# Chapter 11
# Intuitionistic Fuzzy Inventory Model with Pre-payment Scheme for Damageable Item

**Puja Supakar, Sanat Kumar Mahato, and Pintu Pal**

**Abstract** The purpose of this chapter is to present a model for maximizing retail profit through an economic order quantity inventory model. First, we derive the deterministic model for the damageable item under the assumption that advance payment is allowed by the distributor to increase the sales of the items in lots. Since every parameter involves some sort of uncertainty, we developed imprecise model and crispified model in commensurate with the deterministic one and represent this impreciseness in terms of linear triangular intuitionistic fuzzy numbers and crispified these with a well-known signed distance method. We solved both the deterministic and crispified models and results are compared for the numerical experiments. To interpret managerial insights, we carried out the sensitivity studies of the controlling parameters for the crispified model.

**Keywords** Pre-payment · Damageable item · Intuitionistic fuzzy number · EOQ model · Crispification · Signed distance method

## 1 Introduction

In the present market scenario, it is commonly believed that large stockpiles of the products that are kept for display in the mall, shopping centre or street market or the amassed stress of the stocked products will attract the customers to buy more. Due to this storage the products break or damage or this can reduce the actual utility after some time period.

Majumder et al. (2000) have established an imperfect production model with time-varying demand for multi-items. Saha et al. (2010) have proposed a stock dependent EOQ models for breakable products. Rahman et al. (2021) have also evolved a stock and price dependent demand for deteriorating EOQ model. Maragatham and

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Gnanvel (2017) have presented a model for purchasing breakable inventory items with allowable delay in payments. Kundu et al. (2018) have discussed a production model for breakable and imperfect products.

The effect of pre-payment financing scheme is observed as an everyday life. Also, it is one of the effective business tools to encourage the direct customers or to confirm the order or to start new business for the small vendors. It is not possible to arrange full amount at the time of order or supply for the beginners or small vendors. The vendor or customer receives fixed percentage of the total amount as pre-payment during the order and the remainder is adjusted after the delivery. It is further noted that the seller offers a certain percentage discount on the total price depending on the amount purchased. The discount rate may be fixed or it is influenced by the amount of pre-payment, the time period when pre-payment is given and many other factors. However, the customers or small vendors who are not able to pay the pre-payment part of the amount immediately, have to take a loan from a bank or some agency with some rate of interest. In this case, if they are benefited after paying the loan interest, then only they choose the pre-payment financing scheme. Gupta et al. (2009) have suggested a pre-payment EOQ model with uniform demand. Priyan et al. (2014) have expanded a pre-payment EOQ model including fuzzy uncertainties. Zhang et al. (2014) have also presented EOQ model incorporating pre-payment financing scheme. For perishable goods, Tsao et al. (2019) have proposed an optimal pricing and ordering policy using the advance-cash-credit financing scheme. Teng et al. (2016) have devised a deteriorating EOQ model under advance payments scheme. Supakar and Mahato (2018) have discussed a fuzzy-stochastic advance payment EOQ model with linear time varying demand. Khan et al. (2020) have evolved a two-warehouse inventory system with pre-payment financing and demand component based on selling price. Khan et al. (2021) have extended the previous research into an EOQ model considering advance and delay in payments.

Many researchers have considered the parameters to be precise, i.e. every parameter is perfectly determinable. However, in real-life situations, due to insufficient information, lack of evidence, fluctuating financial market, weather change, etc. parameters may not be a fixed value always which can be expressed by several aspects such as interval-valued, fuzzy, intuitionistic fuzzy, stochastic, or combination of these.

Saha et al. (2012) have established a breakable EOQ models with imprecise environment. Supakar and Mahato (2020) have established a pentagonal fuzzy pre-payment EPQ model with time varying deterioration and demand. Chakrabortty et al. (2011) have established intuitionistic fuzzy EOQ model and IF programming technique for solution.

## *1.1 Identification of Research Gaps*

After careful literature review, we have summarized the research findings of some renowned researchers related to this work and presented in a table given below. From this table, the research gaps can easily be viewed. Also, the attempts made in this work are shown in this table from which it can be easily distinguished.

| References | Model | Item type | Payment scheme | Demand rate | Uncertainty |
|---|---|---|---|---|---|
| Majumder et al. (2000) | EPQ | Imperfect | – | Time varying | – |
| Chiang et al. (2005) | – | – | – | – | Fuzzy |
| Gupta et al. (2009) | EOQ | – | Advance | Uniform | Interval valued |
| Saha et al. (2010) | EOQ | Breakable | – | Stock dependent | Trapezoidal Fuzzy |
| Chakrabortty et al. (2011) | EOQ | – | – | Uniform | Intuitionistic fuzzy |
| Saha et al. (2012) | EOQ | Breakable | – | Stock and selling price-dependent | Fuzzy cost and resources |
| Priyan et al. (2014) | EOQ | – | Advance | Constant | Trapezoidal and Triangular fuzzy |
| Zhang et al. (2014) | EOQ | – | Advance | | – |
| Teng et al. (2016) | EOQ | Deteriorating | Advance | Constant | – |
| Maragatham and Gnanvel (2017) | Purchasing Inventory Model | Breakable | Permissible delay | Linear time-dependent | – |
| Supakar and Mahato (2018) | EOQ | – | Advance | Linear time-dependent | Fuzzy-Stochastic |
| Kundu et al. (2018) | EPQ | Breakable Item, Imperfect Production | – | Time sensitive | Triangular fuzzy |
| Tsao et al. (2019) | EPQ | Deterioration | Advance | Price-dependent | - |
| Supakar and Mahato (2020) | EPQ | Deteriorating | Advance | Ramp type | Pentagonal fuzzy Parameters |
| Khan et al. (2020) | two-warehouse inventory system | Deterioration | Advance | Selling price-dependent | – |

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| References | Model | Item type | Payment scheme | Demand rate | Uncertainty |
|---|---|---|---|---|---|
| Khan et al. (2021) | EOQ | – | Advance and delay payment | – | – |
| Rahman et al. (2021) | EOQ | Deterioration | Advance | Price and stock-dependent | – |
| This chapter | EOQ | Damageable | Advance | Selling price-dependent | Intuitionistic fuzzy |

## *1.2 Objectives of This Chapter*

The objectives of this work are:

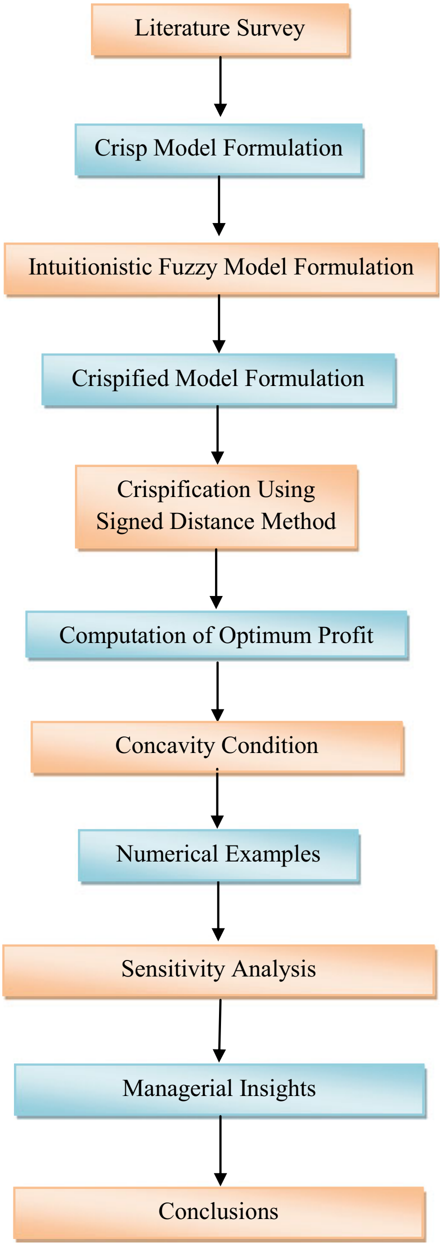
i. to develop realistic models and then find the maximum profit from this model.
ii. to handle the uncertainty from the real data with suitable intuitionistic fuzzy number.
iii. to make the computation easy, use crispified method to convert the intuitionistic fuzzy numbers to a fixed real number.
iv. to present actual market demand, that is not always constant.
v. to consider damageable units in the model, which is a common fact because of the amassed stress of the stocked products kept in heaped or displayed in the mall, supermarket.

## *1.3 Outcomes of the Chapter*

The outcomes of the present work are given below

i. Designing of an EOQ model in crisp and intuitionistic fuzzy environments involving pre-payment scheme, selling price-dependent demand for damageable item.
ii. The output value of the profit function for crispified input data is better than the crisp input data.
iii. To compute intuitionistic fuzzy model is very hard. So we convert the intuitionistic fuzzy model to crispified model using signed distance method.
iv. The profit function is increasing when the parameters $\tilde{t}_c, \tilde{o}_c$ and $n_0$ are increasing and $\tilde{P}_c$ and $\alpha$ are decreasing.
v. The profit function does not behave uniformly with respect to the parameters $\tilde{h}_c$ and $t_1$ for whole cycle length.

**Schematic diagram of proposed study**



# 2 Nomenclatures and Assumptions

## 2.1 Nomenclatures

| Symbol | Meaning | Unit |
|---|---|---|
| $O_c$ | Ordering cost per order | $ |
| $\tilde{O}_c$ | Intuitionistic fuzzy ordering cost per order | $ |
| $d_{avg}(\tilde{O}_c, 0)$ | Crispified ordering cost per order | $ |
| $P_c$ | Purchasing cost per unit item | $ |
| $\tilde{P}_c$ | Intuitionistic fuzzy purchasing cost per unit item | $ |
| $d_{avg}(\tilde{P}_c, 0)$ | Crispified purchasing cost per unit item | $ |
| $t_c$ | Transportation cost per unit item | $ |
| $\tilde{t}_c$ | Intuitionistic fuzzy transportation cost per unit item | $ |
| $d_{avg}(\tilde{t}_c, 0)$ | Crispified transportation cost per unit item | $ |
| $h_c$ | Holding cost per order | $ |
| $\tilde{h}_c$ | Intuitionistic fuzzy holding cost per order | $ |
| $d_{avg}(\tilde{h}_c, 0)$ | Crispified holding cost per order | $ |
| $n_0$ | The mark up of selling price for damaged item | year |
| $I_e$ | Interest charged per annum | $ |
| $\tilde{I}_e$ | Intuitionistic fuzzy interest charged per annum | $ |
| $d_{avg}(\tilde{I}_e, 0)$ | Crispified interest charged per annum | $ |
| $t_1$ | The length of time during which the pre-payments are paid,$0 \le t_1 \le 1$ | Year |
| $t_2$ | The length of time after that system face to damageable item, $t_2 > t_1$ | Year |
| $\alpha$ | Prior to the delivery date, the amount of purchase costs to be prepaid, $0 \le \alpha \le 1$ | Unitless |
| $N$ | The number of equal pre-payments before receiving the order quantity | Unitless |

### Decision variables

| | | |
|---|---|---|
| $S_p$ | Selling price per unit item | *$* |
| $T$ | Length of cycle time | *year* |

### Functions

| | |
|---|---|
| $D(S_p)$ | Annual demand rate depends on unit selling price $S_p$ |
| $Q(t)$ | Inventory level in units at time t |
| $TP(T, S_p)$ | Average of total profit |

(continued)

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| | |
|---|---|
| $B(q)$ | The number of damaged units per unit of time and is based on initial inventory level $q$ |

## 2.2 Assumptions

To formulate the crisp and the intuitionistic fuzzy EOQ model, the following assumptions are under consideration:

i. There is a single item under consideration.
ii. Pre-payment is allowed with $n$ equal instalments.
iii. The products break or damage after some time period. The number of damaged units per unit of time is a function of initial inventory level $q$, i.e. $B(q) = a_1 q^r T,\ a_1$ is a constant.
iv. The demand rate is selling price-dependent, i.e. $D(S_p) = a - bS_p,\ a > bS_p, a$ and $b$ are constants.
v. Intuitionistic fuzzy numbers are used to express the uncertain parameters for the imprecise model.
vi. Transportation cost is applicable.
vii. Time horizon is finite.
viii. Finite numbers of replenishments are allowed.

# 3 Some Important Definitions

## 3.1 Intuitionistic Fuzzy Number (IFN)

An intuitionistic fuzzy number $\tilde{A}$ with the membership function is $\sigma_{\tilde{A}}(x)$ and non-membership function $\delta_{\tilde{A}}(x)$ is:

(i). An intuitionistic fuzzy subset of the real line.
(ii). Normal, i.e., there is a $x_0 \in X$ such that $\sigma_{\tilde{A}}(x_0) = 1$ and $\delta_{\tilde{A}}(x_0) = 0$.
(iii). Convex for the membership function, i.e., for all $x_1, x_2 \in X, \sigma_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min\{\sigma_{\tilde{A}}(x_1), \sigma_{\tilde{A}}(x_2)\}$, where $\lambda \in [0, 1]$.
(iv). Concave for non-membership function $x_1, x_2 \in X, \delta_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \leq \max\{\delta_{\tilde{A}}(x_1), \delta_{\tilde{A}}(x_2)\}$, where $\lambda \in [0, 1]$ (Fig. 1).

## 3.2 Triangular Intuitionistic Fuzzy Number (TIFN)

A triangular intuitionistic fuzzy number (TIFN) $\tilde{A}$ is an intuitionistic fuzzy set in R with following membership function $(\sigma_{\tilde{A}}(x))$ and non-membership function $(\delta_{\tilde{A}}(x))$

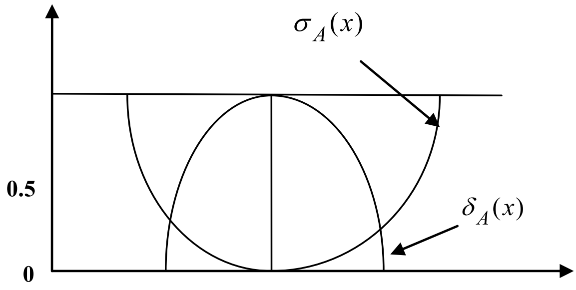


**Fig. 1** Membership and Non-membership functions of IFN

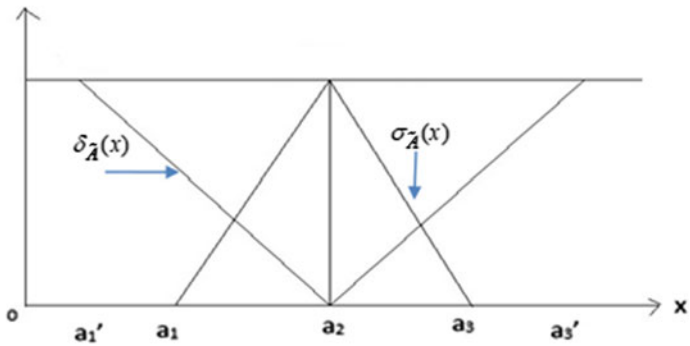


**Fig. 2** Graphical representation of TIFN

$$\sigma_{\tilde{A}}(x) = \left\{ \begin{array}{l} \dfrac{x - a_1}{a_2 - a_1}, \text{ for } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, \text{ for } a_2 \le x \le a_3 \\ 0, \ \text{otherwise} \end{array} \right\} \text{ and } \delta_{\tilde{A}}(x) = \left\{ \begin{array}{l} \dfrac{a_2 - x}{a_2 - a_1'}, \text{ for } a_1' \le x \le a_2 \\ \dfrac{x - a_2}{a_3' - a_2}, \text{ for } a_2 \le x \le a_3' \\ 1, \ \text{otherwise} \end{array} \right\}$$

where, $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$ and $0 \le \sigma_{\tilde{A}}(x), \delta_{\tilde{A}}(x) \le 1$, for $\sigma_{\tilde{A}}(x) = 1 - \delta_{\tilde{A}}(x), \forall x \in R$ (Fig. 2).

This TIFN is denoted by $\tilde{A} = (a_1, a_2, a_3; a_1', a_2, a_3')$.

## 4 Signed Distance Method of Crispification (Chiang et al. 2005)

Suppose $x, 0 \in R$, then $d_0(x, 0) = x$ is stated as the signed distance of $x$ determined from the origin 0. If $x > 0$, the distance between $x$ and 0 is $d_0(x, 0) = x$. Accordingly, if $x < 0$, the distance between $x$ and 0 is $-d_0(x, 0) = -x$. Due to this, $d_0(x, 0) = x$ is defined as the signed distance of $x$ to 0.

Therefore, the signed distance of $A_1(\alpha)$ and $A_2(\alpha)$ determined from 0 are $d_0(A_1(\alpha), 0) = A_1(\alpha)$ and $d_0(A_2(\alpha), 0) = A_2(\alpha)$, respectively.

Similarly, signed distance of $A'_1(\beta)$ and $A'_2(\beta)$ determined from 0 are $d_0(A'_1(\beta), 0) = A'_1(\beta)$ and $d_0(A'_2(\beta), 0) = A'_2(\beta)$.

Thus, the signed distance of the interval $[A_1(\alpha), A_2(\alpha)]$ determined from origin 0 is given by

$$d_0([A_1(\alpha), A_2(\alpha)], 0) = \frac{1}{2}[d_0(A_1(\alpha), 0) + d_0(A_2(\alpha), 0)] = \frac{1}{2}[A_1(\alpha) + A_2(\alpha)].$$

Similarly, the signed distance of the interval $[A'_1(\beta), A'_2(\beta)]$ determined from origin 0 is given by

$$d_0([A'_1(\beta), A'_2(\beta)], 0) = \frac{1}{2}[d_0(A'_1(\beta), 0) + d_0(A'_2(\beta), 0)] = \frac{1}{2}[A'_1(\beta) + A'_2(\beta)].$$

Signed distance of the TIFN $\tilde{A} = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ determined from origin 0 with respect to the membership function is

$$\begin{aligned} d_\sigma(A, 0) &= \frac{1}{2}\int_0^1 [A_1(\alpha) + A_2(\alpha)]d\alpha \\ &= \frac{1}{2}\int_0^1 a_1 + \alpha(a_2 - a_1) + a_3 - \alpha(a_3 - a_2)d\alpha \\ &= \frac{1}{4}(a_1 + 2a_2 + a_3) \end{aligned}$$

Again, the signed distance of the TIFN $\tilde{\mathrm{A}}$ determined from origin 0 with respect to the non-membership function is

$$\begin{aligned} d_\delta(A, 0) &= \frac{1}{2}\int_0^1 [A'_1(\beta) + A'_2(\beta)]d\beta \\ &= \frac{1}{2}\int_0^1 a_2 - \beta(a_2 - a'_1) + a_2 + \beta(a'_3 - a_2)d\beta \\ &= \frac{1}{4}(a'_1 + 2a_2 + a'_3) \end{aligned}$$

Therefore, the average signed distance of the TIFN $\tilde{A}$ measured from origin 0 is given by

$$d_{\text{avg}}(A, 0) = \frac{d_\sigma(A, 0) + d_\delta(A, 0)}{2} = \frac{1}{8}(a_1 + 4a_2 + a_3 + a'_1 + a'_3).$$

# 5 Model Formulation

A single item EOQ model has been developed for damageable item in crisp and intuitionistic fuzzy environments considering allowable pre-payment and selling price-dependent demand. To make the computation easier we have converted the intuitionistic fuzzy model by using very well-known signed distance method.

## *5.1 Crisp Model*

Let us assume that there is no breaking/damage within the period $0 \leq t \leq t_2$. After, that damage is started and continued throughout the whole cycle. If $Q(t)$ indicates the inventory level at any instant t, thus the EOQ model can be represented by the differential equation stated hereunder:

$$\begin{aligned} \frac{dQ(t)}{dt} &= -D(S_p),\ 0 \leq t \leq t_2 \\ \frac{dQ(t)}{dt} &= -D(S_p) - B(q),\ t_2 \leq t \leq T \end{aligned} \tag{1}$$

with the boundary conditions $Q(T) = 0,\ Q(0) = q$.

By solving the differential Eq. (1), we can obtain,

$$Q(t) = \begin{Bmatrix} q - D(S_p)t, 0 \leq t \leq t_2 \\ (D(S_p) + B(q))(T - t),\ t_2 \leq t \leq T \end{Bmatrix} \tag{2}$$

Since $Q(t)$ is continuous at $t = t_2$, utilizing the result of Eq. (2) we get,

$$t_2 = \frac{(D(S_p) + B(q))T - q}{B(q)} \tag{3}$$

Each cycle of profit is comprised of the following components: ordering cost, purchasing cost, holding cost, selling price of fresh item, selling price of for damageable item and interest charged for pre-payment, transportation cost. Evaluation of components is done in the following way:

(a) Total ordering cost for the cycle is: $OC = O_c$
(b) Total purchasing cost for the replenishment cycle is: $A_c = P_c q$.
(c) Total holding cost for the cycle length is:

$$H_c = h_c\left(\int_0^{t_2} Q(t)dt + \int_{t_2}^{T} Q(t)dt\right)$$
$$= h_c\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p)+B(q))(T-t_2)^2}{2}\right)$$

(d) Total selling price:
$$SP = S_p\left(\int_0^{t_2} Q(t)dt + \int_{t_2}^{T} Q(t)dt\right)$$
$$= S_p\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p)+B(q))(T-t_2)^2}{2}\right).$$

(e) Total number of damageable units is:
$\int_{t_2}^{T} B(q)dt = B(q)(T - t_2)$.
Selling price for each damaged item is: $n_0P_c$, $0 \le n_0 \le 1$.
Total selling price for damageable item is: $D_c = B(q)(T - t_2)n_0P_c$.

(f) Total transportation cost for the cycle length: $TC = t_cq$.

(g) Fig. 3 shows the interest that is assessed for pre-payment for the cycle as follows:

$$IC = I_e\left(\frac{\alpha A_c}{n}\left(\frac{t_1}{n}\right)(1+2+3+...+n)\right) = \frac{I_e\alpha t_1(1+n)}{2n}A_c = \frac{P_cqI_e\alpha t_1(1+n)}{2n}$$

From the above arguments, the annual total average profit incurred to the retailer is:

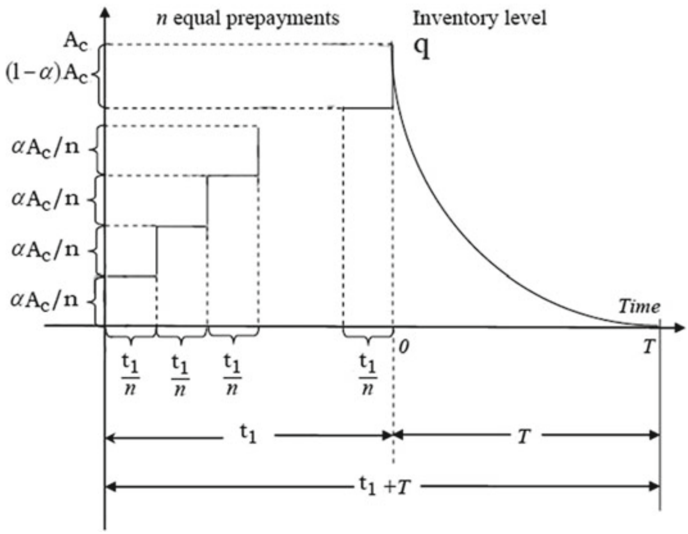


**Fig. 3** Diagrammatic representation of the model

$$TP(T, S_p) = \frac{SP + D_c - OC - A_c - H_c - TC - IC}{T}$$
$$= \frac{S_p}{T}\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p) + B(q))(T - t_2)^2}{2}\right) + \frac{B(q)(T - t_2)n_0P_c}{T} - \frac{O_c}{T} - \frac{P_cq}{T} - \tag{4}$$
$$\frac{h_c}{T}\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p) + B(q))(T - t_2)^2}{2}\right) - \frac{t_cq}{T} - \frac{I_e\alpha t_1(1 + n)}{2n}\frac{P_cq}{T}.$$

## 5.2 Concavity Analysis

From the Fig. 4 it is clearly visible that the objectives function, i.e. total average profit is a concave function of selling price and cycle length. Hence, we can have unique global optimum of total average profit.

To find $T^*$, for any particular $S_p > 0$, calculating the 1st order partial derivative of $TP(T, S_p)$, w.r.t $T$, putting the result at zero gives us,

$$\frac{\partial TP}{\partial T} = -\frac{1}{T^2}\left[\begin{array}{l}(S_p - h_c)\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p) + B(q))(T - t_2)^2}{2}\right) + \\ B(q)(T - t_2)n_0P_c - O_c - P_cq - t_cq - \frac{I_e\alpha t_1(1 + n)P_cq}{2n}\end{array}\right] +$$
$$\frac{1}{T}\left[\begin{array}{l}(S_p - h_c)\left(qt_2' - D(S_p)t_2t_2' - (D(S_p) + B(q))(T - t_2)(1 - t_2') - B'(q)(T - t_2)^2\right) \\ +B(q)(1 - t_2')n_0P_c + B'(q)(T - t_2)n_0P_c\end{array}\right] = 0$$

Additionally, by calculating 1st order partial derivative of $TP(T, S_p)$ w.r.t. $S_p$, for any particular T and putting the result at zero gives us,

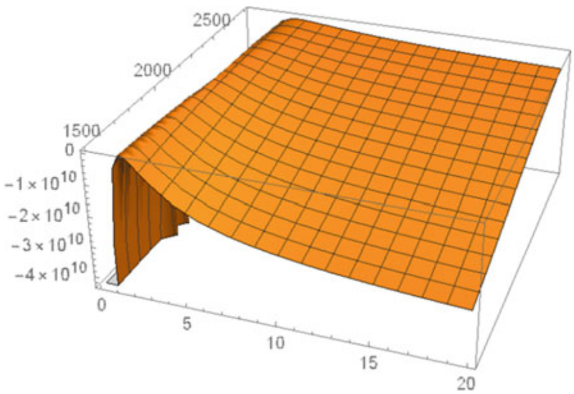


**Fig. 4** Profit graph of crisp model

$$\frac{\partial TP}{\partial S_p} = \frac{1}{T}\left[\begin{array}{l}\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p)+B(q))(T-t_2)^2}{2}\right) + S_p\left(-\frac{D'(S_p)t_2^2}{2} - \frac{D'(S_p)(T-t_2)^2}{2}\right) + \\ h_c\left(-\frac{D'(S_p)t_2^2}{2} - \frac{D'(S_p)(T-t_2)^2}{2}\right)\end{array}\right] = 0$$

$$\begin{aligned} f_1(T) &= S_p\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p)+B(q))(T-t_2)^2}{2}\right) + B(q)(T-t_2)n_0P_c - O_c - P_cq- \\ &h_c\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p)+B(q))(T-t_2)^2}{2}\right) - t_cq - \frac{I_e\alpha t_1(1+n)P_cq}{2n} \end{aligned} \tag{5}$$

$$g_1(T) = T \tag{6}$$

Considering the 1st order and 2nd order derivatives of $f_1(T)$ w.r.t $T$,

$$\begin{aligned} f_1'(T) &= (S_p - h_c)\left(qt_2' - D(S_p)t_2t_2' - (D(S_p)+B(q))(T-t_2)(1-t_2') - B'(q)(T-t_2)^2\right) \\ &+B(q)(1-t_2')n_0P_c + B'(q)(T-t_2)n_0P_c \end{aligned} \tag{7}$$

$$\begin{aligned} f\prime_2(T) &= (S_p - h_c)\begin{pmatrix} qt\prime_2 - D(S_p)(t'^2_2 + Tt\prime_2) + B(q)(T-t_2)t\prime_2 \\ +(B(q)+B'(q))(T-t_2)(1-t'_2) \end{pmatrix} - \\ &B(q)t\prime_2 n_0P_c + 2B'(q)(1-t'_2)n_0P_c \\ &= (S_p - h_c)\left(qt\prime_2 - D(S_p)(t'^2_2 + Tt\prime_2) + (B(q)+B'(q))(T-t_2)(1-t'_2)\right) + \\ &B(q)\left((S_p-h_c)(T-t_2) - n_0P_c\right)t\prime_2 + 2B'(q)(1-t'_2)n_0P_c \end{aligned} \tag{8}$$

Here,

$$t_2 = \frac{\left(D\left(S_p\right)+B(q)\right)T - q}{B(q)} = \frac{D\left(S_p\right)}{a_1q^r} + T - \frac{q}{a_1q^rT} \tag{9}$$

$$1 - t'_2 = -\frac{q}{a_1q^rT^2} < 0, t\prime\prime_2 = -\frac{2q}{a_1q^rT^3} < 0, (t'^2_2 + Tt\prime\prime_2) = 1 + \frac{q^2}{a_1q^rT^4} > 0,\ T > t_2 \tag{10}$$

It can be shown after rearranging the terms that $f_1''(T) < 0$ for suitable choice of parameters, which has been experimented numerically. Therefore, $TP(T, S_p) = \frac{f_1(T)}{g_1(T)}$ is a strictly pseudo-concave in nature w.r.t $T$.

## 5.3 Optimality Analysis

For any fixed T, taking the 2nd order partial derivatives of $TP(S_p, T)$ w.r.t. $S_p$, we get

$$\frac{\partial^2 TP}{\partial S_p^2} = \frac{1}{T}\left[\left(\frac{bt_2^2}{2} - \frac{(-b + B(q))(T - t_2)^2}{2}\right) + (S_p - h_c)\left(\frac{bt_2^2}{2} + \frac{b(T - t_2)^2}{2}\right)\right] \tag{11}$$

For any fixed $S_p$, taking the 2nd order partial derivatives of $TP(S_p, T)$ w.r.t. T, we get

$$\begin{aligned}\frac{\partial^2 TP}{\partial T^2} = \frac{2}{T^3}&\left[\begin{array}{l}(S_p - h_c)\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p) + B(q))(T - t_2)^2}{2}\right) + \\ B(q)(T - t_2)n_0P_c - O_c - P_cq - t_cq - \frac{I_e\alpha t_1(1 + n)P_cq}{2n}\end{array}\right] + \\ \frac{1}{T}&\left[\begin{array}{l}(S_p - h_c)\big(qt''_2 - D(S_p)(t'^2_2 + Tt''_2) + (B(q) + B'(q))(T - t_2)(1 - t'_2)\big) + \\ B(q)\big((S_p - h_c)(T - t_2) - n_0P_c\big)t''_2 + 2B'(q)(1 - t'_2)n_0P_c\end{array}\right] - \\ \frac{1}{T^2}&\left[\begin{array}{l}(S_p - h_c)\big(qt'_2 - D(S_p)t_2t'_2 - (D(S_p) + B(q))(T - t_2)(1 - t'_2) - B'(q)(T - t_2)^2\big) \\ +B(q)(1 - t'_2)n_0P_c + B'(q)(T - t_2)n_0P_c\end{array}\right]\end{aligned} \tag{12}$$

Again,

$$\begin{aligned}\frac{\partial^2 TP}{dS_pdT} = -\frac{1}{T^2}&\left[\begin{array}{l}\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p) + B(q))(T - t_2)^2}{2}\right) + \\ S_p\left(-\frac{D'(S_p)t_2^2}{2} - \frac{D'(S_p)(T - t_2)^2}{2}\right) - \\ h_c\left(-\frac{D'(S_p)t_2^2}{2} - \frac{D'(S_p)(T - t_2)^2}{2}\right)\end{array}\right] + \\ \frac{1}{T}&\left[\begin{array}{l}\left(qt'_2 - D(S_p)t_2t'_2 - (D(S_p) + B(q))(T - t_2)(1 - t'_2) - \frac{B'(q)(T - t_2)^2}{2}\right) \\ +(S_p - h_c)\big(-D'(S_p)t_2t'_2 - D'(S_p)(T - t_2)(1 - t'_2)\big)\end{array}\right]\end{aligned} \tag{13}$$

Now, computing the value below we have,

$$\frac{\partial^2 TP}{\partial S_p^2}\frac{\partial^2 TP}{\partial T^2} - \left(\frac{\partial^2 TP}{\partial T\partial S_p}\right)^2 = 9.44387 \times 10^9 > 0,\ \frac{\partial^2 TP}{\partial T^2} = -2.91676 \times 10^9 < 0. \tag{14}$$

The condition of maximum holds for the objective function.

### 5.4 Intuitionistic Fuzzy Model

Considering the parameters $\tilde{P}_c$, $\tilde{t}_c$, $\tilde{h}_c$, $\tilde{O}_c$, $\tilde{I}_e$ as the triangular intuitionistic fuzzy numbers, the annual total intuitionistic fuzzy average profit incurred to the retailer is:

$$\begin{aligned}\tilde{T}P(T, S_p) = \frac{S_p}{T}\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p) + B(q))(T - t_2)^2}{2}\right) + \frac{B(q)(T - t_2)n_0\tilde{P}_c}{T} - \\ \frac{\tilde{O}_c}{T} - \frac{\tilde{P}_c q}{T} - \frac{\tilde{h}_c}{T}\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p) + B(q))(T - t_2)^2}{2}\right) - \tilde{t}_c q - \frac{\tilde{I}_e \alpha t_1(1 + n)}{2n}\frac{\tilde{P}_c q}{T}\end{aligned} \tag{15}$$

### 5.5 Crispified Model

Now, in order to crispify the intuitionistic fuzzy model given in (15), we used the method described in Sect. 4. The annual total crispified average profit incurred at the retailer is:

$$\begin{aligned}d_{avg}(\tilde{T}P(T, S_p), 0) = \frac{S_p}{T}\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p) + B(q))(T - t_2)^2}{2}\right) + \frac{B(q)(T - t_2)n_0 d_{avg}(\tilde{P}_c, 0)}{T} \\ - \frac{d_{avg}(\tilde{O}_c, 0)}{T} - \frac{\tilde{P}_c q}{T} - \frac{d_{avg}(\tilde{h}_c, 0)}{T}\left(qt_2 - \frac{D(S_p)t_2^2}{2} - \frac{(D(S_p) + B(q))(T - t_2)^2}{2}\right) \\ - d_{avg}(\tilde{t}_c, 0)q - \frac{d_{avg}(\tilde{I}_e, 0)\alpha t_1(1 + n)}{2n}\frac{d_{avg}(\tilde{P}_c, 0)q}{T}\end{aligned} \tag{16}$$

## 6 Numerical Examples and Result Analysis

### 6.1 Numerical Examples

Our aim in this section is to demonstrate theoretical results numerically using Mathematica and obtain the global optimal solution. We have formed two numerical examples corresponding to the crisp and intuitionistic fuzzy environments and are given in Example 1 and Example 2 (Table 1).

**Table 1** Intuitionistic fuzzy input data

| Parameters | Values | Crispified values |
|---|---|---|
| $\tilde{P}_c$ | (1200,1400,1600; 1300,1400,1700) | 1425 |
| $\tilde{t}_c$ | (0.05,0.25,0.30; 0.10,0.25,0.4) | 0.2300 |
| $\tilde{h}_c$ | (0.10,0.12,0.14; 0.11,0.12,0.15) | 0.1225 |
| $\tilde{o}_c$ | (11,13,14; 12,13,15) | 13.000 |
| $\tilde{I}_e$ | (0.01,0.025,0.035; 0.015,0.025,0.04) | 0.02500 |

**Table 2** Output data

| Model | $T^*$ | $S_p^*$ | $TP^*$ |
|---|---|---|---|
| Crisp | 2.24419 | 2218.10 | 43,536.30 |
| Intuitionistic Fuzzy | 2.29722 | 2259.77 | 50,179.40 |

**Example 1**: Crisp model

$$O_c = \$10/\text{order},\ P_c = \$1500/\text{unit},\ h_c = \$0.1/\text{unit/year},\ I_e = 0.02, a = 100,\ b = 0.025,\ \alpha = 0.04,$$
$$n = 10,\ q = 100\ \text{unit},\ n_0 = 0.02\ \text{year}, r = 0.02, a_1 = 0.10, t_1 = 0.5\ \text{year}, t_c = \$0.2/\text{unit/year}.$$

**Example 2**: Intuitionistic Fuzzy Model and Crispified Model

## *6.2 Result Analysis*

From Table 3 we can see that:

i. $TP^*$ is a decreasing function of $\tilde{P}_c$ and $\alpha$ but increasing function of $\tilde{t}_c$, $\tilde{o}_c$ and $n_0$.
ii. $T^*$ is an increasing function of $n_0$, $\tilde{t}_c$ and $\tilde{o}_c$.
iii. $S_p^*$ is an increasing function of $\tilde{t}_c$, $\tilde{o}_c$.
iv. When $\tilde{h}_c$ changes from 0.10 to 0.12, $T^*$, $S_p^*$ and $TP^*$ all are increasing, whereas, when $\tilde{h}_c$ changes from 0.14 to 0.16 they are decreasing.
v. When $t_1$ changes from 0.3 to 0.4 and 0.6 to 0.7, $T^*$, $S_p^*$ and $TP^*$ all are decreasing, whereas, when $t_1$ changes from 0.4 to 0.6 they are increasing.
vi. When $\tilde{P}_c$ changes from 1350 to 1450, $T^*$ and $S_p^*$ are decreasing but when it changes from 1450 to 1500 they are increasing.
vii. When $\alpha$ changes from 0.01 to 0.03, $T^*$ increases but when it changes from 0.03 to 0.07, $T^*$ decreases.

**Table 3** Sensitivity analysis on parameters of crispified model

| Parameters | Parameters change | $T^*$ | $S_p^*$ | $TP^*$ |
|---|---|---|---|---|
| $\tilde{P}_c$ | 1350 | 2.39447 | 2329.97 | 59,617.70 |
| | 1400 | 2.21551 | 2194.84 | 46,327.20 |
| | 1450 | 2.20382 | 2185.42 | 43,000.60 |
| | 1500 | 2.20556 | 2186.85 | 40,834.60 |
| $\tilde{t}_c$ | 0.21 | 2.19428 | 2177.38 | 43,699.60 |
| | 0.22 | 2.22948 | 2206.12 | 46,214.80 |
| | 0.24 | 2.24263 | 2216.90 | 46,659.10 |
| | 0.25 | 2.34362 | 2294.20 | 51,905.10 |
| $\tilde{h}_c$ | 0.10 | 2.25689 | 2228.17 | 47,650.00 |
| | 0.12 | 2.28051 | 2246.62 | 49,004.90 |
| | 0.14 | 2.24817 | 2220.95 | 47,566.30 |
| | 0.16 | 2.23487 | 2210.50 | 46,508.80 |
| $\tilde{o}_c$ | 11 | 2.19428 | 2177.38 | 43,699.60 |
| | 12 | 2.22948 | 2206.12 | 46,514.10 |
| | 14 | 2.24263 | 2216.90 | 46,659.10 |
| | 15 | 2.34362 | 2294.20 | 51,905.10 |
| $\alpha$ | 0.01 | 2.25819 | 2228.85 | 48,253.10 |
| | 0.03 | 2.27256 | 2210.50 | 48,139.50 |
| | 0.05 | 2.24238 | 2216.78 | 46,456.10 |
| | 0.07 | 2.2298 | 2206.60 | 45,869.20 |
| $n_0$ | 6 | 2.18896 | 2172.73 | 43,485.60 |
| | 7 | 2.21585 | 2195.35 | 44,811.60 |
| | 8 | 2.26192 | 2231.94 | 48,301.10 |
| | 9 | 2.35121 | 2299.30 | 53,679.30 |
| $t_1$ | 0.3 | 2.29436 | 2257.37 | 496,430.80 |
| | 0.4 | 2.20405 | 2185.49 | 44,346.80 |
| | 0.6 | 2.38139 | 2321.31 | 54,110.10 |
| | 0.7 | 2.20550 | 2187.04 | 43,630.03 |

viii. When $\alpha$ changes from 0.01 to 0.03 and 0.05 to 0.07, $S_p^*$ decreases but when it changes from 0.03 to 0.05, $S_p^*$ increases.

ix. When $n_0$ changes from 6 to 7 and 8 to 9, $S_p^*$ increases but when it changes from 7 to 8, $S_p^*$ decreases.

## 7 Managerial Insights

In this intuitionistic fuzzy EOQ model, we have incorporated a pre-payment financing scheme with equal instalments based on the selling price-dependent demand rate and initial stock dependent damageable units. Cycle length and selling price play a significant role in decision-making. Here, we are primarily focused on maximizing the average profit. By observing the several characteristics stated below, the average profit can be increased in various companies. Hence, a manager can easily include some essential components for optimizing his investments to earn maximum profit.

(i) The demand rate is considered as selling price-dependent, it is more realistic rather than taking it as constant. The manager will identify the actual profit by taking the selling price-dependent demand rate.

(ii) Here, we considered that the products break or damage after some time period. In real-life situation large stocks of the products displayed in the mall, supermarket, or street market to attract the customers. This type of storage can reduce the actual utility, glamour, and brightness of the products after some time period.

(iii) The manager can commence the pre-payment policy to recover any financial crisis.

(iv) To handle the impreciseness of the parameters, the organization can adopt the parameters as fuzzy, intuitionistic fuzzy, type-2 fuzzy, neutrosophic fuzzy, stochastic or combination of these.

(v) The manager can reduce the parameters $\tilde{P}_c$ and $\alpha$ and increase the parameters $\tilde{t}_c$, $\tilde{o}_c$ and $n_0$ to reach the goal to achieve maximum profit of the inventory system keeping other parameters same.

## 8 Conclusions and Future Scopes

The purpose of this chapter is to examine an inventory model that includes initial stock levels dependent damageable units with the policy of pre-payment with $n$ equal instalments before receiving the order. Demand rates are based on selling prices, and damageability rates are based on the initial stock level to go through real-life scenarios. We first design the crisp model, followed by formulating its intuitionistic fuzzy counterpart with the intuitionistic parameter values. Our crispified model then allowed us to reduce computation time by converting intuitionistic fuzzy parameters to crisp values using the signed distance method. After solving the crisp and crispified model, the sensitivities of the parameters are tabulated. Additionally, we have produced managerial insights of our work so that the business managers can easily recognize the business benefits from our studies. Researchers can apply the concepts discussed here to other inventory models and can see many more prospects for future work in inventory modelling. During the analysis and to find out the solutions, we have used Mathematica 11.2. Nevertheless, one can also use some soft computing algorithms, such as the GA, PSO, ACO, ABC, or an extension of these algorithms, or MATLAB, etc. By using real data, researchers can manage the impreciseness of parameters in stochastic, neutrosophic fuzzy, type-2 fuzzy or any other fuzzy number.

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