Trajectory Tracking Control for An Underactuated Unmanned Surface Vehicle Subject to External Disturbance and Error Constraints

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Abstract. This paper is concerned with trajectory tracking control for an underactuated unmanned surface vehicle (USV) subject to external disturbance and error constraints. Firstly, a disturbance observer is constructed to observe the unknown external disturbance induced by wind, waves, and currents. Secondly, in order to avoid the "explosion of complexity" problem caused by virtual control variable differentiation, a first order filter and a time-varying tan-type barrier Lyapunov function (BLF) are developed. Then, by introducing the backstepping and dynamic surface control techniques, feedback control laws ensuring trajectory tracking are designed such that the errors of position and heading angle between the unmanned surface vehicle and a virtual leader converge to a bounded region. Finally, the effectiveness of the designed control scheme is verified by a simulation example.

Keywords: Time-varying tan-type BLF · Disturbance observer · Underactuated USV · Trajectory tracking control

1 Introduction

As an effective equipment for the exploration and development of ocean resources, USVs are widely used in military and civil fields such as maritime battlefield reconnaissance, scientific experiments, salvage and lifesaving [1]. In recent years trajectory tracking control of USVs has attracted growing attention. However, for the trajectory tracking problem of USVs, there exists some significant challenges, which include

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1) in order to reduce manufacturing costs, most of the deployed and developing USVs are underactuated, while most of the existing methods about trajectory tracking of USVs are designed for fullactuated USVs, which means these methods cannot be applied directly to underactuated USVs;
2) due to the existence of external disturbance induced by wind, waves, and ocean currents, the tracking performance of USVs is inevitably affected;
3) when carrying out tasks such as target tracking or stalking, USVs are usually needed to track the target as soon as possible, which results in a high requirement to the tracking speed and accurace of USVs.

Since the unknown external disturbance can severely affect the trajectory tracking performance of USVs, thus how to compensate the negative influences induced by disturbance is practically valuable and attractive. In order to solve this problem, the disturbance must be accurately estimated first. In [2], a disturbance observer was developed to estimate the external disturbance. However, this developed disturbance observer could only estimate slow time-varying disturbance. By taking into account the coexistence of slow time-varying disturbance and non-slow time-varying disturbance, a more capable disturbance observer was presented in [3]. Moreover, based on the presented disturbance observer in [3], an efficient trajectory tracking control strategy and a high-precision dynamic positioning control strategy were proposed in [4] and [5], respectively. Therefore, constructing a disturbance observer is an effective way to estimate the unknown external disturbance, which gives the first motivation of this paper.

As mentioned above, the trajectory tracking control of USVs has attracted growing attention. Accordingly, a number of notable results have been proposed in the literature. To name a few, under backstepping control methods, a trajectory tracking control strategy was proposed in [6] with double closed-loop structure. However, the introduction of backstepping technique will increase computation and decrease convergence speed. In order to solve these shortcomings, an adaptive dynamic surface control (DSC) method was presented in [7]. Moreover, this DSC method can effectively used to reduce the complexity of the controllers designed. Based on DSC technique, an adaptive updating law, which could effectively improve the computational efficiency of trajectory tracking control algorithms, was proposed in [8]. On the basis of [8], a adaptive control method was proposed in [9] by taking "explosion of complexity" problem into consideration. Consider the advantages of backstepping and DSC techniques synthetically, how to propose a rapid and effective trajectory tracking control scheme for USVs is important, which gives the second and main motivation of this paper.

In addition, by constraining state errors during the process of USV trajectory tracking controller design, the transient and steady-state performance of control system can be significantly improved. An effective way to constrain these errors is to construct a barrier Lyapunov function (BLF). For example, by maintaining the boundedness of a BLF in the closed loop system, the state constraint was implemented in [10]. Motivated by [10], an asymmetric BLF was proposed in [11]. To solve the problem of multiple output constraints in the tracking control, a symmetric BLF was established in [12]. Compared with traditional USV control schemes, constructing a time-varying tan-type BLF to constrain the position errors is more practically valuable and attractive. Thus, how to develop an appropriate error constrain function is the third motivation of this paper.

This paper aims to solve the trajectory tracking problem of underactuated USVs. The main contributions of this paper are highlighted as follows:

1) In order to avoid the "explosion of complexity" problem induced by virtual control variable differentiation, a first order filter and a time-varying tan-type BLF are constructed to reduce the complexity of calculation.
2) An appropriate trajectory tracking control scheme, which can guarantee tracking errors exponentially converge to a bounded region, are proposed for underactuated USV systems.

2 Problem Formulation and Preliminaries

2.1 Modeling for Underactuated USVs

Motivated by [13], the three degrees of freedom (surge-sway-yaw) dynamic model of an underactuated USV can be expressed as

$$\begin{cases} \dot{x} = u\cos\varphi - v\sin\varphi \\ \dot{y} = u\sin\varphi + v\cos\varphi \\ \dot{\varphi} = r \end{cases} \quad \begin{cases} \dot{u} = f_u(\nu) + \frac{1}{m_u}\tau_u + \frac{1}{m_u}\tau_{wu} \\ \dot{v} = f_v(\nu) + \frac{1}{m_v}\tau_{wv} \\ \dot{r} = f_r(\nu) + \frac{1}{m_r}\tau_r + \frac{1}{m_r}\tau_{wr} \end{cases} \tag{1}$$

with

$$\begin{cases} f_u(\nu) = \frac{m_v}{m_u}vr - \frac{d_u}{m_u}u - \frac{d_{u2}}{m_u}\left|u\right|u - \frac{d_{u3}}{m_u}u^3 \\ f_v(\nu) = -\frac{m_u}{m_v}ur - \frac{d_v}{m_v}v - \frac{d_{v2}}{m_v}\left|v\right|v - \frac{d_{v3}}{m_v}v^3 \\ f_r(\nu) = \frac{(m_u - m_v)}{m_r}uv - \frac{d_r}{m_r}r - \frac{d_{r2}}{m_r}\left|r\right|r - \frac{d_{r3}}{m_r}r^3 \end{cases} \tag{2}$$

All the parameters in (1) and (2) can be found in [13].

The reference trajectory of the USV is generated by a virtual leader with the dynamic form as $\eta_d = [x_d,\ y_d,\ \varphi_d]^T$. Without loss of generality, the following assumption is introduced. It is assumed that the reference trajectory η_d and the external disturbance τ_w are bounded.

2.2 Error Constraint Function

By improving the constant BLF proposed in [10], a time-varying tan-type BLF is developed in this paper, which can be expressed as $V_b = \frac{k_b^2}{\pi}\tan\left(\frac{\pi z_1^T z_1}{2k_b^2}\right)$, where k_b is a time-varying upper bound function of $|z_1|$ with $|z_1(0)| \le k_b(0)$.

Remark 1. Note that the initial value of $\|z_1\|$ satisfies $\|z_1(0)\| \le k_b(0)$ and V_b is bounded. If there exist $\lim_{\|z_1\|\to k_b}\frac{k_b^2}{\pi}\tan\left(\frac{\pi z_1^T z_1}{2k_b^2}\right) = \infty$, one can conclude that $|z_1|$ will not exceed k_b. Moreover, If $\|z_1\|$ is not required to be constrained. Then the following equation $\lim_{k_b\to\infty}\frac{k_b^2}{\pi}\tan\left(\frac{\pi z_1^T z_1}{2k_b^2}\right) = \frac{1}{2}z_1^T z_1$ is obtained. It means that when there is no constraint function or the constraint function is infinite, the time-varying BLF will degrade to the commonly form, i.e., $V_b = \frac{1}{2}z_1^T z_1$.

3 Design of Control Scheme

In order to solve the trajectory tracking problem of the underactuated USV, an appropriate trajectory tracking control scheme is proposed in this section by taking into account error constraints.

The position and heading angle errors in the earth-fixed frame are defined as x_e, y_e, and φ_e. Meanwhile, these errors in the body-fixed frame are defined as e_x, e_y, and e_φ. The conversion relationship between the errors in two different frames is:

$$\begin{bmatrix} e_x \\ e_y \\ e_\varphi \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ \varphi_e \end{bmatrix} \tag{3}$$

Taking the time derivative of (3), one has

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_\varphi \end{bmatrix} = \begin{bmatrix} u + re_y - v_p \cos e_\varphi \\ v - re_x + v_p \sin e_\varphi \\ r - \dot{\varphi}_d \end{bmatrix} \tag{4}$$

where $v_p = \sqrt{\dot{x}_d^2 + \dot{y}_d^2}$ and $\varphi_d = \arctan(\frac{\dot{y}_d}{\dot{x}_d})$.

To guarantee the convergence of position errors e_x and e_y, a time-varying tan-type BLF function is constructed as follows:

$$V_1 = \frac{k_{b1}^2}{\pi} \tan\left(\frac{\pi e_x^2}{2k_{b1}^2}\right) + \frac{k_{b2}^2}{\pi} \tan\left(\frac{\pi e_y^2}{2k_{b2}^2}\right) \tag{5}$$

where k_{b1} and k_{b2} are time-varying parameters to be designed later.

Taking the time derivative of V_1 in (5), we have

$$\begin{aligned} \dot{V}_1 = & \frac{2k_{b1}\dot{k}_{b1}}{\pi} \tan\left(\frac{\pi e_x^2}{2k_{b1}^2}\right) + \frac{2k_{b2}\dot{k}_{b2}}{\pi} \tan\left(\frac{\pi e_y^2}{2k_{b2}^2}\right) - \frac{\dot{k}_{b1}}{k_{b1}} \frac{e_x^2}{\cos^2\left(\frac{\pi e_x^2}{2k_{b1}^2}\right)} \\ & - \frac{\dot{k}_{b2}}{k_{b2}} \frac{e_y^2}{\cos^2\left(\frac{\pi e_y^2}{2k_{b2}^2}\right)} + \frac{e_x \dot{e}_x}{\cos^2\left(\frac{\pi e_x^2}{2k_{b1}^2}\right)} + \frac{e_y \dot{e}_y}{\cos^2\left(\frac{\pi e_y^2}{2k_{b2}^2}\right)} \end{aligned} \tag{6}$$

Let $l_1 = \frac{1}{\cos^2\left(\frac{\pi e_x^2}{2k_{b1}^2}\right)}$, $l_2 = \frac{1}{\cos^2\left(\frac{\pi e_y^2}{2k_{b2}^2}\right)}$, $k_{11} = \sup_t \sqrt{(\frac{\dot{k}_{b1}}{k_{b1}})^2 + \sigma_1}, k_{22} = \sup_t \sqrt{(\frac{\dot{k}_{b2}}{k_{b2}})^2 + \sigma_2}$, where $\sigma_1 > 0$ and $\sigma_2 > 0$ are two predefined scalar. Substituting (4) into (6) yields

$$\begin{aligned} \dot{V}_1 < & 2k_{11} \frac{k_{b1}^2}{\pi} \tan\left(\frac{\pi e_x^2}{2k_{b1}^2}\right) + 2k_{22} \frac{k_{b2}^2}{\pi} \tan\left(\frac{\pi e_y^2}{2k_{b2}^2}\right) + k_{11} l_1 e_x^2 + k_{22} l_2 e_y^2 \\ & + l_1 e_x (u + re_y - v_p \cos e_\varphi) + l_2 e_y (v - re_x + v_p \sin e_\varphi) \end{aligned} \tag{7}$$

To guarantee the convergence of yaw velocity error, an auxiliary variable $h = v_p \sin\varphi_e$ is introduced. Meanwhile, a virtual control laws are designed as:

$$\begin{aligned} u_\kappa &= -(2k_{11}+k_1)\tfrac{k_{b1}^2}{\pi}\tfrac{1}{e_x}\sin\left(\tfrac{\pi e_x^2}{2k_{b1}^2}\right)\cos\left(\tfrac{\pi e_x^2}{2k_{b1}^2}\right) - k_{11}e_x - re_y + v_p\cos e_\varphi \\ h_\kappa &= -(2k_{22}+k_2)\tfrac{k_{b2}^2}{\pi}\tfrac{1}{e_y}\sin\left(\tfrac{\pi e_y^2}{2k_{b2}^2}\right)\cos\left(\tfrac{\pi e_y^2}{2k_{b2}^2}\right) - k_{22}e_y - v + re_y \end{aligned} \tag{8}$$

where u_k and h_k are the virtual control signals of u and h, respectively; k_1, k_2, k_{11}, and k_{22} are positive parameters to be designed later; k_{b1} and k_{b2} are the time-varying bound values of $\|e_x\|$ and $\|e_y\|$, respectively. To avoid the derivative of the virtual control signals u_k and h_k, the DSC technique is adopted in the following analysis.

Let u_k and h_k pass through two first-order low-pass filters with the filter time constants λ_1 and λ_2 to obtain u_f and h_f, respectively. There are

$$\begin{cases} \lambda_1 u_f + u_f = u_\kappa \\ u_f(0) = u_\kappa(0) \end{cases} \quad \begin{cases} \lambda_2 h_f + h_f = h_\kappa \\ h_f(0) = h_\kappa(0) \end{cases} \tag{9}$$

Then, define the following new error functions:

$$z_u = u_f - u_\kappa,\ e_u = u - u_f,\ z_h = h_f - h_\kappa,\ e_h = h - h_f \tag{10}$$

Taking the time derivative of e_h in (10), one has

$$\dot{e}_h = \dot{v}_p \sin e_\varphi + v_p \cos e_\varphi (r - \dot{\varphi}_d) + \frac{1}{\lambda_2} z_h \tag{11}$$

Obviously, (7) is converted to

$$\dot{V}_1 < -k_1\frac{k_{b1}^2}{\pi}\tan\left(\frac{\pi e_x^2}{2k_{b1}^2}\right) - k_2\frac{k_{b2}^2}{\pi}\tan\left(\frac{\pi e_y^2}{2k_{b2}^2}\right) + l_1 e_x(e_u+z_u) + l_2 e_y(e_h+z_h) \tag{12}$$

Let $T = -k_1\frac{k_{b1}^2}{\pi}\tan\left(\frac{\pi e_x^2}{2k_{b1}^2}\right) - k_2\frac{k_{b2}^2}{\pi}\tan\left(\frac{\pi e_y^2}{2k_{b2}^2}\right)$, then (12) can be rewritten as

$$\dot{V}_1 < T + l_1 e_x(e_u + z_u) + l_2 e_y(e_h + z_h) \tag{13}$$

Consider the following Lyapunov function candidate:

$$V_2 = V_1 + \frac{1}{2}e_h^2 + \frac{1}{2}e_\varphi^2 \tag{14}$$

Taking the time derivative of (14), one obtains

$$\dot{V}_2 = \dot{V}_1 + e_h(\dot{v}_p \sin e_\varphi + v_p\cos e_\varphi(r - \dot{\varphi}_d) + \tfrac{1}{\lambda_2} z_h) + e_\varphi(r - \dot{\varphi}_d) \tag{15}$$

Let r_k be the virtual control signal of yaw velocity r. Then, design the following virtual control law:

$$r_\kappa = k_3(e_\varphi + e_h v_p \cos e_\varphi) + \dot{\varphi}_d \tag{16}$$

Similar to (9), let r_k pass through a first-order low-pass filter with the filter time constant λ_3 to obtain r_f, that is

$$\lambda_3 r_f + r_f = r_\kappa,\ r_f(0) = r_\kappa(0) \tag{17}$$

Similarly, construct the following new error functions:

$$z_r = r_f - r_\kappa,\ e_r = r - r_f \tag{18}$$

Substituting (16), (17), and (18) into (15) yields

$$\begin{aligned}\dot{V}_2 < T - k_3(e_\varphi + e_h v_p \cos e_\varphi)^2 + l_1 e_x(e_u + z_u) + l_2 e_y(e_h + z_h) \\ + e_h(\dot{v}_p \sin e_\varphi + \tfrac{1}{\lambda_2} z_h) + (e_r + z_r)(e_\varphi + e_h v_p \cos e_\varphi)\end{aligned} \tag{19}$$

According to (10) and (18), one has

$$\dot{e}_u = f_u(\nu) + \frac{1}{m_u}\tau_u + \frac{1}{m_u}\tau_{wu} + \frac{1}{\lambda_1} z_u,\ \dot{e}_r = f_r(\nu) + \frac{1}{m_r}\tau_r + \frac{1}{m_r}\tau_{wr} + \frac{1}{\lambda_3} z_r \tag{20}$$

Consider the following Lyapunov function candidate:

$$V_3 = V_2 + \frac{1}{2} m_u e_u^2 + \frac{1}{2} m_r e_r^2 \tag{21}$$

Taking the time derivative of (21), we have

$$\dot{V}_3 = \dot{V}_2 + e_u(m_u f_u(\nu) + \tau_u + \tau_{wu} + \tfrac{m}{u}\lambda_1 z_u) + e_r(m_r f_r(\nu) + \tau_r + \tau_{wr} + \tfrac{m_r}{\lambda_3} z_r) \tag{22}$$

Substituting (20) into (22) yields

$$\begin{aligned}\dot{V}_3 < T - k_3(e_\varphi + e_h v_p \cos e_\varphi)^2 + e_u(l_1 e_x + m_u f_u(\nu) + \tau_u + \tau_{wu} + \tfrac{m_u}{\lambda_1} z_u) \\ + e_r(e_\varphi + m_r f_r(\nu) + \tau_r + \tau_{wr} + \tfrac{m_r}{\lambda_3} z_r) + l_1 e_x z_u + l_2 e_y z_h \\ + e_h(l_2 e_y + \dot{v}_p \sin e_\varphi + \tfrac{1}{\lambda_2} z_h) + z_r(e_\varphi + e_h v_p \cos e_\varphi)\end{aligned} \tag{23}$$

Then, the actual feedback control laws τ_u and τ_r can be designed as

$$\tau_u = -k_4 e_u - l_1 e_x - m_u f_u(\nu) - \tau_{wu} - \frac{m_u}{\lambda_1} z_u \tag{24}$$

$$\tau_r = -k_5 e_r - e_\varphi - m_r f_r(\nu) - \tau_{wr} - \frac{m_r}{\lambda_3} z_r \tag{25}$$

Since the unmeasurable disturbance τ_{wu} and τ_{wr} in (24) and (25) are unavailable for the control law design. To compensate the negative influences induced by external disturbance, the following disturbance observers are established

$$\hat{\tau}_{wu} = \beta_1 + k_{wu} m_u u,\ \dot{\beta}_1 = -k_{wu}\beta_1 - k_{wu}(m_u f_v(\nu) + \tau_u + k_{wu} m_u u) \tag{26}$$

$$\hat{\tau}_{wr} = \beta_2 + k_{wr} m_r r,\ \dot{\beta}_2 = -k_{wr}\beta_2 - k_{wr}(m_r f_r(\nu) + \tau_r + k_{wr} m_r r) \tag{27}$$

where β_1 and β_2 are the observers state; k_{wu} and k_{wr} are positive parameters to be designed later; $\hat{\tau}_{wu}$ and $\hat{\tau}_{wr}$ are the estimated values of τ_{wu} and τ_{wr}, respectively; Define the observer errors as $\tilde{\tau}_{wu} = \hat{\tau}_{wu} - \tau_{wu}$ and $\tilde{\tau}_{wr} = \hat{\tau}_{wr} - \tau_{wr}$. Then, (24) and (25) can be rewritten as

$$\tau_u = -k_4 e_u - l_1 e_x - m_u f_u(\nu) - \hat{\tau}_{wu} - \frac{m_u}{\lambda_1} z_u \tag{28}$$

$$\tau_r = -k_5 e_r - e_\varphi - m_r f_r(\nu) - \hat{\tau}_{wr} - \frac{m_r}{\lambda_3} z_r \tag{29}$$

Consider the following Lyapunov function candidate:

$$V_4 = V_3 + \frac{1}{2}\tilde{\tau}_{wu}^2 + \frac{1}{2}\tilde{\tau}_{wr}^2 \tag{30}$$

whose derivative is

$$\dot{V}_4 = \dot{V}_3 + \tilde{\tau}_{wu}(\dot{\tau}_{wu} - k_{wu}\tilde{\tau}_{wu}) + \tilde{\tau}_{wr}(\dot{\tau}_{wr} - k_{wr}\tilde{\tau}_{wr}) \tag{31}$$

Substituting (28) and (29) into (30) yields

$$\dot{V}_4 < T - k_3(e_\varphi + e_h v_p \cos e_\varphi)^2 - k_4 e_u^2 - k_5 e_r^2 - k_{wu}\tilde{\tau}_{wu}^2 - k_{wr}\tilde{\tau}_{wr}^2 + \delta \tag{32}$$

where $\delta = l_1 e_x z_u + l_2 e_y z_h + e_h(l_2 e_y + \dot{v}_p \sin e_\varphi + \frac{1}{\lambda_2} z_h) + z_r(e_\varphi + e_h v_p \cos e_\varphi) + e_u \tilde{\tau}_{wu} + e_r \tilde{\tau}_{wr} - \tilde{\tau}_{wu}\dot{\tau}_{wu} - \tilde{\tau}_{wu}\dot{\tau}_{wu}$.

4 Stability Analysis

In this section, we analyze the stability of the designed trajectory tracking control system. Consider a Lyapunov function candidate as

$$V_5 = V_4 + \frac{1}{2}(z_u^2 + z_h^2 + z_r^2) \tag{33}$$

whose derivative is

$$\dot{V}_5 = \dot{V}_4 - \tfrac{1}{\lambda_1} z_u^2 - \tfrac{1}{\lambda_2} z_h^2 - \tfrac{1}{\lambda_3} z_r^2 - z_u \Psi_1 - z_h \Psi_2 - z_r \Psi_3 \tag{34}$$

where $\dot{z}_u = \dot{u}_f - \dot{u}_\kappa = -\frac{1}{\gamma_1} z_u - \Psi_1, \dot{z}_h = \dot{h}_f - \dot{h}_\kappa = -\frac{1}{\lambda_2} z_h - \Psi_2, \dot{z}_r = \dot{r}_f - \dot{r}_\kappa = -\frac{1}{\gamma_3} z_r - \Psi_3$.

It is assumed that Ψ_1, Ψ_2, and Ψ_3 are continuous functions with maximum $\bar{\Psi}_1$, $\bar{\Psi}_2$, and $\bar{\Psi}_3$. Then, substituting (32) into (34) yields

$$\begin{aligned}\dot{V}_5 < T - k_3 e_\varphi^2 - k_3(v_p \cos e_\varphi)^2 e_h^2 - k_4 e_u^2 - k_5 e_r^2 - 2k_3 e_h e_\varphi v_p \cos e_\varphi \\ - k_{wu}\tilde{\tau}_{wu}^2 - k_{wr}\tilde{\tau}_{wr}^2 - \tfrac{1}{\lambda_1} z_u^2 - \tfrac{1}{\lambda_2} z_h^2 - \tfrac{1}{\lambda_3} z_r^2 - z_u\Psi_1 - z_h\Psi_2 - z_r\Psi_3 + \delta\end{aligned} \tag{35}$$

It is assumed that $\bar{\gamma}$, $\bar{\tau}_{wu}$, $\bar{\tau}_{wr}$, $\bar{k}_{b1}$ and $\bar{k}_{b2}$ are continuous functions with maximum γ, $\dot{\tau}_{wu}$, $\dot{\tau}_{wr}$, e_x and e_y, where $\gamma = \dot{v}_p \sin e_\varphi + \frac{z_h}{\lambda_2}$. According to Young's inequality, by expanding δ, we have

$$\begin{aligned}\delta \le \tfrac{l_1}{2} z_u^2 + \tfrac{l_1}{2}\bar{k}_{b1}^2 + \tfrac{l_2}{2} z_h^2 + \tfrac{l_2}{2}\bar{k}_{b2}^2 + \tfrac{l_2}{2} e_h^2 + \tfrac{l_2}{2}\bar{k}_{b2}^2 + \tfrac{v_p}{2} e_h^2 + \tfrac{v_p}{2} z_r^2 + \tfrac{1}{2} e_\varphi^2 + \tfrac{1}{2} z_r^2 \\ + \tfrac{1}{2} e_h^2 + \tfrac{1}{2}\bar{\gamma}^2 + \tfrac{1}{2} e_u^2 + \tfrac{1}{2}\tilde{\tau}_{wu}^2 + \tfrac{1}{2} e_r^2 + \tfrac{1}{2}\tilde{\tau}_{wr}^2 + \tfrac{1}{2}\tilde{\tau}_{wu}^2 + \tfrac{1}{2}\bar{\tau}_{wu}^2 + \tfrac{1}{2}\tilde{\tau}_{wr}^2 + \tfrac{1}{2}\bar{\tau}_{wr}^2\end{aligned} \tag{36}$$

Then, by sorting out the above equation, one can have

$$\begin{aligned}\dot{V}_5 < &-a_1\frac{k_{b1}^2}{\pi}\tan\left(\frac{\pi e_x^2}{2k_{b1}^2}\right)-a_2\frac{k_{b2}^2}{\pi}\tan\left(\frac{\pi e_y^2}{2k_{b2}^2}\right)-a_3e_\varphi^2\\ &-a_4e_u^2-a_5e_r^2-a_6e_h^2-a_7z_u^2-a_8z_h^2-a_9z_r^2-a_{10}\tilde{\tau}_{wu}^2-a_{11}\tilde{\tau}_{wr}^2+\Delta\end{aligned} \tag{37}$$

where $a_1 = k_1$, $a_2 = k_2$, $a_3 = k_3 - k_3v_p - \frac{1}{2}$, $a_4 = k_4 - \frac{1}{2}$, $a_5 = k_5 - \frac{1}{2}$, $a_6 = k_3v_p - \frac{l_2}{2} - \frac{1}{2}$, $a_7 = \frac{1}{\lambda_1} - \frac{l_1}{2} - \frac{1}{2}$, $a_8 = \frac{1}{\lambda_2} - \frac{l_2}{2} - \frac{1}{2}$, $a_9 = \frac{1}{\lambda_3} - \frac{v_p}{2} - 1$, $a_{10} = k_{wu} - 1$, $a_{11} = k_{wr} - 1$, and $\Delta = \frac{1}{2}\bar{\Psi}_1^2 + \frac{1}{2}\bar{\Psi}_2^2 + \frac{1}{2}\bar{\Psi}_3^2 + \frac{1}{2}\bar{\gamma}^2 + \frac{l_1}{2}\bar{k}_{b1}^2 + l_2\bar{k}_{b2}^2 + \frac{1}{2}\bar{\tau}_{wu}^2 + \frac{1}{2}\bar{\tau}_{wr}^2$.

Theorem 1. *Consider underactuated USV (1) satisfying Assumption 1, control laws τ_u in (28) and τ_r in (29), disturbance observers (26) and (27). If given any ς for all initial conditions satisfying $V(0) \leq \varsigma$ with the preselected error constrains satisfying $\|e_x\| \leq k_{b1}$ and $\|e_y\| \leq k_{b2}$, and the design parameters satisfying $a_i > 0$. Then we have the observer errors $\tilde{\tau}_{wu}$ and $\tilde{\tau}_{wu}$ converge to an adjustable neighborhood of the origin by appropriately choosing the design parameters.*

Proof. Let $a = \min\{a_1, a_2...a_{11}\}$, then (37) can be simplified to $\dot{V}_5 \leq -aV_5 + \Delta$.

Solving the above inequality, we have

$$V_5 \leq \left(V_5(0) - \frac{\Delta}{a}\right)e^{-at} + \frac{\Delta}{a}, \forall t > 0 \tag{38}$$

Based on the above analysis, by adjusting parameters k_{11}, k_{22}, k_{b1}, k_{b2}, k_{wu}, k_{wr} and $k_j(j = 1, 2, ...5)$, we can obtain a larger a. When satisfied $a > \frac{\Delta}{V_5}$, we have $\dot{V}_5 < 0$. Hence, it is observed from (38) that V_4 is eventually bounded by $\frac{\Delta}{a}$. e_x, e_y, e_φ, e_u, e_h and e_r are globally uniformly bounded. Theorem 1 is proved.

5 Simulation Results and Discussion

In this section, the effectiveness of the proposed tracking control scheme is verified by a simulation example.

The model parameters in (1) are given as $m_u = 1.956$, $m_v = 2.045$, $m_r = 0.043$, $d_u = 0.0358$, $d_{u2} = 0.0179$, $d_{u3} = 0.0089$, $d_v = 0.1183$, $d_{v2} = 0.0591$, $d_{v3} = 0.0295$, $d_r = 0.0308$, $d_{r2} = 0.0154$, and $d_{r3} = 0.0077$. The design parameters are selected as $k_1 = k_2 = 1$, $k_3 = 3$, $k_4 = 5$, $k_5 = 6$, $k_{wu} = 2$, $k_{wr} = 2$ and $\lambda_1 = \lambda_2 = \lambda_3 = 0.1$. k_{b1} and k_{b2} are constructed as $k_{b1}= 0.43 + 6.35e^{-t}$, $k_{b2} = 0.43 + 3.55e^{-t}$. In addition, the external disturbance are given by $\tau_{wu} = 1 + 0.1sin(0.2t) + 0.2cos(0.5t)$, $\tau_{wr} = 1 + 0.2sin(0.1t) + 0.1cos(0.2t)$. The initial states of the USV are $[x, y, \varphi]^T = [0m, 5m, 0rad]^T$. The initial states of the virtual leader are $[x_d, y_d,\varphi_d]^T = [0m, 0m, 0rad]^T$. The surge reference velocity u_d is chosen as $u_d = 2m/s$, while the yaw reference velocity r_d is designed as $r_d = \frac{\pi}{160}rad/s$ in the first $40s$, $r_d = \frac{\pi}{165}rad/s$ for $40s \leq t < 80s$, $r_d = \frac{\pi}{170}rad/s$ for $80s \leq t < 130s$, and $r_d = 0rad/s$ for $130s \leq t \leq 200s$. The observed and actual external disturbance are shown in Fig. 1.

The real-time tracking performance is shown in Fig. 2. The tracking errors of position and heading angle are depicted in Fig. 3. Meanwhile, the tracking errors

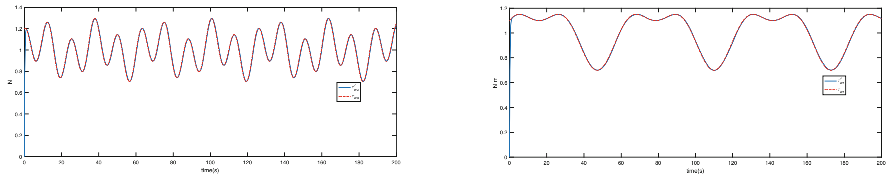

Fig. 1. Estimated and actual disturbance.

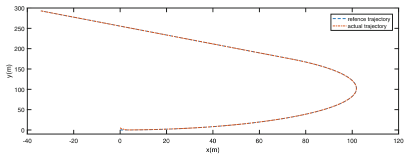

Fig. 2. Trajectory tracking performance.

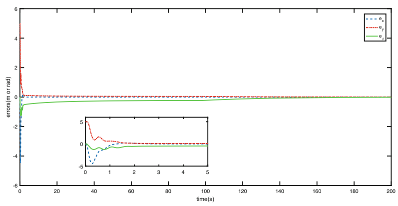

Fig. 3. Tracking errors of position and heading angle.

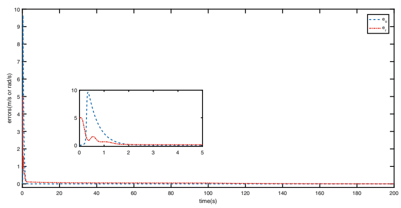

Fig. 4. Tracking errors of velocities.

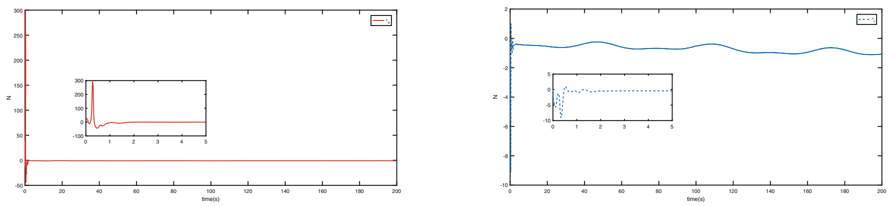

Fig. 5. Control inputs of the USV.

of velocities are shown in Fig. 4. The control inputs of the USV are depicted in Fig. 5. From Fig. 3 and Fig. 4, it can be seen that the proposed trajectory tracking control scheme can guarantee satisfying performance for the underactuated USV. Since the heading angle changes constantly during the process of trajectory tracking, thus the control input τ_r fluctuates in a bounded region. The above simulation results prove the effectiveness of the proposed scheme.

6 Conclusion

In this paper, a trajectory tracking control scheme for underactuated USVs has been proposed. By constructing a disturbance observer, the external disturbance has been estimated effectively. To avoid the problem of "explosion of complexity" induced by the derivation of the virtual control variables, a first order filter and a time-varying tan-type BLF have been established. By introducing the backstepping and dynamic surface control techniques, the trajectory tracking control laws have been designed. Under the designed control laws, the trajectory tracking of underactuated USVs has been achieved. Finally, a simulation example has been presented to verify the applicability of the results derived in this paper.

References

1. Pei, Z.Y., Dai, Y.S., Li, Z.G., Jin, J.C., Shao, F.: Overview of unmanned surface vehicle motion control methods. Mar. Sci. **44**(03), 153–162 (2020)
2. Zhang, Y., Hua, C.C., Li, K.: Disturbance observer-based fixed-time prescribed performance tracking control for robotic manipulator. Int. J. Syst. Sci. **50**(13), 2437–2448 (2019)
3. Van, M.: Adaptive neural integral sliding-mode control for tracking control of fully actuated uncertain surface vessels. Int. J. Robust Nonlinear Control **29**(5), 1537–1557 (2019)
4. Mu, D.D., Wang, G.F., Fan, Y.S., Zhao, Y.: Modeling and identification of podded propulsion unmanned surface vehicle and its course control research. Math. Probl. Eng. 1–13 (2017)
5. Zhao, Y., Mu, D.D., Wang, G.F., Fan, Y.S.: Trajectory tracking control for unmanned surface vehicle subject to unmeasurable disturbance and input saturation. IEEE Access **8**, 191278–191285 (2020)
6. Li, J., Guo, H., Zhang, H.: Double-Loop structure integral sliding mode control for UUV trajectory tracking. Proc. IEEE **7**, 101620–101632 (2019)
7. Swaroop, D., Hedrick, J.K., Yip, P.P., Gerdes, J.C.: Dynamic surface control for a class of nonlinear system. IEEE Trans. Autom. Control **45**(10), 1893–1899 (2000)
8. Pan, C., Zhou, L., Xiong, P., Xiao, X.: Robust adaptive dynamic surface tracking control of an underactuated surface vessel with unknown dynamics. In: 2018 37th Chinese Control Conference, pp. 592–597 (2018)
9. Zhang, C.J., Wang, C., Wei, Y., Wang, J.Q.: Robust trajectory tracking control for underactuated autonomous surface vessels with uncertainty dynamics and unavailable velocities. Ocean Eng. **218**, 108099 (2020)
10. He, W., Yin, Z., Sun, C.Y.: Adaptive neural network control of a marine vessel with constraints using the asymmetric barrier lyapunov function. IEEE Trans. Cybern. **47**(7), 1641–1651 (2017)
11. Yin, Z., He, W., Yang, C.G.: Tracking control of a marine surface vessel with full-state constraints. Int. J. Syst. Sci. **48**(3), 535–546 (2017)
12. Zheng, Z.W., Sun, L., Xie, L.H.: Error-constrained LOS path following of a surface vessel with actuator saturation and faults. IEEE Trans. Syst. Man Cybern. **48**(10), 1794–1805 (2018)
13. Fossen, T.I.: Handbook of marine craft hydrodynamics and motion control (2011)