Chapter 5
Electron Energy Distribution Functions in Ionized Gases: An Engineering Point of View

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Abstract Electron energy distribution functions in low degree ionized gases are of fundamental importance in understanding ionization phenomena in gases and provide a link between theory and measurements. The paper provides a brief historical review of the present state of study in this area, and includes an update of both experimental study of transport phenomena and calculation of the distribution functions. Recent researches in to application of gas discharges in several different areas are briefly included. The need for an analytical expression of the distribution function is pointed out. With this objective a bimodal distribution function is developed and in previous publications its applicability to argon, neon, and xenon have been shown to be successful. In this paper the bimodal distribution is applied to Penning mixtures of neon and argon, and good agreement is obtained between the measured and calculated first ionization coefficients as a function of reduced electric field. Other swarm parameters calculated by this method are also presented for the first time.

5.1 Introduction

Applications underlying the principle of gaseous plasma are many from the common tube lights, sodium vapor lamps, Plasma spray technology developed during 1950s [1], Plasma cutting of metals, and more recent cutting of concrete [2]. Recent applications of cold plasma are in the medical and bio-medical field [3–8].

Computation of electron distribution function as a function of reduced electric field E/N (E = Electric field, N = gas number density) involves various collision cross sections as a function of electron energy. The swarm parameters are the drift velocity (W), diffusion coefficient (D), the characteristic energy D/μ (μ = mobility) and the reduced first ionization coefficient (α/N). The inter-relationships between the cross sections, swarm parameters and electron energy distribution functions are shown in Fig. 5.1.

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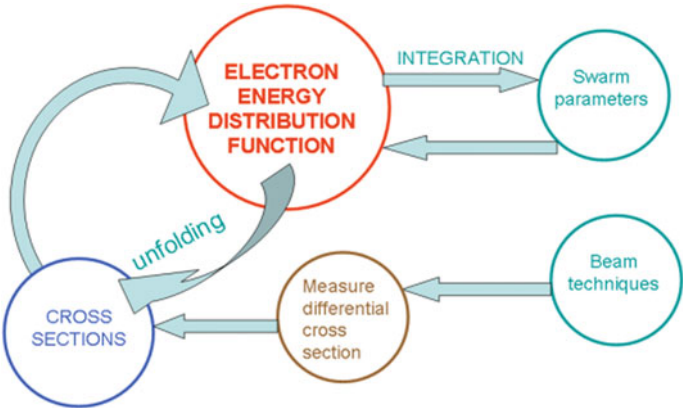

Fig. 5.1 Integration and unfolding procedures with the aid of electron energy distribution functions

The purposes of this paper are:

1. To present a brief review of the success obtained by the use of Boltzmann equation.
2. To present a new method of obtaining analytical expression for energy distribution function and demonstrate its applicability to Penning mixtures.
3. Future challenges.

5.2 Theory

The electron distribution function is calculated using the following methods:

1. Solution of continuity equations. The generation and decay of electrons are calculated using the idea that change of charge in a volume is in accordance with the change of electric current flowing in or out of the volume. The charge particles namely electrons, positive ions, and negative ion densities are represented by partial differential equations with time as the variable. The continuity equations are coupled to Poisson's equation via charge density and the current density is calculated by the equation:

$$J = e\left(N_p W_p - N_n W_n - N_e W_e\right) \tag{5.1}$$

Subscripts e, p, and n represent electrons, positive ions and negative ions respectively, N the number density (m^{-3}) and W the drift velocity (m/s). Particular success has been achieved in SF_6 and other gases, both in one dimensional and three dimensional analysis [9].

2. Monte Carlo Method.

The Boltzmann equation which, in concise form, is stated as

$$\frac{d}{d\varepsilon}\left(\frac{e^2E^2\varepsilon}{3NQ_M}\frac{df}{d\varepsilon}\right)+\frac{2m}{M}\frac{d}{d\varepsilon}\left(\varepsilon^2NQ_M\left[f+kT\frac{df}{d\varepsilon}\right]\right)$$
$$+\sum_j\left[(\varepsilon+\varepsilon_j)f(\varepsilon+\varepsilon_j)NQ_j(\varepsilon+\varepsilon_j)-\varepsilon f(\varepsilon)NQ_j(\varepsilon)\right]$$
$$+\sum_j\left[(\varepsilon-\varepsilon_j)f(\varepsilon-\varepsilon_j)NQ_{-j}(\varepsilon-\varepsilon_j)-\varepsilon f(\varepsilon)NQ_{-j}(\varepsilon)\right]=0 \tag{5.2}$$

Here E = Electric field (V/m), e/m and M have their conventional meaning, Q_M is the momentum transfer cross section (m^2), Q is the cross section, subscripts $-j$ and j are the short form of energy levels, $\varepsilon = -j + \Delta j$, and *f(ε)* is the electron energy distribution.

The equation is the mathematical expression for the statement that the energy gain from the electric field is equal to the energy loss during both elastic and inelastic collisions. Integration of (5.2) results in a second equation having both differentials and integrals, called integro-differential equation. Closed form solution is not possible and numerical techniques are applied to find the solution. Frost and Phelps, in their epochal paper [10] developed the solution using main frame computing facilities at Westinghouse laboratories. It is appropriate to mention that experimental measurements for measuring low energy inelastic collision cross sections were not developed yet, then, and the method adopted was to calculate the transport coefficients from the electron energy distribution functions and compare them with the measured values. Assumed vibrational and rotational excitation cross sections were modified by a technique known as backward prolongation [11]. The choice of gases studied was dictated by the fact that that the transport coefficients at low values of E/N were available in 1962.

The swarm parameters are defined by the following relationships:

$$W=-\frac{E}{3N}\left(\frac{2e}{m}\right)^{1/2}\int_0^\infty\frac{\varepsilon}{Q_M(\varepsilon)}\frac{d}{d\varepsilon}\left[\frac{F(\varepsilon}{\varepsilon^{1/2}}\right]d\varepsilon \tag{5.3}$$

$$D_T=\frac{1}{3N}\int_0^\infty\frac{\varepsilon}{Q_M(\varepsilon)}F(\varepsilon)\,d\varepsilon \tag{5.4}$$

$$\frac{\alpha}{N}=\frac{1}{W}\int_{\varepsilon_i}^\infty\varepsilon^{1/2}Q_i(\varepsilon)F(\varepsilon)\,d\varepsilon \tag{5.5}$$

The mean energy of the swarm is given by

$$\overline{\varepsilon} = \frac{\int_0^\infty \varepsilon F(\varepsilon) d\varepsilon}{\int_0^\infty F(\varepsilon) d\varepsilon} \tag{5.6}$$

In (5.3)–(5.6) the symbols are defined as follows:

D_T = radial diffusion coefficient (m^2/s); e/m = electron charge to mass ratio; *j* = specific inelastic process, k_j = rate constant for jth process; (m^3/s); $Q_{i=}$ ionization cross section (m^2); ε = electron energy, ε_i = ionization threshold energy; ε_j = threshold energy for jth process. The distribution is normalized according to

$$\int_0^\infty F(\varepsilon)\, d\varepsilon = 1 \tag{5.7}$$

Substitution of determined electron energy distribution function (EEDF) into (5.3)–(5.7) yields the respective quantity. This operation is identified with the integration part shown in Fig. 5.1. A large number of publications in the literature show very good correspondence between the theoretically calculated swarm parameters and experimentally determined ones [10].

The detailed collision cross section data for calculation of swarm parameters are not necessary for practical engineering purposes where one can tolerate fewer rigors in accuracy for the benefit of speedier calculation, without the knowledge of extensive physics and computational techniques. The early assumptions that the inelastic collisions can be neglected in finding the solution of (5.2) in closed form, given by Morse, Allis and Lamar [9] results in a unimodal distribution function and is found not satisfactory enough.

The starting point for the simplified approach is the derivation of Morse, Allis and Lamar [9] who derived an analytical equation to the electron energy distribution according to:

$$F(\varepsilon) = G \frac{\varepsilon^{1/2}}{\overline{\varepsilon}^{3/2}} \exp\left[-H\left(\frac{\varepsilon}{\overline{\varepsilon}}\right)^{2(p+1)}\right] \tag{5.8}$$

where $\overline{\varepsilon}$ is the mean energy, *p* is a constant having value of $-1 <$ p and no upper limit seems to exist. G and H are functions given by:

$$G = 2(p+1)\left[\Gamma\frac{5}{4(p+1)}\right]^{3/2} \times \left[\Gamma\frac{3}{4(p+1)}\right]^{-5/2} \tag{5.9}$$

$$H = \left[\Gamma \frac{5}{4(p+1)} \times \frac{1}{\Gamma \frac{3}{4(p+1)}} \right]^{2(p+1)} \tag{5.10}$$

Raju and Hackam [12] have attempted to calculate drift velocity and ionization coefficients by substituting (5.3) and (5.5) into (5.8). For the purpose of integration they approximated the respective cross sections as a function of energy by analytical expressions. In this paper we have used numerical methods to overcome these restrictions and extend the theory to bimodal distribution to fully explore the potential of (5.8).

A bimodal distribution has been shown to be applicable in radio-frequency discharges [13] according to

$$G(\varepsilon) = \gamma F_1(\varepsilon) + (1-\gamma) F_2(\varepsilon) \tag{5.11}$$

In such a discharge electrons are accelerated to high energies due to collisions at the sheath, and this can produce a species of electrons which has considerably higher energy than the bulk component. The bi-modal energy distribution can maintain a discharge at a much lower bulk temperature due to ionization and excitation produced by the warmer component of the plasma [15].

5.3 Application to Penning Gas Mixtures

Penning gas mixtures are mixtures of two gases A and B in which A has a higher ionization potential than B and an excited level of A that is higher than the ionization potential of B exists. When collisions occur between excited level of A and neutral species of B ionization of the latter occurs with a disproportionate increase of ionization of A. This effect, named after the founder during 1930's, is advantageously used where an increase of ionization in gas is desired as in the case of radiation monitoring. Sakai et al. [14, 15] have calculated the electron swarm properties of this mixture where the ratio of argon in the total concentration varied from 10^{-6} to 10^{-1}. Numerical solution of Boltzmann equation was employed to determine the electron energy distributions.

We have calculated, for the first time the swarm coefficients in this mixture using the bimodal distribution function given by (5.11).

To calculate swarm parameters defined by (5.3)–(5.6) we need just the momentum transfer cross sections and ionization cross sections for the two gases under consideration. Argon exhibits pronounced Townsend Ramsauer effect where as neon exhibits slowly increasing momentum transfer cross sections.

The results for argon, neon and xenon using bimodal distribution functions are given elsewhere [16] and the results for neon are checked in this paper [17] before considering Penning mixtures.

Figure 5.2 shows the calculated distribution functions in mixtures of neon and 0.0001% argon calculated using the bimodal distribution according to (5.11). Figure 5.3 appertains to lower values of the reduced electric field, E/N and Fig. 5.4 appertains to higher values. In these figures the curves are shifted to the right successively by two divisions, for purposes of clarity. The curves are neither Maxwellian nor Druyvesteyn distributions. In the region of ionization the tail of the distribution varies with energy according to a power law.

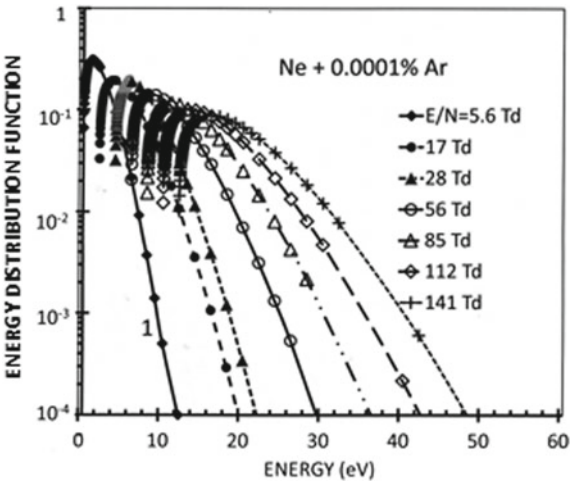

Fig. 5.2 Electron energy distribution function in Ne + 0.0001% Ar Penning mixtures at low E/N. The curves are successively displaced to the right for the sake of clarity

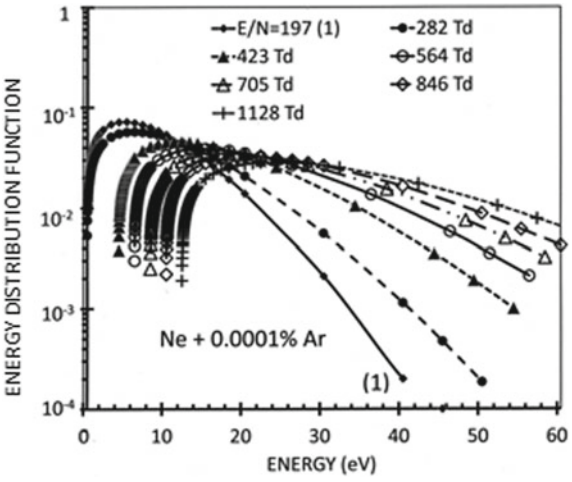

Fig. 5.3 Electron energy distribution function in Ne + 0.0001% Ar Penning mixtures at moderate E/N. The curves are successively shifted to the right for clarity of presentation

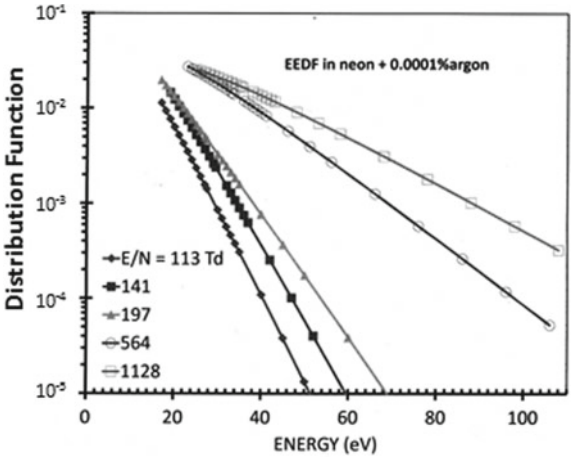

Fig. 5.4 Electron energy distribution function in Ne + 0.0001% Ar Penning mixtures at high E/N. The curves are successively shifted for clarity of presentation

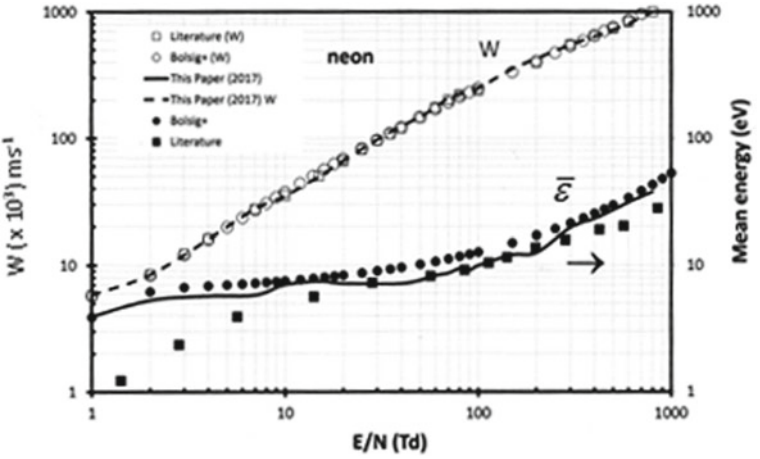

Fig. 5.5 Transport parameters for neon in 100% neon compiled from published literature. Such curves are available from published literature [10, 11] for argon and other gases

5.4 Concluding Remarks

This paper is a first attempt to examine whether there is a way to calculate coefficients by the integration process, by deriving the electron energy distribution functions with a simpler and different approach. It should be understood clearly that the method proposed is not claimed to be more original or more rigorous than the Boltzmann solution. The advantages claimed for the explained procedure are:

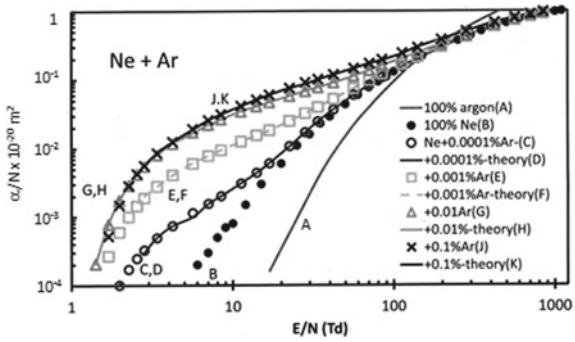

Fig. 5.6 First ionization coefficients in Penning mixtures of neon and argon. Both experimental [17] and theoretical (this paper) values are plotted as shown in the legend. Plotted values unless otherwise specified are experimental

1. It is much simpler to use than the numerical solution of boltzmann equation requiring elaborate collision cross section data and demanding expertise in coding the programs.
2. An analytical expression for energy distribution is necessary for calculation of electron energy distribution functions in non-uniform electric fields, microwave breakdown, breakdown in magnetic fields, and application of plasma science to medical sciences etc., which use the concept of equivalent electric field.
3. The paper highlights some of the challenges in the unfolding procedures for obtaining cross sections from swarm data through the numerical boltzmann solution.

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