilde{R}}\| + \|e_\omega\| \leq (k_1 + 1)L_R \tag{38}$$

Then we consider the boundedness of $\tilde{\tau} = \tau_d - \bar{\tau}$. From the control law, we have,

$$\begin{aligned} \tau_r - \bar{\tau} &= M\dot{\omega}_r - M\dot{\bar{\omega}} + \eta_2(\omega_r) - \eta_2(\bar{\omega}) \\ &\leq Mk_1\|\dot{\tilde{E}}^{\|}\| + L_3 k_1\|\tilde{E}^{\|}\| \\ &\leq Mk_1\|\tilde{\omega}\| + L_3 k_1\|\tilde{E}^{\|}\| \end{aligned} \tag{39}$$

While from (29) it is concluded that $\tau_d - \tau_r = -k_2 e_\omega$, hence combining (37) and (38) we have,

$$\begin{aligned} \|\tilde{\tau}\| \leq &\|\tau_d - \tau_r\| + \|\tau_d - \bar{\tau}\| \\ \leq &\|M\|k_1\|\tilde{\omega}\| + L_3 k_1\|e_{\tilde{R}}\| + k_2\|e_\omega\| \\ \leq &(\|M\|k_1(k_1 + 1) + L_3 k_1 + k_2) b_r \sqrt{\frac{\rho_2}{\min(\beta_1, \beta_2)}} \end{aligned} \tag{40}$$

This completes the proof.

4.3 Tube-Based MPC for Rotational Motion of Rigid Bodies

From the invariant set $\tilde{\Omega}_{E^{\|}}$, we can define the invariant set of $\tilde{R} = R\bar{R}^T$ as $\tilde{\Omega}_R = \{\tilde{R} : \|\frac{(\tilde{R}-\tilde{R}^T)^\vee}{2}\| \leq L_R\}$. From the Rodrigues' formula, it is shown that,

$$\|\tilde{E}^{\|}\| = \sin\|\alpha\| \tag{41}$$

where α is the equivalent angle of the rotation matrix $\tilde{R}$. Then $RR_0^T \in \mathcal{X}$ and $R\bar{R}^T \in \tilde{\Omega}_R$ implies $\bar{R}R_0^T \in \mathcal{X} \oslash \tilde{\Omega}_R$.

It is noted that there is a difference between $\bar{E}^{\|}$ and $\bar{R}R_0^T$. Therefore we need to derive the admissible set of $\bar{E}^{\|}$ from the admissible set of $\bar{R}R_0^T$. Suppose $\bar{E}^{\|} = (a_1, a_2, a_3)^T$, we can derive $\bar{R}R_0^T$ from $\bar{E}^{\|}$ as,

$$\bar{R}R_0^T = f_R(\bar{E}^{\|}) = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{42}$$

where

$$r_{11} = 1 - \frac{\left(a_2\, a_3 - a_2\, a_3 \sqrt{-a_1{}^2 - a_2{}^2 - a_3{}^2 + 1}\right) \left(a_2{}^2 + a_3{}^2\right)}{a_2\, a_3\, \left(a_1{}^2 + a_2{}^2 + a_3{}^2\right)}$$

$$r_{12} = -a_3 - \frac{a_1\, a_2\, \left(\sqrt{-a_1{}^2 - a_2{}^2 - a_3{}^2 + 1} - 1\right)}{a_1{}^2 + a_2{}^2 + a_3{}^2}$$

$$r_{13} = a_2 - \frac{a_1\, a_3\, \left(\sqrt{-a_1{}^2 - a_2{}^2 - a_3{}^2 + 1} - 1\right)}{a_1{}^2 + a_2{}^2 + a_3{}^2}$$

$$r_{21} = a_3 - \frac{a_1\, a_2\, \left(\sqrt{-a_1{}^2 - a_2{}^2 - a_3{}^2 + 1} - 1\right)}{a_1{}^2 + a_2{}^2 + a_3{}^2}$$

$$r_{22} = 1 - \frac{\left(a_2\, a_3 - a_2\, a_3 \sqrt{-a_1{}^2 - a_2{}^2 - a_3{}^2 + 1}\right) \left(a_1{}^2 + a_3{}^2\right)}{a_2\, a_3\, \left(a_1{}^2 + a_2{}^2 + a_3{}^2\right)}$$

$$r_{23} = -a_1 - \frac{a_2\, a_3\, \left(\sqrt{-a_1{}^2 - a_2{}^2 - a_3{}^2 + 1} - 1\right)}{a_1{}^2 + a_2{}^2 + a_3{}^2}$$

$$r_{31} = -a_2 - \frac{a_1\, a_3\, \left(\sqrt{-a_1{}^2 - a_2{}^2 - a_3{}^2 + 1} - 1\right)}{a_1{}^2 + a_2{}^2 + a_3{}^2}$$

$$r_{32} = a_1 - \frac{a_2\, a_3\, \left(\sqrt{-a_1{}^2 - a_2{}^2 - a_3{}^2 + 1} - 1\right)}{a_1{}^2 + a_2{}^2 + a_3{}^2}$$

$$r_{33} = \frac{(a_1{}^2 + a_2{}^2)\sqrt{-a_1{}^2 - a_2{}^2 - a_3{}^2 + 1} + a_3{}^2}{a_1{}^2 + a_2{}^2 + a_3{}^2}$$

From (42) the admissible set of $\bar{E}^{\|}$ can be derived from the admissible set of $\bar{R}R_0^T$. For example, if the admissible set of $\bar{R}R_0^T$ is given by $\mathcal{X} \oslash \tilde{\Omega}_R = \{R : C(\bar{R}R_0^T) \leq 0\}$, then we can express the admissible set of $\bar{E}^{\|}$ as $\bar{\mathcal{X}}^{\|} = \{\bar{E}^{\|} : C(f_R(\bar{E}^{\|})) \leq 0\}$, which is used to define the constraints in the nominal MPC.

Combining the previous results, we are now in the position to derive the nominal MPC for the rotational motion of the rigid body as,

$$\begin{aligned} &\min_{\bar{\delta}_\tau(s)} J(\bar{\zeta}, \bar{\delta}_\tau) = \quad V_r(\bar{\zeta}(t_k + \Gamma)) + \int_{t_k}^{t_k+\Gamma} \left(N_r(\bar{\zeta}(s), \bar{\delta}_\tau(s))\right) ds \\ &s.t. \quad \dot{\bar{E}}^{\|}(s) = R_0 \hat{\bar{e}} R_0^{-1}, \\ &\dot{\bar{e}} = M^{-1}(M\bar{e} \times \omega_0 + M\omega_0 \times \bar{e}) + M^{-1}\bar{\delta}_\tau \\ &(\tilde{E}^{\|}, \tilde{e}) \in \bar{\mathcal{X}}^{\|} \times \bar{\mathcal{V}}, \bar{\delta}_\tau \in \bar{U} \end{aligned} \tag{43}$$

where $\zeta = (E^{\|}, e)$, the state constraint set $\bar{\mathcal{X}} = \mathcal{X} \oslash \tilde{\Omega}_R$, $\bar{\mathcal{V}} = \mathcal{V} \ominus \tilde{\Omega}_\omega$, $\bar{U} = U \ominus \tilde{\Omega}_\tau$.

We then synthesis the tube-based MPC as shown in Algorithm 1. As indicated by Theorem 1, Algorithm 1 combines the feedback control law and the nominal MPC, the constraints on the state/input of the rigid body with uncertainties can therefore be guaranteed to fulfill.

Algorithm 1. Synthesis of the tube-based MPC

Initialization: At time instant t_0, let $\zeta(0) = \bar{\zeta}(0)$.

1: At time instant t_k, solve the nominal MPC problem (43), obtain the nominal state and input $\bar{E}^{\|}(s), \bar{e}(s), \bar{\delta}_\tau(s), s \in [t_k, t_k + \Gamma)$.
2: Calculate $\bar{R}(s), \bar{\omega}, \bar{\tau}, s \in [t_k, t_k + \Gamma)$.
3: **for all** $s \in [t_k, t_{k+1})$ **do**
4: Apply the actual control input $\tau(s)$ to the rigid body, according to (29).
5: **end for**
6: $(\zeta(t_k), \bar{\zeta}(t_k)) \leftarrow (\zeta(t_{k+1}), \bar{\zeta}(t_{k+1}))$, $t_k \leftarrow t_{k+1}$.
7: Go to step 1.

4.4 Simulation

During the simulation, the inertia tensor of the rigid body is $M = \text{diag}(2.263, 2.47, 4.7235) kg \cdot m^2$, the initial attitude of a rigid body is set to $R(0) = \exp(0.65[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0]^T)\hat{}$, and the reference attitude is set to $R_0 = \exp(-0.6[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0]^T)\hat{}$. The angular velocity of the rigid body is under the constraint $\|\omega\| < 1$rad/s. While the attitude constraint of the rigid body is given by $0.65 \leq e_3^T R R_0^T R_0 e_3 \leq 0.95$. The disturbance acting on the rigid body is assumed to uniform distribution $d_r \sim U(-1.75, 1.75)$. The Lipchitz constants are calculated according to the EOM as $L_2 = 1.39$ and $L_3 = 3.34$. The open-source ACADO is adopted to solve the MPC problem [1]. In the simulation, the prediction horizon is set to 0.7 s, and the sampling time is 0.1 s.

From Proposition 1, the tube along the nominal attitude trajectory is calculated as $\{\tilde{E}^{\|} : \|\tilde{E}^{\|}\| \leq 0.1563\}$, from which the constraint for $\bar{R}R_0^T$ is revised as $0.7608 \leq e_3^T \bar{R} R_0^T R_0 e_3 \leq 0.8895$. And the admissible set for $\bar{E}^{\|}$ is further revised as $\{\bar{E}^{\|} : 0.7608 \leq e_3^T f_R(\bar{E}^{\|}) R_0 e_3 \leq 0.8895\}$ in the nominal MPC.

The simulation results are shown in Figs. 1, 2, 3 and 4. The attitude of the rigid body expressed in Euler angles is shown in Fig. 1. It is seen that the attitude of the system evolves from the initial attitude to the reference attitude. The attitude constraint of the rigid body is expressed in Fig. 3, from which it is seen that the attitude constraint is satisfied using the proposed control algorithm, in the presence of uncertainties. It is also noted that because of the attitude constraint, the rotational trajectory from the initial attitude to the desired attitude does not follow the geodesics on $SO(3)$. The angular velocity of the rigid body is depicted in Fig. 2. It is seen that the constraints on the angular velocity are also fulfilled. While the input torque under the proposed control algorithm is presented in Fig. 4. These two figures also show that the velocity and the input torque are all in the admissible sets. From the simulation results, the feasibility of the proposed methodology on attitude control of the rigid body is verified.

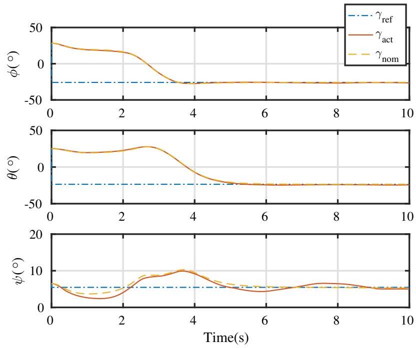

Fig. 1. The nominal and actual attitude expressed in Euler angles. The dot-dashed line represents the reference value. The solid line represents the actual value. While the virtual line represents the nominal value.

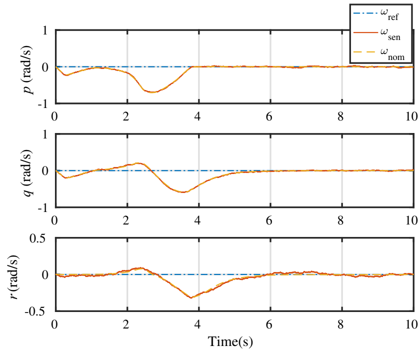

Fig. 2. The reference and actual angular velocity.

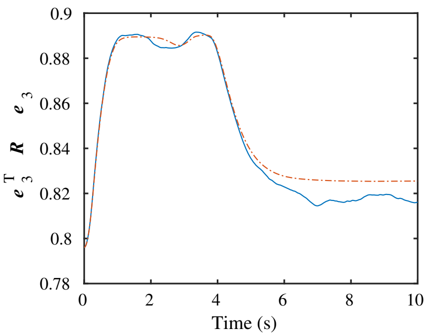

Fig. 3. The attitude constraints in one test trial.

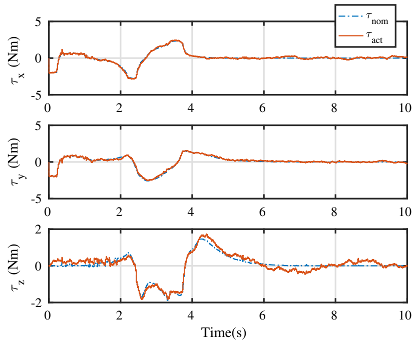

Fig. 4. The actual and nominal input of the vehicle in one test trial.

5 Conclusions

In this paper, we have developed a methodology to design a controller that deals with the state and input constraints for systems on the matrix Lie groups with uncertainties. The methodology is inspired by the Tube-based MPC. By embedding the manifold into Euclidean space, the nominal MPC has been designed on the Euclidean space. As the generated nominal trajectory is restricted on the Lie group, the feedback controller used to track the nominal trajectory has been designed on the manifold directly. The results of the simulation showed that the tracking error in the feedback controller can be bounded into robust invariant sets, which can be used to revise the constraints in the nominal MPC expressed in the Euclidean space. In this way, the nominal MPC in the Euclidean space and the feedback controller on the Lie group can be combined together. Therefore, the proof for the safety of the overall system evolving on the manifold has been obtained. The application example of the proposed methodology on the rotational motion of the rigid body has been presented. The proposed methodology does not rely on any local coordinates of the Lie group and can apply the existing MPC techniques on the Euclidean space.

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