# Modified Multiloop Finite Dimensional Robust Repetitive Controller for Supply Air Pressure Loop of a Heating, Ventilation, and Air Conditioning System

**Sagarika Sarkar, Amrita Mukherjee, Ujjwal Mondal, and Anindita Sengupta**

**Abstract** The objective of this paper is to apply the conventional Proportional Integral Derivative (PID) controller and Repetitive Controller based PID controller (RCPID) for the supply air pressure loop of a Heating, Ventilation, and Air Conditioning (HVAC) system in terms of tracking the given step reference signal and also rejecting periodic disturbances having known periods. The Internal Model Principle (IMP) based Repetitive Controller (RC) gives infinite number of pairs of poles on the imaginary axis in the $s$-plane leading the system to instability. Thus, Finite Dimensional Repetitive Control System (FDRCS) is introduced. Multiple loops Robust Repetitive Controller (RRC) has been used to cope up with the small period uncertainties. Finally, this paper shows the comparison of the conventional PID controller and different models derived Finite Dimensional Robust Repetitive Controller (FDRRC) based PID controllers for the supply air pressure loop of the HVAC system so that the step reference signal is tracked and periodic disturbances get rejected producing minimum error.

**Keywords** PID controller · HVAC system · Supply air pressure loop · IMP · RC · FDRC · FDRRC · FDRRC based PID controllers

## 1 Introduction

In practice, reference signals that are given to any process may be periodic in nature. Even the process may be affected by periodic disturbances. In that case, repetitive controller gives a far better performance in tracking repetitive reference signal and also in rejection of periodic disturbances in comparison with the normal PID controller.

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S. Sarkar (✉) · A. Mukherjee · U. Mondal
Department of Applied Physics, University College of Science and Technology, Kolkata, India

A. Sengupta
Department of Electrical Engineering, Indian Institute of Engineering Science and Technology, Shibpur, Howrah, India

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Repetitive controller was first introduced to control proton synchrotron magnetic power supply. It is also applied to several research areas like tubular heat exchanger, computer hard disk, power supply (UPS), CD player, pulse-width modulated (PWM) inverters, robot control, thickness control in cold rolling, continuous steel casting, noise cancelation and many other industrial applications.

Repetitive Control System (RCS) shows its simplicity on the basis of Internal Model Principle (IMP). This IMP includes a fixed period time delay ($L$) in the positive feedback loop and this time delay is actually the period of the reference signal. Due to the periodic signal generator in the control loop, RCS can track periodic reference signal and reject periodic disturbances properly.

Heating, Ventilation, and Air Conditioning (HVAC) systems have become one of the most essential parts for industrial purposes. Both purposes of heating and cooling are served by this system depending on the requirement.

This paper deals with the comparison of the applications of normal PID controller, Repetitive Controller (RC) and Modified Repetitive Controller in Supply Air Pressure Loop of HVAC system.

## 2 Description of the System

### *2.1 Basic Functions of HVAC System*

HVAC is the technology to provide the indoor and vehicular environmental comfort. An HVAC system provides thermal comfort along with the acceptable air quality. The basic functions of an HVAC system are heating, cooling, humidifying, dehumidifying, ventilating, cleaning and controlling the air movement. This system acts as the air conditioning system in summer and as the heater in winter. The obtained temperature is expected to be constant which is maintained by the supply air pressure loop present in the HVAC system.

### *2.2 Importance of Supply Air Pressure Loop of HVAC System*

Here, Fig. 1 represents a typical HVAC system which includes the supply air pressure loop. Here, the outside air is mixed with the return air of either building or vehicle. A supply fan helps the cooling coil to suck the mixed supply air. The obtained cooled air is delivered to multiple zones. A valve CHWS regulates the off coil air temperature by controlling the supply of chilled water. The supply air pressure should be maintained at a fixed value by regulating the speed of the supply air fan [1] in order to make each zone work properly. Each zone temperature is finely tuned by damper of the Variable Air Volume box in each zone.

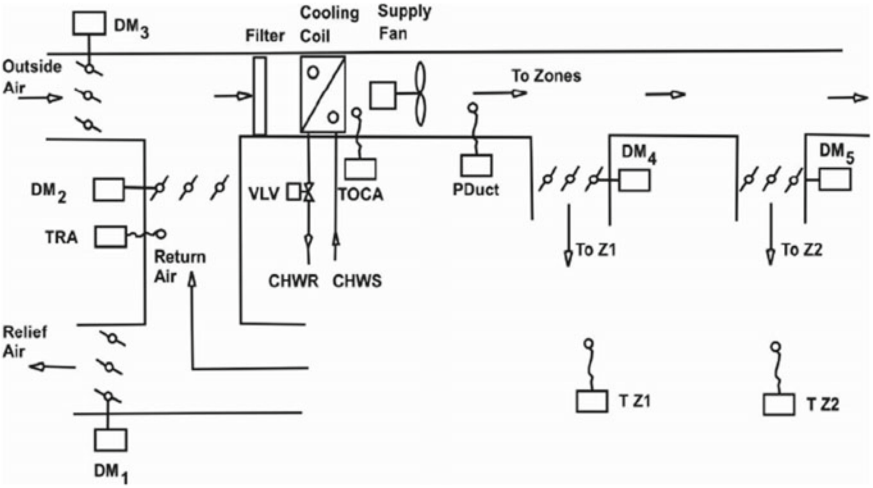


**Fig. 1** A typical HVAC system

One of the most important components needed for a functional HVAC system by avoiding either overload or underload condition is the right amount of air flow. This right amount of air flow can be ensured if speed of air flow be regulated and disturbance can be eliminated. The regulation in air flow and disturbance elimination can be assured using Repetitive Controller (RC). The supply air pressure is directly proportional to the speed of the supply air fan.

The transfer function of the supply air pressure loop, having time delay of 0.5 s, is given as

$$G(s) = \frac{0.81}{(0.97s + 1)(0.1s + 1)} \mathrm{e}^{-0.5s} \tag{1}$$

## 3 Control Strategy

The aim of this paper is to use different control strategies on the process and compare them to have the appropriate one. It is desired to obtain the perfect tracking of step reference signal and rejection of periodic disturbances along with minimum error.

The different controllers used in this paper are discussed below:

### 3.1 Conventional PID Controller

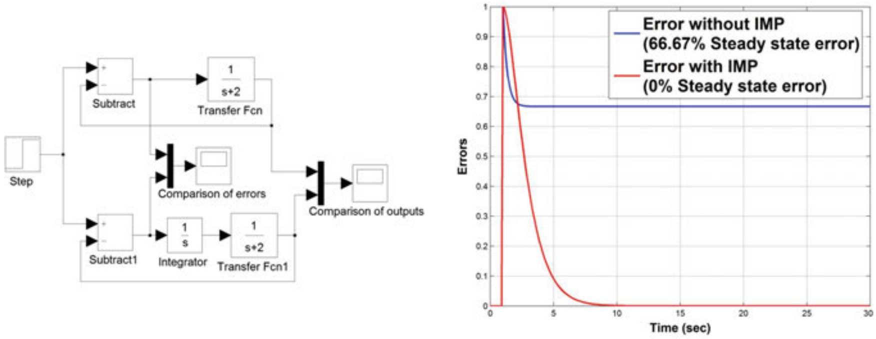
Generally, PID (Proportional-Integral-Derivative) controllers are used in HVAC systems because of the simple structure, hassle free tuning and easy implementation. They are acceptable in many industrial applications but they show inefficiency to meet desired specifications. It is a tough call to obtain a proper tuning of PID controller for supply air pressure loop of an HVAC system in a building [2].

### 3.2 IMP with Periodic Signal Generator and RC Based PID Controller

According to the Internal Model Principle (IMP), proposed by Wonham and Francis, "Perfect tracking of a reference signal can be assured if the generator of the reference signal is included in the stable closed loop system." [3] The inclusion of the generator of reference signal within the closed loop system results in addition of pole(s).

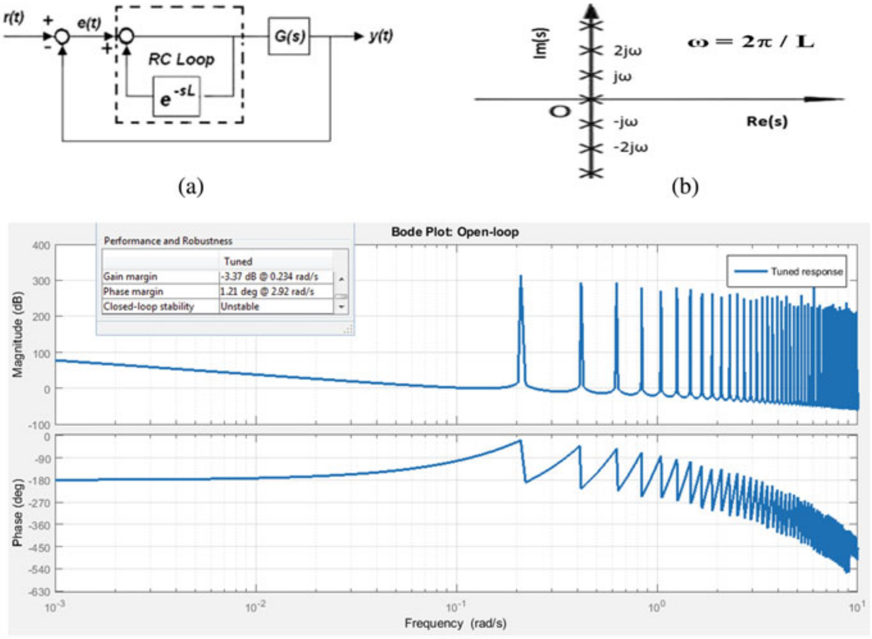
The following example is given for an open loop transfer function $G(s) = 1/(s + 2)$ with a unit step reference signal ($1/s$) in Fig. 2b. The obtained steady-state error ($E_{ss}$) is 66.67% without using IMP, whereas introducing the generator of the reference signal in the stable closed loop system based on IMP gives $E_{ss}$ as 0%.

Repetitive Control System (RCS) designed by Hara et al. [4] in 1985 is based on a simple learning control method mainly to track a periodic reference signal and reject a periodic disturbance signal properly by including periodic signal generator in its control loop. Here, Fig. 3a, b and c shows the basic repetitive control loop, infinite number of open loop pole pairs on the imaginary axis in the s-plane [5] and frequency response of the system with Repetitive Controller based PID (RCPID) Controller.



(a) (b)

**Fig. 2** Tracking of reference step signal without and with IMP; **a** Simulink model, **b** errors without and with IMP



Performance and Robustness

| | Tuned |
|---|---|
| Gain margin | -3.37 dB @ 0.234 rad/s |
| Phase margin | 1.21 deg @ 2.92 rad/s |
| Closed-loop stability | Unstable |

(a) (b)

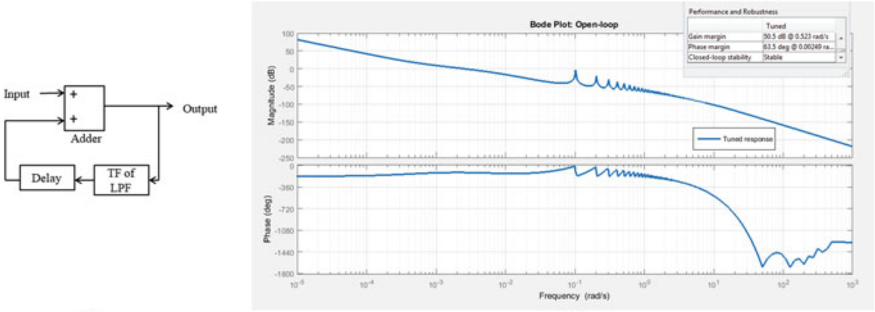
(c)

**Fig. 3** Basic RC loop and unstable frequency response of the process; **a** Basic repetitive control loop, **b** infinite number of open loop pole pairs on the imaginary axis, **c** bode plot of the system with RCPID

The factor $\left(\frac{1}{1-e^{-Ls}}\right)$, included within RCS, produces poles at jp $\left(\frac{2\pi}{L}\right)$ where $p = 0, \pm 1, \pm 2, \pm 3, \ldots. \pm \infty$, corresponding to the harmonic and sub harmonics of the period ($L$) of the reference signal. Thus, the controller is led to track any periodic signal as well as reject any periodic disturbance of period $L$ [4]. In the case of tracking and/or rejecting periodic signal, RCPID controller is used instead of conventional PID controller [6]. RC is sometimes known as Infinite Dimensional Repetitive Controller (IDRC) [7] as use of this controller produces infinite number of poles located on the imaginary axis in the s-plane which cause instability of the system.

### *3.3 Finite Dimensional Repetitive Controller (FDRC) Based PID Controller*

IDRC becomes FDRC by cascading an LPF with the time delay element in the positive feedback loop [8]. This increases the stability of the closed loop system by filtering away the high-frequency modes by this LPF. Here, Fig. 4a, b shows the



(a) (b)

**Fig. 4** FDRC loop and stable frequency response of the process; **a** Basic FDRC loop, **b** bode plot of the system

FDRC loop and system's stable frequency response obtained using FDRC based PID controller.

## *3.4 Multiple Loop RC Based PID Controller*

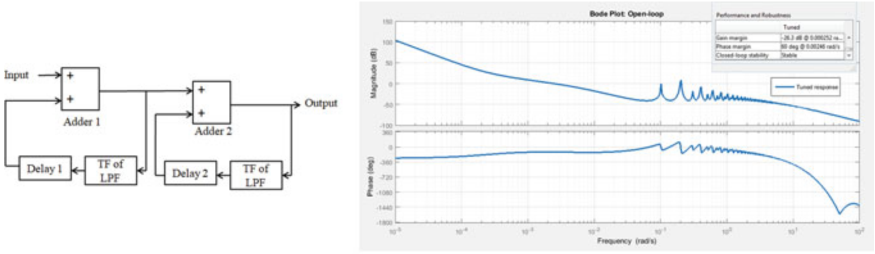
Single loop FDRC gives better performance than RC. But the problem of instability due to the inappropriate estimation of the period of the reference signal and the mismatch of it with the delay of RC loop can be resolved with the help of multiple loop Repetitive Controller [9]. Conventional RC can't handle the uncertainties of the reference signal.

More than one memory loops are used to modify the frequency dynamics of RC and increase its robustness to small period changes. The multiple memory loops act as storing elements of errors for more than one previous trial to implement more robustness and thus RRC is designed [10]. This paper follows the models of Repetitive Controllers given by Singh, Owens, and Ujjwal Mondal.

**FDRRC derived from Singh's Model** According to Singh's model [11], the time delay serves as a feed forward delay element and the delay is added cumulatively for each successive stage in the design of the controller [10]. The representation of FDRRC loop derived from Singh's model and the obtained frequency response are shown in Fig. 5a, b.

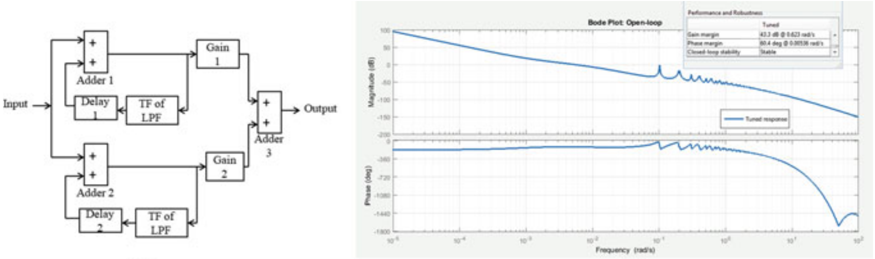
**FDRRC derived from Owens' Model** According to Owens [12], a multi-periodic repetitive controller loop is designed as shown in Fig. 6a, b shows the stable frequency response of the system obtained by this approach.

It is a linear combination of single-periodic repetitive control elements with a stable filter, in each loop, introduced to filter out noise. Here, each internal loop is provided with a gain which represents relative weight.



(a) (b)

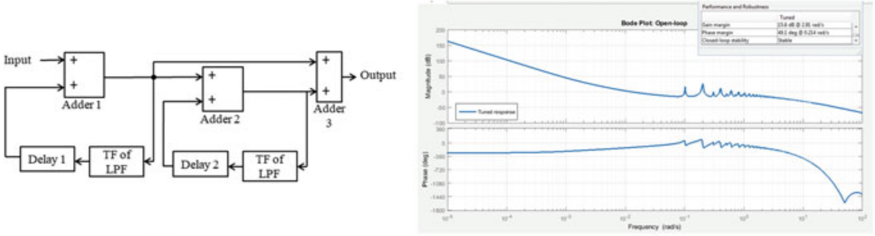
**Fig. 5** FDRRC loop derived from Singh's model and stable frequency response of the process; **a** Simulink model, **b** bode plot of the system



(a) (b)

**Fig. 6** FDRRC loop derived from Owens' model and stable frequency response of the process; **a** Simulink model, **b** bode plot of the system

**FDRRC derived from Mondal's Model** The FDRRC loop derived from the model of multiloop RC given by Dr. Ujjwal Mondal and the system's frequency response by applying this controller are shown in Fig. 7a, b, respectively.



(a) (b)

**Fig. 7** FDRRC loop derived from Mondal's model and stable frequency response of the process; **a** Simulink model, **b** bode plot of the system

## 4 Reference and Disturbance

This paper has dealt with effectiveness of FDRRC based PID controllers for step reference and periodic disturbances over conventional PID controller. The step reference signal is given as

$$r(t) = 2.5u(t) \tag{2}$$

The periodic disturbance is given as the sum of two periodic disturbances having different periods. The disturbance is given as

$$\mathrm{d}(t) = \mathrm{d}_1(t) + \mathrm{d}_2(t) \tag{3}$$

where,

$$\mathrm{d}_1(t) = 0.5\sin\left(\frac{2\pi}{30}\right)t \text{ and } \mathrm{d}_2(t) = 0.5\sin\left[\left(\frac{2\pi}{60}\right)t + \frac{\pi}{2}\right] \tag{4}$$

## 5 Results

Here, the RC based PID controller has been applied to the system to have the comparison with the application of conventional PID controller. But RC leads the system to instability. The response of the system using RCPID controller is given in Fig. 8 along with the reference.

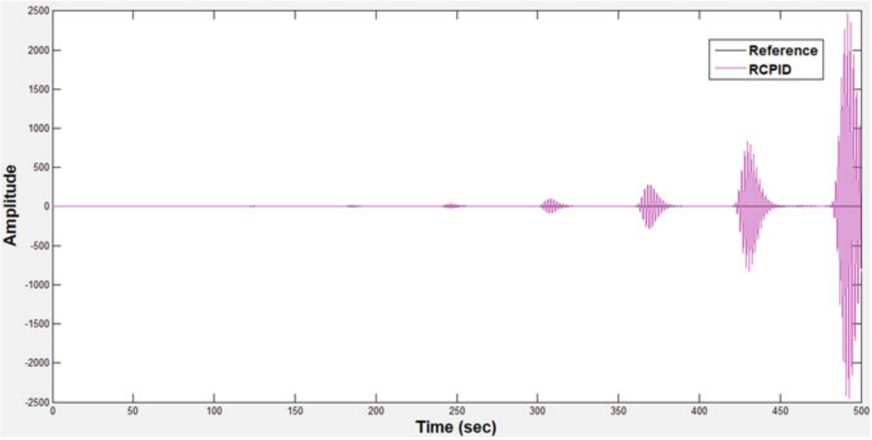


Fig. 8 System's response using RCPID controller with reference

A comparison among the responses of the Supply Air Pressure Loop of an HVAC System obtained by using conventional PID controller, FDRC based PID controller and different models derived FDRRC based PID controllers with the reference is shown in Fig. 9.

Also, a comparison among the tracking errors of the system obtained by using conventional PID controller and different models derived FDRRC based PID controllers is given in Fig. 10.

To have numerical representation of errors obtained in this paper, comparison of the values of Integral Absolute Error (IAE), Integral Time weighted Absolute Error

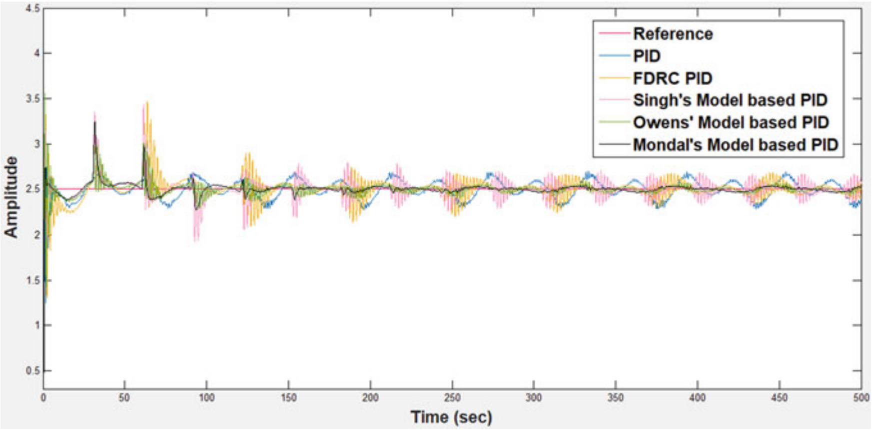


**Fig. 9** Comparison of system's responses using PID controller, FDRC based PID controller and different models derived FDRRC based PID controllers with reference

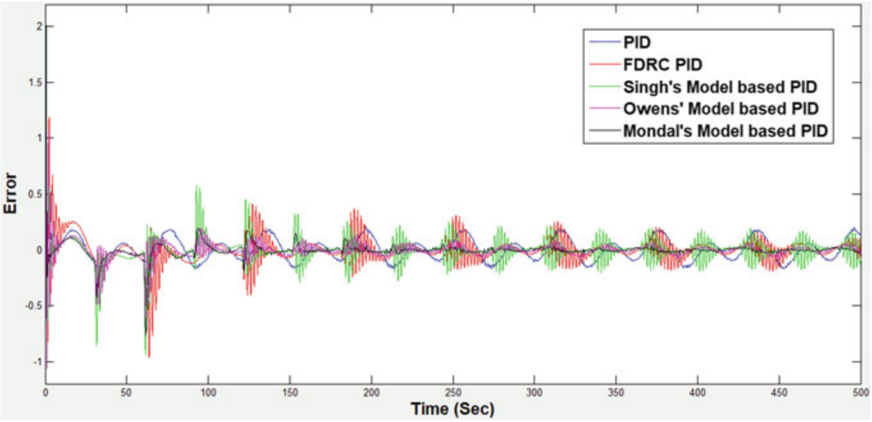


**Fig. 10** Comparison of tracking errors of the system using PID controller, FDRC based PID controller and different models derived FDRRC based PID controllers

**Table 1** Comparison of IAE, ITAE, ISE and ITSE of the system obtained using conventional PID controller, RC based PID controller, single loop FDRC based PID controller and Modified FDRRC based PID controllers

| Control strategies | Comparison of errors (numerical representation) | | | |
|---|---|---|---|---|
| | IAE | ITAE | ISE | ITSE |
| Conventional PID controller | 41.17 | 9815 | 7.038 | 1080 |
| RC based PID controller | $3.752 \times 10^4$ | $1.738 \times 10^7$ | $6.01 \times 10^7$ | $2.925 \times 10^{10}$ |
| Single Loop FDRC based PID controller | 37.55 | 6761 | 10.23 | 819.2 |
| Singh’s model derived FDRRC based PID controller | 31.73 | 6409 | 7.991 | 784 |
| Owens’ model derived FDRRC based PID controller | 20.56 | 3227 | 5.253 | 167 |
| Mondal’s model derived FDRRC based PID controller | 15.94 | 2424 | 4.293 | 116.5 |

(ITAE), Integral Squared Error (ISE) and Integral Time weighted Squared Error (ITSE) [13] is given in Table 1.

## 6 Conclusion

To maintain the healthy and hygienic conditions in terms of temperature, humidity and proper air circulation, HVAC system is used in residential places as well as places like hospitals, office buildings and in vehicles also.

In this paper, basic functions of an HVAC system and importance of its supply air pressure loop have been described in brief. RCS, implemented using IMP, shows infinite dimensional nature and makes the system unstable. Thus, a properly designed Low Pass Filter has been incorporated so that the effect of the positive feedback in the RC loop gets minimized. The stability of the system obtained by suitable design of FDRRCs entails trade(s)-off between stability and tracking performances.

The various models derived FDRRCS have been used with the said system. Comparison among the conventional PID controller and the FDRRC based PID controllers, derived from different models, has been performed in terms of rejecting periodic disturbance and stability. Mondal’s model derived FDRRC gives better result with respect to other approaches used in this paper.

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