Vibrational Analysis of Circular Plates with Square Cutout

AL. Muthuveerappan, C. Ajay, V. Dhakshain Balaji, Varun Gopalakrishnan, and Lokavarapu Bhaskara Rao

Abstract Isotropic materials find wide range of applications in the manufacturing industry. The present work focuses on studying the free vibration analysis in a circular plate made out of isotropic material with and without square cutouts. The system was generated and simulated using CAD modeling software and solved using a finite element method. The variations in the vibrational frequencies of isotropic plates composed of mild steel, cork, and rubber are observed with respect to changes in the height and radius of the plate. The effects of the dimensions of the square cutout on the frequencies of the plates under different modes of vibrations are also tested and tabulated. The frequencies of non-perforated plate increase with increase in height of plate and decrease with increase in radius of plate. In plates with square cutout, the frequencies exhibited a wave-like trend as the cutout dimension was varied.

Keywords Circular plate · Cutout · Vibration analysis · Modal diagram · Finite element analysis

1 Introduction

Various mechanical structures have circular plates with cutouts [1]. These structures have multiple applications in the fields like aerodynamics, aircraft manufacturing, and weapons manufacturing [2]. Cutouts are used for inserting fasteners, accessing

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G. S. V. L. Narasimham et al. (eds.), *Innovations in Mechanical Engineering*, Lecture Notes in Mechanical Engineering,
https://doi.org/10.1007/978-981-16-7282-8_12

ports and other internal structures, setting up electrical lines, ventilation, and system weight reduction [3]. Vibration analyses of such components are performed to identify structural parameters and prevent damages [1]. The inclusion of cutouts in plate-like structures changes the frequency response appreciably. Therefore, there is a need to conduct free vibration analysis of plates with cutouts. Literature review suggest several works done in the field of vibrational analysis of plates with cutouts, and a selected list of those studies is briefly reviewed as follows.

Lee et al. [4] deduced the natural frequencies of a rectangular plate with an arbitrarily placed rectangular cutout using the Rayleigh quotient method. This study was performed under different modes, and it was concluded that this method has the potential to serve as an alternative to finite element analysis. Ramakrishna et al. [5] analyzed the free vibrational characteristics of laminates with circular cutouts using the eight-noded hybrid-stress finite element method under different boundary conditions. By varying the orientation of fiber and the dimensions of the cutout, the different frequency responses were studied. Ju et al. [6] employed the finite element method and studied the free vibrational characteristics of circular and square plates with delamination around cutouts. Through this study, the delamination on the natural frequencies of circular and square plates with cutouts and its mode-dependency was established. Boay [7] studied the free vibrational analysis of symmetrically laminated composite plates with a central hole. The materials and the stacking sequence used by Boay in his study are those which find their applications in aircraft structures. His study also validates the accuracy of the results as obtained by the finite element analysis with experimental data. Sivakumar et al. [8] came up with a genetic algorithm that aids in the optimized design of laminated composite plates. This genetic algorithm worked with any number of variables and any type of design constraints. Turvey et al. [9] conducted experiments to determine the mode shapes and free vibrational frequencies of thick pultruded glass reinforced plastic (GRP) square plates with boundary conditions in varied combinations. Their study confirmed that the prediction of free vibrational response of the GRP plates can be modeled based on the thin homogenous orthotropic/anisotropic plate theory.

Liew et al. [10] conducted a three-dimensional free vibration analysis of perforated superelliptical plates by applying elasticity theory and p-Ritz algorithm. Alongside parametric investigations, the approach provided a complete vibration spectrum for the analysis of given structures. Sai Ram et al. [11] studied free vibrations using finite element method based on higher-order deformation theory in a composite spherical shell cap with and without cutout. The frequency trends with respect to number of plies, boundary conditions, radius to thickness ratio, and cutout size of composite spherical shell were analyzed. Cheng et al. [1] studied the effects of eccentricity, hole size, and boundary conditions on vibration modes of annular-like plates using both numerical and experimental approaches. Kwak et al. [12] employed an independent coordinate coupling method to conduct free vibration analysis of a rectangular plate with hole. The results shown by the novel method were verified using the classical Rayleigh–Ritz method. Jhung et al. [13] performed free vibration analysis of a circular plate with an eccentric hole submerged in fluid based on finite

Fourier–Bessel series expansion and Rayleigh–Ritz method. The study threw light on the effect of fluid and eccentric holes on modal characteristics.

Lee et al. [14] used the indirect boundary integral formulation and addition theorem in an analytical model derived as a coupled infinite system of simultaneous linear algebraic equations to conduct free vibration analysis of a circular plate having multiple circular holes. The numerical results agreed with finite element method (FEM) results with good accuracy and a faster rate of convergence. Maziar et al. [15] utilized finite element method to study the free vibration analysis of functionally graded plates with multiple circular and non-circular cutouts. Effects of boundary conditions and cutout size, location and number were obtained through parametric studies. Choudhary et al. [16] performed the vibrational analysis of a square plate with a circular hole at different boundary conditions using finite element method. Parametric study concluded that an increase in hole diameter resulted in decrease of natural frequency. Bhardwaj et al. [3] conducted free vibration analysis of laminated composite plates with triangular cutouts using Ansys software. Their parametric study revealed the effect of various geometric and material properties on natural frequency. Xue et al. [17] conducted free vibration analysis of porous plates with a centrally located circular hole using isogeometric analysis. The porosity distributions were formulated using non-uniform rational B-spline basis functions and described by first-order shear deformation theory. Investigations into the effects of porosity coefficient, geometric parameters, and boundary conditions on the free vibrations were studied. Vibrations of circular plate with concentric ring and rigid ring support have been studied [18–20]. Exact frequency analysis of annular plates with small core has been studied by Rao and Rao [21, 22]. The frequency of restrained circular plates with weakened interior circle has been studied by Rao and Rao [23–25]. The fundamental frequency of circular plate and annular plate with elastic edge has been studied by Rao and Rao [26–30].

A general conclusion from the above-conducted literature survey is the finite element analysis (FEA) is proven to be an efficient and a reliable method to perform vibrational studies of various composite and isotropic plates, irrespective of the geometry of the plate, and the presence of cutouts. Though numerous works have been done over the years in this field, there is a need to understand the vibrational characteristics of cork-made and rubber-made isotropic plates too. Extending this understanding by considering the presence of cutouts in these plates would enhance their scope of applications in various domains. On being able to provide a comparative analysis between mild steel, cork, and rubber plates with square cutouts, the results of this study suggest a holistic understanding of the different vibrational characteristics of each of these plates, in addition to determining the relations between the geometric specifics of the plate and its vibrational frequencies as well.

2 Materials and Methods

The free vibration analysis of circular plates made of isotropic material with and without square-shaped cutouts was conducted. The materials used in this study are mild steel, cork, and rubber, and their respective properties are given in Table 1. It is to be noted that the materials are selected to cover the spectrum of possible values of Poisson's ratio.

The models were generated using SolidWorks 2020 [31]. Circular plate with different radius values, *R*, and different thickness values, *h*, with square cutouts of different side lengths, a, is taken into consideration. The models were imported into Ansys 19.2 [32], where the natural frequencies were obtained using finite element methods. The non-perforated and perforated plates used in this study for the material sample of mild steel are shown in Figs. 1 and 2, respectively.

Table 1 Material properties

Property	Mild Steel	Cork	Rubber
Density (g/cm^3)	7.85	0.15	0.95
Thermal expansion (K^{-1})	12×10^{-6}	200×10^{-6}	6.1
Young's modulus (Pa)	2×10^{11}	2×10^{7}	1×10^{6}
Poisson's ratio	0.32	0	0.4999

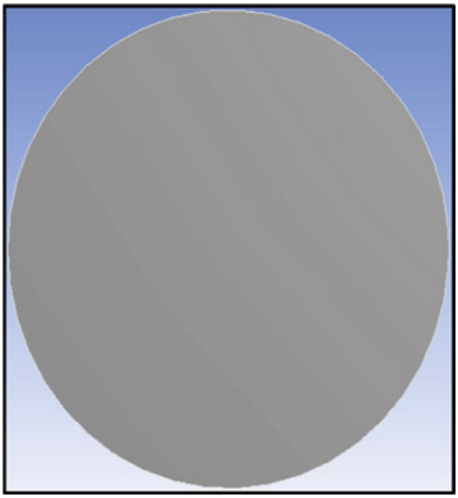


Fig. 1 Non- perforated plate

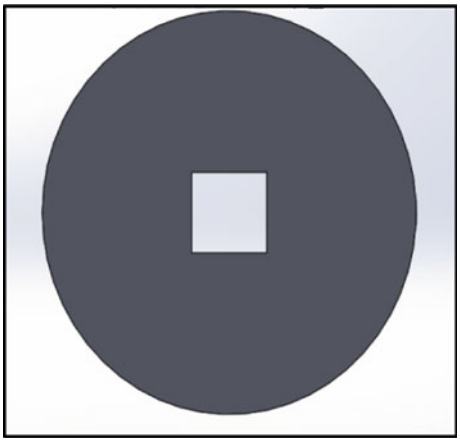


Fig. 2 Perforated plate

3 Validation and Convergence

Free vibrational characteristics of non-perforated mild steel circular plate were analyzed using FEM and compared with the results obtained by Bhatnagar et al. [33]. From Table 2, it is clear that the data as obtained in the present work and Bhatnagar et al. lie within a difference range of 13% which suggests the validity of the method of approach followed us.

Using the same method, the free vibration characteristics of a circular plate composed of cork without any cutouts were studied with the constant parameter being the height of the plate and the variable parameter being the radius of the plate. The results as given in Table 3 suggest the ability of the method to produce converging results. It is also important to note that despite increasing the number of elements analyzed under the method, there is no deviation in the final output obtained in terms of vibrational frequencies under different modes. Thus, the method of approach as followed in the present work is proven to produce reliable converging results, as validated by Bhatnagar et al. [33].

Table 2 Validation of data with reference to Bhatnagar et al.

Mode	Bhatnagar et al. [33]	Present data	%Difference
1	973.43	975.44	0.21
2	1975.5	1860.3	6
3	1976.3	1861.5	5.98
4	3153.3	2812.4	11.42
5	3154.5	2813.2	11.43
6	3578.8	3145.7	12.88

Table 3 Analysis of a cork-made circular plate with no cutouts and varying radius

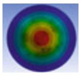
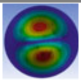
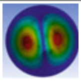
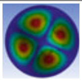
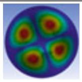
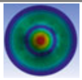
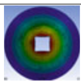
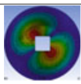
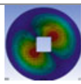
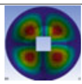
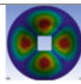
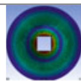
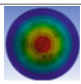
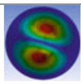
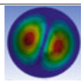
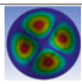
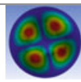
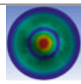
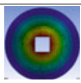
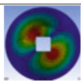
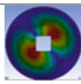
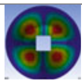
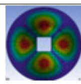
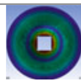
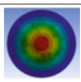
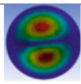
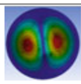
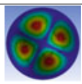
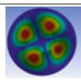
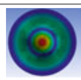
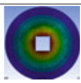
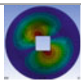
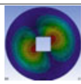
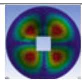
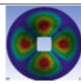
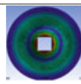
Cork	No. of elements	Frequency mode					
Radius		1	2	3	4	5	6
75	2538	294.43	593.83	593.84	941.85	941.86	1064.9
	4626	294.35	593.55	593.56	941.21	941.22	1064.2
	5370	294.35	593.54	593.55	941.19	941.19	1064.1
	6114	294.35	593.54	593.54	941.16	941.17	1064.1
	7578	294.35	593.53	593.53	941.14	941.14	1064.1
125	4030	108.31	222.66	222.66	360.24	360.24	409.36
	4492	108.31	222.66	222.66	360.23	360.23	409.36
	5190	108.31	222.66	222.66	360.23	360.23	409.35
	5626	108.31	222.66	222.66	360.22	360.23	409.35
	9825	108.29	222.62	222.62	360.13	360.13	409.23
175	4388	55.599	114.97	114.97	187.22	187.22	213.1
	4904	55.599	114.97	114.97	187.22	187.22	213.1
	5788	55.599	114.97	114.97	187.22	187.22	213.1
	6760	55.599	114.97	114.97	187.22	187.22	213.1
	8066	55.599	114.97	114.97	187.22	187.22	213.1
225	5480	33.72	69.903	69.903	114.15	114.15	130.03
	5682	33.72	69.903	69.903	114.15	114.15	130.03
	6652	33.72	69.903	69.903	114.15	114.15	130.03
	7352	33.72	69.903	69.903	114.15	114.15	130.02
	8514	33.72	69.903	69.903	114.15	114.15	130.02
275	3035	22.611	46.947	46.947	76.806	76.806	87.526
	3409	22.611	46.947	46.947	76.806	76.807	87.526
	3510	22.61	46.947	46.947	76.806	76.806	87.526
	8136	22.602	46.915	46.915	76.727	76.727	87.427
	8124	22.603	46.915	46.915	76.727	76.727	87.427

4 Results and Discussion

On performing FEM analysis using Ansys on circular plates composed of mild steel, cork, and rubber, the following results were obtained depending on the presence of a centrally located square cutout.

Table 4 provides a collection of modal diagrams observed in perforated and non-perforated plates for height 10 mm, radius 250 mm, and the area of the cutout being $100 \times 100\ \text{mm}^2$.

Table 4 Modal diagrams for plates with and without perforations under all modes of vibrations

Material	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Mild steel						
Mild steel with cutout						
Cork						
Cork with cutout						
Rubber						
Rubber with cutout						

4.1 Non-Perforated Plate

Non-perforated plate with fine meshing was considered as shown in Fig. 3 for analysis. Having the radius of the plate set at 250 mm, the vibrational frequencies corresponding to the first 6 modes were obtained for mild steel, cork, and rubber, when the height of plate was varied, results of which are shown, respectively, in Tables 5, 6 and 7. In all of the cases, the frequency under a particular mode increases upon increasing the height of the plate as seen, respectively, in Figs. 4, 5 and 6. The aforementioned graphs throw light on the influence of height on the vibrational frequency.

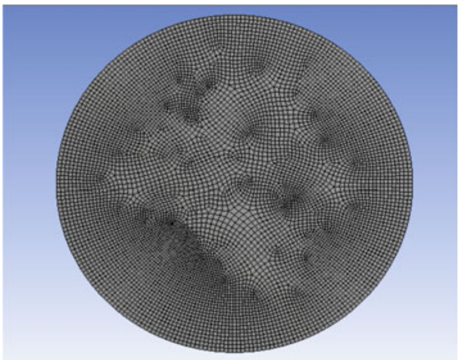


Fig. 3 Meshed non-perforated plate

Table 5 Vibration characteristics of mild steel circular plate with no cutouts and a fixed radius of 250 mm

Mild steel	Frequency mode					
Height (mm)	1	2	3	4	5	6
3	120.17	249.99	250	409.94	409.95	467.42
5	200.06	415.91	415.92	681.44	681.46	776.85
10	398.96	826.78	826.78	1349.7	1349.7	1537.3
25	976.72	1983.4	1983.4	3167.5	3167.6	3588.6
50	1822.3	3506.5	3506.7	5336.3	5336.5	5975.4
100	2963.9	5163.6	5163.9	6660.8	6661	7386

Table 6 Vibration characteristics of cork-made circular plate with no cutouts and a fixed radius of 250 mm

Cork	Frequency mode					
Height (mm)	1	2	3	4	5	6
3	2.6008	5.4112	5.4112	8.8743	8.8743	10.119
5	4.3327	9.0106	9.0106	14.769	14.769	16.838
10	8.6437	17.932	17.932	29.307	29.307	33.39
25	21.254	43.411	43.411	69.742	69.742	79.113
50	40.263	78.734	78.734	121.4	121.4	136.3
100	68.161	121.73	121.74	152.55	152.55	176.25

Table 7 Vibration characteristics of rubber-made circular plate with no cutouts and a fixed radius of 250 mm

Rubber	Frequency mode					
Height (mm)	1	2	3	4	5	6
3	1.205	2.5061	2.5074	4.1097	4.111	4.6866
5	2.0088	4.1751	4.1758	6.8392	6.8399	7.7965
10	4.0091	8.2996	8.3008	13.533	13.535	15.411
25	9.8607	19.879	19.908	31.542	31.555	35.702
50	18.074	34.103	34.136	51.114	51.126	57.114
100	28.2	47.73	47.751	64.622	65.053	65.07

The vibration characteristics were also analyzed by having height fixed at h = 10 mm, while varying the radius of the circular plate without any cutouts. The results show that on increasing the radius of the circular plate, the frequency with which it vibrates decreases, and this has been duly tested under different modes of vibration as well. The results of this test for mild steel, cork, and rubber are shown in Tables 8,

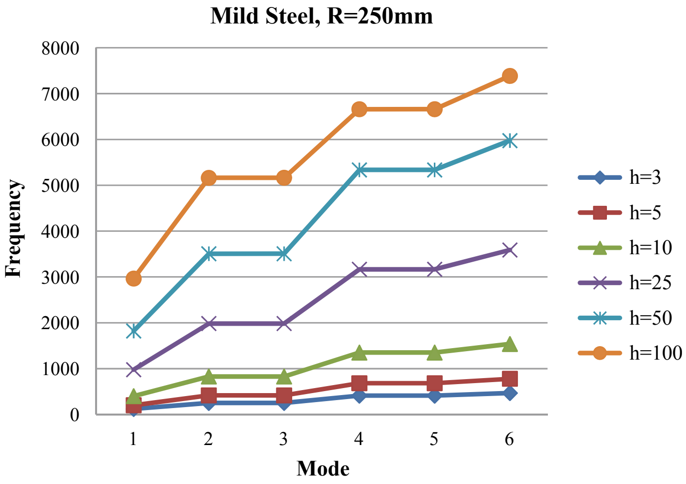


Fig. 4 Vibration analysis of non-perforated mild steel plate with variable heights and a fixed radius of 250 mm

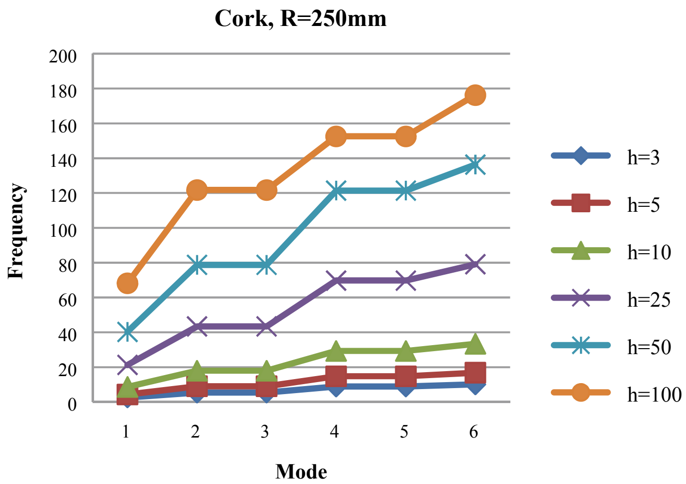


Fig. 5 Vibration analysis of non-perforated cork plate of variable heights and a fixed radius of 250 mm

9 and 10, respectively, and also graphically presented, respectively, in Figs. 7, 8 and 9. The trends can be noticed without fail in the graphs.

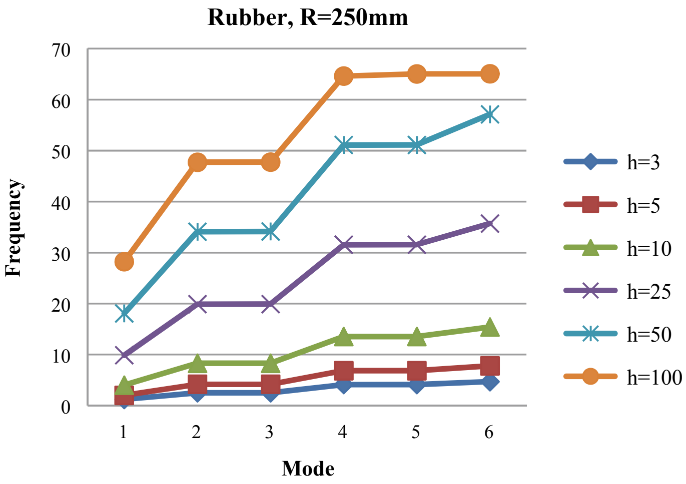


Fig. 6 Vibration analysis of non-perforated rubber plate with variable heights and a fixed radius of 250 mm

Table 8 Vibration characteristics of non-perforated mild steel circular plate and a height of 10 mm

Mild steel	Frequency mode					
Radius (mm)	1	2	3	4	5	6
75	4263.2	8517	8517.7	13,387	13,387	15,108
125	1577.7	3230.7	3230.9	5204.9	5205	5909.4
175	811.62	1674.9	1675	2721.3	2721.3	3095.9
225	492.46	1019.6	1019.6	1662.7	1662.7	1893.3
275	330.18	684.74	684.79	1118.8	1118.8	1274.6

Table 9 Vibration characteristics of non-perforated cork-made circular plate and a height of 10 mm

Cork	Frequency mode					
Radius (mm)	1	2	3	4	5	6
75	294.35	593.53	593.53	941.14	941.14	1064.1
125	108.29	222.62	222.62	360.13	360.13	409.23
175	55.599	114.97	114.97	187.22	187.22	213.1
225	33.72	69.903	69.903	114.15	114.15	130.02
275	22.603	46.915	46.915	76.727	76.727	87.427

4.2 Perforated Plate

Perforated plate with fine meshing was considered as shown in Fig. 10 for analysis. A centrally located square-shaped cutout with variable dimensions was incorporated in

Table 10 Vibration characteristics of non-perforated rubber-made circular plate and a height of 10 mm

Rubber	Frequency mode					
Radius (mm)	1	2	3	4	5	6
75	30.237	59.732	59.802	92.95	92.97	104.73
125	11.247	22.912	22.952	36.757	36.777	41.707
175	5.7921	11.925	11.932	19.331	19.334	21.984
225	3.5123	7.26	7.2667	11.826	11.828	13.464
275	2.3572	4.882	4.8884	7.9719	7.9771	9.083

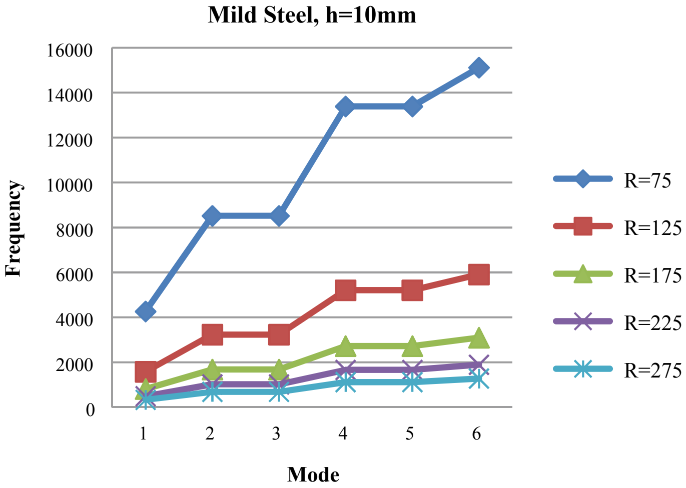


Fig. 7 Vibration analysis of non-perforated mild steel plate of variable radius and a fixed height of 10 mm

the middle of the plate of radius 250 mm and a height of 10 mm. The dimension of the cutout was varied from 2.5 mm to 100 mm for all mild steel, cork, and rubber plates. The vibrational frequencies at first six modes were calculated and given in Tables 11, 12 and 13. In all materials, different modes of frequencies exhibited wave-like trends when the dimension of cutout was varied. This can be observed in Figs. 11, 12 and 13.

5 Conclusion

FEA proved to be an efficient method to perform vibration analysis of three-dimensional isotropic circular plates with and without cutouts. Mild steel, cork, and rubber were the materials used in the study, where the height of the plate as

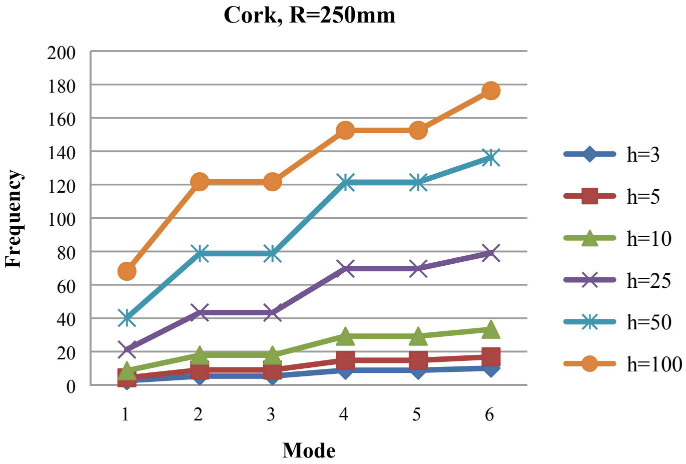


Fig. 8 Vibration analysis of non-perforated cork plate with variable radius and a fixed height of 10 mm

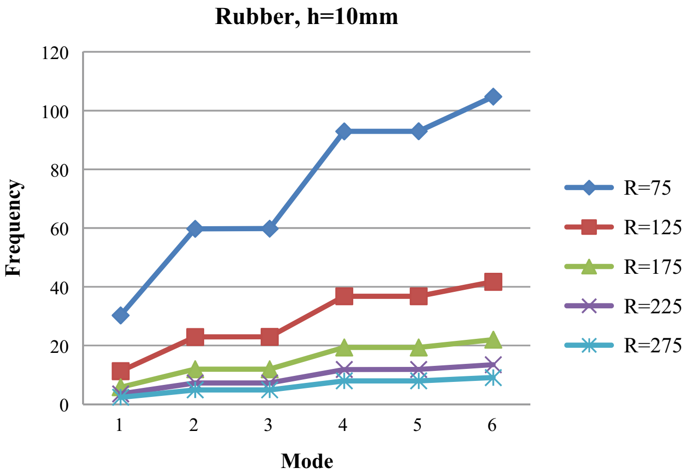


Fig. 9 Vibration analysis of non-perforated rubber plate of variable radius and a fixed height of 10 mm

well as its radius was interchangeably used as the variable and the fixed parameters. Irrespective of the material used, a direct relationship was observed between the frequency of vibration of the plate and the height of the plate, whereas an inverse relationship was noted between the frequency of vibration of the plate and its radius; both of which were tested in a solid circular isotropic plate without any cutouts.

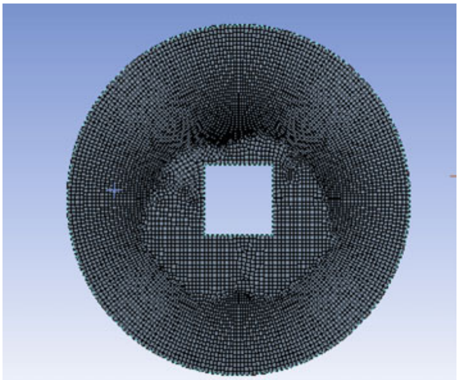


Fig. 10 Meshed perforated plate

Table 11 Vibration characteristics of perforated mild steel circular plate with a centrally located square cutout

Mild steel	Frequency mode					
Cutout area (mm^2)	1	2	3	4	5	6
2.5 × 2.5	399.3	827.56	827.56	1350.9	1350.9	1538.6
5 × 5	399.22	827.5	827.5	1350.6	1350.6	1537.9
7.5 × 7.5	399.12	827.49	827.49	1350.3	1350.3	1536.7
10 × 10	398.93	827.47	827.47	1349.8	1349.9	1534.9
15 × 15	398.52	827.39	827.39	1348.6	1348.8	1530.9
20 × 20	397.96	827.2	827.2	1347.1	1347.3	1526.2
50 × 50	395.04	820.47	820.47	1331.4	1334.5	1523.7
75 × 75	397.51	800.72	800.73	1310.3	1320.6	1583.5
100 × 100	408.41	768.58	768.61	1283.6	1299.3	1704.2

Table 12 Vibration characteristics of perforated rubber circular plate with a centrally located square cutout

Rubber	Frequency mode					
Cutout area (mm^2)	1	2	3	4	5	6
2.5 × 2.5	2.8497	5.9008	5.9009	9.62	9.62	10.96
5 × 5	2.8425	5.8866	5.8866	9.5974	9.5975	10.922
7.5 × 7.5	2.8399	5.8865	5.8865	9.5956	9.5958	10.899
10 × 10	2.8375	5.8863	5.8863	9.5936	9.5938	10.878
15 × 15	2.8301	5.8854	5.8854	9.5876	9.5884	10.816
20 × 20	2.8208	5.8835	5.8835	9.5802	9.5816	10.745
50 × 50	2.7606	5.8313	5.8313	9.5316	9.5539	10.56
75 × 75	2.7294	5.5978	5.5979	9.3598	9.4324	10.891
100 × 100	2.7735	5.2521	5.2528	9.1098	9.2002	11.722

Table 13 Vibration characteristics of cork circular plate with a centrally located square cutout

Cork	Frequency mode					
Cutout area (mm^2)	1	2	3	4	5	6
2.5 × 2.5	27.334	56.705	56.705	92.669	92.669	105.59
5 × 5	27.336	56.705	56.705	92.643	92.644	105.59
7.5 × 7.5	27.338	56.705	56.705	92.604	92.608	105.59
10 × 10	27.34	56.704	56.704	92.544	92.553	105.58
15 × 15	27.348	56.7	56.7	92.413	92.424	105.59
20 × 20	27.36	56.693	56.693	92.243	92.255	105.63
50 × 50	27.629	56.435	56.435	90.608	90.741	107.81
75 × 75	28.252	55.707	55.708	88.83	89.383	113.1
100 × 100	29.398	54.527	54.529	87.204	88.174	121.96

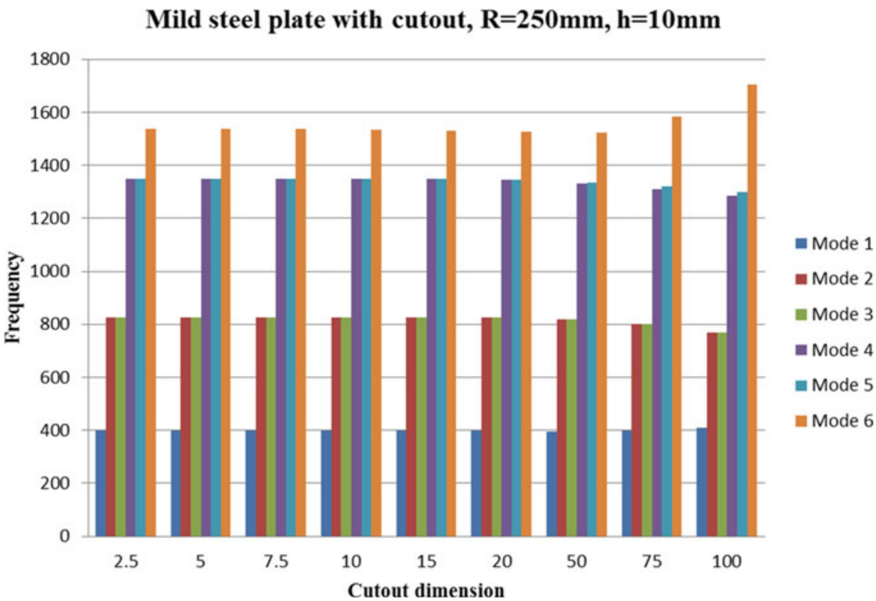


Fig. 11 Vibration analysis of perforated mild steel plate of radius 250 mm and a height of 10 mm

Irrespective of material used, unique wave-like trends were observed in the frequency of plate when the cutout dimension was varied. Table 14 provides a comparative analysis of the isotropic plates for all materials with and without perforations, with a height of 10 mm, radius of 250 mm, and a cutout area of 50 × 50 mm^2. The results obtained for each of the materials in both perforated and non-perforated cases are represented graphically in Figs. 14, 15 and 16 for a better understanding of the trends. From Figs. 14, 15 and 16, we note that the effect of including perforation varies

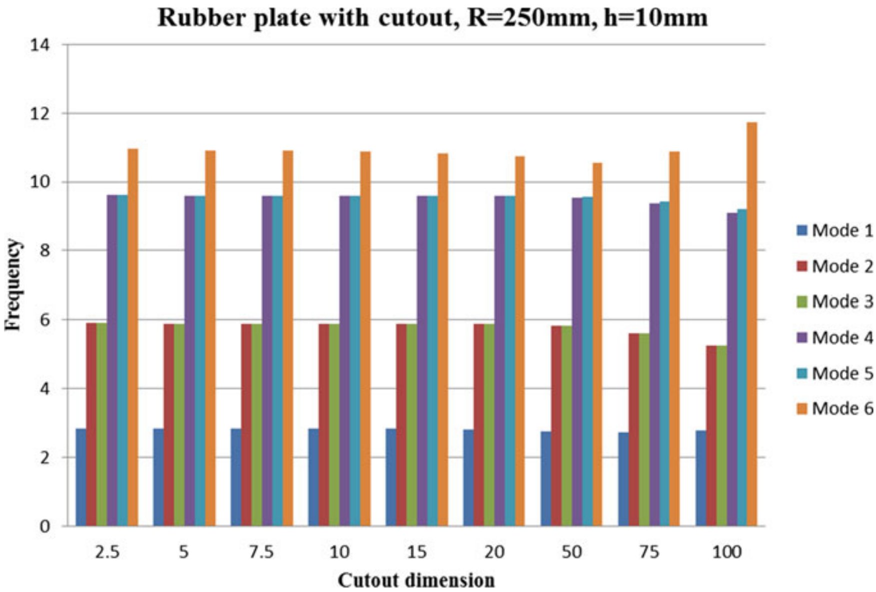


Fig. 12 Vibration analysis of perforated rubber plate of radius 250 mm and a height of 10 mm

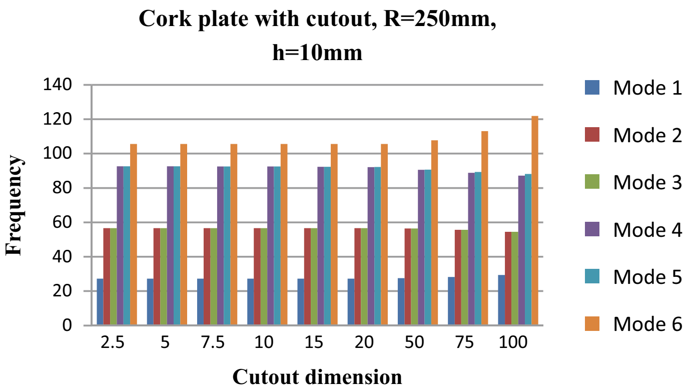


Fig. 13 Vibration analysis of perforated cork plate of radius 250 mm and a height of 10 mm

with material. In cork, the inclusion of cutout has significantly raised the frequency, whereas there is no significant change in frequency in case of mild steel. Also, in rubber, the inclusion of perforation has reduced the frequency. As Poisson's ratio is changed from 0 to 0.5, the effect of inclusion of cutout changes from increasing frequency, producing no considerable change, to decreasing frequency. Hence, it can be concluded that it is essential to improve the number of cases tested in this scenario in order to get a complete understanding of the trends observed and their variations.

Table 14 Comparative analysis of the vibration characteristics of perforated and non-perforated plates for different materials with a plate of fixed dimensions (R = 250 mm, h = 10 mm, a = 50 mm)

Mode	Frequency					
	Non-perforated mild steel	Perforated mild steel	Non-perforated cork	Perforated cork	Non-perforated rubber	Perforated rubber
1	398.96	395.04	8.6437	27.629	4.0091	2.7606
2	826.78	820.47	17.932	54.435	8.2996	5.8313
3	826.78	820.47	17.932	56.435	8.3008	5.8313
4	1349.7	1331.4	29.307	90.608	13.533	9.5316
5	1349.7	1334.5	29.307	90.741	13.535	9.5539
6	1573.3	1523.7	33.39	107.81	15.411	10.56

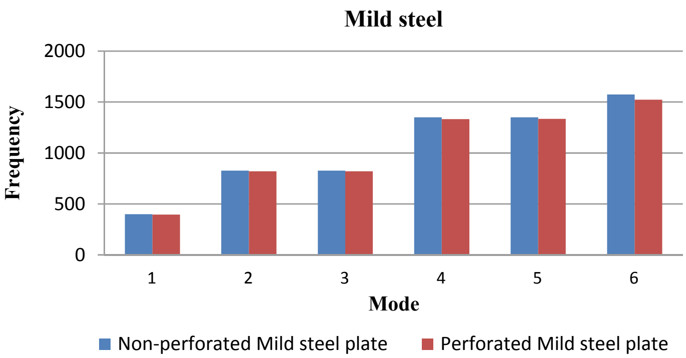


Fig. 14 Comparative analysis of vibration characteristics of non-perforated and perforated mild steel plate

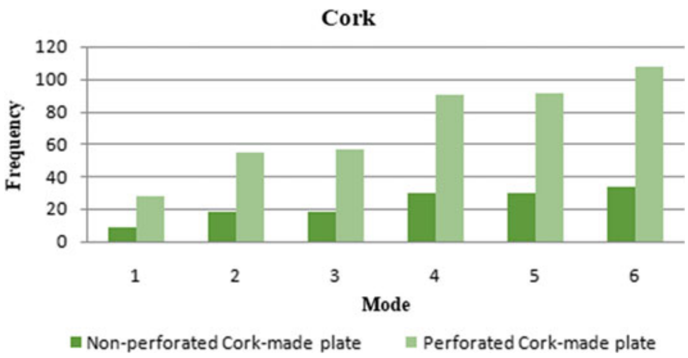


Fig. 15 Comparative analysis of vibration characteristics of non-perforated and perforated cork plate

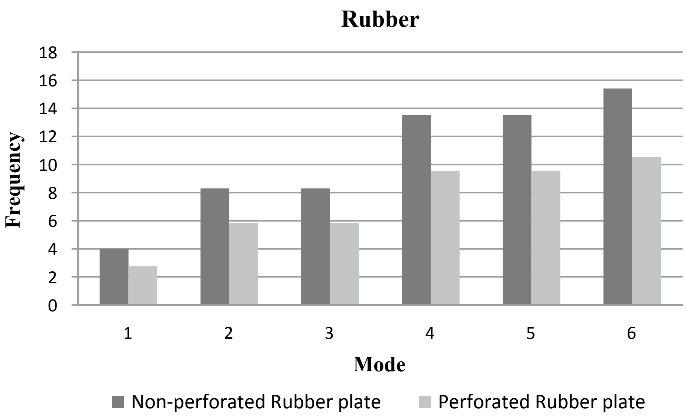


Fig. 16 Comparative analysis of vibration characteristics of non-perforated and perforated rubber plate

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