Approaches to Slope Stability Analysis Considering the Effects of Dilatancy and Strength Non-linearity: A Review

J. Nihar Ranjan, Benu G. Mohapatra, and Manal Alali

1 Introduction

Slope stability analysis is an important problem in geotechnical engineering. The widely used Limit Equilibrium Method (LEM) does not require the dilatancy angle [1] and is usually based on a linear Mohr–Coulomb failure criterion. However, studies by authors have pointed out the limitations of the LEM. These limitations include the possible kinematic inadmissibility of the solutions and an inaccurate Factor of Safety (FoS) [1–4]. In comparison, the Limit Analysis Method (LAM) based on Upper Bound (UB) and Lower Bound (LB) solutions can bracket the true FoS [2, 3, 5]. LAM can be used only if the flow rule is associated ($\psi = \phi$), while practically, soils do not always follow the associated flow rule assumption [2, 5–9]. Strength reduction finite element analyses are increasingly being used in slope stability problems [2]. For these analyses, an associated flow rule overpredicts the factor of safety [1, 10] while a nonassociated flow rule may result in numerical problems if the friction angle is large and the dilatancy angle is very small [2, 11]. In such cases the FoS fluctuates making it difficult to determine a unique value.

The strength envelope nonlinearity of soils in σ-τ stress space is well known [12–18]. Using the linear Mohr–Coulomb strength assumption may result in unsafe assessment of slope stability [12–14, 17, 18]. The present paper reviews the works of previous authors on the effects of dilatancy and nonlinearity and approaches to incorporate these in slope stability analysis.

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2 Dilatancy and Slope Stability Analysis

2.1 Dilatancy

Dilatancy is the behaviour observed in soils whereby volumetric expansion occurs during shearing. It is usually measured in terms of the dilatancy angle ψ. Dilatancy angle ψ is given as the ratio of increase in volumetric strain to increase in major principal strain [18]

$$\tan\psi = \left(\frac{d\varepsilon_v}{d\varepsilon_1}\right) \tag{1}$$

where ε_v= volumetric strain, ε_1= major principal strain.

The dilatancy angle in the triaxial test is measured as [20]

$$\sin\psi = \frac{-d\varepsilon_v}{2d\varepsilon_a - d\varepsilon_v} \tag{2}$$

where $d\varepsilon_v$ = volumetric strain increment, $d\varepsilon_a$ = axial strain increment.

Dilatancy varies with relative density of the soil and becomes more significant for dense sands and overconsolidated clays. Bolton [21] gave an expression for maximum dilatancy angle

$$\psi_{max} = 0.3 I_R[21] \tag{3}$$

where

$$I_R = I_D(10 - \ln p') - 1 \tag{4}$$

I_D is the relative density.

Andersen and Schjetne [20] determined the values of ψ_{max} based on Eq. 3 and compared it with empirically determined values for $p' = 150$ and 600 kPa as shown in Fig. 1.

Dilatancy affects the shear strength of soils thus playing an important role in the understanding and prediction of soil behaviour [10, 18].

2.2 Effects of Dilatancy

The dilatancy angle is not used in the traditional limit equilibrium method (LEM) based on Mohr–Coulomb strength criterion [1]. However, on account of several limitations in the LEM [1–4,21], the finite element strength reduction method and limit analysis method are being increasingly used for slope stability problems.

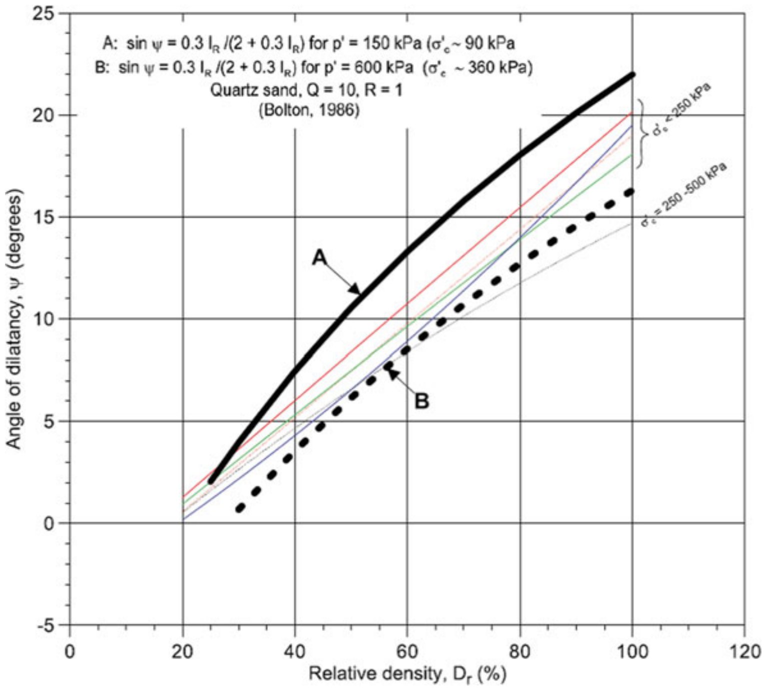

Fig. 1 Tangent dilatancy angle at peak shear stress compared with [21] by Andersen and Schjetne [20]

The limit analysis method is based on the normality condition or the associated flow rule [2, 5–9, 21]. For Mohr–Coulomb failure criterion, in case of an associated flow rule the velocity vector makes an angle of ϕ with the velocity discontinuity line or the rupture surface, where ϕ is the angle of friction for the soil. For the flow rule to be nonassociated the above angle becomes equal to ψ, where ψ varies between 0 and ϕ [3, 9, 24]. A nonassociated flow rule cannot be used in the traditional limit analysis. However, the associated flow rule assumption is not realistic and overpredicts the dilation of soil and safety condition of the slope [2, 5–10].

In a strength reduction finite element analysis based on Mohr–Coulomb yield criterion, the dilatancy angle does not significantly affect the solutions obtained because of the unconfined nature of the slope stability problems [24, 26]. But studies by authors showed that the effect of dilatancy angle becomes significant with high values of ϕ [11, 28]. As per Tschuchnigg et al. [11], the flow rule becomes significant at friction angles larger than 35°, while Tschuchnigg et al. [28] reported a range of 40°–45°. Using the associated flow rule ($\phi = \psi$) leads to overestimations, while setting $\psi = 0°$ gives conservative estimates [1, 10]. Using nonassociated flow rules in a 2D finite element analysis, as the degree of nonassociativity (ϕ-ψ) increased, Tschuchnigg et al. [11] reported numerical difficulties and fluctuating values of FoS as shown in Fig. 2.

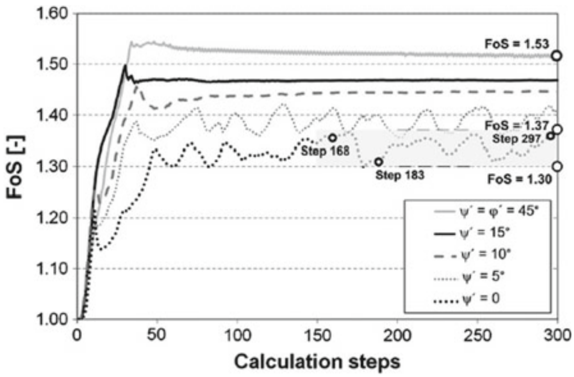

Fig. 2 Predicted factor of safety [11]

Lin et al. [1] investigated the effect of dilatancy angle on FoS using a 3D finite element model and compared the results with Tschuchnigg et al. [11]. As shown in Fig. 3, using the associated flow rule they observed fluctuations in FoS. They concluded that associated flow rule was not suitable for 3D finite element analysis.

Kumar [9] studied the effect of dilatancy on stability factor in the presence of pore-water. Based on data from Kumar [9], Fig. 4 shows the percentage reduction in stability factor when the flow rule is changed from associated to nonassociated ($\psi = 0°$), for slope of angle of 55°, at three different pore-water pressure ratios.

Ganjian et al. [30] investigated the effect of dilatancy angle on 3D safety factors for slopes under static and seismic conditions. Based on their work, Figs. 5 and 6 show the percentage reduction in FoS when the flow rule is changed from associated

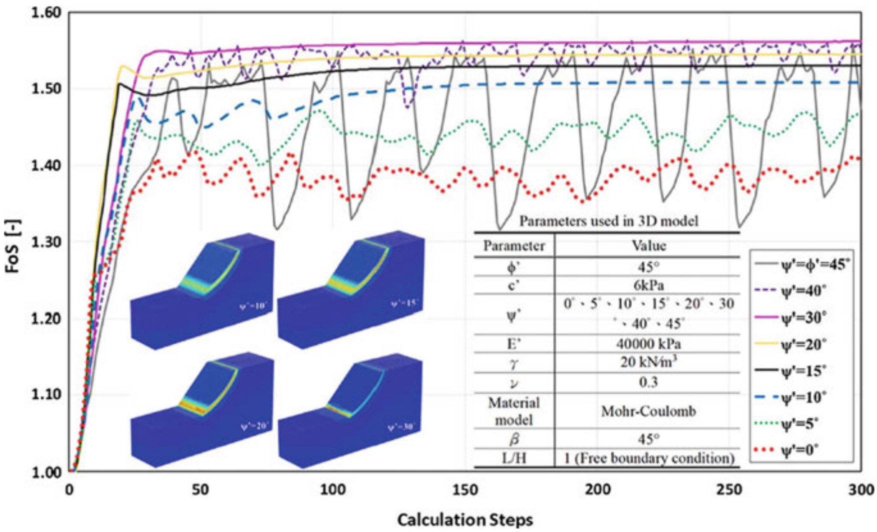

Fig. 3 Comparisons of 3D slope model FoS for different dilatancy angles [1]

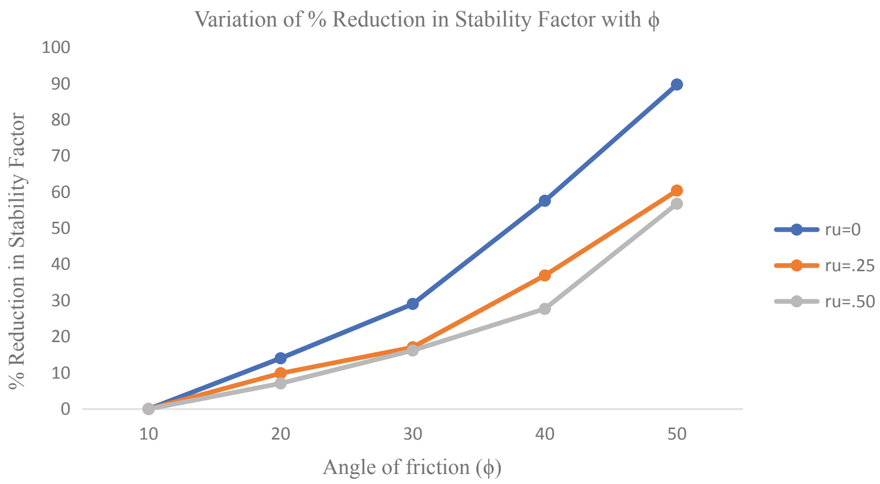

Fig. 4 Effect of dilatancy on stability factor

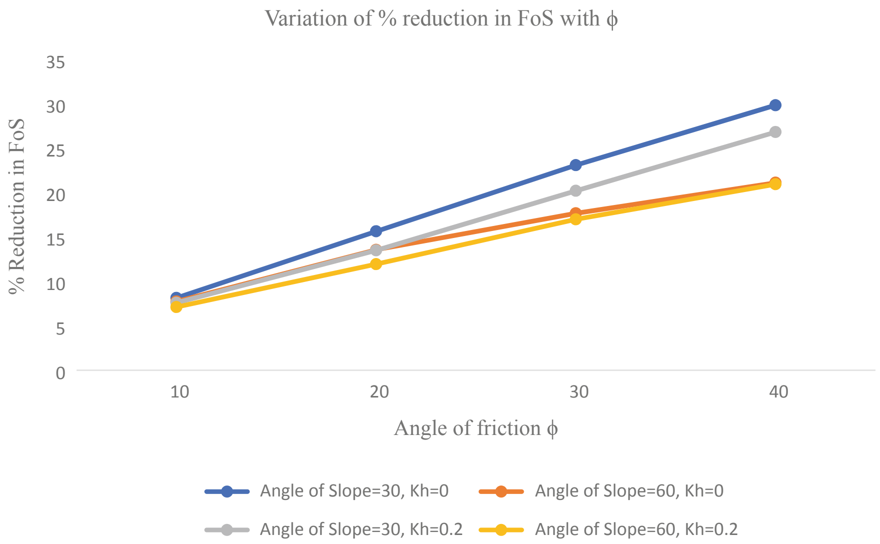

Fig. 5 Effect of dilatancy on 3D factors of safety, without local surcharge load on slope

to nonassociated ($\psi = 0°$). In Fig. 5 no local surcharge load on slope was considered, while Fig. 6 considers local surcharge load on slope. In Figs. 5 and 6, two slope angles 30° and 60° were considered under both static($k_h = 0$) and seismic($k_h = 0.2$) conditions.

Eskandarinejad and Shafiee [7] investigated the seismic stability of reinforced slopes considering nonassociated flow rule. Based on their work, Fig. 7 shows the

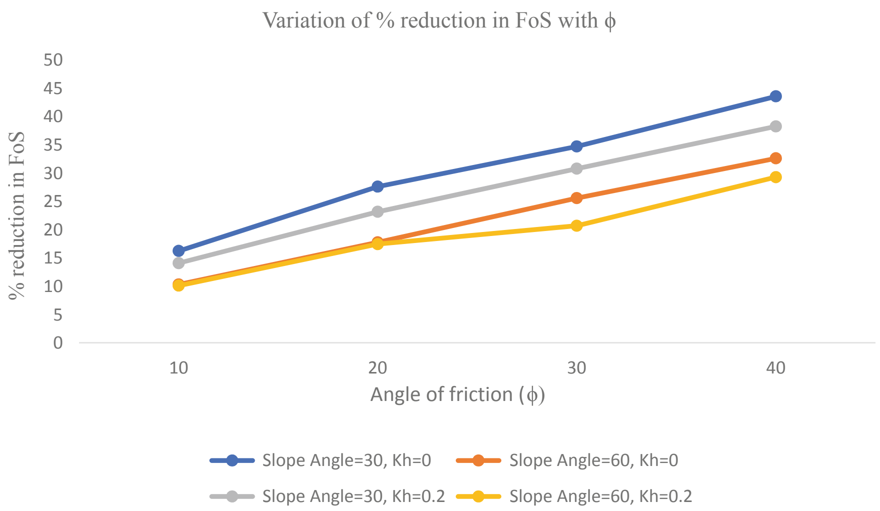

Fig. 6 Effect of dilatancy on 3D factors of safety, with local surcharge load on slope

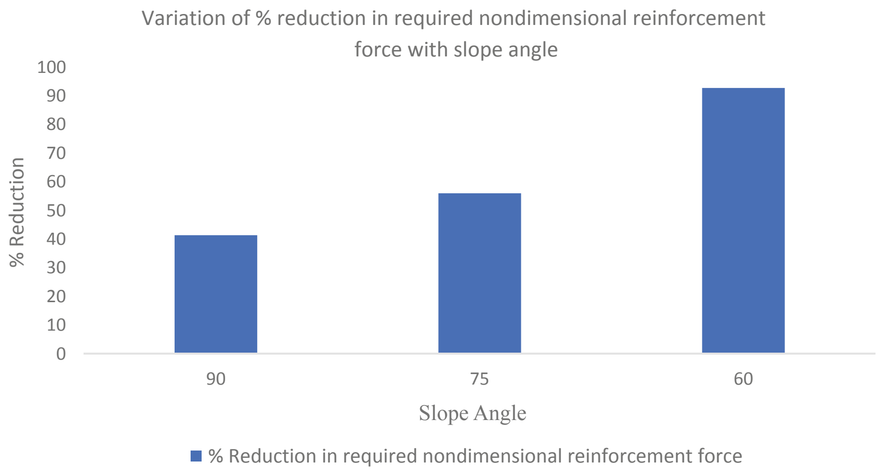

Fig. 7 Effect of dilatancy on required reinforcement force under seismic condition

percentage reduction in the required nondimensional reinforcement force when the flow rule is changed from nonassociated($\psi = 0°$) to associated, at friction angle of 45°. Three slope angles of 90°, 75° and 60° were considered in Fig. 7.

2.3 Incorporating a Nonassociated Flow Rule in Stability Analysis

Several researchers have worked in the area of incorporating nonassociated flow rule in limit analysis. In this regard, reduced effective strength parameters (c^*, ϕ^*) have been suggested to be used along with the associated flow rule to conduct limit analysis of slope stability problems [7, 8]. This method has been referred to as the Davis Approach in the literature [1, 2, 5, 11]. According to the Davis approach [11]

$$c^* = \beta c' \tag{5}$$

$$\tan\phi^* = \beta \tan\phi' \tag{6}$$

$$\beta = \frac{\cos\psi' \cos\phi'}{1 - \sin\psi' \sin\phi'} \tag{7}$$

where ϕ', c', ψ' are given parameters.

c^*, ϕ^* are reduced parameters.

β is the reduction factor.

The original Davis approach is applicable where the FoS is obtained in terms of the maximum load that can be applied for a given soil strength. For strength reduction approach to FoS modified Davis approaches have been recommended to be used [11]. The modified approaches named as Davis B and Davis C depend on iterative procedures to arrive at the relevant reduced strength parameters. The reduction factors in these methods as given in Oberhollenzer et al. [2] are as follows:

Davis B

$$\beta_{failure} = \frac{\cos\left[arctan\left(\frac{\tan\psi'}{FoS}\right)\right]\cos\left[arctan\left(\frac{\tan\phi'}{FoS}\right)\right]}{1 - \sin\left[arctan\left(\frac{\tan\psi'}{Fos}\right)\right]\sin\left[arctan\left(\frac{\tan\phi'}{Fos}\right)\right]} \tag{8}$$

Davis C

$$\beta_{failure} = \frac{\cos\left[arctan\left(\frac{\tan\phi'}{FoS}\right)\right]\cos\psi'}{1 - \sin\left[arctan\left(\frac{\tan\phi'}{FoS}\right)\right]\sin\psi'} \tag{9}$$

Important differences between Davis A, B and C are given in Table 1.

Oberhollenzer et al. [2] have observed that even though the modified approaches are still conservative when compared with the results obtained from strength reduction finite element analysis, they are in better agreement with strength reduction analysis as compared to the original Davis approach. Davis B approach has been

Table 1 Tschuchnigg et al. [11]

Comparison of different procedures			
	Davis A	Davis B	Davis C
β	Constant $\beta_{\text{failure}} = f(\varphi', \psi')$	Varies $\beta_{\text{failure}} = f(\varphi'_{\text{failure}}, \psi'_{\text{failure}})$	Varies $\beta_{\text{failure}} = f(\varphi'_{\text{failure}}, \psi')$
ψ'	Constant	Varies	Constant
Note:	φ^* could theoretically smaller than ψ'	φ^* cannot smaller than ψ'	φ^* could theoretically smaller than ψ', but β increase with decreasing φ'. Limit: $\beta \leq 1.0$

recommended to be used with finite element limit analysis. This approach can also resolve the observed numerical instabilities while using a nonassociated flow rule [2].

3 Nonlinearity and Slope Stability Analysis

3.1 Nonlinearity and Its Effects on Slope Stability

Linear Mohr–Coulomb (MC) failure envelopes, which use a linear approximation to fit the experimental data, are widely used in slope stability analysis. However, it has been observed that strength envelopes of soils have nonlinear forms, especially at low normal stress ranges [12–18]. As Jiang et al. [14] have pointed out, usual experimental investigations for field applications do not cover very low effective normal stresses and extrapolation is done to obtain data not covered in experimental assessment. This may lead to overestimation of soil shear strength at low normal stress ranges. Nonlinearity is important in slope stability calculations since for many practical problems critical slip surfaces are shallow and normal stress acting on such surfaces is small [14, 16–18]. Studies have found that nonlinear parameters have significant impact on problems of slope stability and the predicted slip surface and use of linear Mohr–Coulomb (MC) may yield unsafe solutions [12–14, 17, 18]. Baker [18] observed that the nonlinear (NL) strength envelope gives better description of experimental findings as compared to the linear MC envelope, even for sands and normally consolidated clays which are usually taken to be linear frictional materials. The basic objective of NL strength criterion is not to have a better curve fitting of experimental information, but rather to arrive at a more conservative estimate of strength as compared to the linear MC criterion at normal stresses that are not covered in the experimental normal stress range [18]. An additional advantage of nonlinear strength envelopes in slope stability analysis has been pointed out by Gregory and Bumpas [26]. They observed that with the nonlinear strength envelope explicit consideration of the fully softened zone may not be necessary since the maximum depth of the critical slip surface will be automatically restricted to depths typically observed for shallow slides in highly plastic clays.

Various nonlinear envelopes have been suggested by authors to better fit the experimental data and represent the curved failure envelopes of soils. They include bilinear functions, trilinear functions and various power law relations [14]. Among these the power law failure envelopes have been widely applied to slope stability analysis [12, 16, 17, 28, 30]. One type of power law failure criterion is the simple power law failure criterion, where the failure envelope passes through the origin. Another type is the general power law failure criterion which possesses an initial cohesion at zero normal stress and is more appropriate for cohesive soils [17]. The following form of general power law failure enveloped has been widely used by authors [12]:

$$\tau = c_0\left(1 + \frac{\sigma_n}{\sigma_t}\right)^{1/m} \tag{10}$$

where c_0 = cohesion at zero normal stress.

σ_n = normal stress.

σ_t = tensile stress at zero shear stress.

m = nonlinearity coefficient.

The value of m depends on the geotechnical material involved [30]. Zhao et al. [30] have discussed the range of m suggested by various authors and the widest range is given to be from 1 to 2.

3.2 Incorporating Nonlinearity in Slope Stability Analysis

As has been observed by [14, 16, 32–37], the nonlinear strength criterion cannot be directly used in stability analysis since the shear strength of soils is a nonlinear function of stress. With the power law failure criterion broadly two approaches are used in stability analysis. In one approach the applicable equilibrium and energy dissipation equations are written in terms of the nonlinear power law failure criterion. Then mathematical tools like variational calculus, numerical approaches, iterative procedures, dynamic programming, Taylor series expansion etc. are used to solve the governing nonlinear equations. But these procedures are mathematically relatively more complicated. In the second approach instantaneous Mohr–Coulomb parameters are determined from the nonlinear failure criterion by the tangential technique. These parameters are used in the analysis and then optimization is done to arrive at the required solutions.

4 Conclusion

Study of literature has revealed that dilatancy and soil strength nonlinearity have significant effect on problems of slope stability. Approaches have been developed by authors to use a nonassociated flow rule in limit analysis. In strength reduction method even though for small friction angles the dilatancy angle setting does not have much effect on the solution, as the friction angle increases and assumes high values the significance of flow rule increases and cannot be neglected in the analysis. Nonlinearity of strength envelope has significant effect on the solutions obtained from stability analysis and the corresponding failure mechanisms. This is especially relevant since in many slope problems the normal stresses on the failure surface fall in low ranges. Hence dilatancy and nonlinearity effects should be incorporated into analyses for obtaining better solutions to slope stability problems.

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