Chapter 12
Geometric Error Modeling of a Special NC Process Device for Precision Two-Dimensional Optical Drum

Xuebing Han, Weidong Feng, and Likun Zhao

Abstract In order to improve the precision of the special NC process device for machining two-dimensional optical drum and ensure its quality, an error model of the special NC process device is established. According to the mechanical topology of the NC process device, to obtain the influence degree of each error component on the total error, the geometric error model is derived by using error analysis technology. The model covers the main error sources of the process unit such as the two axes errors, the parts error and encode error. It can analyze, synthesize, and distribute these errors. An error prediction has been done by the model. An experiment has been conducted on a new NC process device to validate the method. An electronic theodolite and a plane mirror are used to measure the repeat positioning accuracy of the two shafts. The predicted repeated positioning accuracy is compared with the measured results. It shows that integrated geometric error modeling method is effective and applicable in Special NC Process Device.

12.1 Introduction

In the machining of ultra-precision two-dimensional optical drum, a two-dimensional indexing table is a necessary device to make the dividing of the drum around the central axis and the inclination between the machining surface and the central axis [1]. The traditional indexing device has a manual two-dimensional turntable to adjust the angle of the drum during processing [2]. The angle of the turntable must be readjusted after each surface is processed. This device has low positioning accuracy and slow processing efficiency. In order to improve the accuracy and the efficiency, a new NC process device is designed [3]. The device uses a computer-controlled motor to drive the turntable to rotate. It can automatically divide and greatly improve the

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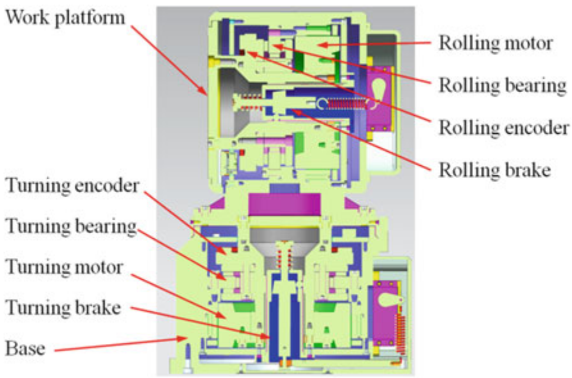
processing efficiency of the drum. Compared with the manual turntable, the new NC process device not only increases the motor and encoder, but also has great differences in parts and structure. In order to improve the positioning accuracy of NC process device and ensure the machining quality of drum, it is necessary to establish the accuracy model as the design basis of the device. At present, multi-body dynamics method is a common method to establish error model [4, 5]. This method has high precision but complex calculation. Especially when there are many errors, it needs to be calculated by computer programming [6]. In this paper, the geometric method is used to establish the error model, which has the characteristics of simple principle and convenient calculation.

12.2 Overall Structure of the Device

The whole process device is composed of two parts: two-dimensional turntable and turntable controller. The two-dimensional turntable provides high-precision absolute position information, shafting accuracy, and interface with the machine tool and the processing workpiece. The turntable controller has turntable power supplying, motor driving, position information collection feedback, and shafting locking functions. The overall structure of the two-dimensional turntable (as shown in Fig. 12.1) can be divided into two parts: horizontal axis system and vertical axis system. The horizontal shaft system consists of rolling bearing, rolling motor, rolling encoder, rolling brake, work platform, and some connecting parts. The vertical shaft system is composed of turning bearing, turning motor, turning encoder, turning brake, base, and some connectors.

Fig. 12.1 Overall structure of the process device



12.3 Error Modeling of the Device

During the process of the drum, the relative position accuracy of the working face is mainly guaranteed by the shafting accuracy of the process device. There are many factors that affect the accuracy of shafting. The factors mainly include machining error of single part, shape error of part, position error of part, fit clearance, temperature change, influence of lubricant, friction, wear, and elastic deformation, etc. All of those have inevitable consequences for the accuracy of shafting. According to the structure and working principle of the process device, the factors that affect the repeated positioning of the shafting mainly include the swaying error of the two shafting, the system accuracy of the encoder, the machining dimension error, and shape error of the two shaft supporting parts, etc.

12.3.1 Swaying Errors Analysis of the Horizontal Axis

12.3.1.1 Principle of the Errors

The swaying errors are mainly caused by the circular runout of the bearing's inner ring and the motor's rotor. The horizontal axis coordinate system is established as shown in Fig. 12.2.

T point is the projection of the center point of the drum shaft on the processing surface of the drum. According to the processing principle of the two-dimensional optical drum, all points on the same process plane have the same angular relationship in the horizontal axis coordinate system, without considering the machining error of the machine tool. That is all of the points have the same horizontal axis angle. Therefore, the horizontal axis angle of *T* point is the horizontal axis angle of the process plane. The angle error of the machined refraction surface can be calculated by the angle error of horizontal axis at *T* point.

In Fig. 12.2, the x-axis is turning center of the horizontal axis. The y-axis is in a same plane with the x-axis and *T* point. The y-axis is perpendicular to the x-axis. *O* point is the left support of horizontal shaft. ρ is a distance between the center point of the drum and *O* point. *H* is the rotation angle of the horizontal shaft. The space

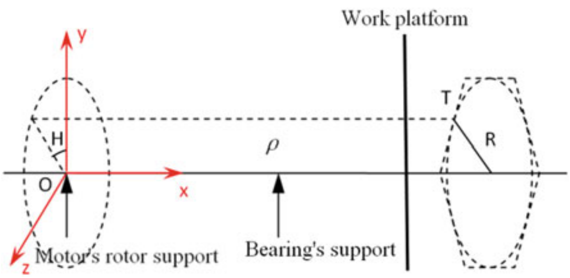


Fig. 12.2 The horizontal axis coordinate system

position of T point can be calculated by the following formula.

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \rho \\ R\cos H \\ R\sin H \\ 1 \end{bmatrix} \tag{12.1}$$

The circular runout of the bearing's inner ring and the motor's rotor can make the horizontal axis coordinate system rotate and translate along y and z axes.

12.3.1.2 Machine Error of the Drum from Rotating Errors

The rotating of the horizontal axis coordinate system around y and z axes can be simplified as shown in Fig. 12.3. L is the distance of the bearing's inner ring and the motor's rotor. δ is the deflection distance produced by the superposition of two errors of the circular runout.

The angle error of the horizontal shaft caused by the bearing's inner ring and the motor's rotor can be calculated by the following formula.

$$\varepsilon = \mathrm{arctg}(\delta/L) \tag{12.2}$$

θ is horizontal axis rotate angle error by ε. The actual space position of T point can be given by homogeneous coordinate equation. Actual position of T point with error rotating around the y-axis.

$$T' = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_y & 0 & -\sin\theta_y & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta_y & 0 & \cos\theta_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \rho\cos\theta_y - R\sin H\sin\theta_y \\ R\cos H \\ \rho\sin\theta_y + R\sin H\cos\theta_y \\ 1 \end{bmatrix} \tag{12.3}$$

where θ_y is the rotation error of the horizontal axis around the y-axis, caused by the radial runout of the bearing inner ring and the motor's rotor. The formula (12.4) is deduced from (12.2) and (12.3). H' is the actual angle of the horizontal axis.

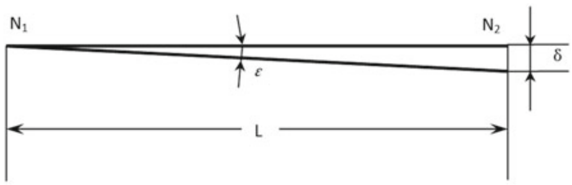


Fig. 12.3 The angle error between actual position and ideal position

$$H = \arctan\left(\frac{z'}{y'}\right) = \arctan\left(\frac{\rho \sin\theta_y + R \sin H \cos\theta_y}{R \cos H}\right) \approx \tan H \cos\theta_y \quad (12.4)$$

Because θ_y is a very small angle and $\cos\theta_y$ is very close to 1, which can be considered approximately θ_y has no effect on the angle of the processing surface of the drum.

The actual space position of T point rotating around the z-axis is

$$T' = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_z & \sin\theta_z & 0 & 0 \\ -\sin\theta_z & \cos\theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \rho\cos\theta_z + R\sin\theta_z\cos H \\ -\rho\sin\theta_z + R\cos\theta_z\cos H \\ R\sin H \\ 1 \end{bmatrix} \quad (12.5)$$

The formula (12.6) can be obtained from (12.1) and (12.5).

$$H' = \arctan\left(\frac{R \sin H}{-\rho \sin\theta_z + R \cos\theta_z \cos H}\right) \quad (12.6)$$

The formula (12.6) is similar to formula (12.4), and it can be considered that the error of the roll around z-axis is approximately considered θ_Z has no effect on the angle of the drum surface.

12.3.1.3 Machine Error of the Drum from Translating Errors

λ_y and λ_z is the translation error of the horizontal axis along the y-axis and the z-axis, caused by the radial runout of the bearing inner ring and the motor's rotor. In order to simplify the calculation, only when the angle error is the largest, that is, when H is 0.

$$\theta_{H-t-z} = H' - H = \arctan(\lambda_z / R) \quad (12.7)$$

At the limit position, H is 90°, the error from the translation error of the horizontal axis along the y-axis is

$$\theta_{H-t-y} = \arctan(R/\lambda_y) - 90^\circ \quad (12.8)$$

12.3.2 Swaying Errors Analysis of the Vertical Axis

The swaying errors of vertical axis are mainly caused by bearing end face runout and radial runout. The end face runout error produces the angle error of H angle, which has a great impact on the machining surface of the drum. It can be calculated by the following formula:

$$\theta_{H-V-e} = \arcsin(\alpha/r) \tag{12.9}$$

In the formula, α is the value of end face runout, and r is the bear's radius.

The radial runout error also produces the angle error of H angle. The formula (12.10) can be obtained using the similar method of the horizontal axis. In the formula, β is the value of radial runout.

$$\theta_{H-V-r} = \arctan\left(\beta/R\right) \tag{12.10}$$

The calculation method of angle error caused by radial runout of vertical axis is similar to that of horizontal axis. The radial runout can produce the translation of the vertical axis coordinate system along y and z axes. The translation along y-axis has no effect on the angle of the drum surface. The error from the translation along z-axis is formula (12.11).

$$\theta_{A-t-z} = \arctan\left(\alpha_z/R_1\right) \tag{12.11}$$

12.3.3 Other Errors Analysis of the Two Shafts

In addition to the error of bearing support, the machining dimension errors and shape errors of parts also affect the accuracy of shaft system. The main part processing errors affecting the shafting accuracy include: part size processing error, roundness error, cylindricity error, coaxiality error, perpendicularity error, etc.

The encoder is an important accessory to ensure the accuracy of the shafting. The resolution, accuracy, and repeatability of the encoder have a great immediate impact on the performance of the device.

12.4 Error Prediction of the Device

The bearings in horizontal and vertical shafts are preliminarily selected. The bearing's end face runout and radial runout are all less than 2 μM in the horizontal shaft. The thrust ball bearing in the vertical shaft, those are all less than 4 μM. Therefore, λ_y and

Table 12.1 The errors value

Error source	Formula	Calculated value
Horizontal axis	$\theta_{H-t-y} = \arctan(R/\lambda_y) - 90°$ $\theta_{H-t-z} = \arctan(\lambda_z/R)$	0.02″ 0.6″
Vertical axis	$\theta_{H-V-e} = \arcsin(\alpha/r)$ $\theta_{H-V-r} = \arctan(\beta/R)$ $\theta_{A-t-z} = \arctan(\alpha_z/R_1)$	0.2″ 4″ 4.6″
Parts	$\theta_{H-P} = \sqrt{\theta_{H-t-y}^2 + \theta_{H-t-z}^2 + \theta_{H-V-e}^2 + \theta_{H-V-r}^2}$ $\theta_{A-P} = \arctan(\alpha_z/R_1)$	1″ 1″
Encode	$\theta_{H-E} = 1''$ $\theta_{A-E} == 1''$	1″ 1″

λ_z are all 2 μM. α and β are all 2 μM. Design dimension of r, R, and R_1 is 180, 200, and 180 mm. Part size processing error, roundness error, cylindricity error, coaxiality error, and perpendicularity error are approximate to 0.3 um ($\lambda_y = \lambda_z = \alpha = \beta =$ 0.3 um) based on the parts processing capacity. The accuracy of the selected encoders is ±1″. The calculated values of the errors are in Table 12.1.

The overall error of horizontal shaft and vertical shaft can be combined with errors in Table 12.1.

$$\delta_H = \sqrt{0.02^2 + 0.6^2 + 0.2^2 + 4^2 + 1^2 + 1^2} = 4.3'' < 5''$$

$$\delta_A = \sqrt{4.6^2 + 1^2 + 1^2} = 4.8'' < 5''$$

12.5 Test of the Model

The developed and manufactured NC process device based on the model was tested. The repeated positioning accuracy of NC process device was tested by an electronic theodolite and a plane mirror. The model of the electronic theodolite is TM5100A, and its measuring precision is 0.04″. The results are the repeated positioning accuracy of horizontal shaft is 2.88″, that of vertical shaft is 3.12″. The accuracy of the measured results is higher than the design accuracy. The main reason is that in order to simplify the calculation, the error values are the maximum value at the limit position. If the standard deviation of the errors is used in the model, the prediction result will be more accurate.

12.6 Conclusions

The accuracy model of the NC process device for ultra-precision machining of two-dimensional optical drum is established. The error sources of the NC process device are analyzed in detail. The errors of the horizontal and vertical shaft system are analyzed and calculated, which have a great impact on the repeated positioning accuracy. The important parts and key control components are designed and selected, and the error synthesis prediction is carried out. Compared with the measured results of the developed device, the model has good calculation accuracy and can be used in the design of NC process device.

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