

# Modified Cowper-Symonds Model for Predicting the Stress–Strain Behavior of SA516 Gr. 70 Carbon Steel



S. Sharma, M. K. Samal, and V. M. Chavan

**Abstract** The flow behavior of SA516 Gr.70 carbon steel under dynamic loading conditions was studied experimentally using the split-Hopkinson pressure bar (SHPB). These tests were performed at room temperature at strain rates ranging from 450/s to 3500/s. Quasi-static tensile tests were performed for comparison with high strain rate test results. The strain rate sensitivity at these dynamic rates was found to be positive. The experimental data were fit to the Cowper-Symonds (CS) model. As the CS model did not fit the high strain rate data satisfactorily, the Cowper-Symonds model was modified. This modified Cowper-Symonds model gave the best fit to the experimental data.

**Keywords** High strain rate · Split Hopkinson bar · Strain rate sensitivity

## 1 Introduction

Material properties under dynamic loading conditions are important for design, safety and structural integrity assessment of structures subjected to high rate of loading. A few examples of such applications are design of armor systems, vehicle crashworthiness tests in automobile industry, impact analysis, drop test of radioactive material shipping cask and high speed machining. Materials respond differently under static and dynamic conditions of loading. The flow behavior of a material is a function of strain rate and varies with change in strain rate. Also, as the strain rate of the test is

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increased from quasi-static to dynamic, conditions change from isothermal to adiabatic. High strain rate tests are, thus, necessary to understand the material response under such loading conditions.

SA516 Gr.70 carbon steel, the material of interest in this paper, is a pressure vessel steel and has ferritic–pearlitic microstructure. As a pressure vessel and reactor-grade steel used in nuclear piping system, the fracture properties of this alloy have been studied at different conditions, from quasi-static to high strain rate, and from sub-room temperature to elevated temperature. The effect of loading conditions, notch root radius, radiation and geometry of sample on fracture toughness has been studied [1–3]. These studies show that the fracture properties of the material are significantly influenced by the loading rate and temperature. It was stated that serrations are observed in quasi-static flow stress data, and DSA is found to deteriorate the fracture properties. However, to the best of our knowledge, the flow behavior of SA516 Gr.70 at high strain rates and the corresponding constitutive material model for the prediction of the flow stress of this steel at high strain rate are very rare in literature.

This paper presents the high strain rate compression flow behavior of SA516 Gr.70 pressure vessel steel using the SHPB. A comparison of stress–strain curves at different strain rates has been made, and a constitutive equation to predict the flow behavior at high strain rates is also presented.

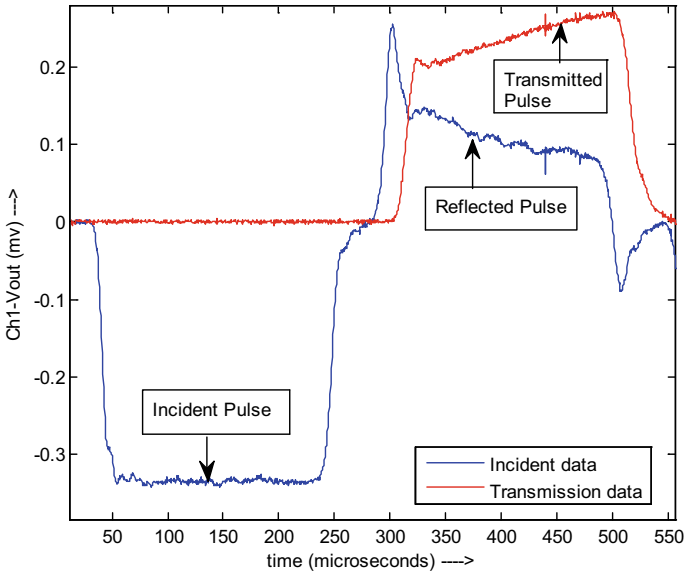
## 2 Experimental Procedure

The present study was carried out on SA516 Gr. 70 carbon steel plate material with the chemical composition as given in Table 1. The rest is that of Fe in the Table 1 for chemical composition of the material. The material was obtained as rolled and normalized in the form of a block of size 310 mm × 120 mm × 90 mm. Cylindrical sample of 5 mm diameter and 5 mm length was used for compression tests. The high strain rate tests were done at strain rates ranging from 450/s to 3500/s at room temperature. Molybdenum disulfide grease was used as lubricant to reduce friction at the bar–specimen interfaces. These high strain rate tests were carried out using the Split-Hopkinson Pressure Bar (SHPB) test set-up [4–6].

The output of a high strain rate SHPB test is the elastic strain generated in incident and transmission bar, as shown Fig. 1, measured by strain gauges in volts. The volt signal is converted to equivalent strain using the gauge factor and excitation voltage values. These elastic strains, namely, the reflected strain and transmitted strain are used to calculate the stress and strain in the test specimen. The expression given below provides the relation between the elastic strain generated in SHPB and specimen

**Table 1** Composition of A516 Gr.70 Carbon steel (wt %)

C	Mn	Si	P	S	Ni	Cr	Cu
0.24	1.14	0.2	0.016	0.022	47 ppm	30 ppm	180 ppm



**Fig. 1** Raw data output from SHPB test showing the incident and transmitted signal in volts

stress, strain and strain rate.

$$\text{Stress, } \sigma_s = \frac{A_0 E \varepsilon_t}{A_s} \quad (1)$$

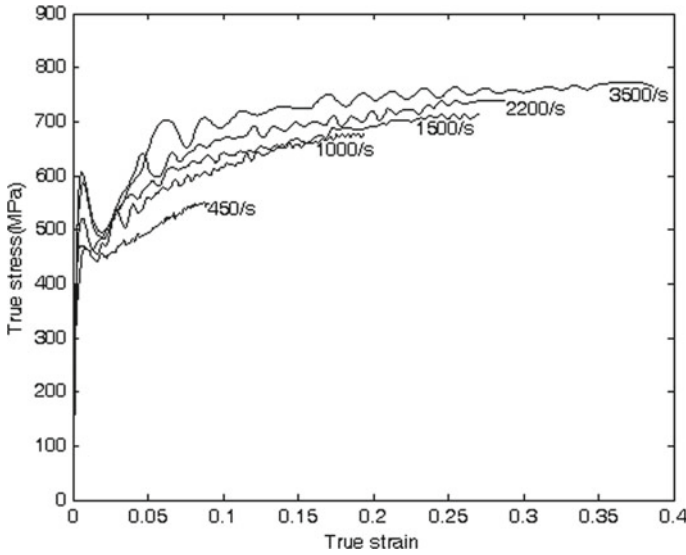
$$\text{Strain, } \varepsilon_s = \frac{-2C_0}{L_s} \int_0^t \varepsilon_r dt \quad (2)$$

$$\text{Strain rate, } \dot{\varepsilon}_s = \frac{-2\varepsilon_r C_0}{L_s} \quad (3)$$

where  $A_0$  and  $A_s$  are the cross-section area of bar and specimen,  $C_0$  is stress wave velocity for a wave of infinite wavelength,  $E$  is young's modulus of bar,  $L_s$  is length of specimen,  $\sigma_s$ ,  $\varepsilon_s$  and  $\dot{\varepsilon}_s$  are specimen stress, strain and strain rate, respectively.  $\varepsilon_r$  is the reflected strain in incident bar and  $\varepsilon_t$  is the transmitted strain in transmission bar.

### 3 Results and Discussion

The test data were processed using the equations mentioned in the previous section. The stress–strain curves obtained at different strain rates are presented in Fig. 2. The



**Fig. 2** Plot of adiabatic true stress versus true strain curves at room temperature. Nominal strain rate during the test are shown

flow stress of SA516 Gr.70, at high strain rates, increases with increasing strain rate. As the strain rate increases, the strain also increases. This is typical in compression SHPB. The oscillations seen in the curves are due to wave dispersion effects and not due to material property. All high strain rate tests are adiabatic in nature, i.e. the heat generated during high strain rate deformation does not have sufficient time to dissipate. This increases the specimen temperature.

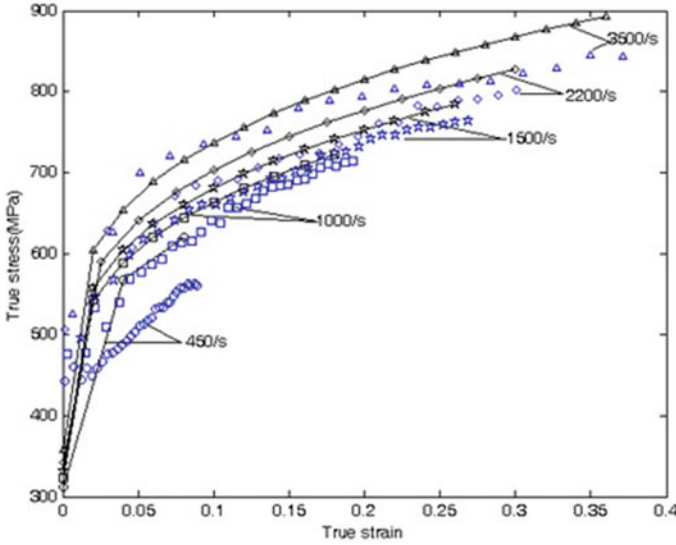
To determine the isothermal stress–strain, the heat generated during the test has to be calculated and adjusted in terms of stress. The increase in specimen temperature is calculated as,

$$T = T_0 + \frac{\beta}{\rho C_p} \int_0^{\epsilon} \sigma \partial \epsilon, \quad (4)$$

Or

$$\Delta T = T - T_0 = \frac{\beta}{\rho C_p} \int_0^{\epsilon} \sigma \partial \epsilon, \quad (5)$$

where,  $T_0$  is the initial temperature of the test,  $\Delta T$  is rise in the specimen temperature due to adiabatic nature of the test,  $\beta$  is the fraction of heat dissipation due to plastic deformation assumed as 1 [7],  $\rho$  is the density of the material and  $C_p$  is the specific



**Fig. 3** CS material model fit to the isothermal stress–strain curves. Solid lines with marker are the constitutive model fit to experimental data represented by markers

heat at constant pressure. For SA516 Gr.70, the value of  $C_p$  is 490 J/kg.K and  $\rho$  is 7800 kg/m<sup>3</sup>. The isothermal stress  $\sigma_{iso}$  is estimated from the adiabatic stress  $\sigma_{adbt}$  as,

$$\sigma_{iso} = \sigma_{adbt} - \frac{\partial \sigma}{\partial T} \Delta T, \tag{6}$$

where, the later term on RHS is the temperature sensitivity of stress as determined from the high strain rate experimental data of  $\sigma$  versus T. The experimental isothermal flow stress as determined from Eq. 6 of the high strain rate tests is shown in Fig. 3 onwards, in markers. The flow curves at different strain rates appear parallel to each other only being shifted vertically.

### 3.1 Fit to Constitutive Model

The use of a constitutive equation for material deformation is essential during FEM calculations in the plastic domain. There are numerous constitutive equations used to describe the flow behavior in both high and low strain rate regimes. The simplest are the ones at constant temperature and strain rates, where the only variation of flow stress is with strain. These are usually described using either power laws or an exponential relation. The strain rate effect on flow stress is also usually described using a power law and in some cases a logarithmic form. The effect of temperature

**Table 2** Constants of CS model

$A'$	$B'$	$p$	$D$	$q$
295	580	0.27	35,000/s	1.5

on the flow stress of the material is either clubbed together with the strain rate term in models based on thermally activated deformation, or treated separately as a power law. A combination of the effect of strain, strain rate and temperature on the flow stress when represented as one equation is referred to as the constitutive model for plastic deformation of that material. The present high strain rate data were fit to the Cowper-Symonds (CS) [8, 9] model and a modified form of the Cowper-Symonds model. Quasi-static tension test data at room temperature were used to determine some of the constants in the equation.

### 3.2 Cowper-Symonds Model

The CS model is similar to the JC model in that the strain and strain rate terms are multiplicative; however, the effect of temperature is not included in this. The general form of CS model is,

$$\sigma = (A' + B' \varepsilon^p) \left( 1 + \left( \frac{\dot{\varepsilon}}{D} \right)^{\frac{1}{q}} \right), \quad (7)$$

where, the first bracket is the static flow stress of the material as a function of plastic strain, and the second bracket is the effect of strain rate hardening on flow stress. The first term (work hardening) is of the power law type. This second term, referred as the dynamic hardening factor, is a function of power of strain rate rather than a logarithm term. The procedure for the determination of the parameters  $A'$ ,  $B'$ , and  $p$  of Eq. 7 is similar to the one used in Ref. [6]. The constants  $A'$  is the yield stress of the material experimentally determined along with  $B'$  and  $p$  from quasi-static data. Using these constants and the high strain rate test data, constants  $D$  and  $q$  were calculated. The high strain rate data were fit to Eq. 7 (Fig. 3) using the constants as listed in Table 2. The fit of the data to the CS model was not satisfactory.

### 3.3 Modified Cowper-Symonds Model

As the Cowper-Symonds models did not fit the high strain rate data well, a modification to the dynamic hardening factor of the Cowper-Symonds model was made and used to fit high strain rate test data. A simple modification of removing the power

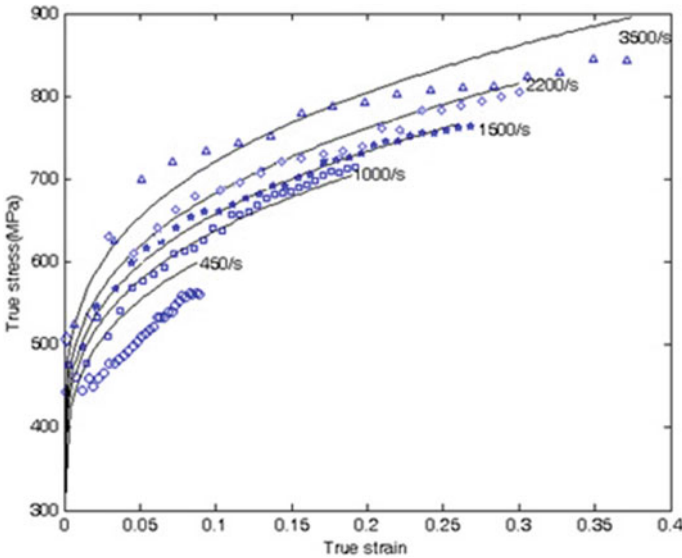
**Table 3** Constants of modified CS model

$A'$	$B'$	$p$	$C'$	$r$
295	580	0.30	1900/s	0.20

from the strain rate to a power to the full strain rate term was used as the modified Cowper-Symonds equation,

$$\sigma = \left( A' + B' \varepsilon^p \right) \left( 1 + \frac{\dot{\varepsilon}}{C'} \right)^r, \tag{8}$$

where,  $A'$ ,  $B'$ ,  $p$ ,  $r$  and  $C'$  are the constants to be calculated.  $A'$  is the yield stress of the material experimentally determined along with  $B'$  and  $p$  from quasi-static data. The constants  $C'$  and  $r$  are determined using high strain rate test data. The high strain rate data were fit to Eq. 8 using the constants as listed in Table 3 (Fig. 4). It is observed that the high strain rate data fit the modified Cowper-Symonds model better than the Cowper-Symonds model.



**Fig. 4** Modified CS material model fit to the isothermal stress–strain curves. Solid lines are the constitutive model fit to experimental data represented by markers

## 4 Conclusion

The study of flow behavior of SA516 Gr.70 carbon steel material under dynamic loading conditions (strain rate range from 450/s to 3500/s) was carried out using SHPB at room temperature. The material is showing positive rate sensitivity in the strain rate range of the tests. Material constants for Cowper-Symonds and modified Cowper-Symonds model were determined. It was observed that the modified Cowper-Symonds model fitted the high strain rate experimental data better than the Cowper-Symonds model.

## References

1. Yoon JH, Lee BS, Hong JH (1999) Effects of loading rate and temperature on J-R fracture resistance of an SA516-Gr.70 steel for nuclear piping. *Int J Press Vessels Pip* 76:663–670
2. Seok CS, Murty KL (1999) Effect of dynamic strain aging on mechanical and fracture properties of A516 Gr. 70 steel. *Int J Press Vessels Pip* 76:945–953
3. Seok CS, Kim SY (2002) Effect of specimen configurations on the fracture resistance curve. *Nucl Eng Des* 214:47–56
4. Bazle AG, Lopatnikov SL, Gillespie JW (2004) Hopkinson bar experimental technique: a critical review. *Appl Mech Rev* 57:223–250
5. Nemat-Nasser S (2000) Mechanical testing & evaluation. *ASM Handbook*, Ohio 8:427–436
6. Qianab X, Pengc X, Songc Y, Huang J (2020) Dynamic constitutive relationship of CuCrZr alloy based on Johnson-Cook model. *Nuclear Mater Energy* 24
7. Kapoor R, Nemat-Nasser S (2000) Comparison between high and low strain-rate deformation of tantalum. *Metall Mater Trans A* 31:815–823
8. Jiang B, Wangb Z, Wang M, Yamb MCH, Ding F (2020) Effects of loading mode on mechanical properties of high strength steel Q690 and their application in coupon test. *Constr Build Mater* 253
9. Mahallea G, Kotkunda N, Gupta AK, Singh SK (2019) Cowper-Symonds strain hardening model for flow behaviour of inconel 718 Alloy. *Mater Today: Proceed* 18:2796–2801