Numerical Analysis of Buckling in Rectangular Plates with Different Cut-Outs

Mahendhar Kumar, Akash Venkateshwaran, Chamala Vaishnavi, and Lokavarapu Bhaskara Rao

Abstract This article examines the impact of boundary conditions on the buckling load for rectangular uniform isotropic plates of different aspect ratios and cut-out shapes. Plates with cut-outs have been extensively used in many applications like an aircraft fuselage, wings, etc. So proper understanding of their buckling is a crucial step before implementing them in various applications. Tests have been completed on aluminium alloy plates with circular holes and notches under different boundary conditions comprising of clamped, fixed and their combinations are considered. For complex geometries like such, analytical methods are tiresome and time-consuming, hence numerical methods are enforced to obtain very close results to what is expected from an analytical solution. Analysis by the numerical method is led and the effect of aspect ratio, boundary conditions, and cut-out shape on the buckling behaviour of isotropic plates under in-plane axial compression load is investigated and discussed. Buckling analysis is performed by employing finite element analysis software ANSYS. The numerical results received are in true agreement with the formerly posted data.

Keywords Finite element analysis · Buckling · Ansys · Circular holes · Notches

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1 Introduction

Reddy defines a plate as: “A plate is a structural element with plan form dimensions that are large compared to its thickness and is subjected to loads that cause bending deformation in addition to stretching [1]”. These thin structural elements are predominantly used in automobiles, ships, planes, bridges and buildings. When these slender components are subjected to axial compressive loads, they fail suddenly due to a phenomenon known as buckling instability [2]. Buckling of a plate is defined as the loss of its stability under compressive loading [3].

Buckling of plates is an imperative topic in structural engineering, be it mechanical, civil, marine, or aircraft structures. The prediction of buckling of structural members restrained laterally is crucial for designing numerous engineering components [4]. An important feature of this elastic buckling is that the instability may also occur at a stress degree that is extensively lower than the yield strength of the material.

Too often, it is almost inevitable to have holes in these plate elements due to their practical requirements like maintenance, inspection and service and also to produce lighter and more efficient structures. These perforations cause a redistribution of the plate membrane stresses accompanied by changes within the mechanical behaviour of the plate with notable modification in their stability [5, 6]. Loss of stability implies that shape of the buckled structure vagaries into a different configuration when the loads reach a critical value. At a certain given critical load, the plate will all of a sudden present a large deflection in the out-of-plane transverse direction [7]. Buckling occurrence relies on the shape of the structure, properties of the material, loading configuration and boundary conditions. Different bodies buckle in different ways. Flat plates experience bifurcation buckling, aka classical buckling [8].

A large number of researches have been done to study the buckling behaviours of perforated plates in the last decade. The first theoretical examination of buckling of plates was attributed to Bryan. Using the energy criterion of stability, he was able to perform the buckling analysis of rectangular plates subjected to uniaxial and uniform compression loads [9, 10]. The analysis methods espoused in the published articles can be segregated into two classes, i.e. linear elastic buckling and nonlinear elasto-plastic buckling. Amongst the elastic buckling studies category, El-Sawy and Nazmy explored how aspect ratio affected the critical buckling loads of rectangular plate with rectangular and eccentric circular holes under uniaxial and uniform compression load [11].

El-Sawy and Martini used the FEM techniques to find the elastic buckling stresses of biaxial loaded perforated rectangular plates with circular holes located in the longitudinal axis [12]. Finite element analysis (FEA) is by far the most effective and widely used numerical method in structural engineering. That is due to its profound theory and its ability to analyze complicated geometries and include nonlinearities. ANSYS® [27] provides ready-to-use general-purpose FEA software that has the capability of coupling different analysis fields. Most of the known exact buckling loads of plates have been summarized in the text by Timoshenko and Gere [13]. On the other hand, Moen and Schafer formulated and authorized analytical expressions for assessing the impact of single or multiple holes on the elastic buckling critical stress of plates on compression [14]. Subsequently, Paik examined the ultimate strength properties of perforated plates under edge shear loading, axial compressive loading, and the combined edge shear loads and biaxial compression, and put forward empirical formulae to speculate the ultimate strength of the same, derived from the regression analysis of the nonlinear FEA results [15–17]. Rao and Rao explored the buckling load responses of circular plates with internal elastic ring support and extended their study by restraining the edges against any translational and rotational movements [18].

Buckling of circular and annular plates with guided edges [19–21] and with elastic/rigid ring support [22] and elastic edges [23, 24] studied by Rao and Rao. However, they have not considered the cut-out in their study. In the pool of studies devoted to the problem of elasto-plastic buckling, El-Sawy et al. studied the elasto-plastic buckling of uniaxially loaded square and rectangular plates with circular cut-outs by using FEM techniques [25].

This article deals with buckling analysis of uniform isotropic aluminium plates under diverse boundary conditions. The effects on critical buckling load by the number of cut-outs, aspect ratio and specific boundary conditions are studied. First, the problem statement is described. Next, the test specimens and final element model are presented. Then, the consequences of different parameters are discussed in the light of the results.

2 Problem Statement

In this study, buckling loads of aluminium alloy have been determined numerically; effects of different parameters like boundary conditions, length to thickness ratio, and cut-out shape were studied. The rectangular plates made of aluminium alloy with and without cut-outs chosen for study are shown in Fig. 1. The elastic properties of aluminium alloy used in this study are given in Table 1. The cut-out chosen are central hole, central notch, 3 holes and 6 notches with diameter D. The diameter D of the cut-outs are 2.5 mm, 5 mm, 10 mm, 15 mm, and other dimensional parameters chosen for this study are given in Table 2.

3 Finite Element Model

In this investigation, the commercial finite element code ANSYS was used as a tool for performing numerical analysis. The investigation included five different types of rectangular plates (with and without cut-outs as seen in Fig. 1), four different lengths, 2 different thicknesses which contributed to eight different *L*/*T* ratios, four different

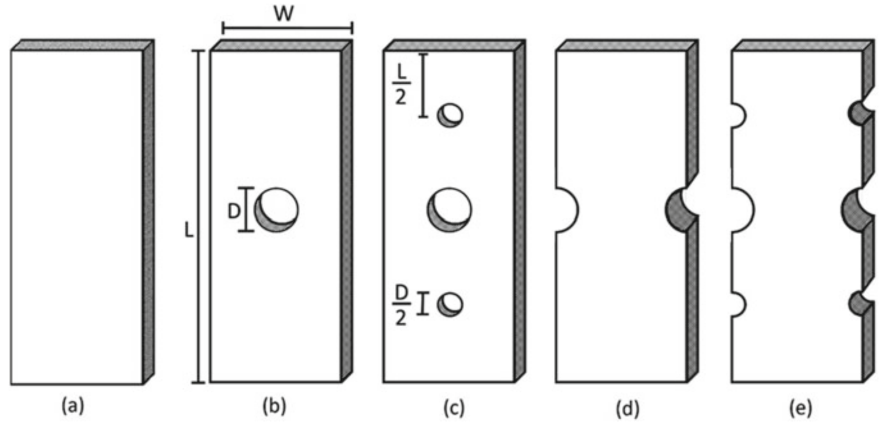


Fig. 1 **a** Simple plate **b** plate with central hole **c** plate with 3 holes **d** plate with central notch **e** plate with 6 notch

Table 1 Material property of aluminium alloy

Elastic properties		Values
Density	kg m^−3	2770
Young's modulus	Pa	7.10E + 10
Poisson's ratio		0.33
Bulk modulus	Pa	6.90E + 10

Table 2 Plates considered in this study

Width (mm)	Thickness (mm)	Length (mm)	Length/thickness ratio
30, 40	3, 4.5	75, 150, 250, 300	16.66, 25, 33.33, 50, 55.55, 66.66, 83.33,100

radii of cut-outs and finally 2 sets of boundary conditions. The boundary conditions implemented in this study were as follows:

- Both opposite edge of the plates is clamped, whilst the remaining edges are free (CC)
- Lower end of the plate is clamped, whilst all the remaining three edges are free (CF).

Figure 2 represents the boundary conditions, and Fig. 3 shows a typical finite element mesh used for plates with cut-outs. The element used in this study is eight-noded SHELL 91 multi-layered shell elements which pose 3 rotational degrees of freedom and 3 displacement degree of freedom. In the presence of cut-outs, large numbers of nodes were used around the vicinity of cut-outs for proper results. From Fig. 2, it can be noticed that a compressive load was applied to the rectangular plates in the *y*-direction.

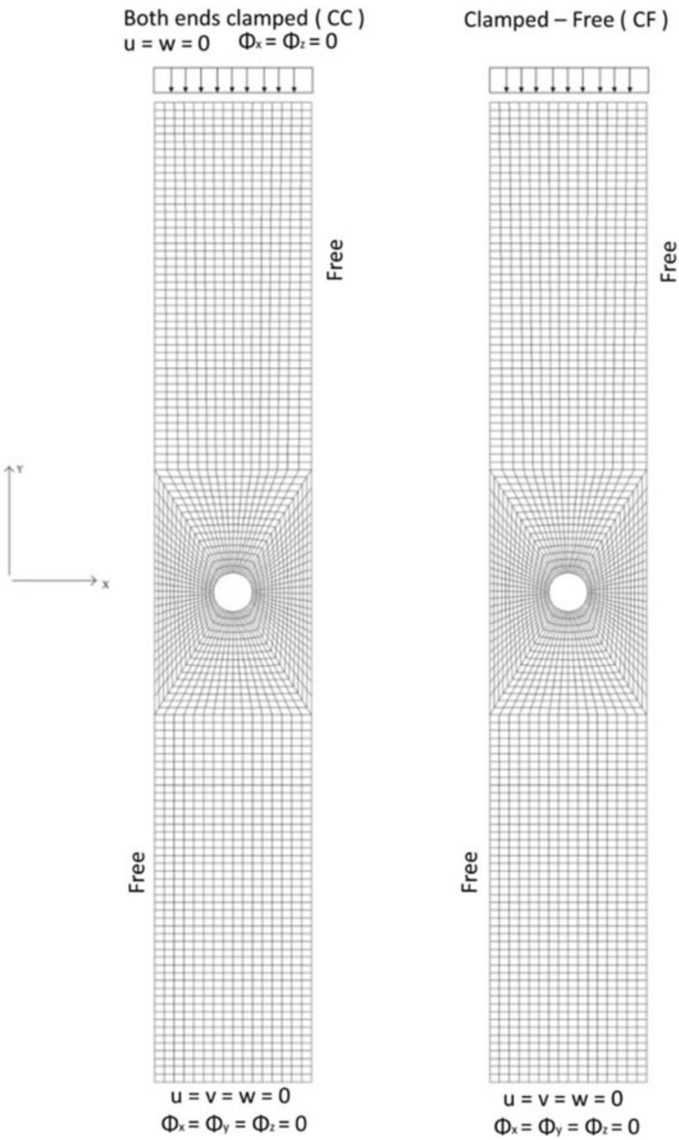


Fig. 2 Boundary conditions used on rectangular plates

4 Validation of the Model

The verification of the present model was acknowledged through a paper published by Baba, [26]. The reason behind referring to this particular paper was it has very similar cut-outs and loading conditions with the present work. Table 3 shown below

Percentage error = (Pmodel buck - Preference buck) × 100 / Preference buck

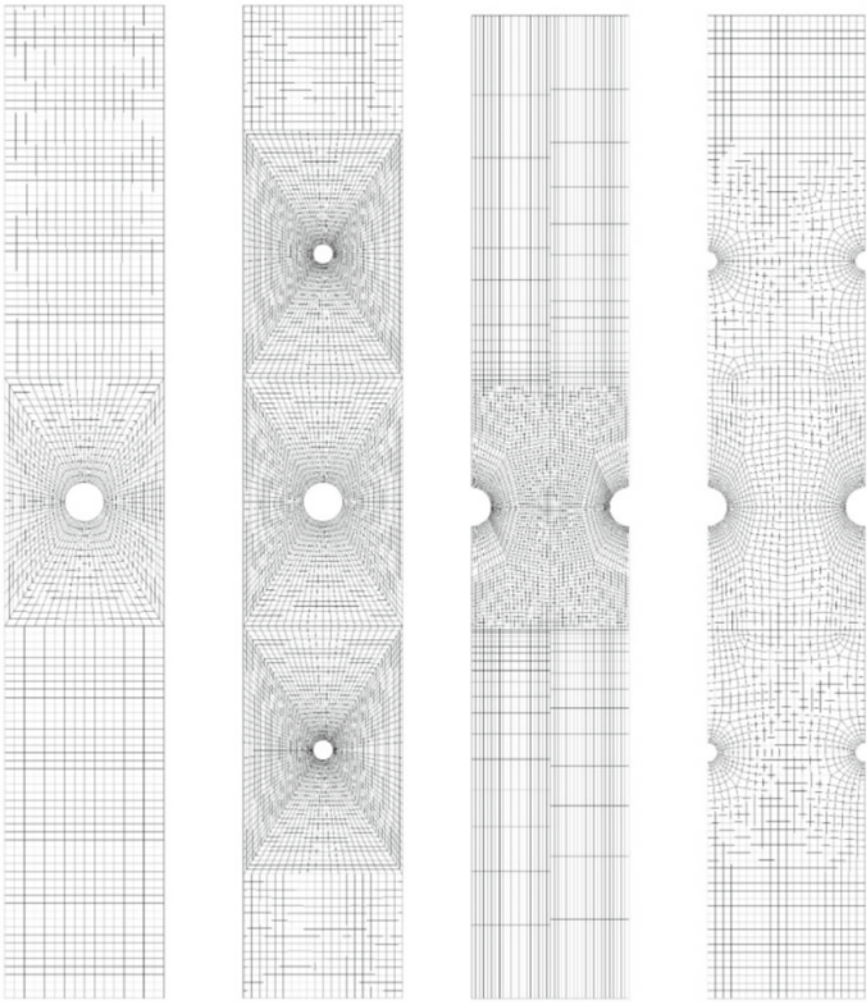


Fig. 3 Finite element mesh incorporated in the buckling analysis

Table 3 Buckling loads for validation and material properties

	Plate without hole		Plate with circular hole		Plate with semi-circular hole	
	CC	CP	CC	CP	CC	CP
Buckling load [17]	234.81	118.64	228.78	116.65	229.25	116.81
Buckling load (FEA model)	235.82	120.79	230.92	119.18	228.26	119.17
Percentage error %	0.43	1.81	0.93	2.16	0.43	2.02

provides buckling load for all three types of plates which are simple plates, a plate with a central hole and a plate with semi-circular holes. The composite material property used in the study are shown in Table 3 as well.

Mechanical properties	Unit	Values
E1	GPa	39
E2 = E3	GPa	8.2
G12 = G13 = G23	GPa	2.9
V12 = v13 = v23	GPa	0.29
t	mm	0.1875

Buckling load for the stacking sequence 8 [0/45/-45/90] as with two boundary conditions (CC, CP) for all types of plates was calculated using the present FEA model. The validation process shown in Table 3 has an average percentage error of 1.29%. Thus, it can be observed there is a good agreement between the present study and the values available in the literature.

$$\text{Percentage error} = (\text{Pmodel buck} - \text{Preference buck}) \times 100/\text{Preference buck}$$

5 Result and Discussion

This study aims to understand the effect of cut-outs, boundary conditions and L/T ratio on the buckling load under compression loading. The buckling loads (2 Modes) for different aluminium alloy plates are tabulated below. Buckling loads with 2 Modes for a plate of chosen length $L = 75$ mm is tabulated in Table 4. Whilst the 2 Modes of buckling loads for a plate of length $L = 150$ mm are compiled in Table 5. Similarly, Tables 6 and 7 speak for the buckling load of a plate of length $L = 250$ mm and 300 mm respectively. Graphs are used in this study to compare the effects of different dimensions of cut-outs and L/T ratio. In Fig. 4, Mode 1 buckling loads for a plate of $L = 75$ mm are plotted against the different diameter of cut-outs, and graphs for Mode 2 buckling loads can be found in Fig. 5, the graphs are plotted after considering the possible L/T ratio which can be found above each plot. Similarly, Figs. 6, 7, 8, 9, 10 and 11 represent Mode 1-Mode 2 buckling loads of a rectangular plate of length $L = 150$ mm, 250 mm, 300 mm, respectively. To study the effect of the L/T ratio, buckling loads from simple plates have been used. Figure 12 (Mode 1) and Fig. 13 (Mode 2) show the influence of the L/T ratio on buckling loads for different boundary conditions considering both widths $W = 30$ mm, 40 mm into consideration. A particular sample model was chosen and the visualization of buckling of the sample model for both modes is presented in Fig. 14. The following section deals with the buckling results of aluminium alloy plates.

Table 4 Buckling load for all aluminium alloy plates with length $L = 75$ mm

	Simple plate	Plate with centre hole				Plate with 3 holes				Plate with 2 notch				Plate with 6 notch			
		d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15
Mode 1																	
Height = 3, Width = 40																	
CC	47,092	46,437	45,496	42,472	38,749	46,439	45,469	42,309	38,297	46,486	40,547	38,330	35,151	46,498	45,654	38,059	36,683
CF	2923.8	2915	2884.1	2764.4	2565.8	2909.8	2863.8	2694.5	2445.2	2914.7	2889	2770.1	2573	2908.7	2861.8	2698.2	2443.8
Height = 4.5, Width = 40																	
CC	1.56E + 05	1.55E + 05	1.52E + 05	1.42E + 05	1.29E + 05	1.55E + 05	1.52E + 05	1.41E + 05	1.28E + 05	1.55E + 05	1.53E + 05	1.44E + 05	1.31E + 05	1.55E + 05	1.52E + 05	1.42E + 05	1.29E + 05
CF	9843.3	9805.4	9695.9	9282.5	8603.8	9787.5	9624.5	9040.6	8194.4	9803.6	9693.6	9281.9	8608.2	9782.8	9616.7	9030.3	8186.3
Height = 3, Width = 30																	
CC	34,899	34,636	33,420	30,595	26,601	34,551	33,390	30,441	26,171	34,601	33,806	27,544	24,245	34,640	33,742	30,769	23,775
CF	2170.4	2161.4	2130.5	2006.7	1794.4	2153.7	2110.6	1939.4	1685.1	2159	2128.1	2013	1800.1	2159.5	2106.2	1937.2	1687.1
Height = 4.5, Width = 30																	
CC	1.16E + 05	1.15E + 05	1.11E + 05	1.01E + 05	88,746	1.15E + 05	1.11E + 05	1.01E + 05	87,096	1.15E + 05	1.12E + 05	1.03E + 05	90,401	1.15E + 05	1.12E + 05	1.02E + 05	88,065
CF	7305.7	7262.3	7158.7	6736.3	6003.7	7243.2	7089.2	6504.4	5631	7261.4	7157.8	6736.4	6008.2	7240.4	7082.8	6495.9	5623.2
Mode 2																	
Height = 3, Width = 40																	
CC	67,529	67,210	66,044	60,911	51,711	67,032	65,330	58,622	48,369	67,840	68,991	71,156	73,640	67,951	68,958	72,077	73,901
CF	26,266	26,188	25,904	24,846	23,216	26,139	25,709	24,188	22,058	26,205	26,048	25,126	23,574	26,156	25,792	24,511	22,379

(continued)

Table 4 (continued)

Mode 1																	
Height = 3, Width = 40																	
	Simple plate	Plate with centre hole				Plate with 3 holes				Plate with 2 notch				Plate with 6 notch			
		d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15
Height = 4.5, Width = 40																	
CC	2.20E + 05	2.19E + 05	2.15E + 05	1.98E + 05	1.68E + 05	2.18E + 05	2.13E + 05	1.90E + 05	1.56E + 05	2.21E + 05	2.23E + 05	2.30E + 05	2.37E + 05	2.21E + 05	2.24E + 05	2.32E + 05	2.39E + 05
CF	87,852	87,516	86,527	82,902	77,354	87,345	85,854	80,639	73,434	87,558	86,724	83,478	78,119	87,389	86,083	81,328	74,342
Height = 3, Width = 30																	
CC	71,004	71,124	70,045	67,454	52,733	70,691	69,181	64,799	49,199	70,941	70,534	61,037	55,524	69,955	69,738	65,251	51,800
CF	19,536	19,462	19,183	18,122	16,494	19,385	18,994	17,492	15,420	19,445	19,195	18,283	16,627	19,473	19,007	17,597	15,582
Height = 4.5, Width = 30																	
CC	2.31E + 05	2.31E + 05	2.29E + 05	2.18E + 05	1.68E + 05	2.31E + 05	2.26E + 05	2.10E + 05	1.55E + 05	2.31E + 05	2.29E + 05	2.18E + 05	1.95E + 05	2.31E + 05	2.26E + 05	2.10E + 05	1.83E + 05
CF	65,318	64,940	64,033	60,461	54,855	64,760	63,380	58,310	51,210	64,958	64,118	60,665	55,000	64,779	63,477	58,575	51,439

Table 5 Buckling load for all aluminium alloy plates with length $L = 150$ mm

	Simple plate	Plate with centre hole				Plate with 3 holes				Plate with 2 notch				Plate with 6 notch			
		d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15
Mode 1																	
Height = 3, Width = 40																	
CC	11,537	11,505	11,384	10,936	10,246	11,505	11,383	10,927	10,221	11,507	10,702	10,339	9745.6	11,507	11,395	10,313	10,301
CF	717.14	715.81	712.06	697.3	671.42	715.17	709.52	687.93	652.47	715.79	712.7	698.01	672.32	715.14	709.37	688.48	652.19
Height = 4.5, Width = 40																	
CC	38,863	38,730	38,308	36,768	34,422	38,729	38,302	36,734	34,329	38,741	38,364	36,979	34,793	38,741	38,344	36,894	34,594
CF	2416.5	2411.9	2398.5	2347.1	2258.1	2409.6	2389.5	2314.1	2191.7	2411.7	2398.3	2347.1	2258.4	2409.4	2388.9	2313	2190.5
Height = 3, Width = 30																	
CC	8565.3	8530.9	8412.1	7957.7	7219.4	8530.7	8410.7	7951.2	7196.9	8002.4	7906	7523.1	6875.5	8038.3	7900.9	7506.7	6834.6
CF	534.1	533.19	529.44	514.09	484.89	532.54	526.89	504.91	466.87	533.62	529.9	514.65	485.65	533.04	527.31	505.43	467.51
Height = 4.5, Width = 30																	
CC	26,655	28,716	28,298	26,731	24,218	28,695	28,292	26,705	24,131	28,724	28,337	26,881	24,472	28,725	28,321	26,807	24,296
CF	1802.9	1796.5	1783.1	1729.6	1628.9	1793.6	1774.1	1697	1565.8	1796.3	1783	1729.5	1629.1	1794.1	1773.7	1696.6	1565.4
Mode 2																	
Height = 3, Width = 40																	
CC	23,725	23,740	23,718	23,579	23,192	23,712	23,605	23,150	22,324	23,738	22,351	22,193	21,795	23,716	23,608	22,816	22,356
CF	6481.7	6470.2	6437.2	6307.9	6088.1	6464.1	6413.2	6220.2	5912.4	6470.4	6446.5	6321.8	6106.5	6464.6	6414.9	6237.4	5926.2
Height = 4.5, Width = 40																	
CC	79,519	79,528	79,444	78,926	77,513	79,433	79,055	77,451	74,558	79,525	79,410	78,737	77,131	79,447	79,048	77,426	74,496
CF	21,800	21,760	21,643	21,195	20,441	21,739	21,560	20,887	19,830	21,760	21,646	21,209	20,466	21,739	21,562	20,907	19,865

(continued)

Table 5 (continued)

Mode 1																	
Height = 3, Width = 40																	
	Simple plate	Plate with centre hole				Plate with 3 holes				Plate with 2 notch				Plate with 6 notch			
		d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15
Height = 3, Width = 30																	
CC	17,585	17,597	17,582	17,473	17,126	17,570	17,471	17,055	16,281	16,585	16,558	16,395	16,011	16,624	16,455	16,026	15,244
CF	4825	4813.5	4780.6	4647.4	4405	4807.5	4757.1	4563.2	4241.6	4819	4786.9	4656.2	4416.4	4813.6	4763.6	4573.4	4254.3
Height = 4.5, Width = 30																	
CC	54,939	58,944	58,887	58,472	57,188	58,816	58,505	57,027	54,309	58,932	58,813	58,153	56,559	58,853	58,450	56,831	53,936
CF	16,253	16,186	16,070	15,606	14,773	16,160	15,987	15,310	14,204	16,185	16,070	15,611	14,784	16,165	15,987	15,316	14,215

Table 6 Buckling load for all aluminium alloy plates with length $L = 250$ mm

Mode 1																	
Height = 3, Width = 40																	
	Simple plate	Plate with centre hole				Plate with 3 holes				Plate with 2 notch				Plate with 6 notch			
		d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15
CC	4118.5	4098.9	4073.4	3974.3	3807.5	4098.9	4073.3	3973.7	3805.5	4099	4075	3981.7	3822.9	4099.8	4074.3	3978.9	3816.4
CF	256.24	255.93	255.09	252	246.38	255.74	254.49	249.89	242.04	255.91	255.16	252.04	246.52	255.87	254.56	250	242.19
Height = 4.5, Width = 40																	
CC	13,868	13,823	13,732	13,389	12,816	13,823	13,732	13,386	12,808	13,824	13,738	13,414	12,870	13,825	13,736	13,404	12,845
CF	864.94	863.75	860.84	849.65	830.5	863.26	858.89	842.44	815.37	863.72	861.04	850.09	831.17	862.62	858.35	842	814.97
Height = 3, Width = 30																	
CC	3068.5	3051.7	3026.2	2924	2738.7	3051.7	3063.3	2923.5	2736.7	3052.1	3027.1	2928.6	2748.5	3051.9	3027.9	2926.6	2743.3
CF	191.15	190.84	190.04	186.73	180.22	190.7	189.49	184.68	176	190.84	190.03	186.72	180.24	190.7	189.48	184.67	176
Height = 4.5, Width = 30																	
CC	10,332	10,201	9845.1	9208.2	10,292	10,201	9843.3	9200.3	10,292	10,205	9861.4	9242.3	10,293	10,203	9853.8	9222.8	10,292
CF	644.54	640.6	629.03	606.54	642.97	638.66	621.77	591.67	643.44	640.57	629	606.54	642.95	638.58	621.69	591.59	643.46
Mode 2																	
Height = 3, Width = 40																	
CC	8466	8443.4	8442	8432.2	8403.5	8437.6	8418.3	8341	8205.3	8442.4	8438.4	8417.4	8369.4	8438.5	8415.5	8332.1	8186
CF	2314.7	2312	2304.6	2277.5	2228.7	2310.2	2298.9	2258	2189.2	2311.8	2305.3	2278.2	2230.7	2311.5	2299.9	2259.7	2191.8
Height = 4.5, Width = 40																	
CC	28,450	28,421	28,415	28,379	28,272	28,400	28,333	28,058	27,583	28,419	28,404	28,323	28,143	28,402	28,323	28,021	27,500
CF	7810.5	7799.7	7774	7676.6	7510.6	7794.3	7755.5	7609.6	7372.3	7799.5	7776.5	7681.3	7518.7	7787.6	7749.8	7605.9	7370.4

(continued)

Table 6 (continued)

Mode 1																	
Height = 3, Width = 40																	
	Simple plate	Plate with centre hole				Plate with 3 holes				Plate with 2 notch				Plate with 6 notch			
		d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15
Height = 3, Width = 30																	
CC	6295.4	6278.7	6277.8	6270.4	6245.3	6273	6270.8	6180.6	6048.8	6278.7	6274.5	6257.2	6216	6272.7	6254.4	6170.8	6028.8
CF	1723.3	1720.6	1713.5	1684.4	1628.4	1719.3	1708.5	1665.9	1590.6	1720.7	1713.4	1684.5	1629	1719.3	1708.4	1666.1	1591.3
Height = 4.5, Width = 30																	
CC	21,156	21,136	21,132	21,104	21,010	21,116	21,051	20,787	20,323	21,134	21,121	21,054	20,899	21,117	21,040	20,748	20,244
CF	5806.3	5796.8	5772.4	5669.7	5476.1	5792.4	5753.9	5604.5	5344.2	5796.8	5771.4	5670.2	5477.8	5792.2	5753.4	5604.4	5345.1

Table 7 Buckling load for all aluminium alloy plates with length $L = 300$ mm

	Simple plate	Plate with centre hole				Plate with 3 holes				Plate with 2 notch				Plate with 6 notch			
		d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15
Mode 1																	
Height = 3, Width = 40																	
CC	2842	2700.5	2690.1	2634.9	2543.9	2701.6	2685	2635.3	2543.7	2706.8	2690.1	2638.9	2552.6	2706.7	2700.1	2636.6	2548.9
CF	177.22	177.21	176.76	174.95	171.68	177.14	177.55	173.79	169.19	177.21	176.75	174.95	171.7	177.13	176.45	173.77	169.2
Height = 4.5, Width = 40																	
CC	9590.1	9090.8	9056	8865.1	8553.9	9087.6	9040.4	8857.6	8547.4	9554.2	9497.5	8867.1	8567.2	9551.4	9503.6	8863	8556.3
CF	597.68	597.64	595.61	589.73	578.48	597.39	594.98	585.64	569.72	597.27	595.58	589.71	578.48	597	594.54	585.53	569.7
Height = 3, Width = 30																	
CC	2119.9	2015.8	2002.4	1948	1927.7	2022	2002.5	1957.8	1845.3	2047.9	2034.3	1942.4	1876.9	2056.4	2034.4	1978.6	1874.6
CF	132.5	132.44	131.99	130.08	126.19	132.26	131.69	128.79	123.85	132.39	131.93	129.95	126.26	132.31	131.61	128.84	123.77
Height = 4.5, Width = 30																	
CC	7154.2	6786	6739.5	6546.9	6196.5	6784	6721	6545	6192.2	6820	6740.1	6552.9	6210.4	6820.7	6773.2	6545	6201.3
CF	446.85	446.66	445.07	438.4	425.34	446.42	443.54	434.3	416.69	446.67	445.03	438.37	425.35	446.4	443.97	434.23	416.72
Mode 2																	
Height = 3, Width = 40																	
CC	5837.3	5572.2	5571.5	5566.8	5554	5571.3	5564.4	5517.5	5442.8	5583.2	5574.1	5562.3	5543.2	5580.1	5580.7	5511.7	5435.6
CF	1599.2	1598.8	1594.9	1579	1550.5	1598.2	1602.8	1568.4	1527.9	1598.8	1594.8	1579.1	1551	1598.1	1592.2	1568.4	1528.5
Height = 4.5, Width = 40																	
CC	19,670	19,740	19,619	18,725	18,679	18,724	18,685	18,535	18,281	19,630	19,612	18,686	18,606	19,615	19,578	18,515	17,256.1

(continued)

Table 7 (continued)

Mode 1																	
Height = 3, Width = 40																	
	Simple plate	Plate with centre hole				Plate with 3 holes				Plate with 2 notch				Plate with 6 notch			
		d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15	d = 2.5	d = 5	d = 10	d = 15
CF	5389.8	5389.5	5370.6	5320	5221.9	5387.2	5365.7	5282.7	5142.9	5385.2	5370.4	5320.3	5223	5382.7	5360.9	5282.5	5144.1
Height = 3, Width = 30																	
CC	4339.4	4154.6	4153.5	4150.1	4128.5	4337.6	4286.3	4141.2	4026.8	4214.3	4212.6	4201.5	4187.9	4227.6	4200	4154.8	4077.2
CF	1194	1193.7	1189.7	1172.9	1138.9	1191.9	1187	1161	1118.2	1193.2	1189.2	1171.7	1139.8	1192.5	1186.3	1161.8	1117.8
Height = 4.5, Width = 30																	
CC	14,649	13,974	13,971	13,951	13,910	13,960	13,915	13,773	13,512	14,035	13,967	13,932	13,868	14,026	13,985	13,750	13,484
CF	4024.8	4023.8	4009.8	3951.1	3837.7	4021.6	3995.2	3914.3	3760.9	4023.9	4009.5	3951.1	3838.5	4021.6	3999.9	3913.8	3761.9

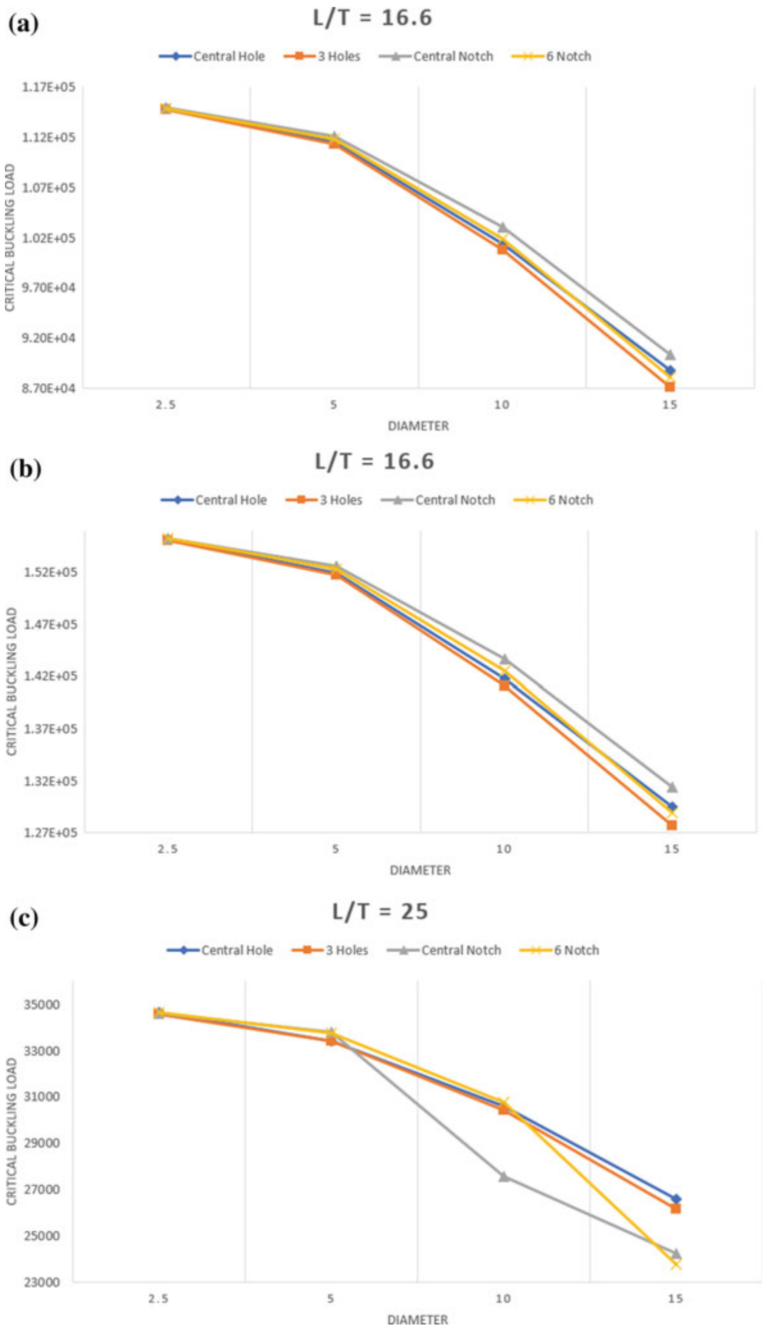


Fig. 4 Variation of buckling load for a plate of length 75 mm-mode 1 **a** CC condition width 30 **b** CC condition width 40 **c** CC condition width 30 **d** CC condition width 40 **e** CF condition width 30 **f** CF condition width 40 **g** CF condition width 30 **h** CF condition width 40

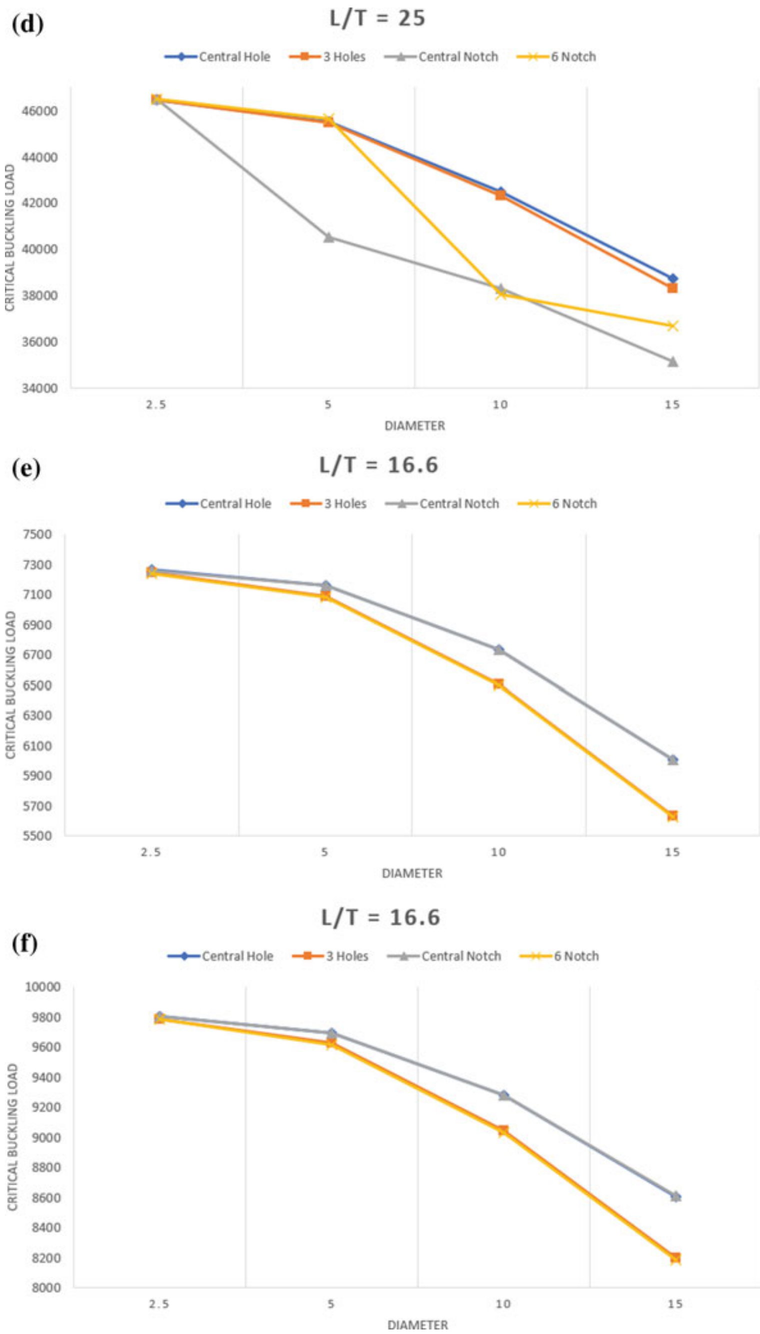


Fig. 4 (continued)

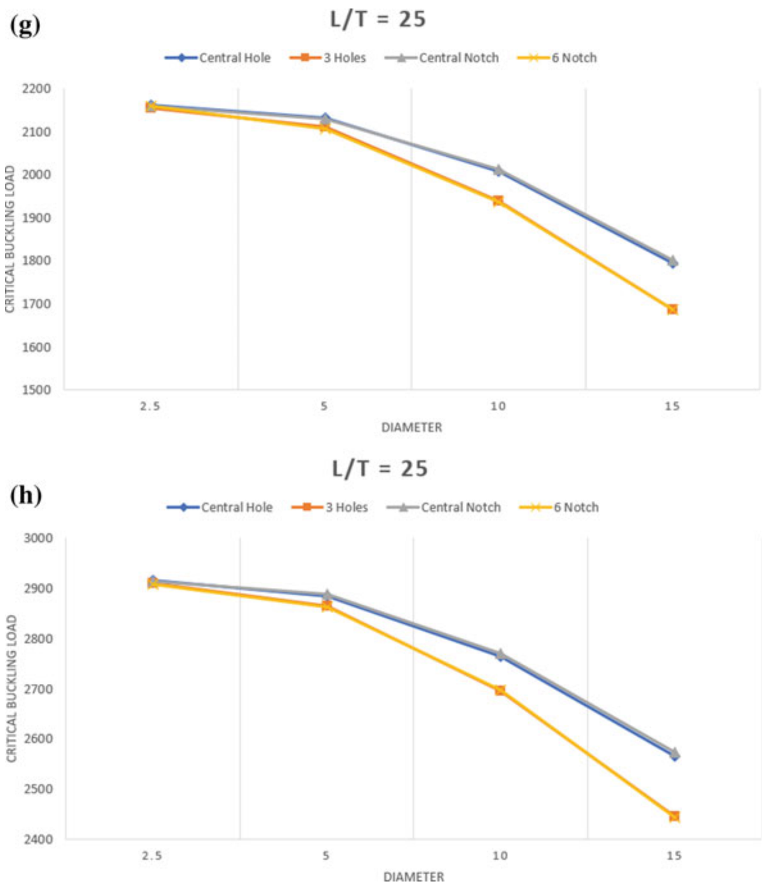


Fig. 4 (continued)

5.1 Effect of Cut-Outs

Aluminium alloy plates with and without cut-outs are extensively used in industries for their easy manufacturability and low-cost purposes. To meet the design requirement, it is important to understand the buckling response of such plates. In this section, the effect of the central hole, 3 holes along with the central notch, and 6 notches are taken into account.

It can be seen that the buckling load gradually decreases with the introduction of holes and notches into the simple geometry (Figs. 4 and 5). Comparing the result obtained for the plate of length $L = 75$ mm in Mode 1, we can notice that as the diameter of the cut-out increased there's a drop of about 17% in buckling load for a plate with a central hole, 18% drop for a plate with 3 holes, 25% drop for a plate with central notch and 22% drop for a plate with 6 notches in the CC boundary

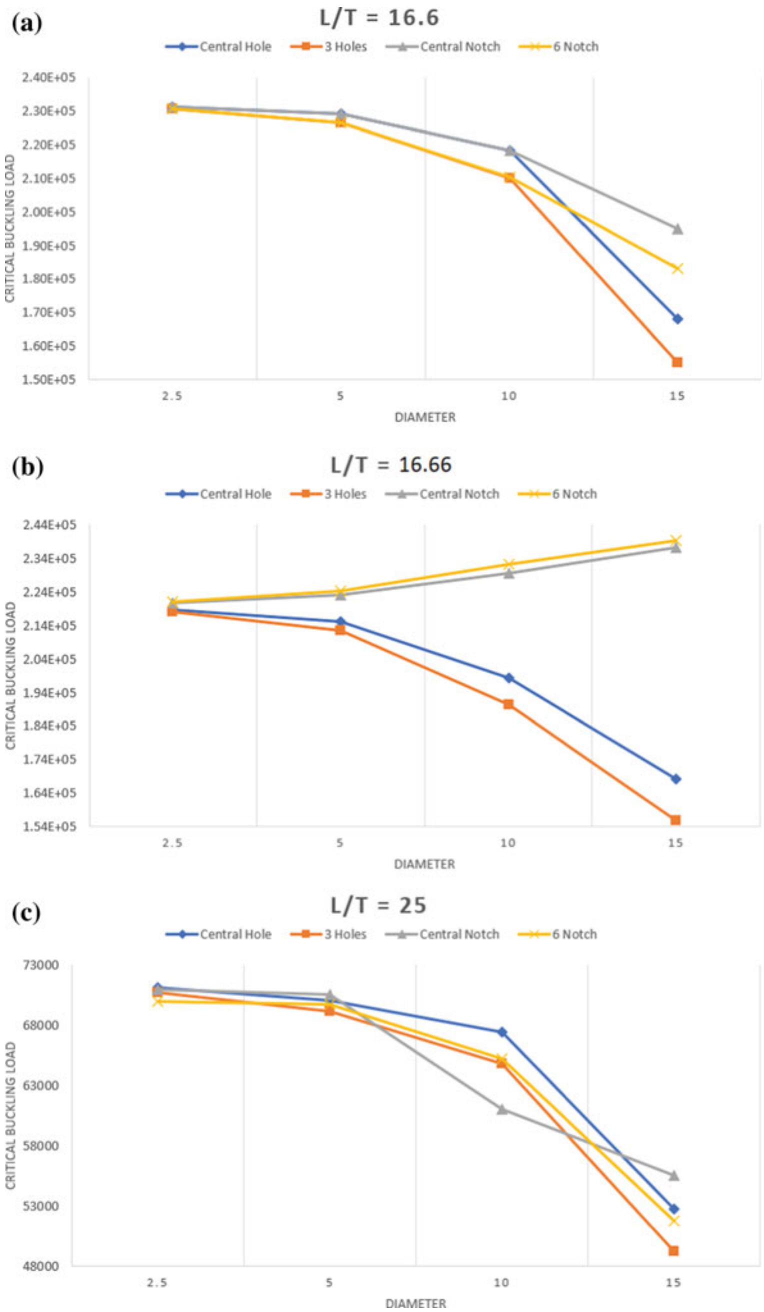


Fig. 5 Variation of buckling load for a plate of length 75 mm-mode 2 **a** CC condition width 30 **b** CC condition width 40 **c** CC condition width 30 **d** CC condition width 40 **e** CF condition width 30 **f** CF condition width 40 **g** CF condition width 30 **h** CF condition width 40

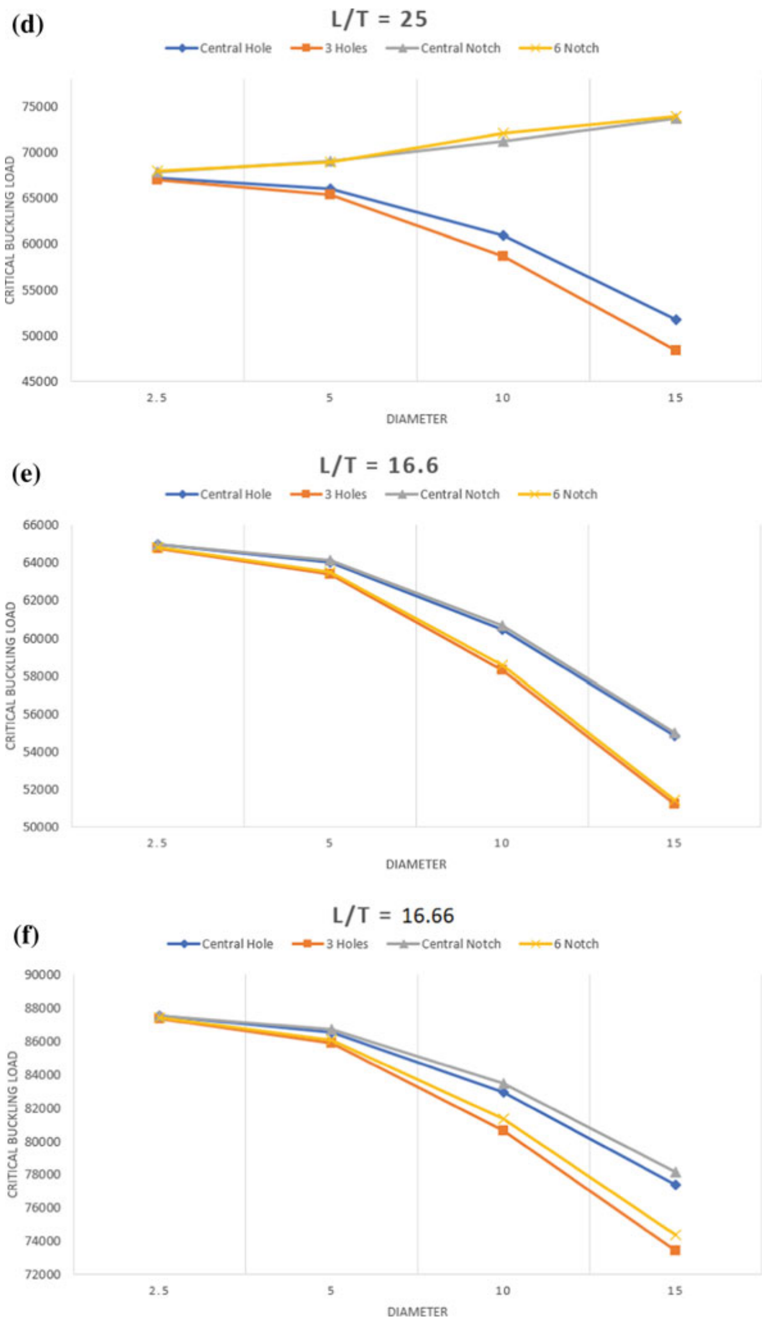


Fig. 5 (continued)

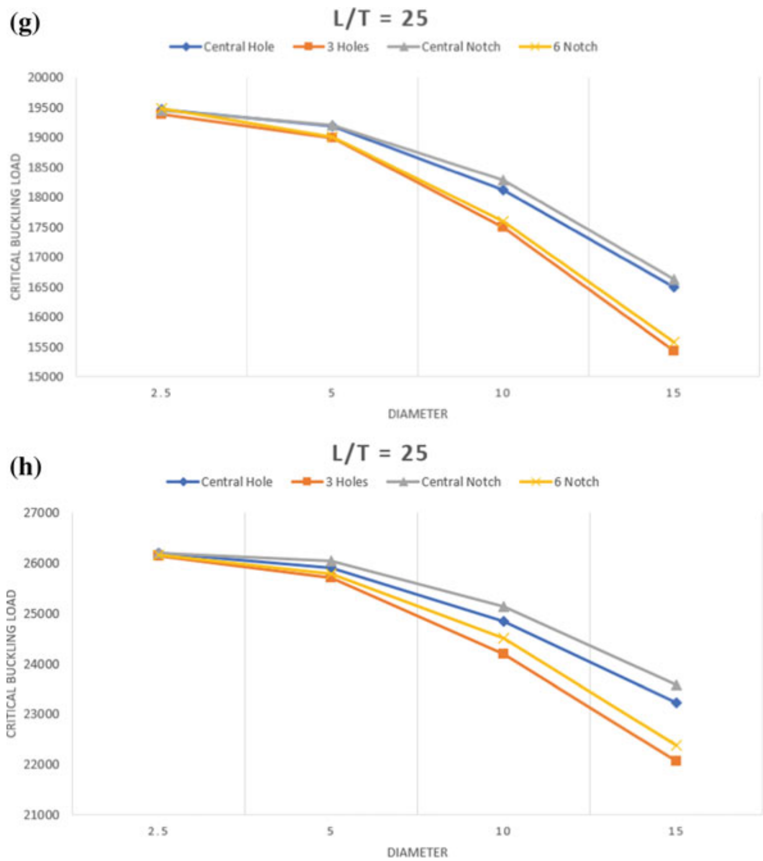


Fig. 5 (continued)

conditions. A similar decrease in buckling load for increasing diameter of cut-outs can be observed in all other cases of varying lengths and other boundary conditions (CF), especially in Mode 1 buckling analysis (Figs. 6, 8 and 10). As the investigation is carried out with all possible *L*/*T* ratios it has become easy to compare the effect of different cut-outs with varying *L*/*T* ratios.

With the increase in the *L*/*T* ratio, the influences of cut-outs on buckling loads are boosted. There is a drastic drop in buckling load under CC boundary conditions for different cut-outs as the *L*/*T* ratio increased. This can be noticeably seen in the Mode 1 table for a plate of Length $L = 75$ mm (Table 5).

Similar trends are followed in all cases of varying lengths. After tabulating all the buckling loads, it merits referencing that even though plates with central hole patterns have somewhat less buckling load than those with a central notch pattern at a lesser diameter range, but the diameter increased the buckling load of holes was a little higher than the notches. The conduct of plates with holes and notch patterns is

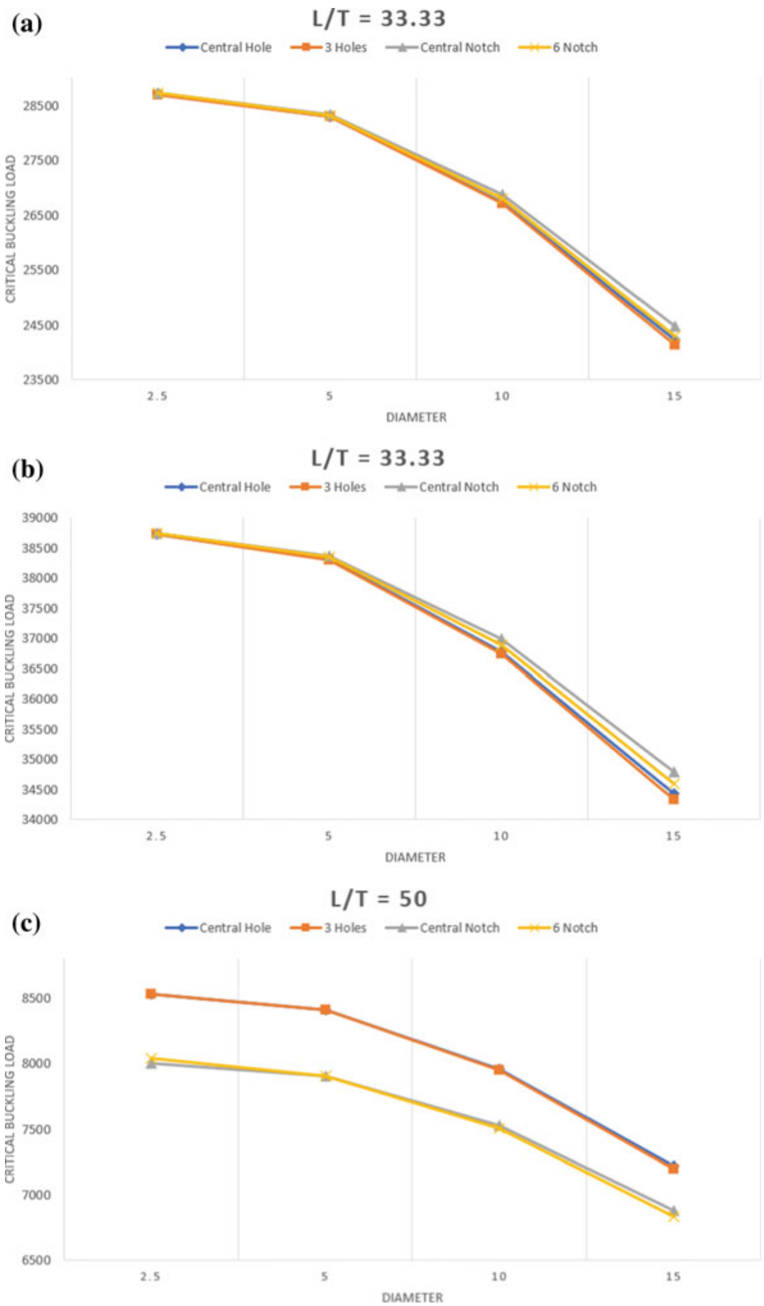


Fig. 6 Variation of buckling load for a plate of length 150 mm-mode 1 **a** CC condition width 30 **b** CC condition width 40 **c** CC condition width 30 **d** CC condition width 40 **e** CF condition width 30 **f** CF condition width 40 **g** CF condition width 30 **h** CF condition width 40

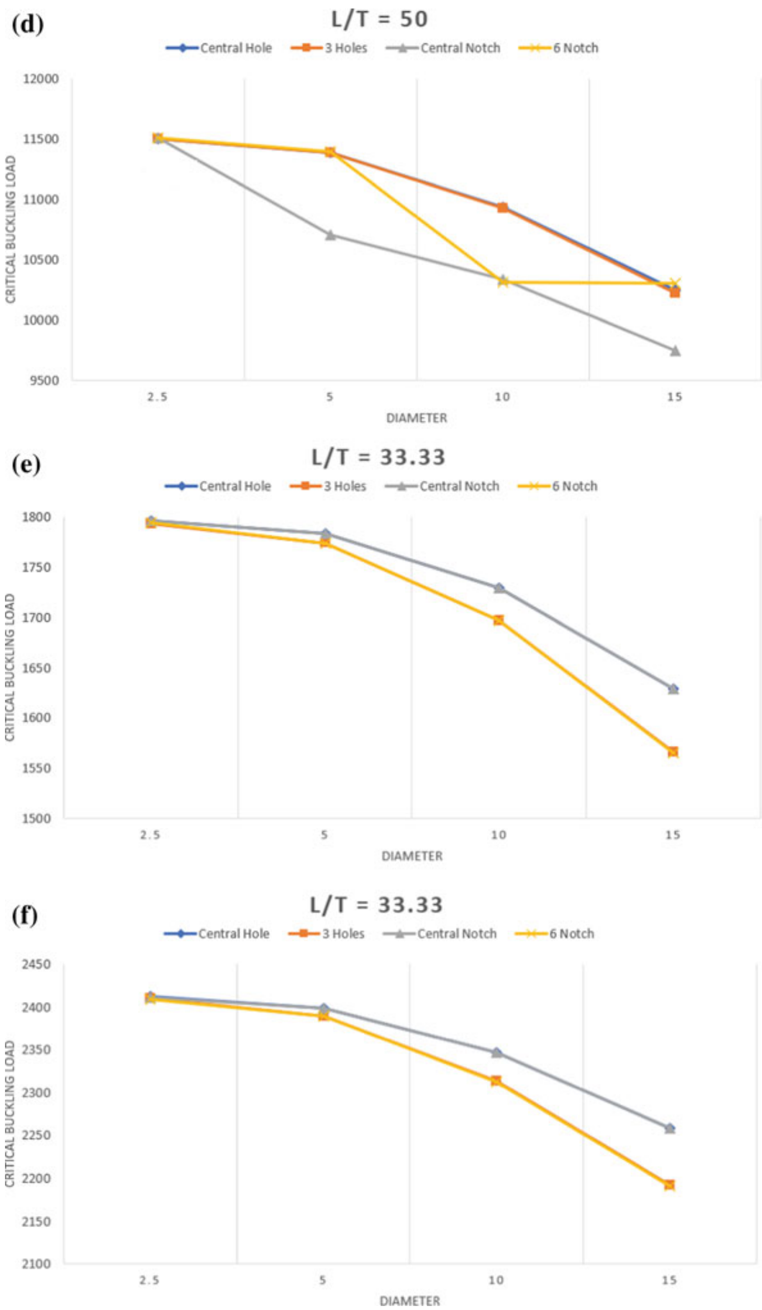
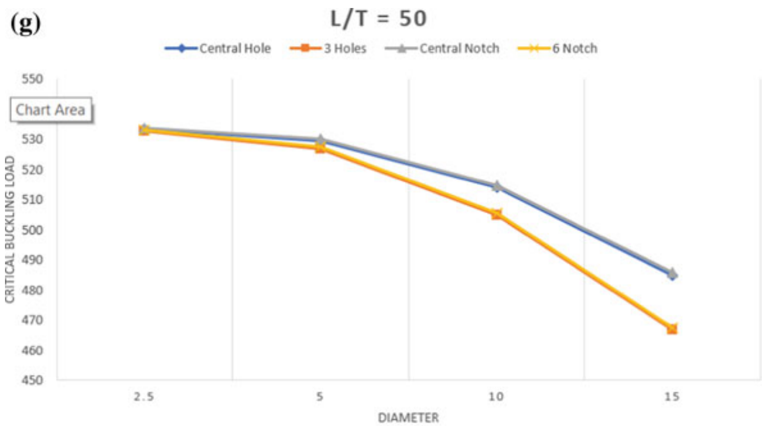


Fig. 6 (continued)

(g)



(h)

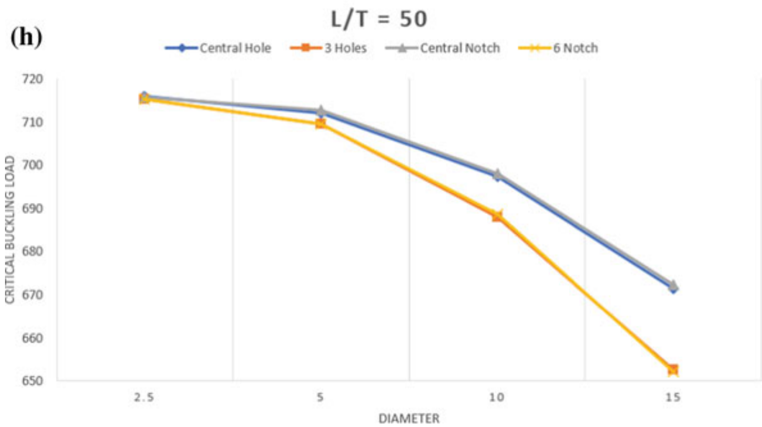


Fig. 6 (continued)

very much alike, regarding buckling load. This can be clarified by the way that the regions of holes and notch patterns are equivalent.

When the Mode 2 buckling analysis is considered, the same decrease in buckling load has been noticed for increasing diameters of cut-outs for nearly all cases of varying length, width and L/T ratio. However, at one special case, the buckling loads increased as the diameter of the cut-out increased, these special cases were noticed under the plate of length $L = 75$ mm, width $W = 40$ (Fig. 5). This increase in buckling load was only observed in this case and all other types of the plate had their buckling load decreasing for an increase in diameter of cut-outs (Figs. 7, 9 and 11).

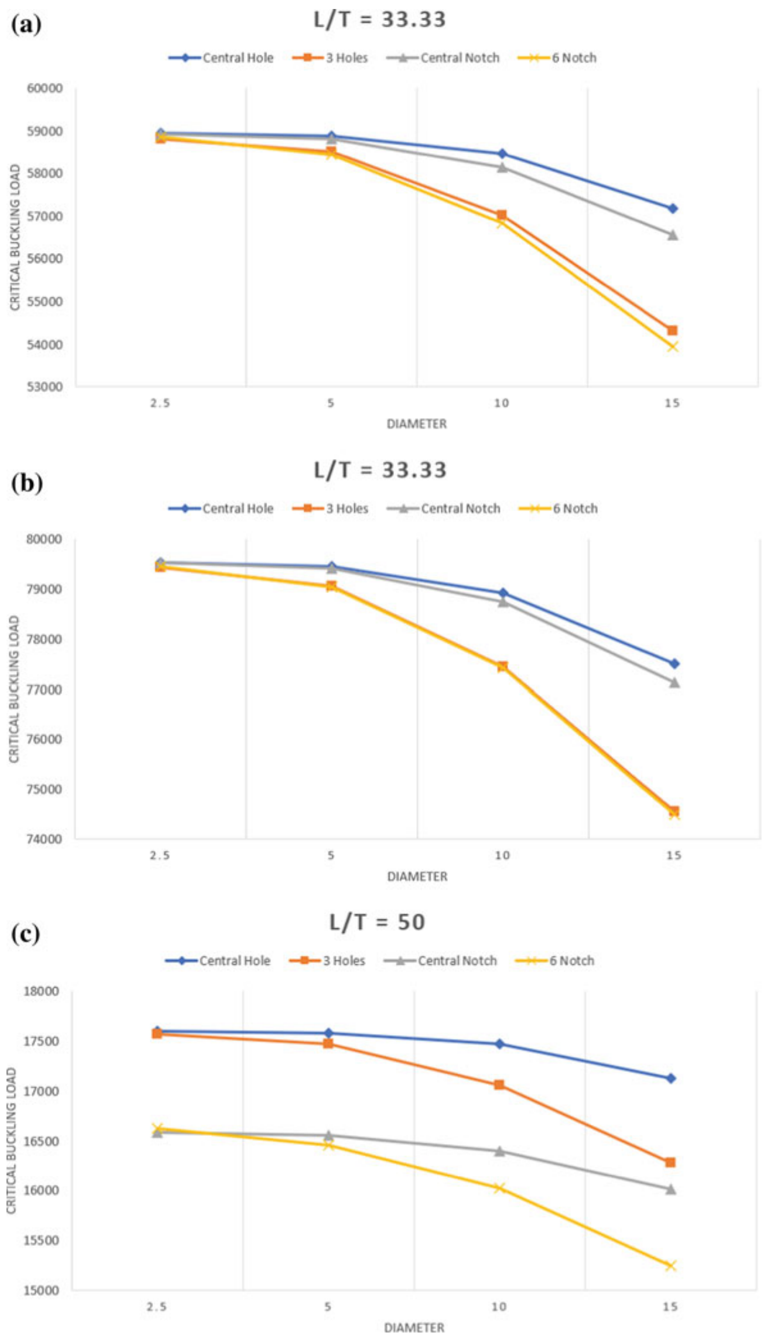


Fig. 7 Variation of buckling load for a plate of length 150 mm-mode 2 **a** CC condition width 30 **b** CC condition width 40 **c** CC condition width 30 **d** CC condition width 40 **e** CF condition width 30 **f** CF condition width 40 **g** CF condition width 30 **h** CF condition width 40

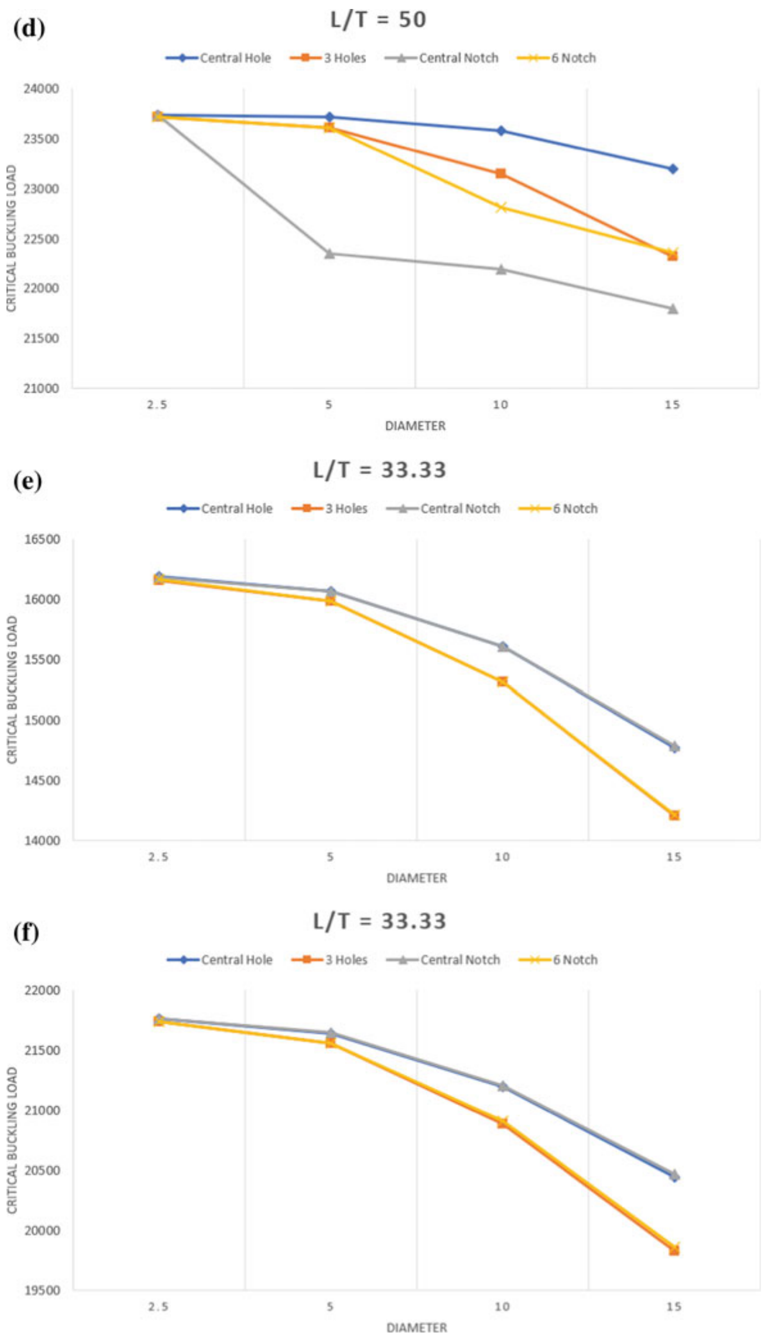


Fig. 7 (continued)

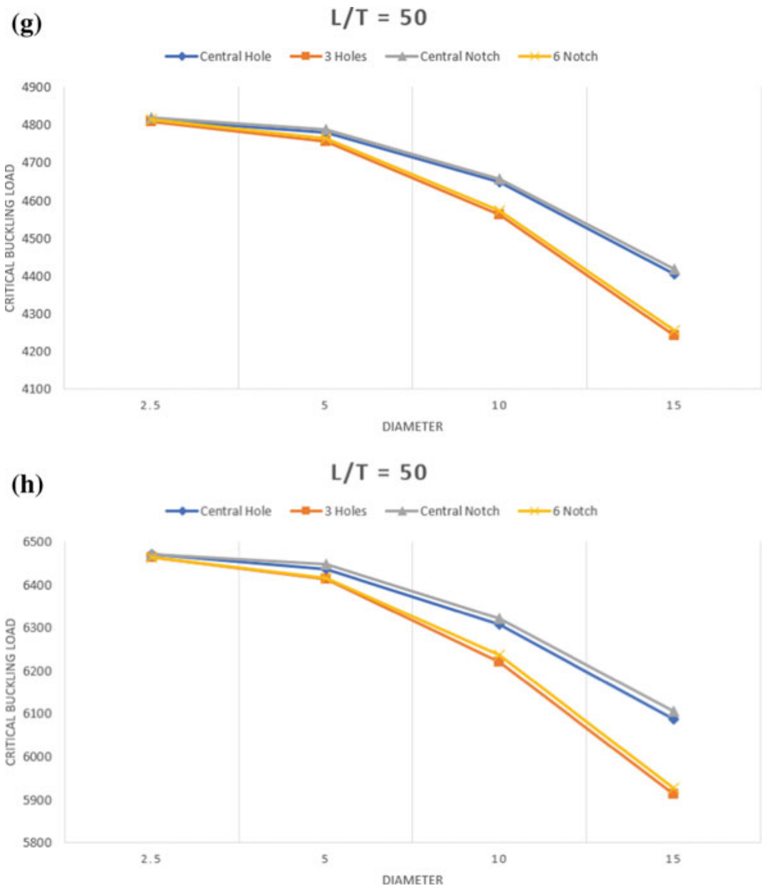


Fig. 7 (continued)

5.2 Effect of Length to Thickness Ratio

This part of the paper deals with the effect of length to thickness ratio on buckling load for different plate dimensions. As mentioned earlier in Table 2, totally eight different *L*/*T* ratios were used in this study. All the possible buckling loads in both Mode 1 and Mode 2 for different *L*/*T* ratios were tabulated in Tables 4, 5, 6, and 7. As mentioned earlier, buckling loads from simple plates are used to study the effect of the *L*/*T* ratio. As expected, the numerical analysis showed the reduction in buckling load for increasing *L*/*T* ratio which can be noticed in both Fig. 12 for Mode 1 and Fig. 13 for Mode 2. The reduction in buckling load as the *L*/*T* ratio increased from 16.66 to 100 was about 98% in all the cases on which the buckling analysis is studied. This drop has to be considered, whilst design components which include

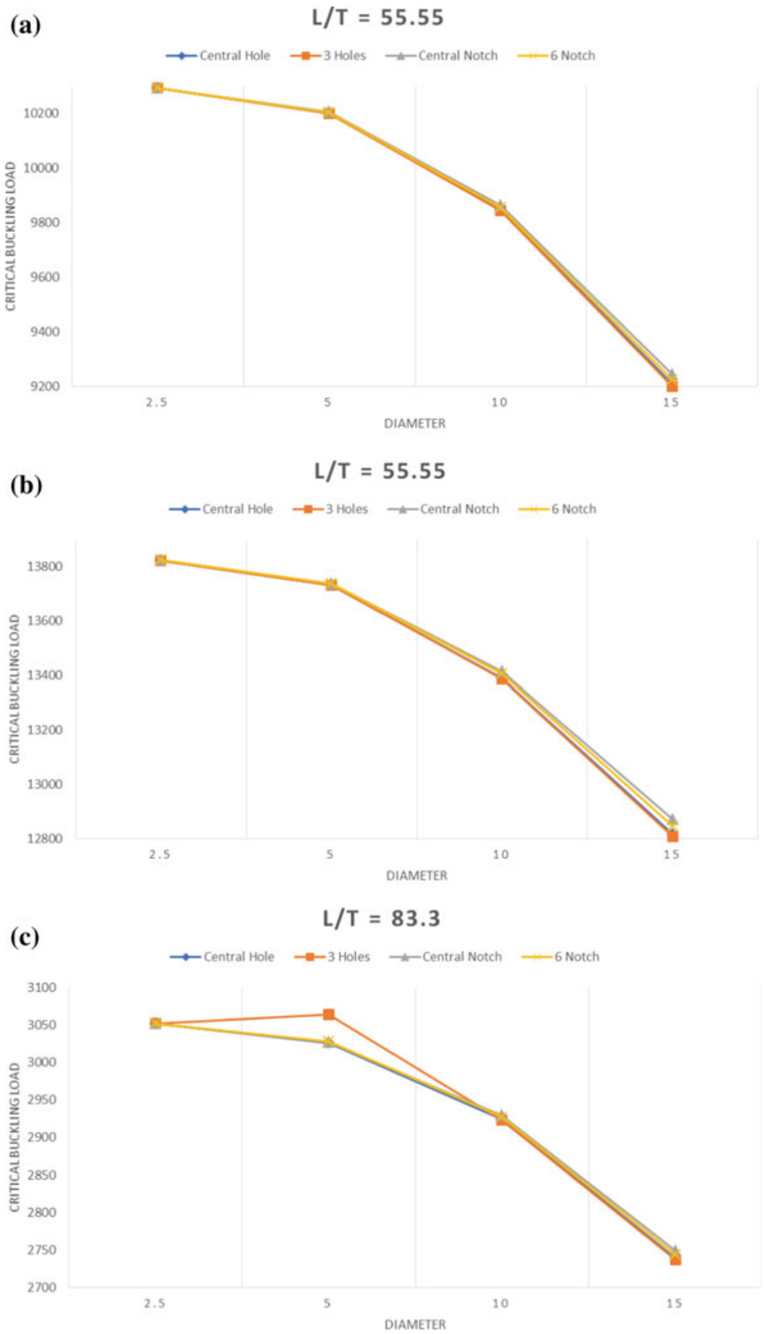


Fig. 8 Variation of buckling load for a plate of length 250 mm-mode 1 **a** CC condition width 30 **b** CC condition width 40 **c** CC condition width 30 **d** CC condition width 40 **e** CF condition width 30 **f** CF condition width 40 **g** CF condition width 30 **h** CF condition width 40

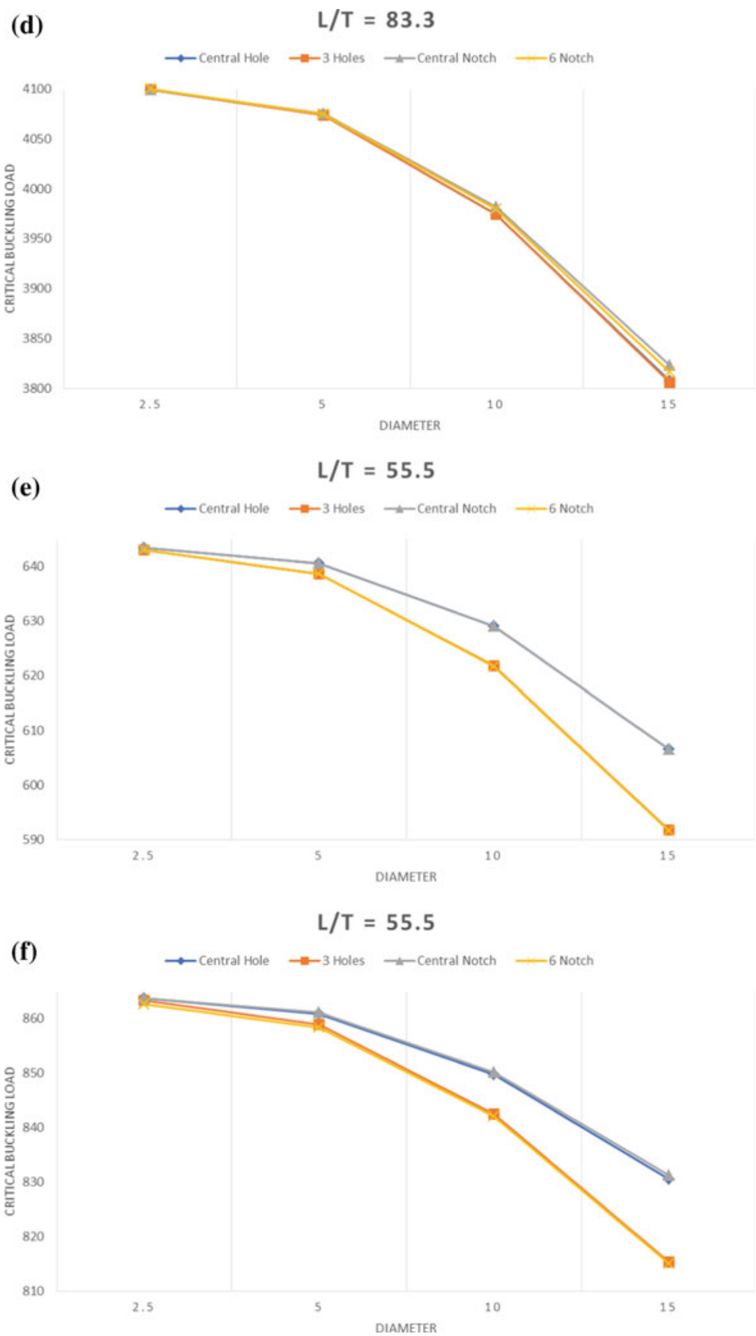


Fig. 8 (continued)

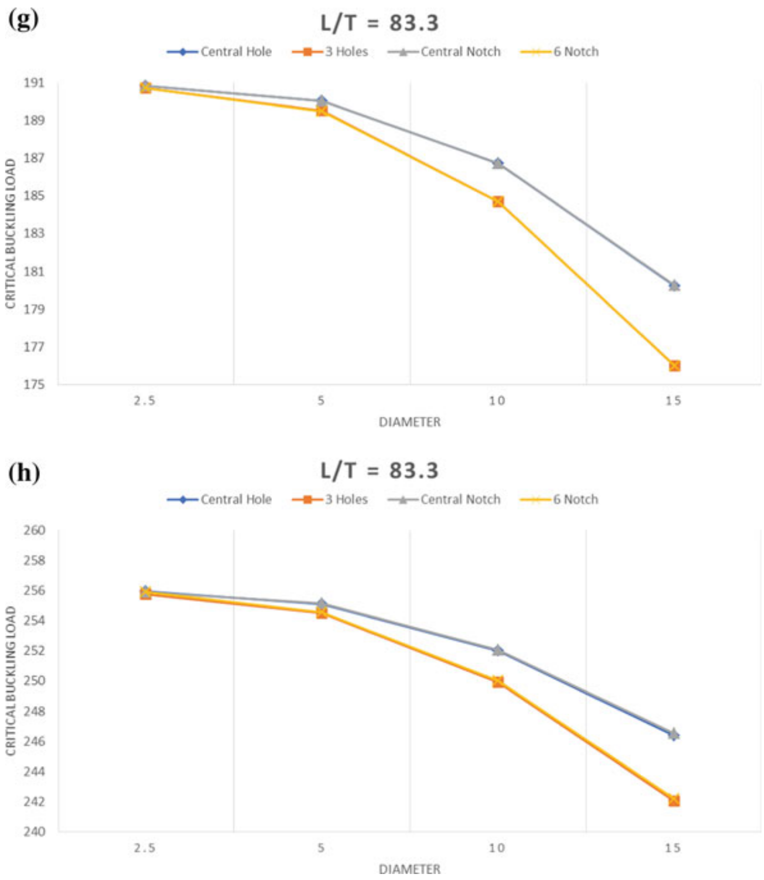


Fig. 8 (continued)

plates because the *L*/*T* ratio plays a major role in buckling load and an optimum *L*/*T* ratio must be picked in any case.

In many cases, the buckling loads of different cut-outs of the same diameter have very similar values under the same *L*/*T* ratio. Another special case can be noticed from Figs. 12 and 13, i.e. the buckling load slightly increased for all boundary conditions despite the width, when the *L*/*T* ratio was around 55.5 and then decreased gradually as the *L*/*T* ratio increased further.

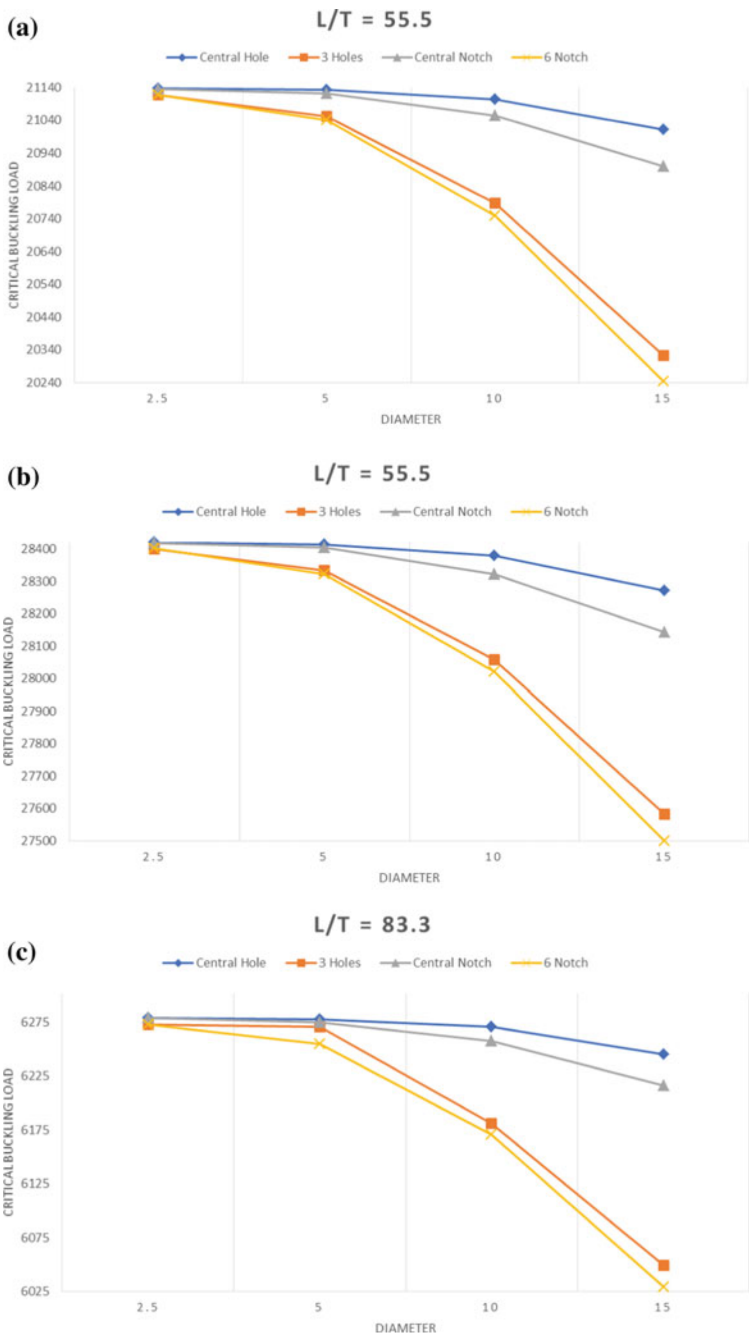


Fig. 9 Variation of buckling load for a plate of length 250 mm-mode 2 **a** CC condition width 30 **b** CC condition width 40 **c** CC condition width 30 **d** CC condition width 40 **e** CF condition width 30 **f** CF condition width 40 **g** CF condition width 30 **h** CF condition width 40

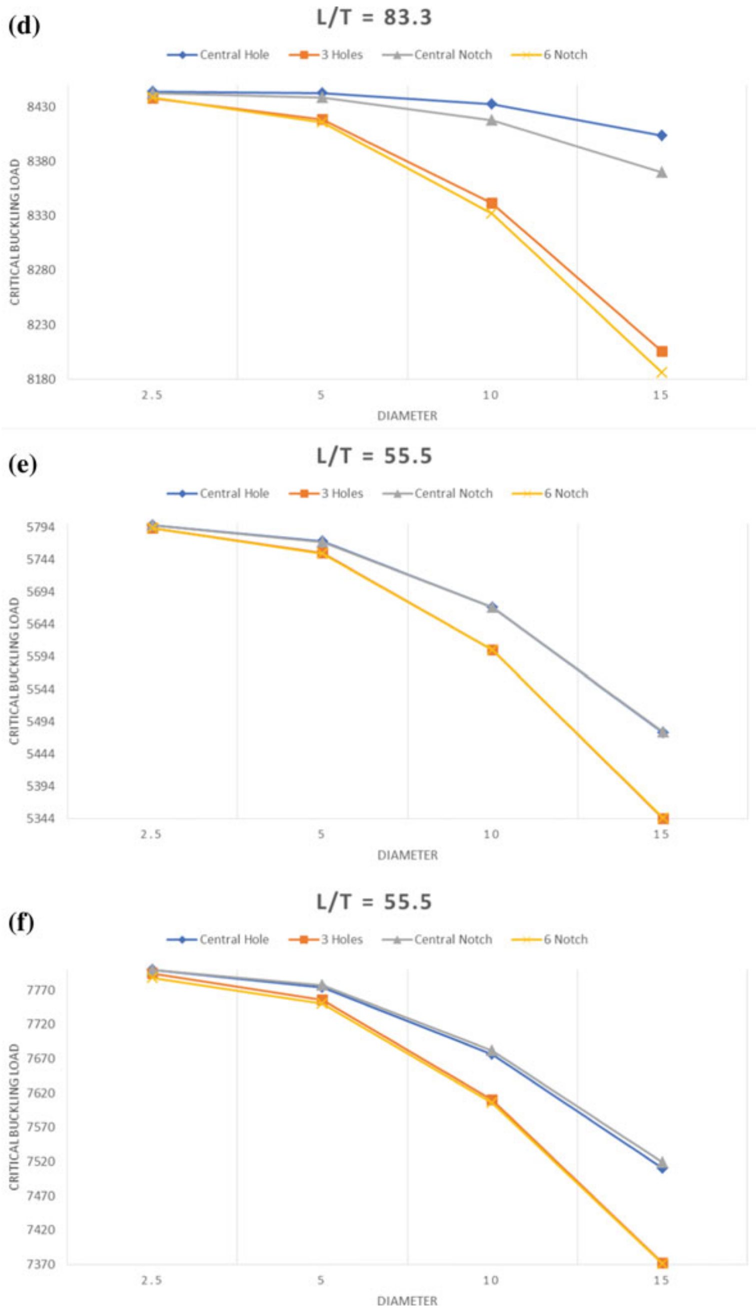


Fig. 9 (continued)

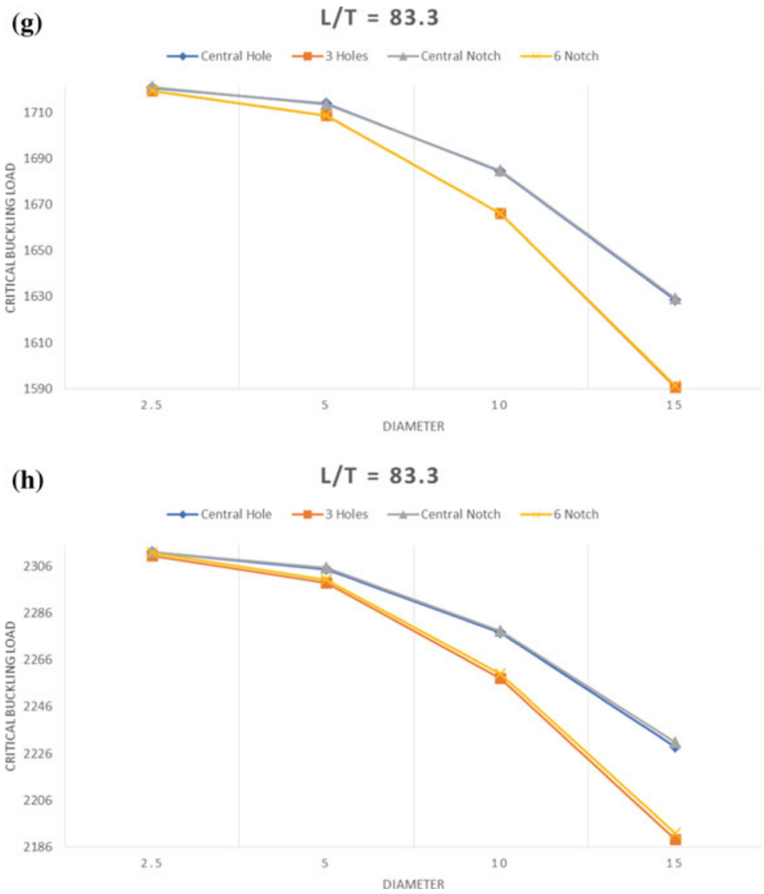


Fig. 9 (continued)

5.3 *Effect of Boundary Conditions*

In this study, the aluminium alloy plates are investigated with 2 different boundary conditions to exhibit different characteristics under buckling load. Boundary conditions have the highest effect on buckling load when compared to other parameters like cut-outs and *L*/*T* ratio. Tables 4, 5, 6, and 7 tabulates the buckling load of different plates under both CC and CF boundary conditions.

Figures 4, 5, 6, 7, 8, 9, 10 and 11 show the effect of boundary condition on rectangular plates under compression loads. It can be noticed from both table and figure that different boundary condition has a different influence on buckling load. In general, it can also be observed that the buckling loads under CC boundary conditions are much higher than the CF boundary conditions in all studied cases.

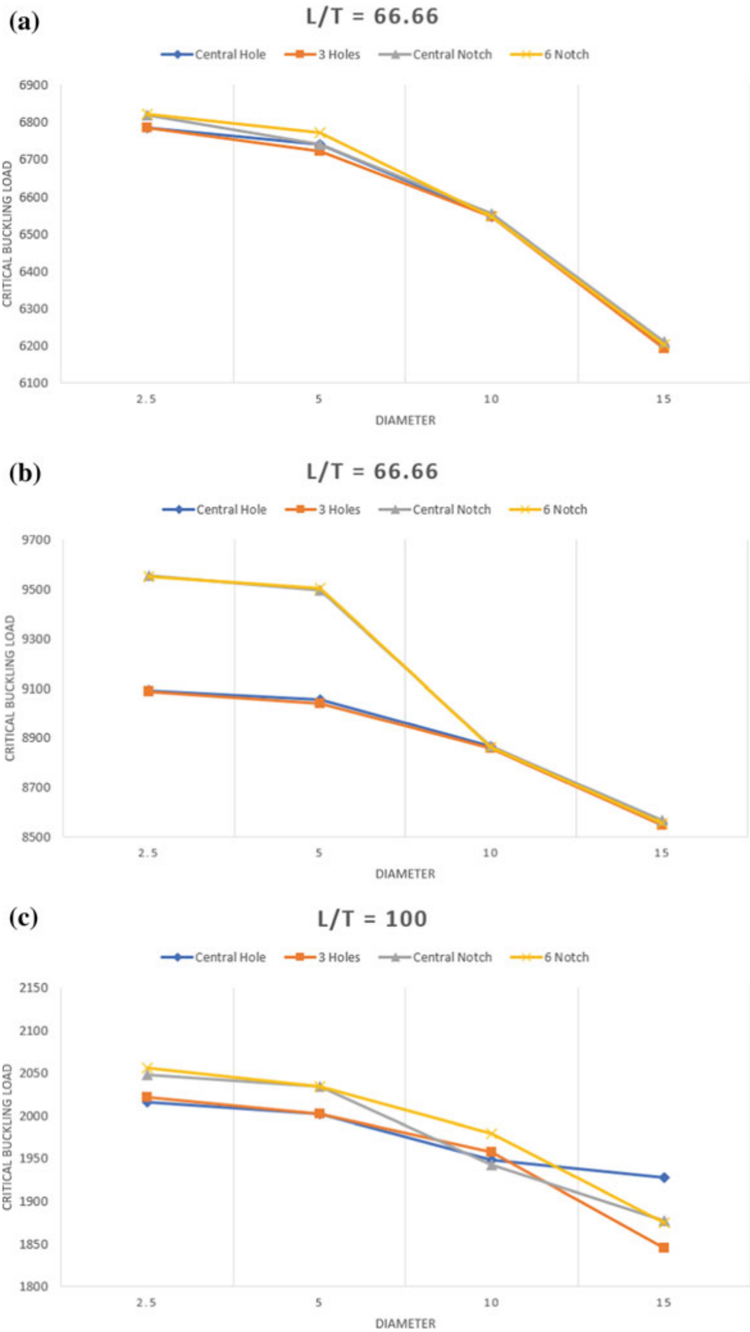


Fig. 10 Variation of buckling load for a plate of length 300 mm-mode 1 **a** CC condition width 30 **b** CC condition width 40 **c** CC condition width 30 **d** CC condition width 40 **e** CF condition width 30 **f** CF condition width 40 **g** CF condition width 30 **h** CF condition width 40

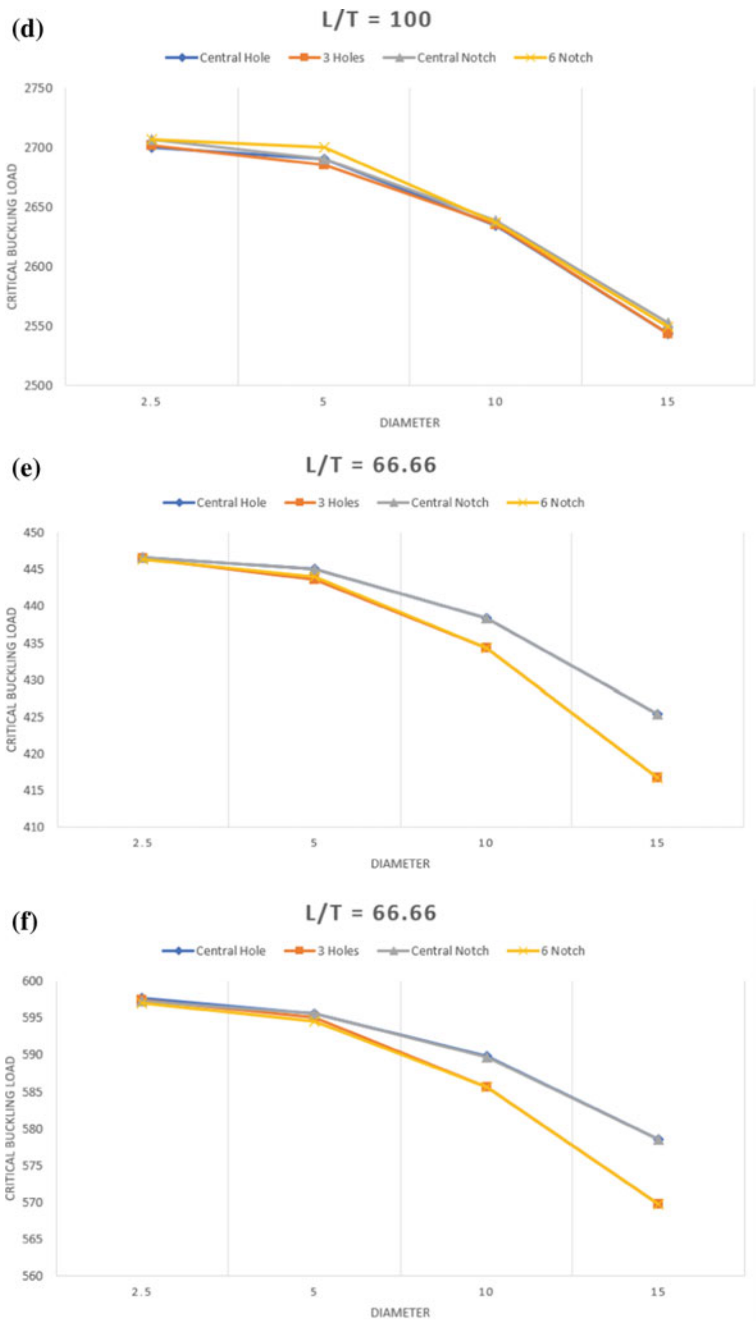
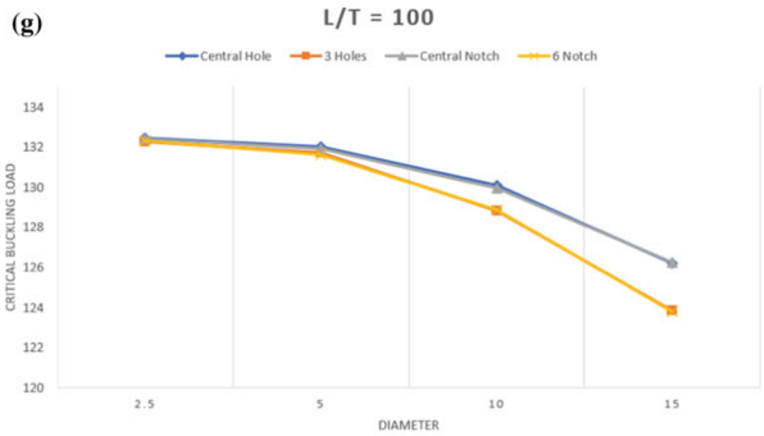


Fig. 10 (continued)



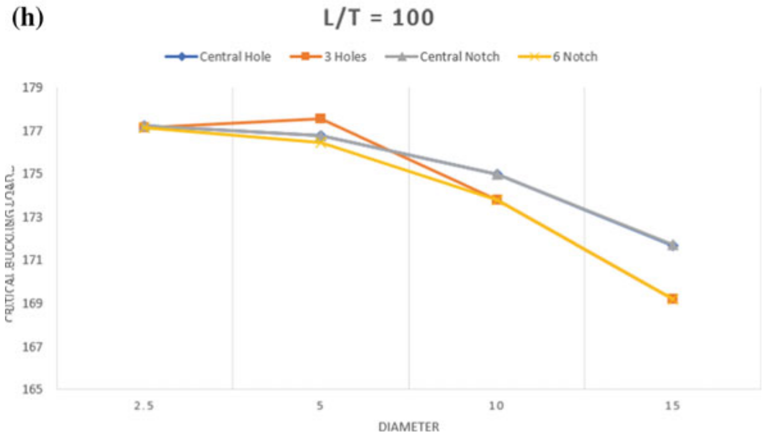


Fig. 10 (continued)

Results show that the buckling load of the plates impressively increased under all boundary conditions as the *L*/*T* proportion diminished. The distinction in buckling load brought about by the changing of *L*/*T* proportion is almost the equivalent for every one of the boundary conditions. The test results likewise foresee essentially a similar pattern. Due to the rigidity of the CC boundary condition, they usually possess a higher buckling load than the CF boundary condition despite the influence of other effects of cut-outs and *L*/*T* ratio.

From Fig. 12 and Fig. 13, we can notice that the buckling load for CC conditions is much greater than the CF conditions for chosen width and a similar trend is followed in both Modes of buckling analysis. When compared, there is a constant drop of about 70–75% in buckling load from CC boundary condition to CF boundary conditions for all *L*/*T* ratios.

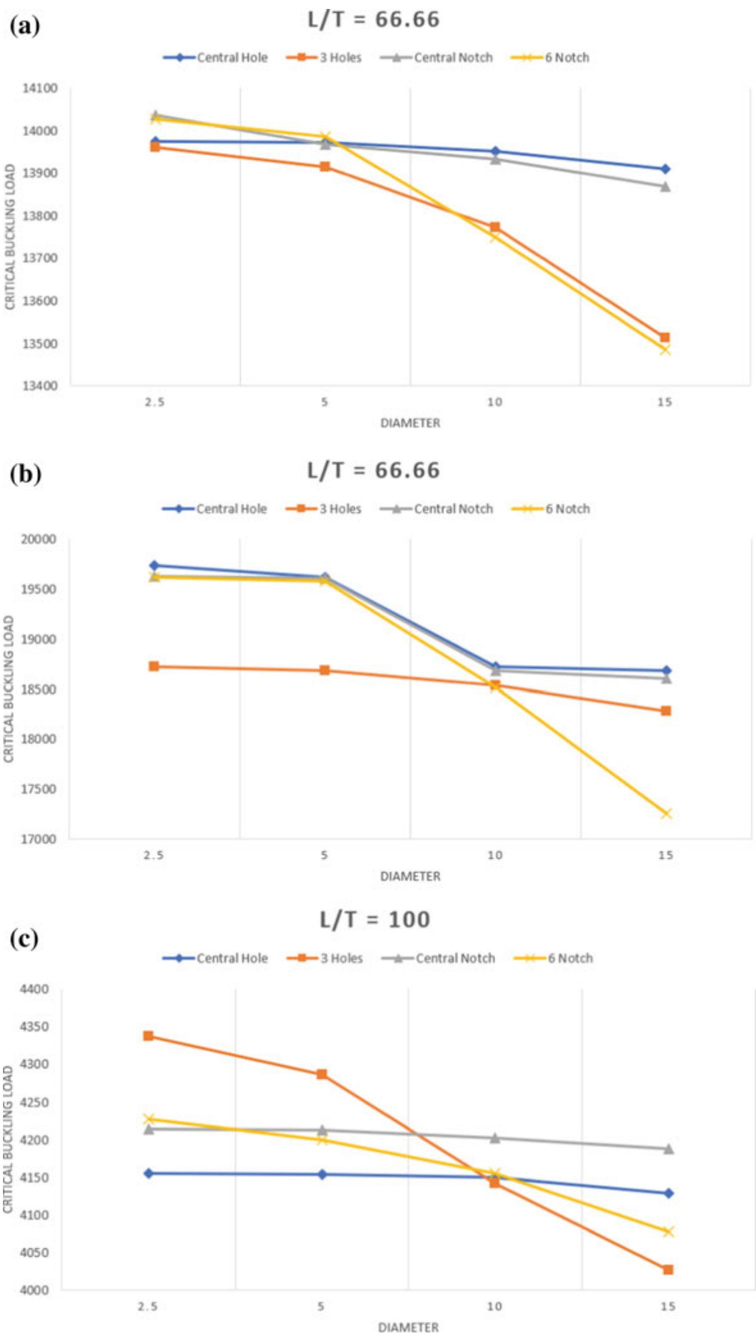


Fig. 11 Variation of buckling load for a plate of length 300 mm-mode 2 **a** CC condition width 30 **b** CC condition width 40 **c** CC condition width 30 **d** CC condition width 40 **e** CF condition width 30 **f** CF condition width 40 **g** CF condition width 30 **h** CF condition width 40

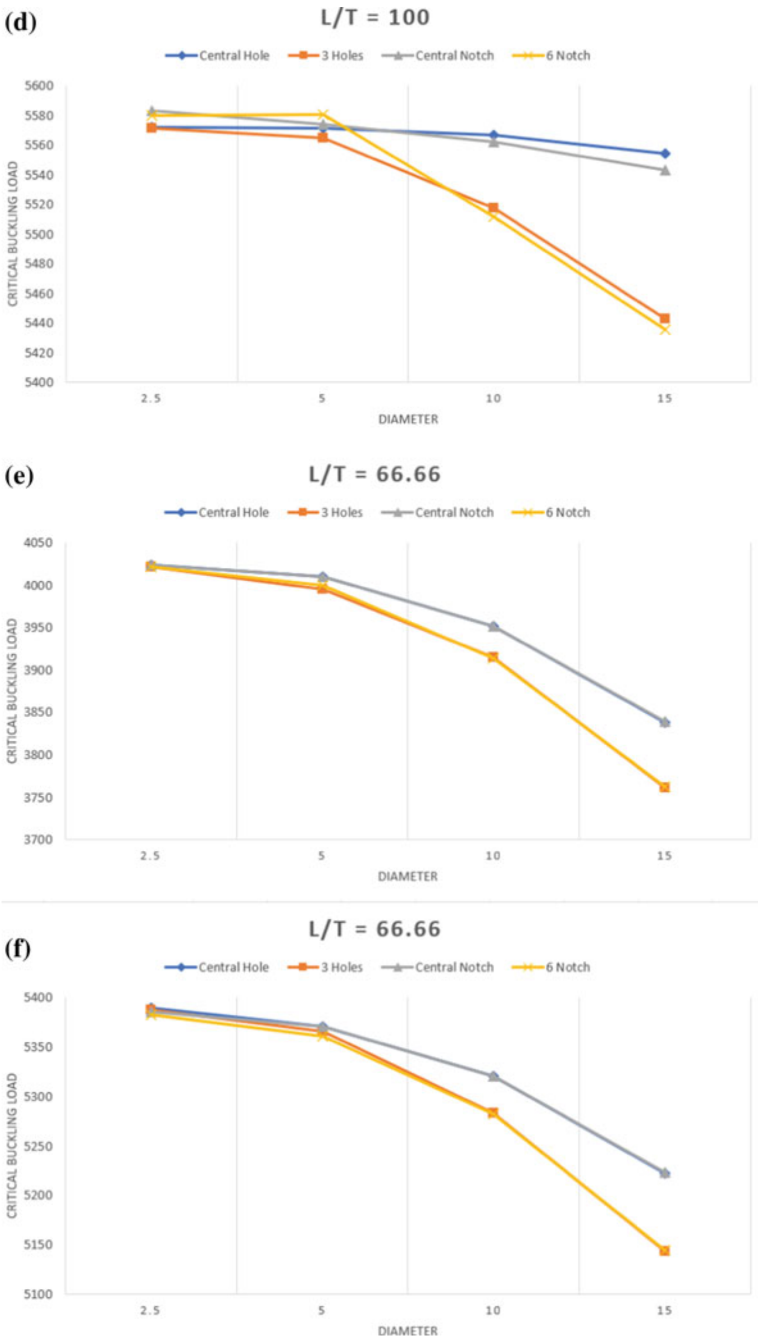


Fig. 11 (continued)

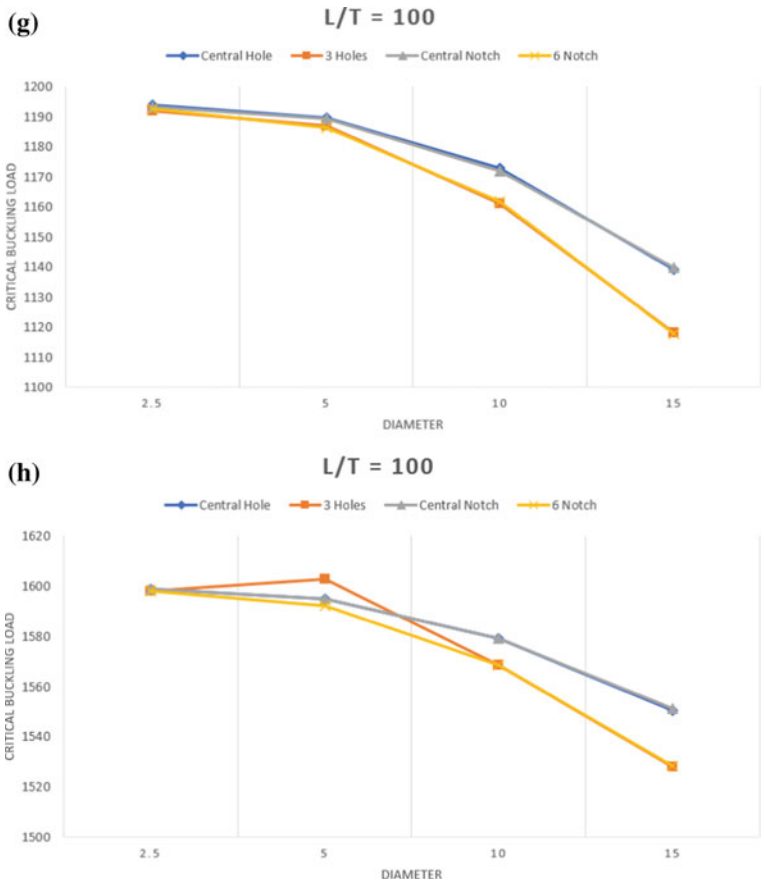


Fig. 11 (continued)

6 Conclusion

In this article, the buckling reaction of aluminium alloy rectangular plates with two different boundary conditions is considered. The considered rectangular plates have changed the aspect ratio, cut-out shape. The test results for the numerical analysis are obtained from ANSYS finite element code after proper validation of the integrity of this tool.

From the present numerical analysis study, the following conclusions can be made.

1. The diminishing buckling load for both Mode 1 and Mode 2 due to the presence of cut-outs was found to be prominent. With the increase in diameter of the various cut-outs, the buckling load was found to be dropping exponentially. This proves that the presence of cut-outs lowers the buckling load. From the

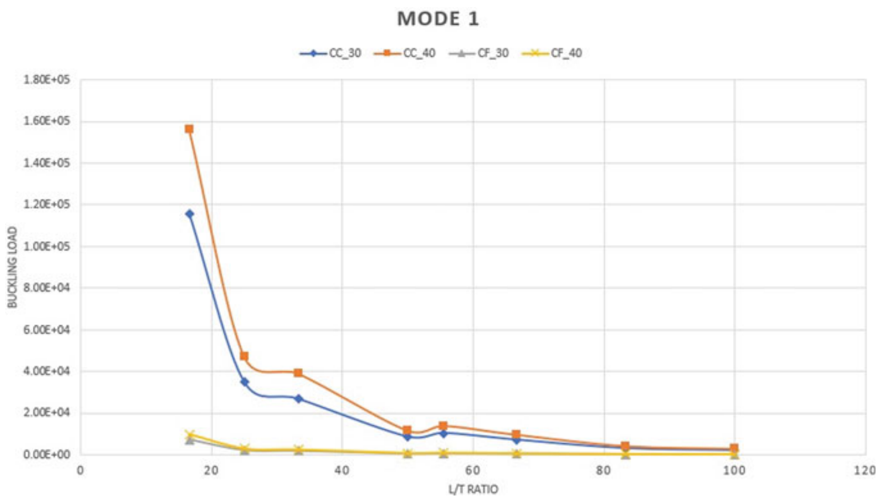


Fig. 12 Effect of *L*/*T* ratio on the rectangular plate for different boundary condition and widths (Mode 1)

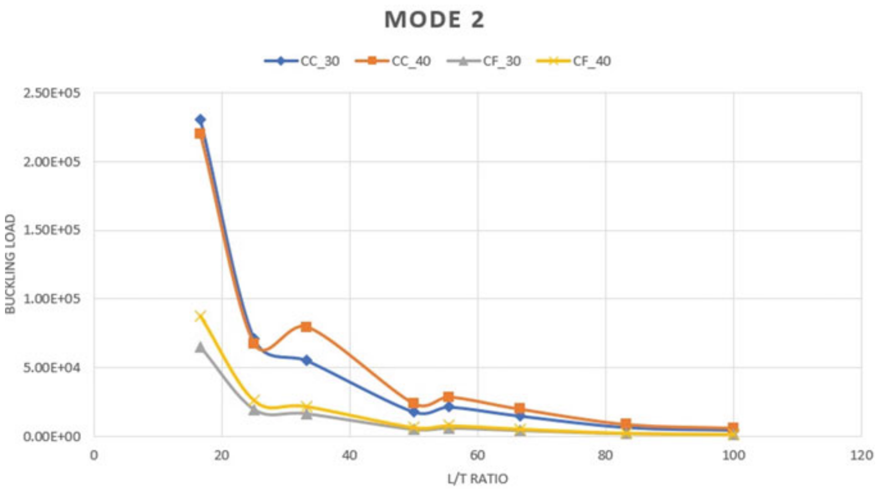


Fig. 13 Effect of *L*/*T* ratio on the rectangular plate for different boundary condition and widths (Mode 2)

discussion on the effect of cut-outs, we can also conclude that the effect of both holes and notches behaves similarly.

2. The increase in buckling load in Mode 2 for $L = 75$ mm and $W = 40$ mm in CC boundary condition was noticed because of the close dimensional parameters, as the length and width dimensions are close to each other and also the rigidity

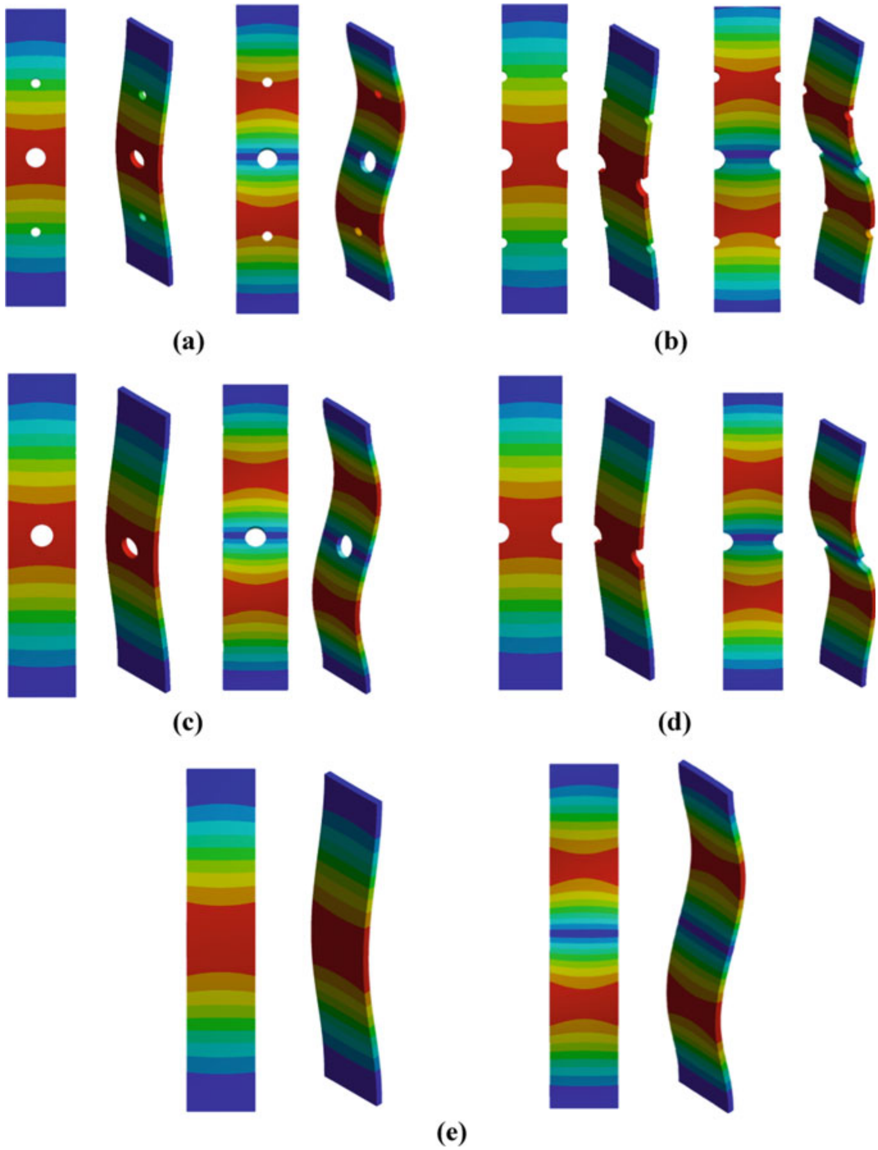


Fig. 14 Visualization of both mode 1 and mode 2 buckling in a rectangular plate with different cut-outs **a** cut-out with 3 holes **b** cut-out with 6 notches **c** cut-out with central hole **d** cut-out with central notch **e** simple plate with no cut-outs

increase for smaller plates in CC boundary conditions, combined effect of these two reasons are the conclusion behind the increase in buckling load.

3. As the *L*/*T* ratio increased, the buckling load in both Modes decreased drastically. From the earlier discussion, we can conclude that the buckling load decreased about 98% when the *L*/*T* ratio increased from 16.66 to 100.
4. The clamped boundary condition showed the highest buckling load amongst the other boundary conditions. As already mentioned, it is only because of the rigidity that arises when components are clamped at ends. The buckling loads of CF conditions are much smaller when compared to CC boundary conditions. We can notice a drop of the buckling load of about 70–75% from CC to CF boundary condition for all widths despite the change in *L*/*T* ratio.

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