



# Cantilever Secant Pile Constructed in Sand: Numerical Comparative Study and Design Aids

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**Abstract.** Civil engineering projects include excavation work, therefore some needs retaining structures. Cantilever secant pile wall is an economical supporting system up to 5.0-m depths. The parameters controlling wall tip displacement are the focus of this paper. Three analysis techniques have been arbitrated. They are the conventional method, spring analogy and finite element. Hence, three computer programs have been used, Excel sheet, SPW-2006 and Plaxis-2D. Two soil densities have been considered, i.e. loose and dense sand. Ten wall rigidities have been analyzed covering ranges flexible up to rigid walls. Three excavation depths, i.e. 3.0, 4.0 and 5.0-m were tested to cover the practical range. This work submits beneficial hints about secant piles to assist designers. Also, finite element analysis is recommended to be the fair judge when two designs conflict. A rational procedure using empirical equations has been suggested to upgrade the conventional method to predict wall tip displacement  $\delta$ . Designer must keep ' $\delta$ '  $\leq 0.8\%$  of the excavation depth, 'h', for safety.

**Keywords:** Secant pile · Numerical analysis · Design aids

## 1 Introduction

The manuscript theme is soil retaining structures, with a special emphasis on in cantilever secant pile walls. The wall tip displacement as well the comparison between the methods of analysis that used to handle this problem presents the target of this study. The paper skeleton starts by hints on the literature leads to research questions. Then the research strategy presented by figures, tables, flowchart, the flowchart is the key figure. Then, the results presented, analyzed, discussed, and the conclusions offered.

Worku (2007) studied the use of springs to account for subgrade deformation. Bekdas *et al.* (2020) investigated the analysis of cantilever soldier pile walls in sandy soil based on the finite element and provides an optimal design. Ramadan *et al.* (2018) evaluated the design parameters needed to design contiguous pile supporting shallow excavations in clay. 3D- finite element model is used. Mahesh *et al.* (2019) examined diameter effects on pile capacity, bending moment and wall displacement. The case study involves deep excavation supported by secant piles of reinforced and plain concrete piles. Liao *et al.* (2014) performed comprehensive model tests to study the failure modes of secant faces.

Khater (1991) published a case of sheet pile failures in clay, reported: “*safety factor penetration ratio 20% is insufficient in clayey soil*”.

The research statement could be presented by four research questions. Does wall penetration depth have a peak value Q1? Which analysis method is reliable: conventional method, Winkler or finite element, Q2? Is it reasonable to suggest empirical equations to predict the wall tip displacement ‘ $\delta$ ’, Q3? What is a ‘ $\delta$ ’ limitation for safe design? Q4. *Satisfactory answer of those questions is the aim and importance of this work.*

## 2 Methods Employed

Methodology is the strategy that outlines the way in which the research is to be carried out. Here, it is numerical analysis. Methods are the tools chosen to create quality data. Herein, they are the conventional method, Winkler analogy and 2D-FEA.

### 2.1 Parametric Study

Figure 1 is the variable interactions flowchart. Its basis is simple, 1<sup>st</sup> level ‘1-2-3’ is the analysis methods. The three dotted rectangles present excavation depths. The numbers on the box show ‘routes’, e.g. ‘4-7-12-18’, is  $h = 5.0\text{-m}$ , loose soil S1, five wall rigidities  $I_6: I_{12}$ . This repeated three times, i.e. (1-4), (2-4) and (3-4). Box 20; explains that every single case has been investigated 10 times by increasing  $D_0$  in steps of 10% each,  $D_0$  is the minimum penetration depth calculated by the conventional method.

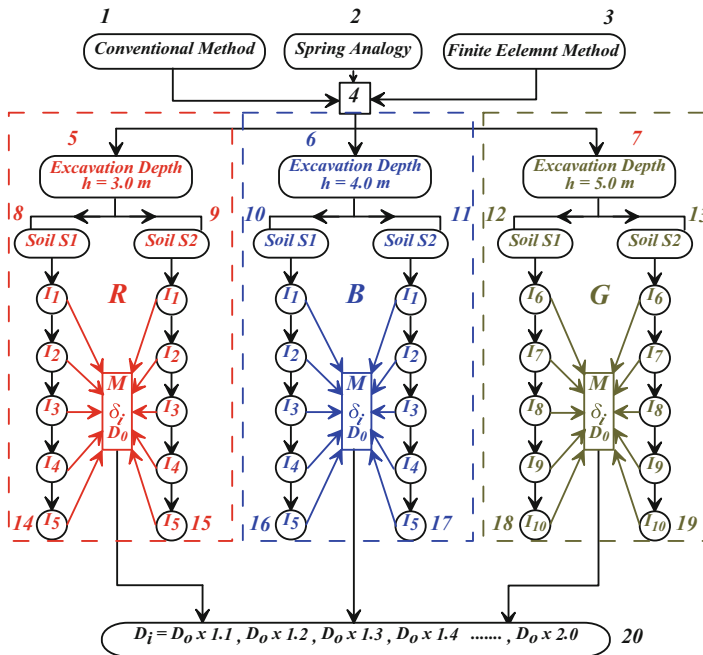


Fig. 1. Flow chart and research algorithm.

Quality data is defined as a set of ‘focused-on’ data induced by analyzing suggested variables intentionally oriented to achieve the research goal. So, the parametric study technique is appropriate for that achievement. Briefly, two soils have been considered, loose and dense well graded dry sand, S1 and S2 respectively. Three excavation depths “h” have been studied, h = 3.0-m, 4.0-m and 5.0-m. This cover the practical range of cantilever secant pile. Ten wall rigidities “ $I_n$ ”, integer “n” increases from 1 to 10, cover flexibility up to very rigid walls. The above have been analyzed three times, i.e. conventional method, then Winkler analogy and finite element analysis. Figure 1, Table 1 and Table 2 give a detailed description of every single variable used.

Table 1, summarizes the mechanical and physical properties. Even if the soil is dry sand, some parameters such as,  $\gamma_{sat}$ ,  $\psi^o$  and c are needed for Plaxis-2D, by default. They have been used but never affect the physical meaning or behavior of dry sand.

**Table 1.** Physical and mechanical properties of the used soil.

Soil type and notations	$\gamma_d$	$\gamma_{sat}$	$\phi^o$	$\psi^o$	C	$\mu_s$	$E_s$
SW, loose dry sand, “S1”	14.0	20.0	29	1	1.0	0.30	7500
SW, dense dry sand, “S2”	16.0	20.0	37	7	1.0	0.38	11500

$\gamma_d$  dry density ( $kN/m^3$ ) -  $\gamma_{sat}$  saturated density ( $kN/m^3$ ) -  $\phi^o$  internal friction (deg) -  $\psi^o$  dialancy angel (deg) - c cohesion ( $kN/m^2$ ) -  $\mu_s$  Poisson’s ratio -  $E_s$  elasticity modulus ( $kN/m^2$ ).

Table 2, summarizes the mechanical properties of the wall per meter run. The overlap of RC pile on bentonite pile is kept constant as 0.10-m. The  $D_{eqv}$  stands for the equivalent pile diameter induced from neglecting bentonite piles mechanical properties.

**Table 2.** Physical and mechanical properties of the secant piles.

Secant pile structural parameters	$I_n$	i	EI	EA	Wt/m’	$D_{P-Eqv}$ .
For excavation depths H = 3.0 and 4.0-m; Soil types: S1 & S2	$I_1$	0.07490	19260	3433751	7.60	0.25
	$I_2$	0.08739	28554	3738973	8.28	0.30
	$I_3$	0.09987	40599	4069631	9.01	0.35
	$I_4$	0.11236	55733	4414822	9.78	0.40
	$I_5$	0.12484	74328	4769098	10.56	0.45
For excavation depth H = 5.0-m and Soil type: S1 & S2	$I_6$	0.14482	112150	5347749	11.84	0.50
	$I_7$	0.17228	182757	182757	13.64	0.60
	$I_8$	0.20224	288411	7051280	15.62	0.70
	$I_9$	0.23096	422127	7913813	17.53	0.80
	$I_{10}$	0.25964	592019	878001	19.44	0.90

$I$  moment of inertia ( $m^4$ ) –  $i$  radius of gyration (m) –  $EI$  flexural rigidity ( $kN.m^2/m$ ) –  $W_t$  wall weigh ( $kN/m'$ ) – “ $E$  elasticity modulus =  $1942 \cdot 10^4$   $kN/m^2$  -  $\mu_s$  Poisson’s ratio = 0.20”; concrete.

**2.2 Methods and Tools**

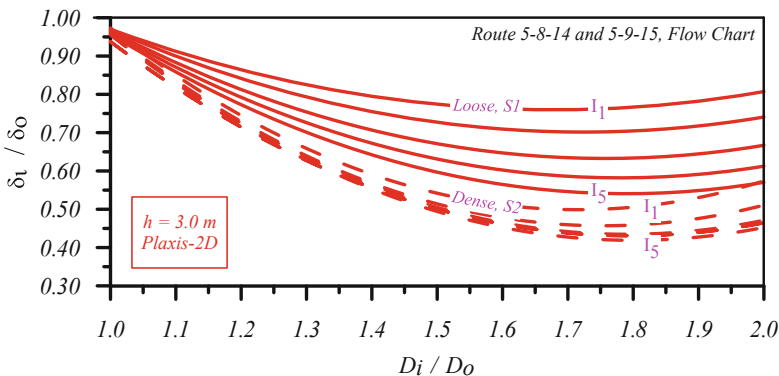
The conventional method formulations described by Das (1987) and incorporated into an Excel sheet is developed to fulfill research needs. Spring analogy used here simulates Elasto-plastic soil response. Beam element models the sheet piles. This behavior has been formulated and runs with 2D noncommercial software ‘SPWALL20006’. It has been coded by Prof. Arnold Verruijt for Delft University of Technology (2006). The finite element analysis used is Plaxis-2D. Triangular 15-node elements were used to model the soil with Mohr-Coulomb material model. Plate and interface element were used for the sheet pile modeling. Staged construction procedure was used.

**3 Presentation and Analysis of Results**

Figure 2 and Fig. 3 demonstrates the case of an excavation depth  $h = 3.0\text{-m}$ , the other two cases, i.e.  $h = 4.0\text{-m}$  and  $5.0\text{-m}$  having typical trend with different values. Figure 2 presents the FEA results, while Fig. 3 reports the spring analogy, comparison wise. Loose sand S1 and dense S2 have been investigated, with each, five wall rigidities,  $I_n$  have been inspected.  $D_0$  is the minimum possible penetration depth (no safety factor).  $D_i$  is a penetration depth with increasing factor of safety by steps 10% of  $D_0$ . Hence, x-axis is the dimensionless ratio  $D_i/D_0$ .  $\delta_0$  is the horizontal displacement of the wall tip corresponding to penetration depth  $D_0$ . The  $\delta_i$  is variable displacement of the wall tip at the penetration depth  $D_i$ . The y-axis is the dimensionless ratio  $\delta_i/\delta_0$ , too.

**3.1 Effective Penetration Depth and Design Hints**

Figure 2 and Fig. 3, as  $I_n$  or soil density increases,  $\delta_i$  decreases for the same  $D_i/D_0$  ratio. The effect of “ $I_n$ ” on  $\delta_i$  is more clear for S1 than for S2. ‘ $\delta_i$ ’ decreases as  $D_i/D_0$  increases, dramatically up to  $D_i/D_0$  is 1.50. Then curves get flatter, almost horizontal. It is concluded, for  $D_i/D_0$  beyond 1.5, no significant reduction of  $\delta_i$ . Increasing  $D_0$  by “design factor to satisfy safety”, of 20% appears not to be that generous! The later “point of view” is previously reported in a published case history, Khater (1991).



**Fig. 2.** Effect of penetration depth on wall tip displacement, [Plaxis cases]

Figure 3 shows the results of the spring analogy for the same cases previously studied based on the Plaxis. The curve behaviors are typical in both techniques, but the Winkler approach has numerical values are higher. The numerical deviation is expected due to the lack of soil shear that leads to high values of  $\delta_0$ , thus forcing all curves to be compacted. The two comments about  $D_i/D_0 = 1.5$  and  $D_i/D_0 = 1.2$  are valid herein and through the other two excavation depths. Again, Winkler overestimated ‘ $\delta$ ’.

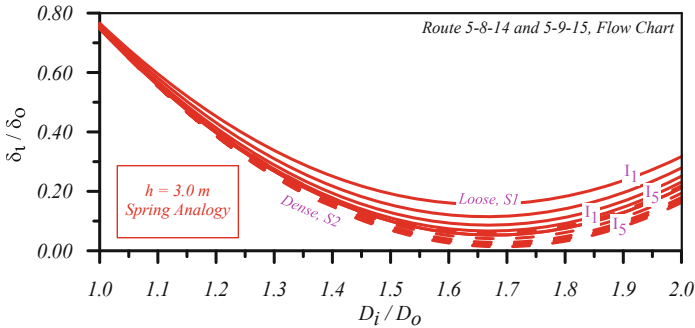


Fig. 3. Effect of penetration depth on wall tip displacement, Winkler cases

Table 3 displays the result variations among the three used methods. Conventional method results are the base of comparison. The ratio (% $D_0$ ) definition is  $D_0$  computed by spring analogy (or Plaxis) to  $D_0$  calculated by the conventional method. The ratio (% BM) definition follows the same (% $D_0$ ) concept, and both are decimals. Columns (2) and (6) show the values of (% $D_0$ ) for the spring analogy and FEA- Plaxis 2D. The values range from 1.10 to 1.17. Columns (4) and (8) show the values of (%BM), for the spring analogy is 1.0; and for FEA- Plaxis 2D it ranges from 0.51 to 0.68. It is concluded that, the conventional method and the spring analogy underestimate the minimum possible penetration depth  $D_{0-min}$  and overestimate the maximum expecting bending moment  $BM_{max}$ , in general. This leads to higher values of  $\delta_0$  predicted by spring analogy, and in comparison reasonable ones for FEA- Plaxis 2D.

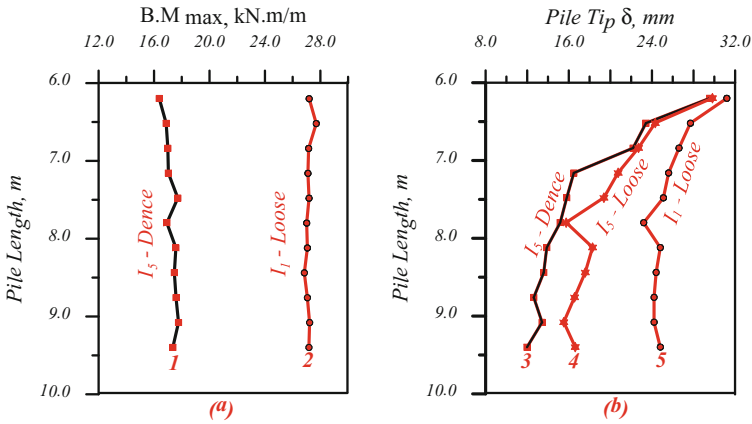
Figure 4-a, curves (1) and (2), for all soil densities or wall rigidities, increasing pile penetration depth does not affect maximum bending moment. Figure 4-b, curve (3), (4) and (5), for all soil densities or wall rigidities increases the pile penetration depth reduces the wall tip horizontal displacement. Most of the reduction induces from the increase of wall rigidity. To reduce excessive  $\delta_i$  it is advised to increase the penetration depth up to 1.5  $D_0$  then increase the rigidity  $I_n$ , then change the structural system.

**Table 3.** Comparison between the three analysis methods

Excavation depth vs. analysis method		Excavation depth <sup>a</sup>							
		h = 3.0 m				h = 4.0-m			
		D <sub>O</sub>	% D <sub>O</sub>	BM	% BM	D <sub>O</sub>	% D <sub>O</sub>	BM	% BM
Conventional method	S1	2.92	1.00	51.29	1.00	3.89	1.00	121.57	1.00
	S2	1.96	1.00	31.75	1.00	2.61	1.00	75.25	1.00
Spring analogy	S1	3.20	1.10	51.25	1.00	4.30	1.10	124.5	1.02
	S2	2.15	1.10	31.75	1.00	2.85	1.10	75.25	1.00
FEA plaxis – 2D	S1	3.20	1.10	27.67	0.54	4.30	1.10	71.52	0.59
	S2	2.30	1.17	16.25	0.51	3.00	1.15	46.80	0.62
Notes		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

*h* and *D<sub>O</sub>* (m), *BM* (kNm/m')

<sup>a</sup>N.B: Case 5, *h* = 5.0-m gives nearly same results, e.g. FEA % D<sub>O</sub> = 1.18 and % BM = 0.68



**Fig. 4.** Hints may enhance the secant pile design.

### 3.2 Rational Empirical Equations

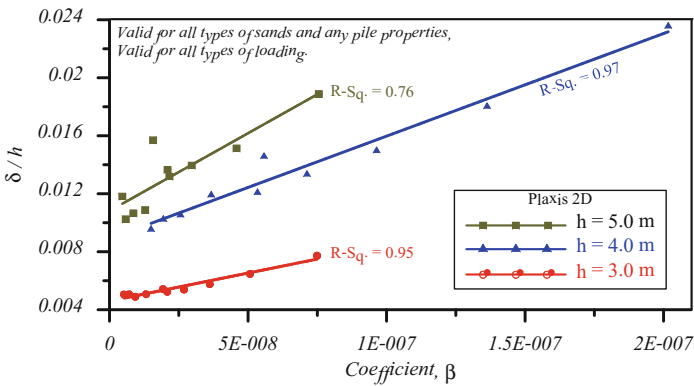
The variables that affect the displacement of the wall tip “ $\delta_i$ ” could be grouped in three categories. The first one will be ignored, here-in, because it is believed its effect is small, as well as its range of variation, i.e. soil and pile Poisson’s ratio.

Second category variables are; soil modulus of elasticity  $E_S$ , pile flexural rigidity  $EI$  and Rankin’s earth pressure coefficient difference  $(k_p - k_a)$ . Increases in the value of these variables reduce the expected  $\delta_i$  i.e. inverse proportionally. Last category consists of only one parameter, the maximum bending moment  $M_{max}$ . Increases in the bending moment increases  $\delta_i$ , i.e. direct proportionally. The effect of soil density ‘ $\gamma$ ’ is indirectly covered by the existence of  $E_S$ ,  $k_p$ ,  $k_a$  and  $M_{max}$ .

Then, the above parameters have been re-arranged in a common fraction denoted as ‘ $\beta$ ’, Eq. (1). Its denominator is second group variables, while the numerator is  $M_{max}$ . It has been assumed, with some confidence, the powers of all variables equal to “1”, which is rational and matches the engineering sense.

$$\beta = \frac{M_{max}}{E_S EI (k_p - k_a)} \tag{1}$$

Consider  $\beta$  is x-axis of and  $\delta/h$  is its y-axis, Fig. 5. For each excavation depths the input data is fed into Plaxis, run by run, and have been used to calculate  $\beta_i$ , and the corresponding output per each run used to calculate  $\delta_i/h$ . Then, Fig. 5 graphs have been plotted containing three curves presenting three excavation depths; linear curve fitting is the best fit. Equations (2), (3) and (4) present each curve trend.



**Fig. 5.** Prediction of wall tip horizontal displacement, first approximation.

Case of  $h = 5.m$ ,  $R-sq = 0.76$

$$\delta/h = 107202.4\beta + 0.010815 \tag{2}$$

Case of  $h = 4.m$ ,  $R-sq = 0.97$

$$\delta/h = 70791.446\beta + 0.00889 \tag{3}$$

Case of  $h = 3.m$ ,  $R-sq = 0.95$

$$\delta/h = 38582.394\beta + 0.0046 \tag{4}$$

**Unit's Key.** The above equations units are:  $M_{max}$  (kN.m/m),  $E_s$  (kN/m<sup>2</sup>),  $EI$  (kN.m<sup>2</sup>/m),  $\beta$  (m/kN),  $\delta$  (m),  $h$  (m), the constants "107202.4 - 70791.446 - 38582.394" (kN/m) while "0.010815 - 0.00889 - 0.0046" (dimensionless).

Those equations present an improvement to the conventional method of design cantilever secant pile walls, as first approximation. The procedures are: (a) calculate penetration depth "D<sub>O</sub>" and  $M_{max}$  based on the conventional method. (b) Increase D<sub>O</sub> by 50% and reduce  $M_{max}$  by 35%. (c) Calculate "β", then pick up the equation matches the excavation depth to get δ. At this point, one can acquire reasonable bending moment, safe penetration depth and approximate wall-tip displacement. It could be used for dry sand densities, pile rigidities and any type of loading because the above equations use  $M_{max}$ , regardless of loads or soil conditions produced it. This upgrade is limited to homogeneous sand. A look back at Eqs. 2, 3 and 4 says minimum 'δ' occurs if 'β' is "zero". This will never happen unless  $M_{max} = 0.0$ , not logical, or 'β' denominator is "infinity". The only reason to accept this infinity is the existence of rigid capping beam and corners struts. The capping beam effect is the running research by me and research assistant. Until then, 'δ' is acceptable if it is ≤ 5.0-cm, 3.5-cm or 1.5-cm for wall heights 5.0-m, 4.0-m, and 3.0-m respectively. That is to say acceptable limit is  $\delta_i \leq 0.8\%$  excavation depth, 'h'.

## 4 Conclusions

Within the limits and assumptions of this study the following facts are concluded:

1. As the wall penetration depth increases, the wall tip displacement 'δ' decreases. Beyond 50D<sub>O</sub>% increase, no weighty 'δ' improvement is noticed.
2. It is suggested to avoid the 20% D<sub>O</sub> depth increases and to replace it by 50%.
3. High flexural rigidity and all cases of dense soils decreases δ.
4. Conventional method and spring analogy underestimate penetration depth and overestimate bending moment. Also, spring analogy gives higher values of 'δ' if all the above compared to finite element analysis.
5. Generally, increases penetration depth, does not change the values of maximum bending moment. But, it reduces wall displacement, effectively.
6. To reduce excessive δ<sub>i</sub> first increase penetration depth up to 50%, and then increase wall rigidity, if neither insufficient, introduce anchors or struts.
7. The acceptable limit of 'δ<sub>i</sub>' is ≤ 0.8% 'h', unless stringent values are imposed by adjacent structure. This 'δ<sub>i</sub>' is the one that calculated by 2D-FEA model.
8. A rational technique has been suggested using empirical equations to upgrade the capability of conventional method to predict wall tip displacement.

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