

Thermo-Economic Optimization of Spiral Plate HX by Means of Gradient and Gradient-Free Algorithm



Musunuru Hari Krishna, Abhishek Dasore , Upendra Rajak ,
Ramakrishna Konijeti , and Tikendra Nath Verma 

Nomenclature

A	Heat exchange area (m^2)
B	Width of the plate (m)
C_p	Specific heat (J/K)
C	Cost
D	Diameter (m)
GA	Gradient algorithm
GFA	Gradient-free algorithm
h	Heat transfer coefficient (W/m^2K)
H	Hours
K_{plate}	Thermal conductivity of the plate (W/mK)
L	Plate length (m)
\dot{m}	Mass flow rate of fluid (kg/s)
i	Annual discount rate
Δp	Pressure drop
Q	Heat flow rate (W)

M. H. Krishna

Department of Mechanical Engineering, SRK Institute of Technology, Vijayawada 521108, India

A. Dasore · U. Rajak

School of Mechanical Engineering, RGM College of Engineering and Technology, Nandyal 518501, India

M. H. Krishna · R. Konijeti

Department of Mechanical Engineering, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur 522502, India

T. N. Verma (✉)

Department of Mechanical Engineering, Maulana Azad National Institute of Technology, Bhopal 462003, India

R_f	Fouling factor ($m^2 K / W$)
S	Plate spacing (m)
T	Temperature (K)
U	Overall heat transfer coefficient ($W / m^2 K$)
x	Thickness of the plate (m)
η	Pump efficiency
ρ	Density of the fluid (kg / m^3)

Subscripts

mc	Manufacturing cost
oc	Operating cost
ec	Energy
m	Mean
min	Minimum
max	Maximum
h	Hydraulic
f	Fluid
hi	Hot in
ci	Cold in
co	Cold out
ho	Hot out
Outer	Out side
Plate	Wall/plate
Hot	Hot side
Cold	Cold side

1 Introduction

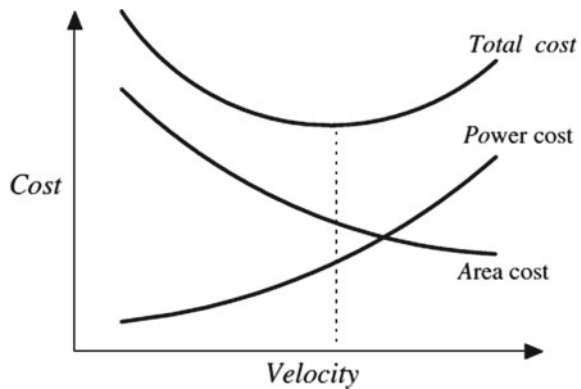
HXs are the devices utilized for the efficient transmission of heat energy from one system to the other. A basic heat exchanger consists of two flow streams of hot and cold fluids which are parted by a solid wall. The quantity of heat transfer relies on factors such as fluid flow type, heat transfer area and thermal conductivity of the wall [1]. SPHX refers to a pair of long flat plates that are looped to form two conduits in a counter-flow arrangement. SPHX is simple, sophisticated, the concentric shape of the flow passageways, and the studs generate turbulence even at the lower Reynolds number [2, 3]. SPHXs are common in petrochemical, food, paper industries for evaporation and condensation processes. These are ideal for cooling slurries and fluids with high viscosities [4, 5].

Optimization of heat exchangers is a challenging area. Optimization quest may be viewed as a design procedure, in which any number of feasible parameters will be examined according to the requisites. Diminishing the capital costs of material and running costs of energy requirements are major goals for industrial applications of heat exchangers [6]. But simultaneously, design of HXs includes intricate procedures, viz. choosing geometrical and dynamic constraints for design, cost approximation and maximization. Usually, the design job is a complex examination method (trial and error process), due to the combined values of constraints of geometry, in addition to this, the designed HX is assessed with reference to the state of specific requirements like exit temperature, heat load and pressure decrement [7]. From this viewpoint, there is a continuous scope that the intended outcomes are not the optimal. Therefore, investigators attempt to enhance thermal equipment via optimization methods and many thought-provoking and efficient works have been reported in recent time [8, 9].

Wang et al. [10] created a new procedure for optimizing HX designs, and they validated the proposed technique by an industrial circumstance application. A novel and efficient software has been developed by Jia et al. [11, 12] to optimize a heat exchanger in which data procurement and process diagrams are encompassed. Reneaume et al. [13] developed mathematics-based formulations for optimizing HXs and proposed a tool for the CAD. They also elucidated the solution approaches under several designs and working constraints. Recently, GA and GFA have gained much consideration in thermal engineering for resolving real-life problems [14]. Applications of these algorithms into HXs optimization have a robust capability of exploration and joint maximization and can effectively enhance and envisage thermal problems.

In the optimization of HXs, a trade-off between heat transmission and pressure diminution should be considered as shown in Fig. 1. In general, high flow velocities indicate a large heat transfer coefficient and hence a lesser heat transfer area and subsequently lower initial cost. However, high velocities will usually lead to large pressure fall and hence a high-power consumption and large operational costs [15, 16]

Fig. 1 Economic optimization of HXs



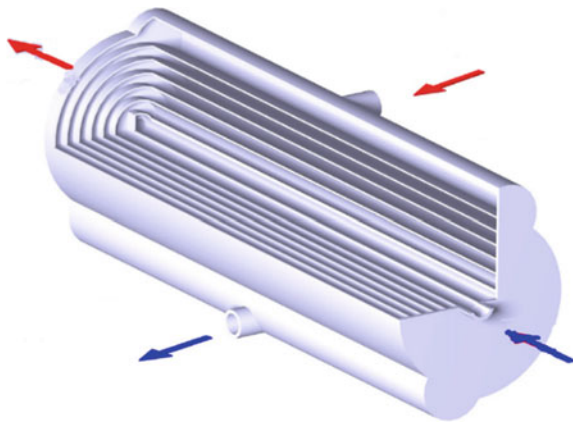
In the current examination, GA technique ‘fmincon’ and GFA method ‘genetic algorithm’ are used to obtain optimal design of the SPHE with the design variables being pressure drop on hot and cold fluids, outer spiral diameter, length and width of the plate. Minimization of the total cost, i.e. the operating cost and the initial investment, is considered as cost function. In order to demonstrate the potentiality of fmincon and genetic algorithm case, studies reported by Segundo et al. [17] were considered. The proposed methods of fmincon and genetic algorithm are not novel, but the application of these into SPHE maximizations for different objectives with various restraints is attractive, and the outcomes are expectantly interesting which are beneficial for further exploration.

2 Physical Model and Design Conditions

From the literature analysis, it is noticed that high effectual, small space, less weight and low price are the common goal in SPHX design. In practicality, there are two design prerequisites. The first one is to reduce the space and annual price of the SPHX as much as attainable under a specified efficacy and a permissible pressure loss. The second requisite is to enhance the efficacy significantly. Before the start of optimal design of SPHX is conducted, the optimization goal should be examined based on the distinct requisites. Predominately, the lowest total space and annual price are analysed in the present investigation. In addition, the lowest pressure falls of both hot and cold sides of SPHX are also taken as an objective.

In the present work, a SPHX is examined. A typical cross-sectional view of SPHX is shown in Fig. 2. For such a SPHX, the two fluids are in counter-flow arrangement with various mass flow rates under designated heat load. There are numerous geometrical constraints which may be considered as maximization variables like pressure fall on both sides, exit diameter of spiral passages, length and width of spiral plate.

Fig. 2 Schematic of SPHX with the indication of inlets and outlets for both fluids



The materials of the plate are stainless steel (*UNSS30400*) with thermal conductivity of $14W/mK$. A thumbnail description of the GA, GFA and selection of variables and constraints are elucidated in subsequent section for better understanding.

Wind-driven optimization adopted by Segundo et al. [17] uses Lagrangian description of flow process due to its infinitesimal fluid particle collection motion. In the present work, *fmincon* and genetic algorithms are adopted. The *fmincon* a GA technique is the best approach for nonlinear optimization problems, since, even if run with random start, it is still faster than other methods and often results in fewer function cells. It employs a Hessian as an optional feed in [18]. The genetic algorithm a GFA is maintained by a population of parent individuals that represent the latent solutions of a real-life problem. For example, the design engineer may inscribe the design constraints into corresponding binary string that is represented as individual [19]. An exact variety of sets of design constraints correspondingly become a population of parent individuals. Figure 3 shows the flowchart of the GFA [20]. Every individual is assigned a fitness supported on how well every individual fit a given ambient and then is assessed by survival of fitness. Fit individuals undergo the method of survival

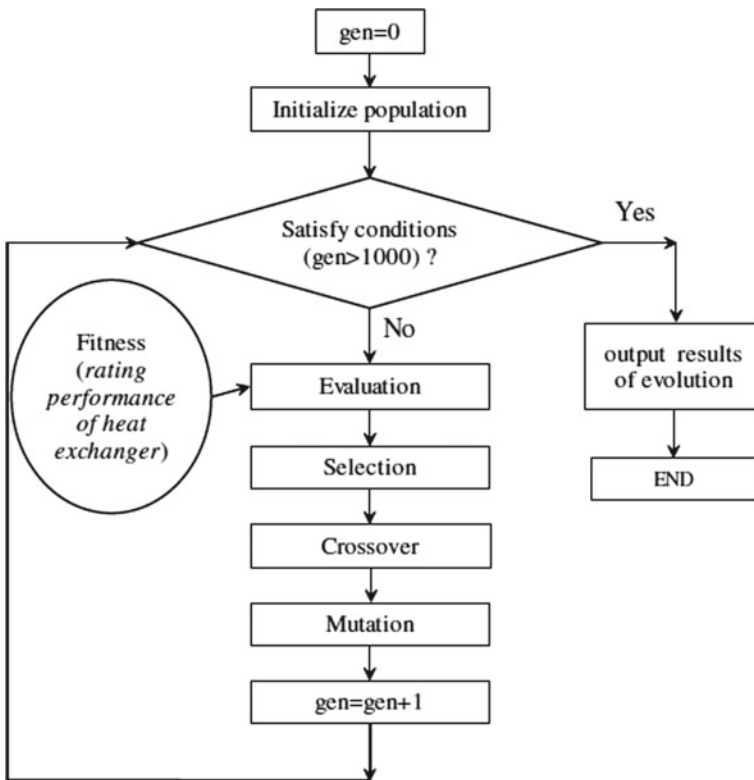


Fig. 3 Flowchart of genetic algorithm

choice, crossover and mutation, leading to make next generation, called kid individuals. A novel population is therefore established by selecting good individuals from parent and kid individuals. After some generations, the algorithm has converted to the best individual, that in all probability indicates the best resolution of the given drawback [21].

3 Optimization

3.1 Mathematical Modelling

The thermal equilibrium for a heat exchanger is given by Eq. (1).

$$Q = m_h C_h (T_{hi} - T_{ho}) = m_c C_c (T_{co} - T_{ci}) \quad (1)$$

The physical principle is law of conservation of energy, with an assumption of adiabatic boundary condition with the surrounding. When the temperatures at inlet and outlet are pre-defined, novel method to evaluate the overall heat transfer coefficient (U) is to utilize the concept of logarithmic mean temperature difference (LMTD), and the heat duty is calculated as given in Eqs. (2–5).

$$Q = U \times A \times LMTD \quad (2)$$

$$U = \frac{1}{\frac{1}{h_{hot}} + \frac{1}{h_{cold}} + \frac{x}{k_{plate}} + R_f} \quad (3)$$

$$LMTD = \frac{(T_{hi} - T_{co}) - (T_{ho} - T_{ci})}{\ln\left(\frac{T_{hi} - T_{co}}{T_{ho} - T_{ci}}\right)} \quad (4)$$

$$A = 2 \times L \times B \quad (5)$$

The Reynolds number (Re), Prandtl number (Pr) and Nusselt number (Nu) are attained from following Eqs. (6–10)

$$Re = \frac{m D_h}{B S \mu} \quad (6)$$

$$Pr = \frac{\mu C_p}{k} \quad (7)$$

$$Nu = 0.239 \times \left(1 + 5.54 \frac{D_h}{D_m}\right) Re^{0.806} Pr^{0.268} \quad (8)$$

$$D_m = \frac{D_{\min} + D_{\max}}{2} \quad (9)$$

$$D_h = \frac{2BS}{B + S} \quad (10)$$

By evaluating the Nu, it is feasible to find the heat transfer coefficient by Eq. (11)

$$h = \frac{k_f \times \text{Nu}}{D_h} \quad (11)$$

The pressure drop is obtained by Eq. (12).

$$\Delta p = 0.00085 \times \frac{Lm^2\rho}{B^2S^2} \quad (12)$$

The outer diameter of the spiral is given by Eq. (13)

$$D_o = \sqrt{[1.28L(S_h + S_c + 2x) + D_{\text{core}}^2]} \quad (13)$$

The total cost is obtained by summing up the manufacturing cost (C_{mc}) and operational cost (C_{oc}) expressed by Eq. (14)

$$C_{\text{total}} = C_{mc} + C_{oc} \quad (14)$$

The manufacturing cost (C_{mc}) is a function of surface of the heat exchanger and is given by Hall equation

$$C_{mc} = 5973 \times A^{0.59} \quad (15)$$

The operational cost can be calculated as follows

$$C_{oc} = \sum_{k=1}^{ny} \frac{C_o}{(1+i)^k} \quad (16)$$

where $C_o = P \times C_{ec} \times H$; and

$$P = \frac{1}{\eta} \left(\frac{m_{\text{hot}}}{\rho_h} \times \Delta p_h + \frac{m_{\text{cold}}}{\rho_c} \times \Delta p_c \right) \quad (17)$$

3.2 Objective Function

The total of a SPHE includes the manufacturing and operating cost, and this is to be minimized.

$$\text{Minimize } J = C_{\text{total}} = C_{mc} + C_{oc} \quad (18)$$

The following are the four constraints in this optimization study, and they are with regard to the pressure drop that occurs as the fluid stream through hot and cold sides, the total heat load limitation and the size limitation of the SPHE.

$$\Delta p_{\text{hot}} - 0.00085 \times \frac{(Lm_{\text{hot}}^2 \rho)}{B^2 S^2} = 0 \quad (19)$$

$$\Delta p_{\text{cold}} - 0.00085 \times \frac{(Lm_{\text{cold}}^2 \rho)}{B^2 S^2} = 0 \quad (20)$$

$$D_{\text{outer}} - [1.28L(S_h + S_c + 2x) + D_{\text{core}}^2]^{0.5} = 0 \quad (21)$$

$$Q - (U \times 2LB \times LMTD) = 0 \quad (22)$$

All these constraints are highly nonlinear. The design constraints in the investigation to be optimized are pressure drop in the hot as well as cold fluids, outer diameter, length and width of SPHE. The values of their upper and lower limits of all the design parameters are given in Table 1.

The optimized results from the present study are compared with the wind-driven optimized results. The physical properties of hot and cold streams are adapted from Moretta et al. [22] and cross-verified in Bidabadi [23] and are mentioned in Table 2.

Additional information needed to calculate the operating costs and heat transfer coefficients are mentioned in Table 3.

Table 1 Upper and lower limits of the design constraints

Property	Hot fluid stream	Cold fluid stream
Pressure drop in hot fluid	127.75	18.92
Pressure drop in cold side	298.60	273.15
Outer diameter	298.15	285.93
Length	4186.80	3786.12
Width	0.05	2.3

Table 2 Property values of hot and cold fluid streams

Property	Hot fluid stream	Cold fluid stream
Mass flow rate (Kg/s)	127.75	18.92
Inlet temperature (K)	298.60	273.15
Outlet temperature (K)	298.15	285.93
Specific heat capacity (J/kgK)	4186.80	3786.12
Density (kg/m^3)	1350	1000
Viscosity ($Pa.s$)	0.10353	0.00122
Velocity (m/s)	4.4006	3.2597

Table 3 Details for optimization

Parameter	Value
Number of years, N_y	15
Discount rate, I	0.1
Energy cost per watt hour	0.00012
Annual work hours ($h/year$)	8000
Thermal conductivity of material (W/mK)	14.532
Fouling factor (m^2K/W)	0.000347
Core diameter (m)	0.3048

4 Results and Discussion

For a specified heat duty (Q), gradient and gradient-free optimization techniques ‘fmincon and genetic algorithm’, respectively are used to obtain the optimized design variables stated in Table 1. The stream properties for the simulation were taken from case study Moretta et al. [22] as stated in Table 2. MATLAB optimization toolbox was used to perform optimization, and tolerance for constraints was set to $1e^{-06}$.

It can be noticed from Table 4 that GFA outperformed wind-driven algorithm and fmincon. GA converges to a cost of 20, 572 where a fmincon converges to a cost of

Table 4 Comparison of present results with reported works

Parameter	Moretta et al. [22] (case study)	Segundo et al. [17] (WDO)	Present work (fmincon)	Present work (genetic algorithm)
$(\Delta P)_{hot}$	110.190	111.198	107.254	163.78
$(\Delta P)_{cold}$	85.430	45.195	43.592	66.56
D_s	0.849	2.280	1.5	1.5
L	7.8170	5	5	7.367
B	0.9144	0.6720	0.855	0.56
U	1113	75	1551.314	1606.6
C_{total}	44,813	18,186	20,941.279	20,572

20941.279, which is about 13.5 and 11.5% higher than the wind-driven algorithm. Even though the goal of the optimization was to minimize the cost, it can be observed that the overall heat transfer coefficient (U) which signifies the physical process of interest is about 95% higher than WDO and 30.7% higher than the case study.

The increase in the cost in the GA as compared to WDO can be quantified to the increased overall heat transfer coefficient. However, the computation time for genetic algorithm is higher than fmincon. For a preliminary analysis with constraint on computation time and facility, it is advisable to utilize gradient-based optimizer fmincon.

From Figs. 4, 5 and 6, it can be observed that fmincon converges to optimized value in two iterations but iterations continued up to four in order to satisfy the specified tolerance constraints on functional and constraint values, whereas GA searches for

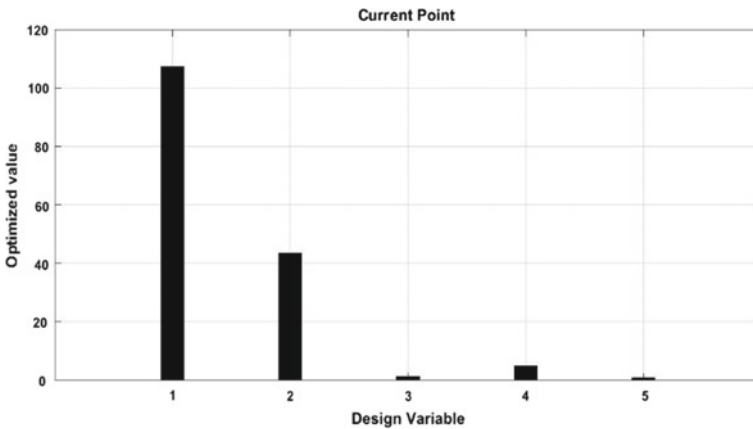


Fig. 4 Optimized parameter plot for fmincon

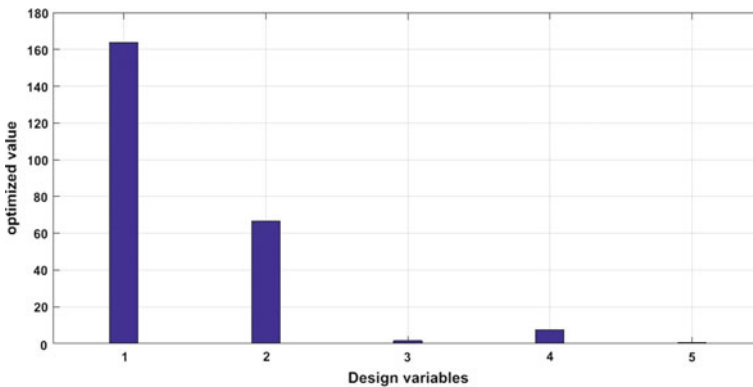


Fig. 5 Optimized parameter plot for GA

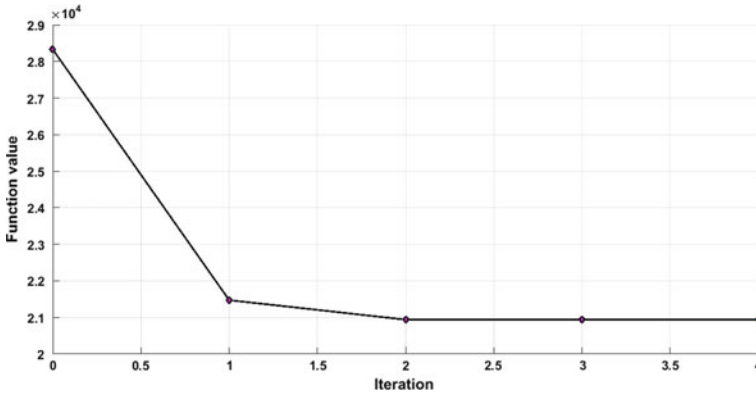


Fig. 6 Variation of objective function with iteration for fmincon

the optimized values in the complete range with random updates of design variables, thereby increasing the number of functional evaluations.

5 Conclusions

In the current investigation, thermo-economic modelling and maximization techniques are employed to attain the optimum values of design constraints of SPHX. Therefore, the extensive thermodynamic modelling of this SPHX is carried out utilizing MATLAB software program and subsequent inferences are drawn:

1. A novel objective function which comprises manufacturing and operating costs is defined.
2. The fmincon and gradient algorithm are applied effectively to the multi-objective maximization of SPHX.
3. The cost of SPHX obtained by fmincon is 53.2% lesser than the ones reported in case study and 13.1% higher than WDO.
4. The cost of SPHX attained by GA is 54% lesser than the one reported in case study and 11.5% higher than WDO.
5. It is concluded that the genetic algorithm can be employed in optimizing the design configurations of SPHX according to various design constraints, viz. minimum surface area and cost.
6. The present analysis can form a basic paradigm in optimizing the designs of various types of HXs.

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