Estimation of Wave Characteristics from the Measured Pressures on Support Structure of Offshore Wind Turbine

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Abstract. Waves at sea are very important for designing an offshore wind turbine support structure. There are several techniques used to observe waves at sea. These techniques can be in-situ technique and remote-sensing technique. This study is going to present an in-situ technique, in which pressure sensors are mounted on a support structure of offshore wind turbine and they are deployed in the water. Wave characteristics have been recovered from the measured pressures on the structure applying the spectral analysis method. Results show that the significant wave height can be well recovered from the subsurface pressure at the angle of 150, 180 and 210° (the angle of the orientation of the pressure sensor with respect to the incident wave direction). There is slightly effects of the submergent level of the pressure measurement on the recovered wave heights.

Keywords: Wave recovery · Wave pressures · Offshore wind turbine · Physical model

1 Introduction

Wave climate is a vital important in estimation of wave loading on coastal and offshore structures. There are various techniques used to observe waves at sea. These techniques can be divided into two main techniques. The first one is in-situ technique in which instruments are deployed in water such as wave buoys, wave poles, inverted echo-sounders, pressure transducers and current meters. The second is remote-sensing technique, in which instruments are deployed at some distance above the water surface such as imaging techniques and altimetry.

The pressure transducer is one of the in-situ techniques. Traditionally, a pressure transducer can measure wave-induced pressure fluctuations at various depths below the water surface. These fluctuations, in combination with the linear wave theory (Airy theory) can be used to estimate wave characteristics. There have been many studies on application of this technique in estimation of wave parameters since 1947 (Folsom 1947; Grace 1978; Cavaleri 1980; Bishop and Donelan 1987; Mai and Schlurmann 2012; Clamond 2013; Oliveras et al. 2012; Kogelbauer 2015). Grace (1978) found that the Airy theory can be used to predict individual wave height from concurrent pressure variations

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at or near the sea floor with an empirical correction factor is included. Disturbance of the orbital motion of water particles under waves on the wave measurements by pressure transducers was found by Cavaleri (1980) and a measurement system has been devised to avoid the effect of dynamic pressure by consisting of a water filled plastic sheath surrounding the captive part of the transducer. His result shows that the dynamic effect of water motion has been cancelled and the pressure transducers were picking up just the pressure field due to the presence of waves. Gabriel et al. (1986) investigated the effects of currents on interpretation of sub-surface pressure spectra. Their study shows that the Doppler effect should be taken into account when recovering surface elevation spectra from subsurface pressure measurement taken in the presence of currents, particularly when the pressure transducer is deployed close to the sea bed. Their study indicates that there is better agreement between the measured water elevation spectra and the one recovered from the pressure measurement at level closer to the water surface. Clamond (2013) derived some new exact relations for the recovery of the steady free surface wave profile from pressure measurements at the flat sea bed, with the introduction of the holomorphic function which yields a reformulation of the problem into a much simplified form. Oliveras et al. (2012) presented a new nonlocal nonlinear equation relating the pressure and the surface elevation, and the solvability properties of this nonlocal equation are rigorously analyzed using the Implicit Function Theorem. They derived various new approximate formulas and these approximations are compared with the nonlocal equation by using numerical and experimental data. The nonlocal equation consistently outperforms its different approximations. Kogelbauer (2015) obtained a relation between the bottom pressure and the free water surface, expressed in terms of Fourier coefficients by using a conformal hodograph transform and the governing equations. Application of the relation in Kogelbauer (2015) to experiment has not been made yet.

This study is going to apply the linear transfer function to recover wave heights from concurrent pressure measurements at various levels below mean water level. The pressure transducers were mounted on a mono-pile to measure pressure distribution around as well as along the mono-pile under wave condition. A polar coordinate has been applied to obtain transfer function which also depends on the location of pressure measurement on the mono-pile. Significant wave heights are recovered from pressure measurements using frequency domains. The theoretical and experimental results are then compared.

2 Methodology

2.1 Physical Model

A small scale 1:40 physical model was constructed in the new wave basin in Hannover, Germany to examine different multi-directional waves interacting with a large mono-pile structure (the diameter of the mono-pile is large in comparing with wave length). The wave basin has dimensions of 40 m wide and 24 m long and the basin can be filled up to 1 m of water depth for experiment. The 3D wave maker has 72 individual wave paddles and each wave paddle has 0.40 m wide and 1.8 m high. Maximum stroke of each paddle is ±0.6 m. The tested mono-pile has its diameter of 0.3 m and height of 0.9 m (see Fig. 1a).

The tested wave condition (UI4) was a unidirectional irregular wave with an underlying JONSWAP spectrum. The experimented wave characteristics had the significant wave height of 0.12 m and the peak wave period of 2.0 s. The diffraction parameter, kr_0, was 0.22 in which k is the wave number and r_0 is the radius of the mono-pile. The tests were performed within a water depth of 0.6 m. The pressures upon the surface of the mono-pile were measured by applying pressure sensors mounted at various levels ($z/h = -0.07$ to -0.77; z is the submergence of pressure sensor, h is the water depth in the basin) on the surface of the structure. Water elevations were measured synchronously next to the structure (see Fig. 1b).

(a) (b)

Fig. 1. (a) Location of pressure transducers on the mono-pile (Note units in cm); (b) Configuration of physical model in the 3D-wave basin.

2.2 Theoretical Aspects

2.2.1 Subsurface Pressure Under Water Wave (Bishop and Donelan 1987)

The pressure beneath a progressive wave can be expressed from Bernoulli's theorem:

$$p(t) = -\rho g z + \rho \frac{\partial \phi}{\partial t} - \frac{1}{2}\rho\left(u^2 + w^2\right) + p_s + \rho\gamma(t) \tag{1}$$

in which, p is the total pressure, ρ is the water density, g is the gravity acceleration, z is the depth of submergence of the pressure transducer (measured positive upward from the mean water level), ϕ is the velocity potential, u is the horizontal velocity, w is the vertical velocity, p_s is the atmospheric pressure at the surface and $\gamma(t)$ is a function of time (see Bishop and Donelan 1987).

The first three terms on the right hand side of Eq. 1 are the hydrostatic pressure (This hydrostatic pressure is independent of the presence of the wave), the pressure due to the passage of the wave form (or the hydrodynamic pressure due to the wave and therefore this term represents the wave-induced pressure) and the pressure due to the local kinetic energy. We are normally interested in the hydrodynamic terms:

$$p(t) = \rho \frac{\partial \phi}{\partial t} - \frac{1}{2}\rho\left(u^2 + w^2\right) + \rho\gamma(t) \tag{2}$$

According to the Airy wave theory, pressure due to the passage of a wave form, at a level z under the water surface, can be estimated by:

$$p(t) = \rho \frac{\partial \phi}{\partial t} = \rho g \frac{H}{2} \frac{\cosh k(d+z)}{\cosh kd} \cos(kx - \omega t) \quad (3)$$

in which, p is the pressure at a level z below still water level, ρ is the water density, g is the gravitational acceleration, H is the wave height, d is the water depth, k is the wave number, ω is the angular frequency and $(kx - \omega t)$ is the phase of wave.

The water elevation can be estimated by using Airy theory as:

$$\eta(t) = \frac{H}{2} \cos(kx - \omega t) \quad (4)$$

Substitute Eq. 4 into Eq. 3 then:

$$p(t) = \rho g \frac{\cosh k(d+z)}{\cosh kd} \eta(t) \quad (5)$$

Therefore, the water fluctuation can be estimated from the fluctuation of pressure:

$$\eta(t) = \frac{1}{\rho g} \frac{\cosh kd}{\cosh k(d+z)} p(t) \quad (6)$$

Since pressures upon a vertical mono-pile have been measured, the water fluctuation can also be estimated by these pressures via the linear theory function which will be presented in the following sections.

2.2.2 Pressures upon a Vertical Circular Cylinder (Sumer and Fredsøe 2006)

From the linear feature of potential flow, the total potential function (ϕ) can be written as the sum of two potential functions:

$$\phi = \phi_i + \phi_s \quad (7)$$

in which, ϕ_i is the potential function of the undisturbed incident wave and ϕ_s is the potential function of the scattered (reflected and diffracted) wave by presence of the vertical mono-pile.

In the case of a wave passing a small vertical mono-pile ($D/L < 0.2$, D is the diameter of the mono-pile and L is the wave length), only potential function of the undisturbed incident wave is taken into account.

$$\phi = \phi_i \quad (8)$$

Therefore, according to MacCamy & Fuchs (Sumer and Fredsøe 2006) the dynamic pressure exerts on the surface of the mono-pile can be expressed as a function of the time, frequency, submerge level and the angle of orientation of the point on the surface of the mono-pile with respect to the incident wave direction:

$$p(t, f, z, \theta) = \frac{\rho g H}{2} \frac{\cosh k(h+z)}{\cosh(kh)} \cdot \left[J_0(kr_0) + \sum_{p=1}^{\infty} 2i^p J_p(kr_0) \cos(p\theta) \right] \cdot e^{-i\omega t} \quad (9)$$

in which, ρ is the water density, g is the gravity acceleration, H is the wave height, k is the wave number, h is the water depth, z is the depth of pressure sensor's submergence which is measured positive upward from still water level, r_0 is the radius of the mono-pile and θ is the angle of orientation of the pressure sensor with respect to the incident wave direction such as definition in Fig. 2. $J_0(kr_0)$ and $J_p(kr_0)$ are the first and second kind of Bessel functions, respectively.

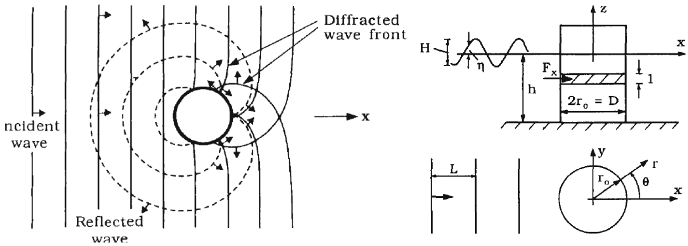

Fig. 2. Sketch of the incident, diffracted and reflected wave fronts for a vertical circular cylinder (Sumer and Fredsøe 2006).

In spectral analysis, the transfer function, $TF(f, z, \theta)$ for the dynamic pressure at any point on the surface of the structure can be expressed as a function of the frequency f, submergence level z and angle θ (Sumer and Fredsøe 2006):

$$TF(f, z, \theta) = real\left\{\frac{\cosh k(d+z)}{\cosh(kd)} \cdot \left[J_0(kr_0) + \sum_{p=1}^{\infty} 2i^p J_p(kr_0)\cos(p\theta)\right]\right\} \tag{10}$$

Spectral density of the dynamic pressure at any point on the surface of the mono-pile can be expressed as:

$$S_{pp}(f, z, \theta) = |\rho \cdot g \cdot TF(f, z, \theta)|^2 S^p_{\eta\eta}(f, z, \theta) \tag{11}$$

in which, $S_{pp}(f, z, \theta)$ is the spectral density of the measured dynamic pressures upon the mono-pile and $S^p_{\eta\eta}(f, z, \theta)$ is the predicted spectral density of the water surface elevation from the measured pressure spectral density.

An empirical factor, $N(f, z, \theta)$, is introduced to account for the differences between the measured water surface wave and the one predicted from dynamic pressure. The empirical factor is expressed in frequency domain as:

$$N(f, z, \theta) = \sqrt{\frac{S^m_{\eta\eta}(f)}{S^p_{\eta\eta}(f, z, \theta)}} \tag{12}$$

in which, $S^m_{\eta\eta}(f)$ is the spectral density of the measured water surface elevation.

3 Results and Discussions

The experimental data of the tested wave UI4 are applied to recover the wave heights from subsurface pressures on the monopile. The tested wave spectrum from wave gauge

WP1 and the spectra of concurrent pressures upon the mono-pile at various levels ($|z/h|$ = 0.01 to 0.76) on the surface of the mono-pile and at an angle θ, are presented in Fig. 3.

Figure 4 presents the wave spectra recovered from pressures measured at a submergence level $|z/h| = 0.095$ on the complete circumference ($\theta = 0°$ to 360° in step of 30°) of the mono-pile under the tested wave UI4 (two top plots). It can be seen from Fig. 4 that the wave spectral densities recovered from pressures at angles $\theta = 150°$, 180° and 210° have the same peak as the measured one from wave gauge WP1. The recovered wave spectral densities at these three angles are lower than the measured one in the low frequency range ($f < f_{peak}$) and are higher than the measurement in the high frequency range ($f > f_{peak}$). In the higher frequency range ($f > 1.25 f_{peak}$) these recovered spectral densities are much higher than the measurement.

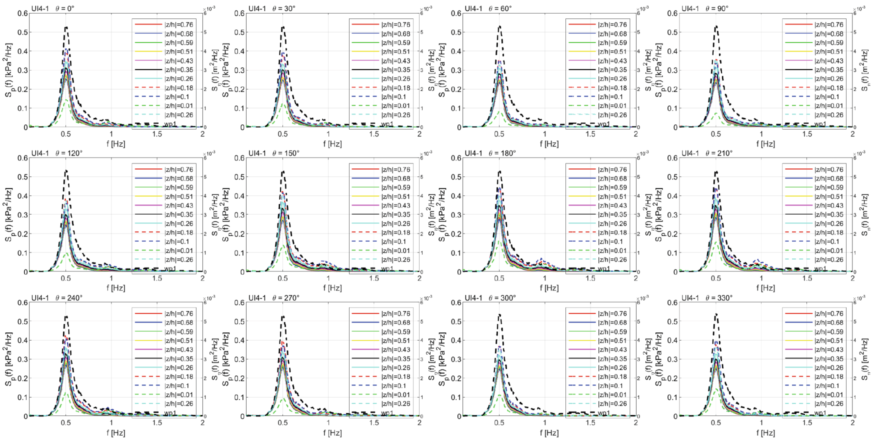

Fig. 3. The tested wave spectrum and concurrent pressures upon the mono-pile.

The theoretical transfer functions *TF(f)* at the submergence level $|z/h| = 0.095$ and on the complete circumference ($\theta = 0°$ to 360° in step of 30°) of the mono-pile are presented in two middle plots in Fig. 4. As it has been mentioned in Sect. 2 that $TF(f, z, \theta) = TF(f, z, 180° - \theta)$ because we get the real part of Eq. 10. It is shown that the higher submergence level z the higher transfer function values at all angles $\theta = 0°$ to 360° (Fig. 4). Two last plots of Fig. 4 present the empirical factors *N(f)* from all pressure data at the submergence level $|z/h| = 0.095$. In the low frequency range ($f < 0.43$ Hz), the empirical factors of all pressure data are very sensitive values. In frequency range $f > 0.43$ Hz the empirical factor trend lines are quite smooth and they vary gradual up to frequency of about 0.83 Hz then they have been reduced dramatically in frequency range of 0.83 Hz to 0.95 Hz. In the frequency range $f > 0.95$ Hz, the empirical factors from most of pressures are nearly horizontal developments. The empirical factors from pressure data at position $\theta = 150°$, 180° and 210° are quite similar and in frequency range 0.43 Hz $< f <$ 0.83 Hz the empirical factors from those positions θ vary from 0.85 to 1.2. The empirical factors from pressures at different submergence level and at a position θ are quite similar to each other. Their difference is about of 10% therefore the submergence level ($|z/h| > 0.095$) has not affected much to the empirical factor *N(f)*.

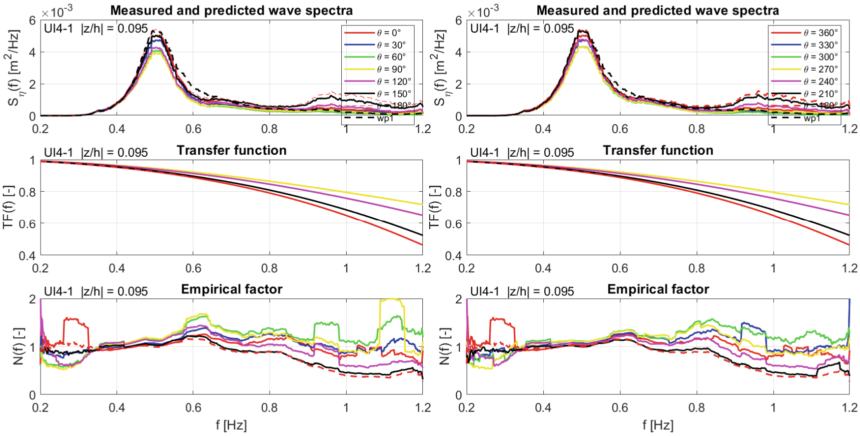

Fig. 4. $S_\eta(f)$, *TF(f)* and *N(f)* at different angles θ and at level $|z/h| = 0.095$ (P9).

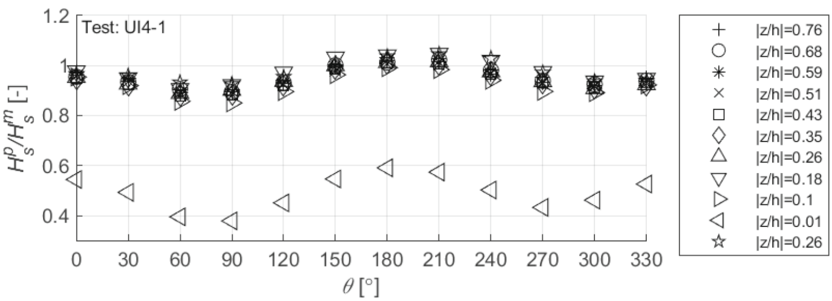

Fig. 5. Ratio of predicted and measured significant wave heights.

The ratios of the recovered significant wave heights $H_s{}^p$ and the measured one $H_s{}^m$ are presented in Fig. 5 for the tested wave UI4. As we can see from Fig. 5 that the values from pressures at level $|z/h| = 0.012$ (mostly at the water surface) are much lower than the values from the higher levels ($|z/h| = 0.095$ to 0.76). Therefore, the pressure data at the level $|z/h| = 0.012$ cannot be used to recover the wave heights. The significant wave heights recovered from pressure records at different submergence levels have small differences. Figure 5 shows the maximum difference of about 10% for the tested wave UI4. Also from Fig. 5, the recovered significant wave heights $H_s{}^p$ from pressure records at position $\theta = 150°$, 180° and 210° are mostly similar to the measured/tested significant wave heights $H_s{}^m$. Therefore, the ratios of $H_s{}^p/H_s{}^m$ at those positions θ are nearly unity. It can be seen that the significant wave height can be recovered quite well from pressure records at positions $\theta = 150°$, 180° and 210° (at all submergence levels $|z/h| > 0.095$), in comparing with the measured one ($H_s{}^m$).

4 Conclusions

This study has applied the linear transfer function to recover the significant wave heights from the tested subsurface pressures under waves. The physical model was carried out in a wave basin in Hannover, Germany. The results show that the significant wave height can be recovered from the tested pressures upon a small vertical mono-pile at the angle $\theta = 150°$, $180°$ and $210°$ (θ is the angle of orientation of the pressure sensor with respect to the incident wave direction) which give very good results in comparing with the measured one. In addition, it is found that the recovered significant wave heights have not been affected much by the submergence level ($|z/h| > 0.095$) of the pressure measurement.

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