

Design Analysis and Optimization of a Rotating Disc with Variable Thickness



Prerana D. Rao, Prathamesh Dehadray, Alampally Sainath,
Lokavarapu Bhaskara Rao, and Ch. Rajesh

Abstract Rotating disc finds its application in gas turbines. Turbine discs are designed with variable thickness in order to prevent stress concentration, and closed form solutions for stress distribution are not reported extensively in literature. Numerical methods have been developed to provide approximate solution for the problem. In the present study, FEM approach is adopted to obtain a stress distribution in a rotating disc of variable thickness made of NIMONIC 901 and NIMONIC 105 super alloys. The comparison of stress distribution resulted in selecting NIMONIC 105 for the manufacture of disc with minimal induced maximum stress. Further, the optimum cost of production was obtained for NIMONIC 901 which further highlights the complexity of striking balance between the cost of production and stress levels induced in the disc. The study reveals that NIMONIC 901 is the better choice due to its low cost of production.

Keywords Rotating disc · Variable thickness · Shrinks fit · Natural frequency · NIMONIC 105

1 Introduction

The rotating disc is an important part of any application, ranging from automobiles to aerospace applications. Rotating discs are used extensively in many applications such as flywheels, IC engines, ship propellers, centrifugal compressors and aircrafts. The stress and strain analysis of rotating disc has been investigated by many authors and researchers. The most frequently used is the disc with circular hole. Closed form solutions are available for disc with circular holes and constant thickness. Therefore, variable thickness discs are the areas of current research. In the advancement

P. D. Rao (✉) · P. Dehadray (✉) · A. Sainath · L. Bhaskara Rao
School of Mechanical Engineering, Vellore Institute of Technology, Chennai Campus,
Vandalur-Kelambakkam Road, Chennai, Tamil Nadu 600127, India

Ch. Rajesh
Department of Mechanical Engineering, Pragati Engineering College, Chennai Campus,
Vandalur-Kelambakkam Road, Surampalem, Tamil Nadu 533437, India

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of technology, better tools are available for the analysis of any engineering problem. The stress and strain developed in the elastic plastic region have been studied, and mathematical models have been developed [1]. Attempts have been made to develop analytical solutions for discs with variable thickness. The thickness is varied hyperbolically, parabolically along the radius, and the subsequent parametric study has been carried out subjecting it to different boundary conditions [2–5]. Various aspects of 2D axisymmetric analysis of the disc have been studied. The study is carried out by applying various boundary conditions, and the subsequent stress developed was studied [6–13]. The study gave a more realistic working of a turbine in actual operating conditions. Turbine discs are mounted on shafts. Turbine discs are manufactured with a circular hole for the mounting of the shaft. A design is studied with non-circular bolt clearance hole profile to check the stress distribution and to study the effect of stress by varying the profile of the hole [14]. Optimization of the design is the next important step in the analysis in order to minimize the stress concentration factor in the disc. Various optimisation techniques have been used with ANSYS to minimize the stress distribution [14–16]. Modal analysis has been of utmost importance. Many studies have been undertaken to study the effects of assembled components in the turbine [17–19]. This paper attempts to find out the mode shapes of the disc rotating at high speeds.

2 Description of Disc

2.1 Disc Geometry

Figures 1 and 2 describe the geometry of the disc. The disc thickness varies from 24 mm at centre to 12 mm at the periphery. The outer diameter is 190 mm. The elliptical hole of major axis of 93 mm and minor axis of 91 mm is considered [6].

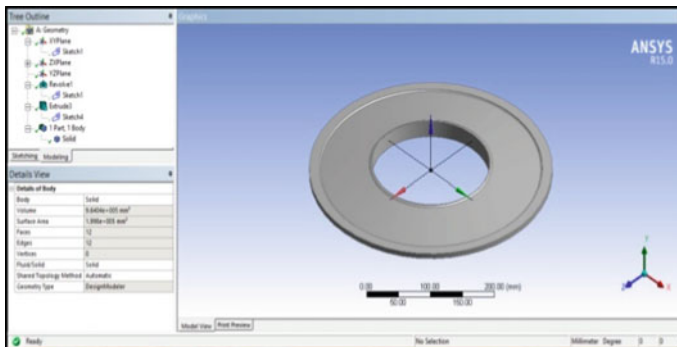


Fig. 1 3D model

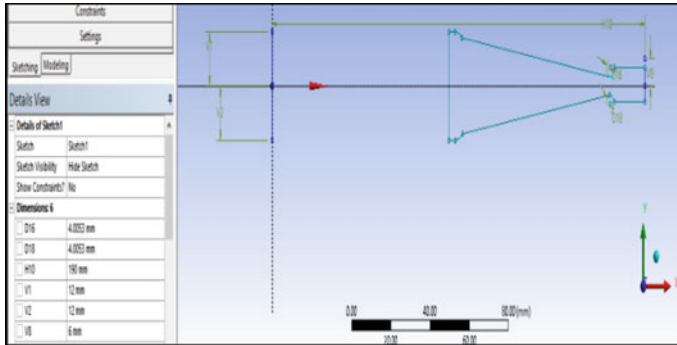


Fig. 2 Dimensions of the disc in mm

2.2 *Disc Material*

Two super alloys of nickel NIMONIC 105 and NIMONIC 901 were selected. Young's modulus and Poisson's ratio are assumed to be linear.

2.3 *Disc Operating Parameters*

In the FE simulation, the shrink fit load arising due the mounting of the disc on to the shaft is considered as contact pressure at the inner surface of the disc. Due to the blades being mounted on the periphery of the disc and blades conducting heat to the disc and the shaft, these are also considered in the analysis as blade load and thermal loads.

3 **Finite Element Modelling**

3.1 *Mesh of Model*

The analysis is carried out using ANSYS Workbench [20]. The disc is modelled as a 3D disc and meshed using SOLID186 which is a 3D six node element exhibiting a quadratic displacement behaviour. Pyr13 is a pyramidal variant of SOLID186 used for interfacing purposes wherever both hexahedral and tetrahedral mesh elements are required (Fig. 3).

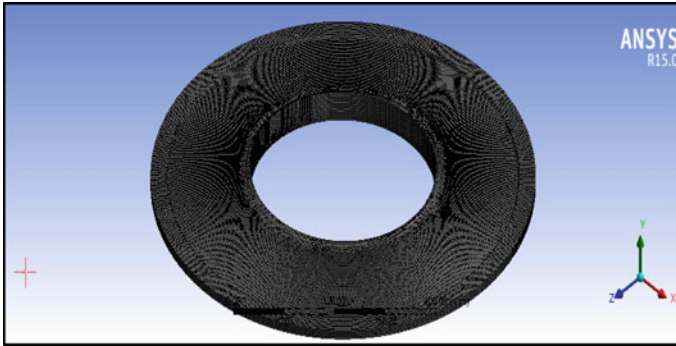


Fig. 3 Meshed model

3.2 Loads and Boundary Conditions

The major loads on the disc considered are (a) shrink fit load, (b) blade load, (c) thermal load, (d) rotational load and (e) combined loading (combination of loads a–d). The shrink fit load is represented as a pressure of 26 MPa at the inner surface of the disc and the blade load as a pressure of 45 MPa at the outer surface of the disc [7]. The temperature of 800 °C is applied at the outer surface and 500 °C at the hub. The rotational velocity of 15,000 rpm [6] is applied about Y-axis which results in a centrifugal force equivalent to the rotational load. Perpendicular motion of the disc is constrained by use of frictionless support (Fig. 4).

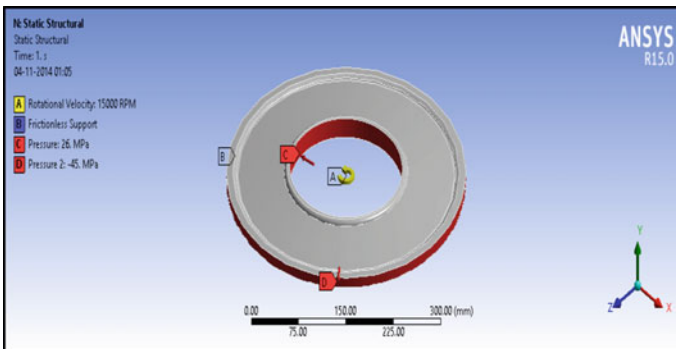


Fig. 4 Loads and boundary conditions applied

4 Results and Discussions

4.1 Analysis

ANSYS 15 was used to obtain stress distribution for the load cases (a)–(d) mentioned earlier. The von Mises and radial stress distribution is obtained considering loading individually and in combination. The following results are for the material NIMONIC 105.

Load case (a) Shrink fit load for NIMONIC 105

For the shrink fit load, the radial stress developed is 14.78 MPa located at the inner surface, and the equivalent stress found is 106.85 MPa occurring at the hub of the disc. Figure 5a shows the radial stress distribution, and Fig. 5b shows the von Mises stress distribution.

Load case (b) Blade load for NIMONIC 105

For the blade load, the radial stress developed is 123.25 MPa located at the inner surface, and the equivalent stress found is 166.27 MPa occurring at the hub of the disc. Figure 6a shows the radial stress distribution, and Fig. 6b shows the von Mises stress distribution.

Load case (c) Rotational load for NIMONIC 105

For the rotational load, the radial stress developed is 118.64 MPa located at the inner surface, and the equivalent stress found is 510.58 MPa occurring at the hub of the disc. Figure 7a shows the radial stress distribution, and Fig. 7b shows the von Mises stress distribution.

Load case (d) Thermal load for NIMONIC 105

For the rotational load, the radial stress developed is 66.858 MPa located at the inner surface, and the equivalent stress found is 602.56 MPa occurring at the hub of the disc. Figure 8a shows the radial stress distribution, and Fig. 8b shows the von Mises stress distribution.

Load case (e) Combined load for NIMONIC 901

For the combined load, the radial stress developed is 264.52 MPa located at the middle, and the equivalent stress found is 1070.7 MPa occurring at the hub of the disc (Fig. 9).

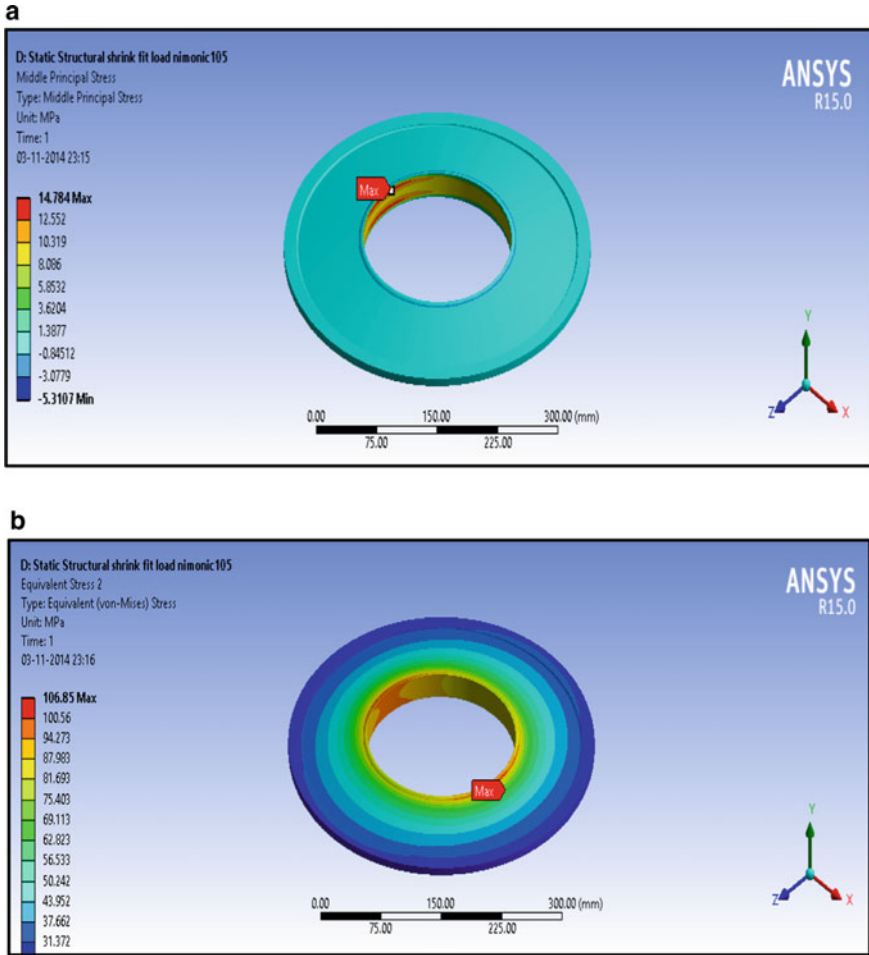


Fig. 5 a Radial stress distribution for shrink fit load. b Equivalent stress distribution for shrink fit load

4.2 Modal Analysis

Modal analysis has been carried out for the frequency range 1000–3000 Hz to find the natural frequencies of the rotating disc to prevent resonance. Figure 10a, b show the natural frequencies at the operating speed of 15,000 rpm for NIMONIC 105 and NIMONIC 901, respectively. The stresses for nimonic 105 under different load cases are presented in Table 1.

From the results presented in Table 1, it can be seen that the maximum load is contributed by the thermal load for NIMONIC105. It contributes 50% of the combined load. From Table 2, it can be seen that maximum load in NIMONIC 901 is

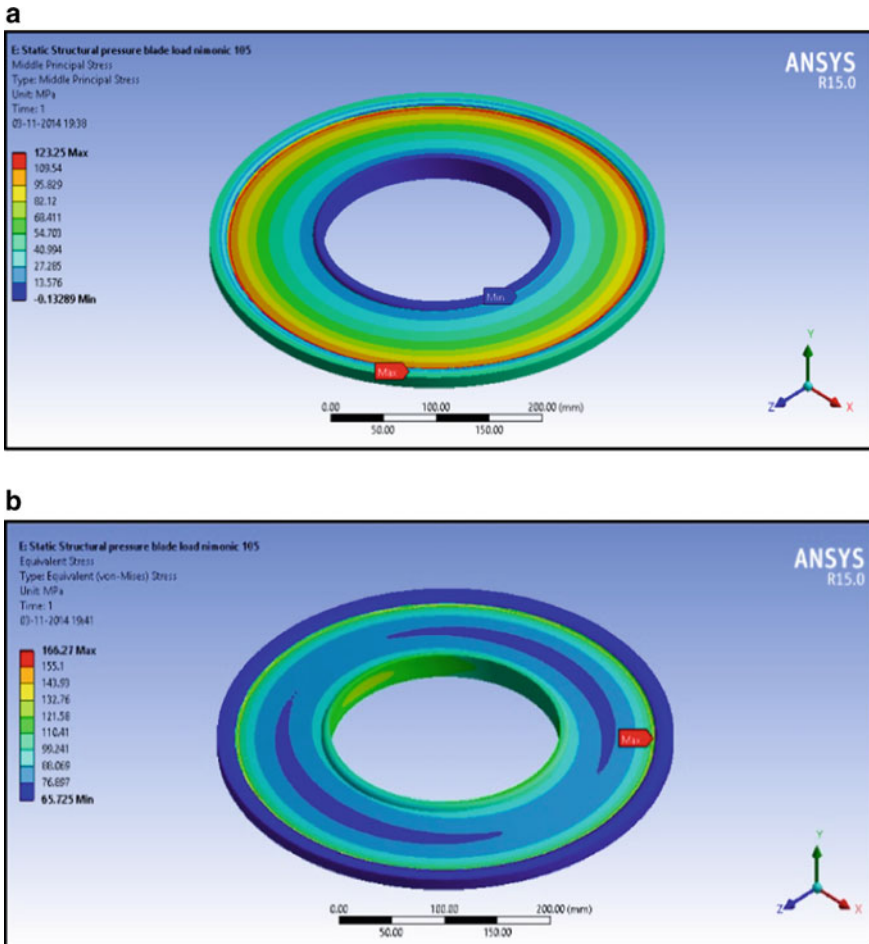


Fig. 6 a Radial stress distribution for blade load. b Equivalent stress distribution for blade load

contributed by the rotational loading. It can be concluded that thermal and rotational loads contribute to most of the stresses in the disc. It can be seen that NIMONIC 901 develops higher stresses when compared to NIMONIC 105. The modal analysis gives the maximum natural frequencies of 2809.2 Hz for NIMONIC 105 and 2493.3 Hz for NIMONIC 901, which helps in avoiding resonance.

4.3 Optimization

Optimization of elliptical hole geometry (a/b) has been carried out to reduce the stress distribution in the disc. Optimization is done based on a dimensional-less ratio

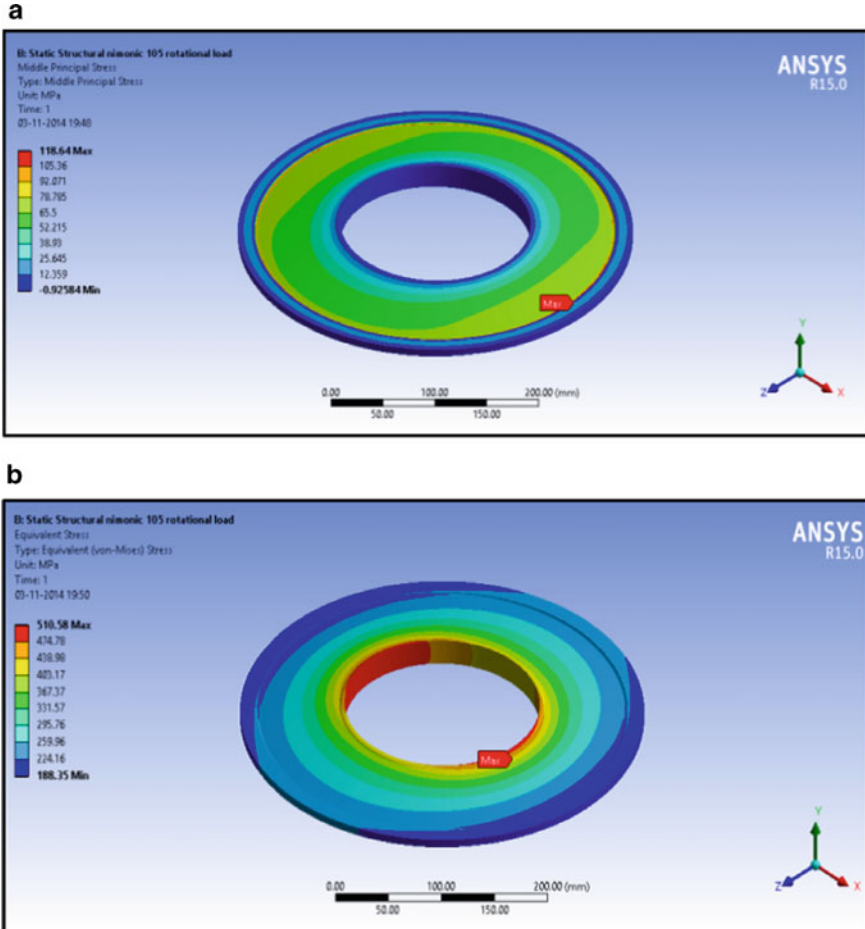


Fig. 7 **a** Radial stress distribution for rotational load. **b** Equivalent stress distribution for rotational load

a/b of the ellipse in which $a = 91$ mm is the semi-minor axis and $b = 93$ mm is the semi-major of the ellipse. In this paper, the optimization is carried out in terms of the non-dimensionless ratio a/b .

From Fig. 11, it can be seen that as the a/b ratio decreases, the value of the equivalent stress also decreases and reaches a lowest value at the ratio 1.01, and with further increase in the ratio, the stress value starts to increase. Thus, it can be seen that at the ratio of 1.01, the stresses induced will be less, and hence, the model is optimized at that value. The value of 1.01 is same for both the materials.

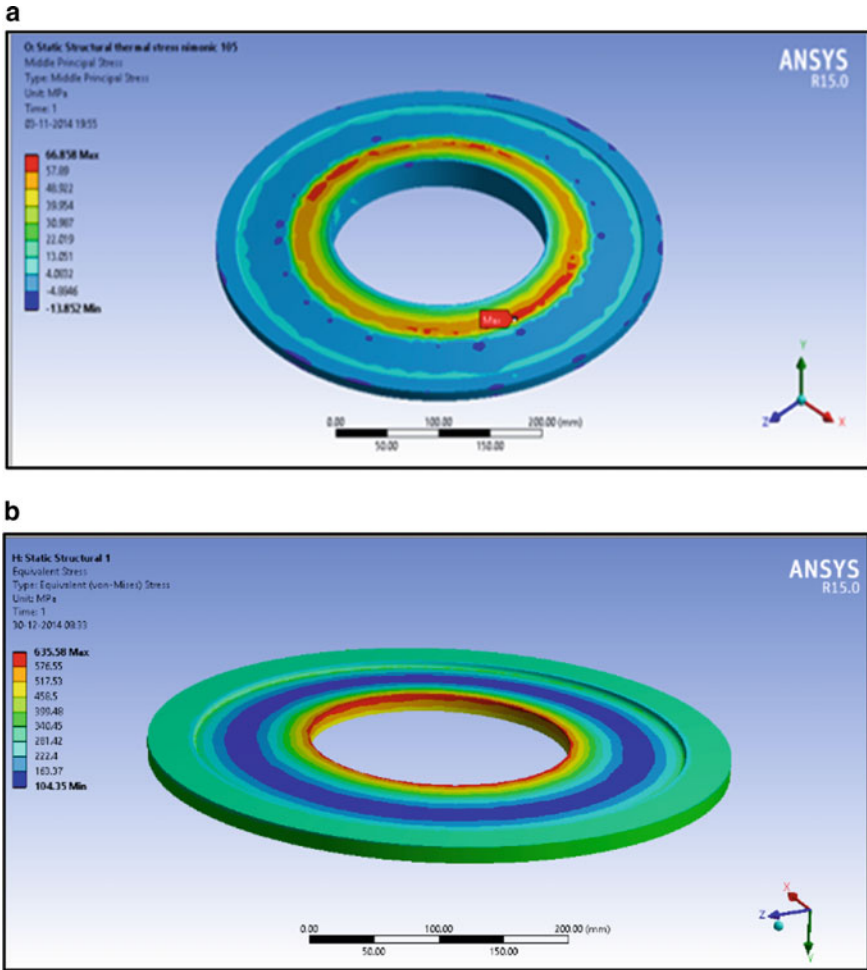


Fig. 8 a Radial stress distribution for thermal load. b Equivalent stress distribution for thermal load

5 Conclusion

This paper uses FEA as a tool to study the effects of different type of loading on a turbine disc. This method proves to be a very effective method as the closed form solution is not available for complex loadings and geometry. As disc is an integral part of the turbine, necessary precaution should be taken to prevent failure. Various loadings have been taken into account including the shrink fit load, blade load, thermal load and the rotational load. Modal analysis was under taken in order to find out the natural frequencies at the operating speed. Static structural analysis was carried along with modal analysis.

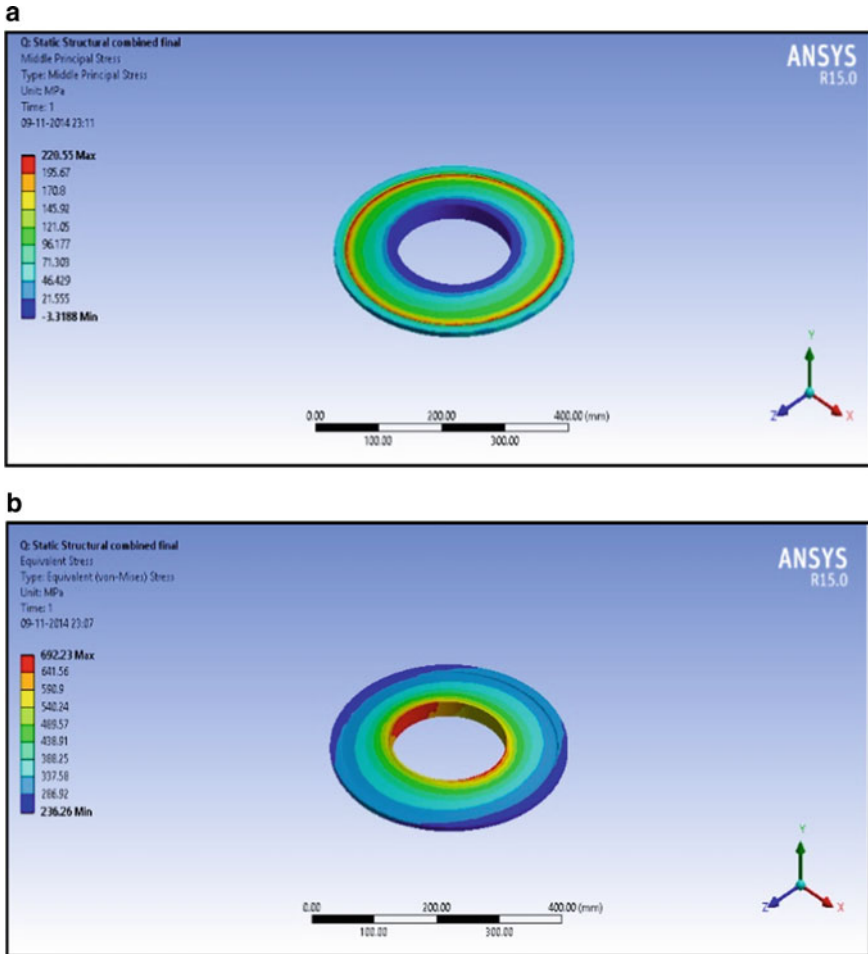
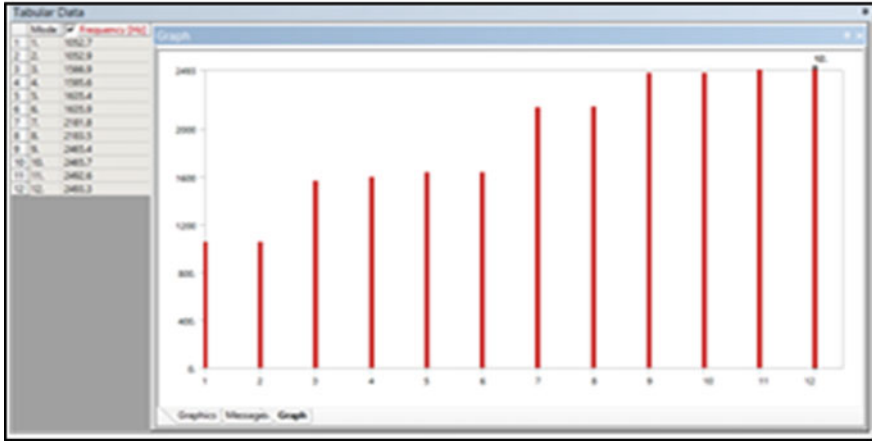


Fig. 9 a Radial stress distribution for combined load. **b** Equivalent stress distribution for combined load

It was seen that among the two materials chosen for the study, NIMONIC 105 develops lesser stresses than NIMONIC 901. It can be concluded that NIMONIC 105 is better material for the manufacturing of the disc. The optimization is carried out, and it was found that any ratio around 1.01 is preferable as the stress developed is the least. Hence, an optimal design should be considered keeping in mind the application and the cost.

a



b

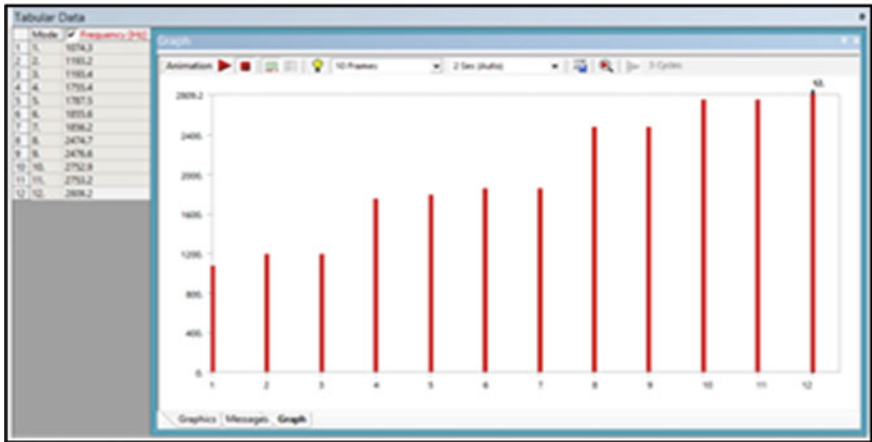


Fig. 10 **a** Natural frequencies for NIMONIC 105. **b** Natural frequencies for NIMONIC 901

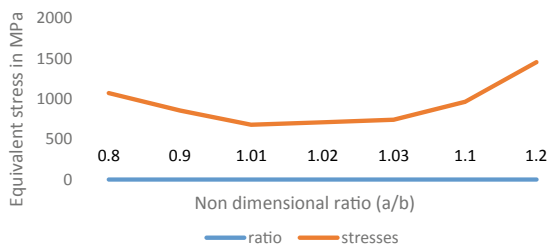
Table 1 The stresses for nimonic 105 under different load cases

Load case	NIMONIC 105		
	Loading	Stress (MPa)	Radial stress (MPa)
a	Shrink fit	106.85	14.78
b	Blade load	166.27	123.25
c	Rotational load	510.58	118.64
d	Thermal	635.58	66.858
e	Combined	692.23	220.55

Table 2 Stresses for nimonic 901 under different load cases

Load case	NIMONIC 901		
	Loading	Equivalent stress (MPa)	Radial stress (MPa)
a	Shrink fit	110.83	22.109
b	Blade load	127.77	192.68
c	Rotational load	554.29	182.23
d	Thermal	372.59	81.65
e	Combined	773.39	359.22

Fig. 11 Variation of stress distribution versus non-dimensional ratio (a/b)



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