

Chapter 8

Irrigation Planning with Fuzzy Parametric Programming Approach



J. B. Gurav, R. U. Kamodkar, and D. G. Regulwar

Abstract The present book chapter deals with tackling of uncertainty and vagueness involved in the irrigation planning problem with the concept of sustainability. Multi Objective Fuzzy Linear Programming (MOFLP) models have been developed using Fuzzy Parametric Programming (FPP) approach and applied it to the case study of Jayakwadi Project Stage-I, Maharashtra State, India. The various objectives such as NB, CP, EG, and MU, which are maximized. The involvement of Decision Maker (DM) is permitted in the proposed model to handle the fuzziness in the resources, technological coefficients, and coefficients of objective function. The results of the study show that the level of satisfaction (λ) is maximum when the precision level (μ) is minimum and vice versa. It is also seen that for each level of precision which is varying from 0 to 1 with an interval of 0.1, the aggregated level of satisfaction (λ) is above 68%. The obtained results help to provide insight into various alternative optimal cropping patterns with different degree of precision and allow the DM to take judicious decision(s) in the context of the socio-economic development of a particular region.

Keywords Sustainability · Irrigation planning · Uncertainty · Membership functions: Monotonously increasing · Linear concave · Linear convex

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8.1 Introduction

8.1.1 General

The irrigation planning problem can be formulated with reference to the availability of various resources such as land availability, availability of water, cropping pattern at the time of inception of a particular project, cropping seasons, climatic conditions requirement of the food for a particular region, and socio-economic development, etc.

8.1.2 Sustainable Irrigation Planning

The formulation of irrigation planning problem and its execution with reference to proper use of the natural resources available for the betterment of mankind and other living creatures, which may be recognized as sustainable irrigation planning. In the mathematical sense, this refers to the adoption of irrigation practice of sustainable objectives and constraints. The sustainable objectives can be clubbed together as maximization of manure utilization, employment generation, crop production, net benefits, green energy, etc. These objectives can be subjected to various constraints/stipulations such as cultivable land, availability and quality of water, labor availability, manure availability, fixed water demand for industrial and drinking purpose, cropping pattern, etc. The active involvement of decision (policy) maker plays a vital role in all phases of the irrigation planning and its execution.

8.1.3 Uncertainty and Vagueness in Irrigation Planning

In the real-world scenario, various parameters are related to irrigation planning such as hydrology, hydrogeology, soil and its characteristics, climatic variables, hydro-meteorological parameters, management practice data, crop data, and basin data. Uncertainty and vagueness are associated with these parameters, which needs proper attention to work out adequate irrigation planning model and its solution. In the case of handling the number of objectives in case of irrigation planning, focusing only on a single objective does not result in optimal cropping pattern planning because various purposes are conflicting with each other. Similarly, dealing with the aim of net benefits maximization, the irrigation policymaker needs to think from different aspects, such as the generation of employment, sufficient food availability, etc.

8.1.4 Necessity of Fuzzy Logic for Uncertainty and Vagueness

The fuzzy set theory/concept can be used to express mathematically the uncertainty and vagueness related to a variety of parameters of sustainable irrigation planning. Zadeh (1965), the founder of the term fuzzy, developed the fuzzy set theory which has given the substitute to classical set theory. This theory involves the infinite range of alternatives between zero and one. It also involves the parameters and its grade of membership in the form of a fuzzy set, which can be used to deal with uncertainty and vagueness.

8.1.5 Literature Review

The methodology has developed and suggested for the solution of fuzzy linear programming (Gasimov and Yenilmez 2002). Jimenez et al. (2007) have developed a number of LP problems with a variety of parameters as fuzzy numbers however whose decision variables are crisp in nature and proposed an interactive approach for the solution of LP with fuzzy numbers. A two-phase method has introduced for solving MOFLP problems with the involvement of DM (Arikan and Gungor 2007). The fuzzy dynamic programming model has developed with the use of fuzzy inference system, to include experience and professional judgments of DM and farmers, which helps to work out optimal cropping pattern with conjunctive use of surface water and groundwater to a wide range of climatic conditions (Safavi and Alijanian 2010).

A number of LP models have been developed with the use of fuzzy logic. These models tackle uncertainty and vagueness associated with irrigation planning with the concept of sustainability (Regulwar and Gurav 2011, 2012; Gurav and Regulwar 2012). The LP model with robust optimization method has been proposed and utilized it for agricultural water resource management under uncertainty to the case study of the irrigation network in the province of Isfahan, Iran (Sabouni and Mardani 2013). An inexact fuzzy chance-constrained nonlinear programming method has proposed for the management of agriculture water resources under various uncertainties, which is an improvement over the existing stochastic programming methods (Guo et al. 2013).

A bi-level programming model involving the fuzzy inputs, which has developed and utilized it for the solution to regional water resources allocation problem (Xu et al. 2013). Kumari and Mujumdar (2015) have developed fuzzy a stochastic dynamic programming model and applied to the case study of Bhadra reservoir Karnataka, India.

The biobjective programming model with fuzzy inputs has been developed and applied successfully with its capability to increase agricultural water productivity and to reduce the shortage of irrigation water, along with proper justification to

the various concerns of DM and farmers (Li et al. 2016). The particle swarm optimization techniques along with the fuzzy approach have been proposed to deal with uncertainty for irrigation planning problem with the use of hyperbolic and exponential membership functions (Morankar et al. 2016). Fuzzy optimization model along with fuzzy inference system, which has proposed for conjunctive use of surface and groundwater resources and applied to the case study in the Astaneh-Kouchesfahan Plain in the north of Iran (Sami et al. 2018).

The fuzzy multi objective heuristic model has been proposed along with the uncertainties involved in water resources and economic parameters in a basin. The results of the study show that optimal operation policies provide better results than the deterministic model (Banihabib et al. 2019). Yue et al. (2020) have developed a full fuzzy-interval credibility-constrained nonlinear programming (FFICNP) model to deal with uncertainties in planning irrigation water allocation and found that, higher net system benefit and system efficiency for lower credibility level.

From the above literature review, it is found that various models of LP and MOFLP have been used to tackle the uncertainty and vagueness associated with parameters of irrigation planning.

8.2 Methodology and Model Development

8.2.1 Objectives

The various objectives are considered in the formulation of irrigation planning model depending upon the requirement of the particular region and national importance. In the present study, four objectives of maximization type are considered from the analysis point of view.

Net Benefits (NB)

Most of the time, the farmers have the tendency to get the maximum net benefits with the cultivation of particular crops for the economic-prosperity, which insists the DM to incorporate this as part of irrigation planning policy.

In the present study, discussion with farmers, agricultural and field experts is the basis for deciding the input cost. The input cost is taken as of twenty percent of gross benefit for each crop in the particular crop season. On the basis of average yield of a particular crop per ha and the current market price of a crop, the gross benefits are worked out for each crop.

$$\text{Maximize NB} = \left[\left(\sum_{i=1}^2 J_i^K F_i^K + \sum_{i=1}^3 J_i^R F_i^R + \sum_{i=1}^2 J_i^{TS} F_i^{TS} + \sum_{i=1}^2 J_i^P F_i^P + \sum_{i=1}^1 J_i^{HW} F_i^{HW} \right) \right]$$

$$-\left(\sum_{i=1}^2 J_i^K G_i^K + \sum_{i=1}^3 J_i^R G_i^R + \sum_{i=1}^2 J_i^{TS} G_i^{TS} + \sum_{i=1}^2 J_i^P G_i^P + \sum_{i=1}^1 J_i^{HW} G_i^{HW}\right) \quad (8.1)$$

- i crop index, 1 ha = 10,000 m².
 J_i^K Area (ha) for various crops in Kharif Season;
 J_i^R Area (ha) for various crops in Rabi Season;
 J_i^{HW} Area (ha) for various crops in Hot Weather Season;
 J_i^P Area (ha) for Perennial crops;
 J_i^{TS} Area (ha) for Two Seasonal crops;
 F_i Coefficient of benefit for i th crop in particular season;
 G_i Input cost for i th crop in particular season.

Crop Production (CP)

To expect maximum production of a particular crop is the natural tendency of every farmer, which is essential for the DM to consider it for the irrigation planning with the objective of maximization of crop production. The average yield for crop per ha is considered as crop production coefficient.

$$\text{Maximize (CP)} = \left[\left(\sum_{i=1}^2 J_i^K U_i^K + \sum_{i=1}^3 J_i^R U_i^R + \sum_{i=1}^2 J_i^{TS} U_i^{TS} + \sum_{i=1}^2 J_i^P U_i^P + \sum_{i=1}^1 J_i^{HW} U_i^{HW} \right) \right] \quad (8.2)$$

U_i Average yield of i th crop (Tons per ha).

The food sufficiency for a particular region can be thought of with the help of maximization of crop production, which mainly focus on the survival of people with satisfaction food demand of the particular region, with of view of this the second objective can be thought of sustainability-related.

Employment Generation (EG)

Socio-economic development of a region cannot be possible without availing employment generation to people. This suggests the DM focus on the need for maximization of employment generation related to irrigation planning.

$$\text{Maximize(EG)} = \left[\left(\sum_{i=1}^2 J_i^K M_i^K + \sum_{i=1}^3 J_i^R M_i^R + \sum_{i=1}^2 J_i^{TS} M_i^{TS} + \sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^1 J_i^{HW} M_i^{HW} \right) \right] \quad (8.3)$$

M_i Requirement of Man Days for i th crop per ha.

The required labor/Man Days for a particular crop per ha have been considered based on the basis of discussions with farmers and experts from the agricultural field. This objective related to sustainability with the view of socio-economic development

for developing countries like India, where there is uneven distribution of agricultural land.

Manure Utilization (MU)

$$\text{Maximize (MU)} = \left[\left(\sum_{i=1}^2 J_i^K M_i^K + \sum_{i=1}^3 J_i^R M_i^R + \sum_{i=1}^2 J_i^{\text{TS}} M_i^{\text{TS}} + \sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^1 J_i^{\text{HW}} M_i^{\text{HW}} \right) \right] \quad (8.4)$$

M_i Manure Utilization Requirement (Tons/ha for i th crop).

The discussion with farmers and agricultural field experts is used to fix up the manure utilization requirement per ha for the various crop in respective seasons. The concept of utilization of green manure has introduced in the present study with a view to promote the natural ability of soil to sustain micronutrient sufficiency for various crops in the respective crop season. The waste generated out of various activities of farming and livestock, which can be used as a green manure after decomposition. Such prepared manure is free from harmful chemicals, fertilizers, and pesticides. This attracts the DM to include it in the form of sustainability for irrigation planning.

8.2.2 Constraints

Total Sowing Area

It can be represented as below:

$$\left(\sum_{i=1}^2 J_i^K + \sum_{i=1}^3 J_i^R + \sum_{i=1}^2 J_i^{\text{TS}} + \sum_{i=1}^2 J_i^P + \sum_{i=1}^1 J_i^{\text{HW}} \right) \leq \text{TJ} \quad (8.5)$$

TJ Total command area.

Maximum Sowing Area

It can be represented as below:

Kharif

$$\left(\sum_{i=1}^2 J_i^P + \sum_{i=1}^2 K_i^{\text{TS}} + \sum_{i=1}^2 J_i^K \right) \leq \text{TJ}_i^P + \text{TJ}_i^{\text{TS}} + \text{TJ}_i^K \quad (8.6)$$

Rabi

$$\left(\sum_{i=1}^2 J_i^P + \sum_{i=1}^2 K_i^{\text{TS}} + \sum_{i=1}^3 K_i^R \right) \leq \text{TJ}_i^P + \text{TJ}_i^{\text{TS}} + \text{TJ}_i^R \quad (8.7)$$

Hot Weather and Perennial

$$\left(\sum_{i=1}^2 J_i^P + \sum_{i=1}^1 J_i^{\text{HW}} \right) \leq \text{TJ}_i^P + \text{TJ}_i^{\text{HW}} \quad (8.8)$$

$\text{TJ}_i^K, \text{TJ}_i^R, \text{TJ}_i^{\text{HW}}, \text{TJ}_i^P, \text{TJ}_i^{\text{TS}}$ Command area for i th crop (ha) in different seasons.

Affinity

Perennial

$$J_1^P \leq \text{TJ}_i^P \quad (8.9a)$$

J_1^P Area under Sugarcane (Perennial)

$$J_2^P \leq \text{TJ}_i^P \quad (8.9b)$$

J_2^P Area under Banana (Perennial).

Two Seasonal

$$J_3^{\text{TS}} \leq \text{TJ}_i^{\text{TS}} \quad (8.9c)$$

J_3^{TS} Area under Chillies (Two seasonal)

$$J_4^{\text{TS}} \leq \text{TJ}_i^{\text{TS}} \quad (8.9d)$$

J_4^{TS} Area under LS Cotton (Two seasonal).

Kharif

$$J_5^K \leq \text{TJ}_i^K \quad (8.9e)$$

J_5^K Area under Sorghum (Kharif)

$$J_6^K \leq \text{TJ}_i^K \quad (8.9f)$$

J_6^K Area under Paddy (Kharif).

Rabi

$$J_7^R \leq TJ_i^R \quad (8.9g)$$

J_7^R Area under Sorghum (Rabi)

$$J_8^R \leq TJ_i^R \quad (8.9h)$$

J_8^R Area under Wheat (Rabi)

$$J_9^R \leq TJ_i^R \quad (8.9i)$$

J_9^R Area under Gram (Rabi).

Hot Weather

$$J_{10}^{HW} \leq TJ_i^{HW} \quad (8.9j)$$

J_{10}^{HW} Area under Groundnut (Hot Weather).

Labor Availability Constraint

To deal with issue of unavailability of labor for the duration of cultivation period of different crops in different season, it is proposed that to address the issue of uncertainty concerned with the labor availability; the labor requirement should not be more than the whole labor availability for the duration of that specific crop season.

Kharif

$$\left(\sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^2 J_i^{TS} M_i^{TS} + \sum_{i=1}^2 J_i^K M_i^K \right) \leq \sum_{i=1}^2 AM_i^P + \sum_{i=1}^2 AM_i^{TS} + \sum_{i=1}^2 AM_i^K \quad (8.10)$$

Rabi

$$\left(\sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^2 J_i^{TS} M_i^{TS} + \sum_{i=1}^3 J_i^R M_i^R \right) \leq \sum_{i=1}^2 AM_i^P + \sum_{i=1}^2 AM_i^{TS} + \sum_{i=1}^3 AM_i^R \quad (8.11)$$

Perennial and Hot Weather

$$\left(\sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^1 J_i^{HW} M_i^{HW} \right) \leq \sum_{i=1}^2 AM_i^P + \sum_{i=1}^1 AM_i^{HW} \quad (8.12)$$

AM_i Availability of labor for particular crop;
 M_i Man Days Requirement for particular crop per ha.

Manure Availability

It can be represented as below:

Kharif

$$\left(\sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^2 J_i^{TS} M_i^{TS} + \sum_{i=1}^2 J_i^K M_i^K \right) \leq \sum_{i=1}^2 AM_i^P + \sum_{i=1}^2 AM_i^{TS} + \sum_{i=1}^2 AM_i^K \quad (8.13)$$

Rabi

$$\left(\sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^2 J_i^{TS} M_i^{TS} + \sum_{i=1}^3 J_i^R M_i^R \right) \leq \sum_{i=1}^2 AM_i^P + \sum_{i=1}^2 AM_i^{TS} + \sum_{i=1}^3 AM_i^R \quad (8.14)$$

Perennial and Hot Weather

$$\left(\sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^1 J_i^{HW} M_i^{HW} \right) \leq \sum_{i=1}^2 AM_i^P + \sum_{i=1}^1 AM_i^{HW} \quad (8.15)$$

AM_i Availability of Manure for particular crop;
 M_i Manure Utilization Requirement for particular crop per ha.

Water Availability

It can be represented as below:

$$\left(\sum_{i=1}^2 J_i^K W_i^K + \sum_{i=1}^3 J_i^R W_i^R + \sum_{i=1}^2 J_i^{TS} W_i^{TS} + \sum_{i=1}^2 J_i^P W_i^P + \sum_{i=1}^1 J_i^{HW} W_i^{HW} \right) \leq AW_i^j \quad (8.16)$$

$j = 1, 2, 3, 4, 5$ (No of crop Seasons)

W_i Irrigation water requirement for i th crop;
 AW_i^j Total water availability for i th crop (all crops) for j th interval (all seasons).

Non-negativity Constraint

$$\begin{aligned} & J_i^K, J_i^R, J_i^{TS}, J_i^P, J_i^{HW}, U_i, M_i, T J, T J_i^K, T J_i^R, T J_i^P, \\ & T J_i^{TS}, T J_i^{HW}, AM_i^P, AM_i^{TS}, AM_i^K, AM_i^R, \\ & AM_i^{HW}, M_i^P, M_i^{TS}, M_i^K, M_i^R, M_i^{HW}, \\ & W_i^P, W_i^{TS}, W_i^K, W_i^R, W_i^{HW}, AW_i^j \geq 0 \quad \forall i, j \end{aligned} \quad (8.17)$$

8.2.3 Fuzzy Linear Programming Problem with Fuzzy Parameters

The proposed model, for fuzziness associated with various parameters to deal with uncertainty and vagueness, which can be expressed as below in the form of Eq. (8.18).

$$\begin{aligned}
 \max &= \sum_{j=1}^n \tilde{c}_j x_j \\
 \text{s.t.} & \quad \sum_{j=1}^n \tilde{a}_{ij} x_j \leq \tilde{b}_i, (i \in \mathbb{N}_m) \\
 & \quad x_j \geq 0 (j \in \mathbb{N}_n)
 \end{aligned} \tag{8.18}$$

where $\tilde{c}_j, \tilde{a}_{ij}, \tilde{b}_i =$ fuzzy numbers can be represented by membership functions (linear); $x_j =$ variables whose states are fuzzy numbers ($i \in \mathbb{N}_m, j \in \mathbb{N}_n$); the operations of addition and multiplication are treated as the fuzzy arithmetic operations, and fuzzy numbers ordering denoted by \leq .

The FLP approach can be used to solve the above problem (Eq. 8.18). While using such approach, the best solutions are used to formulate the intervals of membership functions (linear). Because of this, it doesn't provide the scope to include the fuzziness acquired from the DM. However, such difficulty can be tackled by the utilization of Fuzzy Parametric Programming (FPP) method. In the present study, this method is proposed and used to develop the multiple objective decision problems for various grades of precision suggested by the DM based on his/her expertise in the relevant field, which helps to reflect the fuzziness in every model. Such a designed model is used to find out the optimal solutions for all competing objectives.

8.2.4 Algorithm for MOFLP Model Using FPP Approach

The algorithm for the proposed MOFLP model using the approach of Fuzzy Parametric Programming, which has represented as below:

- i. Design of mathematical programming model, which allows the numeral of contending objectives and various types of constraints.
- ii. Include the input from DM based on his/her experience and rational view regarding fuzziness allowed in the various values of the parameters (i.e., $a, b,$ and c), and different possible combination(s) of parameters.
- iii. It is using the FPP concept, the imprecision/fuzziness included in the various values of the parameters (i.e., $a, b,$ and c) as discussed in the above step (ii), which is used to fix up the type of membership function along with its construction. The trade-off between the membership function is formulated

in any one of the forms of different types of membership functions such as linear/nonlinear (convex/concave) and/or piecewise linear. It is considered that, $[c^0, c^1]$, $[a^0, a^1]$, $[b^0, b^1]$ are the possible interval values of fuzzy parameters designated by the DM. c^0 represent “practically acceptable value (PAV)” and c^1 as “practically unacceptable value (PUAV)” value and similarly for b^0 and b^1 . a^0 represent “practically unacceptable value (PUAV)” value and a^1 as “practically acceptable value (PAV).” The lower bounds represent “practically acceptable value (PAV),” which indicate that the solution is implementable. In contrast, the upper bound shows that the obtained solution cannot be implemented practically on the field, as these bounds show that its values are in the form of unrealistic and impossible. The formulation of multi objective decision problem concerning various levels of precision (μ) for multiple types of membership functions. The FPP model is developed for a different level of precision based on formulated membership functions for fully trade-off membership ($\mu = (\mu_{ck} \text{ or } \mu_{zk}) = \mu_A = \mu_b$), and which are considered to be linear in nature. Consequently, the developed models using FPP approach can be represented in the form of Eq. (8.19), which is shown below.

$$\begin{aligned} \text{Max } Z_k &= [C_{kj}^1 - \mu c_{kj} (C_{kj}^1 - C_{kj}^0)]x_j \\ \text{s.t. } [A_{ij}^1 + \mu A_{ij} (A_{ij}^0 - A_{ij}^1)]x_j &\leq b_i^1 - \mu b_i (b_i^1 - b_i^0), \forall i, j \text{ and } \mu \text{ level} \\ x_j &\geq 0. \end{aligned} \tag{8.19}$$

- iv. The solution of the above Eq. (8.19), which leads to the various values for each objective functions, based these values a pay-off table is prepared, and with the help of which upper (Z_u) and lower (Z_l) values for each objective are decided.
- v. The membership function, which is considered as linear, is constructed with the help of values represented in the form of pay-table as obtained in the above step (v).
- vi. The developed MOFLP model, which is represented in the form of Eq. (8.20), it is having the new objective function of maximization type and represented with a new dummy variable (λ) for a different level of precisions (i.e., $\mu = 0, 0.1, 0.2, \dots, 1.0$). This proposed model is subjected to newly added constraints because of fuzziness involved in the objective function values along with original fuzzy constraints.

$$\begin{aligned} \text{Max } \lambda \\ \text{s.t. } (Z_k - Z_{lk}) / (Z_{uk} - Z_{lk}) &\geq \lambda \\ [A_{ij}^1 + \mu A_{ij} (A_{ij}^0 - A_{ij}^1)]x_j &\leq b_i^1 - \mu b_i (b_i^1 - b_i^0), \\ 0 &\leq \lambda \leq 1, \\ x_j &\geq 0. \end{aligned} \tag{8.20}$$

where $\lambda = \mu_k(x) = (Z_k - Z_{lk}) / (Z_{uk} - Z_{lk})$, $\mu = \mu_s = \mu_A = \mu_b = \mu_c$ and $Z_k = [C_{kj}^1 - \mu c_{kj} (C_{kj}^1 - C_{kj}^0)] x_j$.

- vii. Obtain the solution of the above problem (Eq. 8.20) for various precision levels.
- viii. The DM can choose the most appropriate solution from the solution obtained in the above step (vii), and if not satisfied, then return to step (ii) or (iii) and reiterate the entire process.

Figure 8.1 shows the algorithm for the solution using the above procedure.

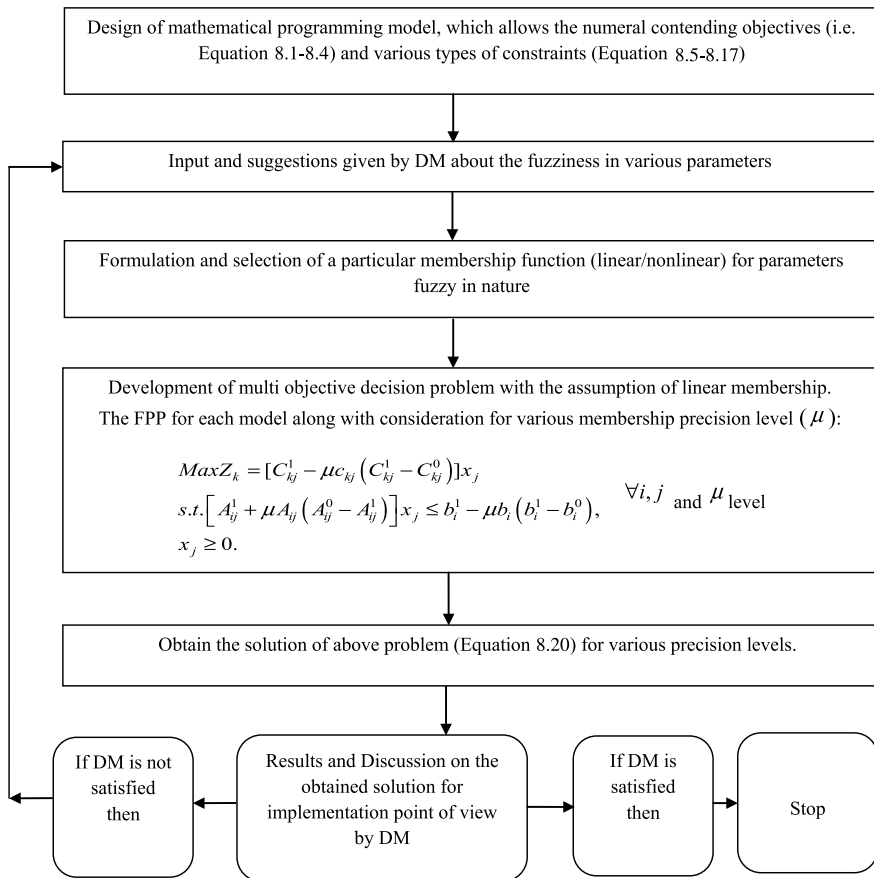


Fig. 8.1 The FPP approach-based algorithm for MOFLP with fuzziness for various parameters (i.e., a , b , and c)

8.3 Results and Discussion

The fuzziness involved in various parameters (a , b , and c) has taken into account in the proposed MOFLP model. The solution has been obtained concerning the algorithm and procedure mentioned in the above section. Figures 8.2, 8.3 and 8.4 shows the construction of the membership function for various fuzzy parameters (a , b , and c). The membership functions (Figs. 8.2, 8.3 and 8.4) associated with a_{ij} , b_i , and c_{kj} parameters, which are assumed as linear monotonously increasing and linear concave/convex, where $\mathbf{A} = [a_{ij}]$, $\mathbf{b} = [b_i]$, $\mathbf{c}_k = [c_{kj}]$, $i = 1 \dots 17$, and $j = 1 \dots 10$, $k = 1 \dots 4$.

$$\mu_{a_{ij}} = (a_{ij} - a_{ij}^1) / (a_{ij}^0 - a_{ij}^1),$$

Fig. 8.2 Membership function-monotonously increasing for technological coefficients (a^1 —practically acceptable value and a^0 —practically unacceptable value)

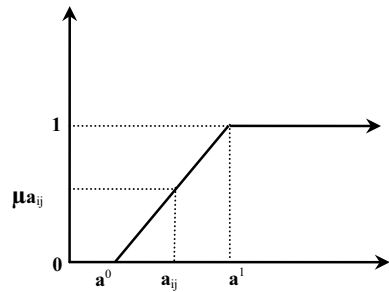


Fig. 8.3 Membership function-linear concave for cost coefficients (c^1 —practically unacceptable value and c^0 —practically acceptable value)

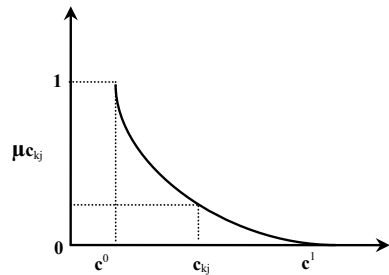
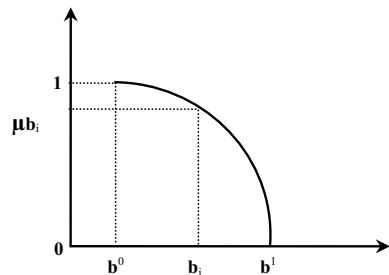


Fig. 8.4 Membership function-linear convex for stipulations/resources (b^0 —practically acceptable value and b^1 —practically unacceptable value)



$$\begin{aligned}\mu b_i &= 1/(1 - \exp(-0.8))[1 - \exp(-0.8(b_i - b_i^1)/(b_i^0 - b_i^1))], \\ \mu c_{kj} &= 1/(1 - \exp(3))[1 - \exp(3(c_{kj} - c_{kj}^1)/(c_{kj}^0 - c_{kj}^1))],\end{aligned}$$

The membership functions are used to determine the various model parameters; these parameters are represented as follows:

$$\begin{aligned}a_{ij} &= a_{ij}^1 + \mu a_{ij}(a_{ij}^0 - a_{ij}^1), \\ b_i &= b_i^1 - (1/0.8) \ln[1 - \mu b_i(1 - \exp(-0.8))](b_i^0 - b_i^1), \\ c_{kj} &= c_{kj}^1 + (1/3) \ln[1 - \mu c_{kj}(1 - \exp(3))](c_{kj}^0 - c_{kj}^1),\end{aligned}$$

After this, all the coefficients can be parameterized with reference to its membership functions. The parametric multiple objective linear programming problem can be formulated and expressed as below:

$$\begin{aligned}\text{Max } Z_k &= [c_{kj}^1 + (1/3) \ln[1 - \mu c_{kj}(1 - \exp(3))](c_{kj}^1 - c_{kj}^0)]x_j, \\ &[A_{ij}^1 + \mu A_{ij}(A_{ij}^0 - A_{ij}^1)]x_j \leq b_i^1 - (1/0.8) \ln[1 - \mu b_i(1 - \exp(-0.8))](b_i^0 - b_i^1), \forall i, j \text{ and } \mu \text{ level} \\ &x_j \geq 0.\end{aligned}$$

The best value for the objective function is found out when $\mu s = \mu a = \mu b = \mu c$ for all i, j, k , because every membership function replicates the degree of precision and the precision evaluated from the optimal solution equals the precision of the most risky of the parameters ($\mu s = \min(\mu a, \mu b, \mu c)$). In other words, at a particular level of precision, the best value of the objective function can be determined using values of parameters at the same specific level of precision.

It is recommended that one should not use the developed MOLP model as it is to find out the optimal solutions due to the different nature of membership functions; therefore, it is essential to carry out sequences of the model run for different values of membership with precision level (i.e., $\mu s = \mu a = \mu b = \mu c$) varying in between zero to one with an equal interval of 0.1. Such a developed Multi Objective Linear Programming model has analyzed for each precision level with priority to one objective among the four objectives, based on these ideal solutions obtained, the pay-off table (Table 8.1) has prepared. With reference to the comparisons of values presented in Table 8.1, values (bold figures) for upper bound (Z_u) and lower bound (Z_l) have been decided for each level of precision (μ) and every objective function, with the help of these values and using FLP approach, various membership functions are constructed which are linear. The Eqs. 8.21–8.24 shows the formulated membership functions when the precision level is zero (i.e., $\mu = 0$). In case of irrigation planning and management, the DM decides the level of precision for all decision parameters which are fuzzy, with the help of his/her expertise and field experience associated with the implementation of irrigation policy.

Table 8.1 Pay-off table

Precision level (μ)	NB (Million Rupees)	CP (Tons)	EG (Million Man Days)	MU (Tons)	Precision level (μ)	NB (Million Rupees)	CP (Tons)	EG (Million Man Days)	MU (Tons)
0	1976.02	517,825.50	37.43	119,208.40	0.6	1231.27	337,904.40	22.22	74,212.58
	1950.05	516,103.30	34.10	99,319.41		1206.91	336,721.90	19.31	58,983.19
	1606.95	177,867.20	31.47	82,930.79		977.19	86,736.79	17.80	51,732.58
0.1	1875.09	506,347.90	32.02	101,499.90	0.7	1160.73	329,662.90	17.96	60,422.87
	1736.53	458,701.10	32.61	104,684.70		1157.18	319,811.30	20.74	69,544.24
	1709.47	457,046.10	29.57	87,243.55		1135.35	318,770.60	17.82	54,655.98
0.2	1398.39	139,973.00	27.13	72,896.98	0.8	917.60	81,399.37	16.52	48,593.92
	1644.72	448,300.80	27.61	88,306.19		1085.79	310,017.10	16.68	56,455.39
	1596.59	424,969.40	29.74	96,181.25		1085.50	302,036.70	19.31	64,982.39
0.3	1567.35	423,318.60	26.84	80,178.75	0.9	1066.00	301,124.80	16.42	50,562.09
	1273.56	114,388.70	24.49	67,041.67		860.01	76,245.92	15.31	45,558.86
	1509.45	415,127.60	24.92	80,405.94		1012.61	290,396.20	15.49	52,655.20
0.4	1487.84	399,023.10	27.47	89,586.07	1.0	1014.86	284,224.70	17.93	60,517.18
	1457.06	397,389.80	24.67	74,469.34		997.50	283,430.50	15.09	46,628.38
	1183.91	105,265.80	22.42	62,506.09		803.50	71,179.32	14.14	42,569.73
0.5	1404.53	389,597.50	22.81	74,367.40	1.0	941.29	271,093.20	14.33	48,951.97
	1393.76	376,840.80	25.53	83,954.67		943.91	266,021.40	16.56	56,071.37
	1363.48	375,323.90	22.68	68,730.80		928.56	265,335.50	13.79	42,787.96
0.5	1108.08	98,467.73	20.66	58,619.26	1.0	746.55	66,106.50	13.00	39,571.06
	1315.04	367,785.10	21.01	69,229.96		896.74	260,858.60	13.25	45,279.42
	1309.39	356,729.20	23.81	78,899.24		746.55	66,106.50	13.00	39,571.06
0.5	1282.23	355,389.20	20.91	63,633.50	1.0	896.74	260,858.60	13.25	45,279.42
	1040.10	92,373.79	19.17	55,043.33		746.55	66,106.50	13.00	39,571.06
	1234.84	348,090.60	19.42	64,642.42		896.74	260,858.60	13.25	45,279.42

Regulwar and Gurav (2012)

$$\mu z_1(x) = \begin{cases} 1 & z_1 \geq 1976.02 \\ \frac{(Z_1 - 1606.95)/(1976.02 - 1606.95)}{0} & 1606.95 < z_1 < 1976.02 \\ 0 & z_1 \leq 1606.95 \end{cases} \tag{8.21}$$

$$\mu z_2(x) = \begin{cases} 1 & z_2 \geq 517,825.50 \\ \frac{(Z_2 - 177,867.20)/(517,825.50 - 177867.20)}{0} & 177,867.20 < z_2 < 517,825.50 \\ 0 & z_2 \leq 177,867.20 \end{cases} \tag{8.22}$$

$$\mu z_3(x) = \begin{cases} 1 & z_3 \geq 37.43 \\ \frac{(Z_3 - 31.47)/(37.43 - 31.47)}{0} & 31.47 < z_3 < 37.43 \\ 0 & z_3 \leq 31.47 \end{cases} \tag{8.23}$$

$$\mu z_4(x) = \begin{cases} 1 & z_4 \geq 119208.40 \\ \frac{(Z_4 - 82,930.79)/(119,208.40 - 82,930.79)}{0} & 82,930.79 < z_4 < 119208.40 \\ 0 & z_4 \leq 82,930.79 \end{cases} \tag{8.24}$$

A dummy variable λ is newly added as the fuzzy achievement function ($\lambda = \min[\mu Z_1(x), \mu Z_2(x), \mu Z_3(x), \mu Z_4(x)]$), a final modified form of the optimization problem as a MOFLP model has represented as:

Maximize λ

s.t.,

$$(Z_1 - 1606.95 \times 10^6)/(1976.02 \times 10^6 - 1606.95 \times 10^6) \geq \lambda$$

$$(Z_2 - 17,7867.20)/(517,825.50 - 17,7867.20) \geq \lambda$$

$$(Z_3 - 31.47 \times 10^6)/(37.43 \times 10^6 - 31.47 \times 10^6) \geq \lambda$$

$$(Z_4 - 82,930.79)/(119,208.40 - 82,930.79) \geq \lambda$$

Along with other constraints involving fuzziness (i.e., Eqs. (8.5)–(8.17)) in the model; $1 \geq \lambda \geq 0$.

According to each precision level (μ), the various solutions of the above model is shown in Table 8.2. In the present study, the application of the above MOFLP model using the FPP approach has been shown with the help of a case study of Jayakwadi Project Stage-I, Maharashtra State, India.

The results presented in Table 8.2, which suggests various decision plans in the form of optimal cropping pattern planning. Also, it is seen that for each level of precision which is varying from 0 to 1, the aggregated level of satisfaction (λ) is above 68%. These values of the level of satisfaction (λ) represent that all four objectives under consideration are analyzed at a time and out of which one objective is satisfied at λ level, and other objectives are satisfied at least at λ level. The highest value of the level of satisfaction is 0.727 for the lowest value of μ (i.e., zero), and the lowest value of the level of satisfaction is 0.682 for the highest value of μ (i.e., one), such a

Table 8.2 The MOFLP optimization model solutions for various level of precision ($\mu = 0.0, 0.1, 0.2, \dots, 1.0$)

Precision level (μ)	Level of satisfaction (λ)	NB (Million Rupees)	CP (Tons)	EG (Million Man Days)	MU (Tons)	Acreages for different crops (ha)	Total area for Irrigation (ha) and irrigation intensity (%)
0	0.727	1875.27	381,720.70	35.80	109,306.00	$X_1 = 2536.84$ $X_2 = 2048.17$ $X_3 = 4472.84$ $X_4 = 5665.60$ $X_5 = 18,885.33$ $X_6 = 15,737.78$ $X_7 = 12,878.92$ $X_8 = 47,213.33$ $X_9 = 7454.73$ $X_{10} = 0$	116,893.09 (82.52)
0.1	0.718	1641.40	369,034.80	31.06	95,991.77	$X_1 = 3202.12$ $X_2 = 1597.70$ $X_3 = 4370.66$ $X_4 = 5385.79$ $X_5 = 18,347.20$ $X_6 = 15,289.10$ $X_7 = 9105.37$ $X_8 = 44,881.58$ $X_9 = 7246.50$ $X_{10} = 0$	109,426.02 (77.25)

(continued)

Table 8.2 (continued)

Precision level (μ)	Level of satisfaction (λ)	NB (Million Rupees)	CP (Tons)	EG (Million Man Days)	MU (Tons)	Acreages for different crops (ha)	Total area for Irrigation (ha) and irrigation intensity (%)
0.2	0.715	1504.54	340,704.60	28.44	87,878.35	$X_1 = 3140.23$ $X_2 = 1506.88$ $X_3 = 4264.85$ $X_4 = 5117.82$ $X_5 = 17,801.12$ $X_6 = 14,833.79$ $X_7 = 7457.26$ $X_8 = 42,648.51$ $X_9 = 7034.80$ $X_{10} = 0$	103,625.26 (73.16)
0.3	0.710	1399.96	329,196.70	26.00	81,756.59	$X_1 = 3298.90$ $X_2 = 1339.78$ $X_3 = 4154.85$ $X_4 = 4859.91$ $X_5 = 17,244.87$ $X_6 = 14,370.00$ $X_7 = 5153.98$ $X_8 = 40,499.32$ $X_9 = 6818.76$ $X_{10} = 0$	97,740.37 (69.00)

(continued)

Table 8.2 (continued)

Precision level (μ)	Level of satisfaction (λ)	NB (Million Rupees)	CP (Tons)	EG (Million Man Days)	MU (Tons)	Acreages for different crops (ha)	Total area for Irrigation (ha) and irrigation intensity (%)
0.4	0.707	1310.23	321,893.20	24.10	76,546.99	$X_1 = 3494.90$ $X_2 = 1146.85$ $X_3 = 4040.01$ $X_4 = 4610.37$ $X_5 = 16,675.82$ $X_6 = 13,895.52$ $X_7 = 2933.66$ $X_8 = 38,419.78$ $X_9 = 6597.33$ $X_{10} = 0$	91,814.24 (64.82)
0.5	0.701	1228.98	309,244.90	22.42	71,776.64	$X_1 = 3529.21$ $X_2 = 1015.79$ $X_3 = 3919.55$ $X_4 = 4367.50$ $X_5 = 16,090.79$ $X_6 = 13,407.72$ $X_7 = 1386.23$ $X_8 = 36,395.83$ $X_9 = 6369.27$ $X_{10} = 0$	86,481.89 (61.05)

(continued)

Table 8.2 (continued)

Precision level (μ)	Level of satisfaction (λ)	NB (Million Rupees)	CP (Tons)	EG (Million Man Days)	MU (Tons)	Acreages for different crops (ha)	Total area for Irrigation (ha) and irrigation intensity (%)
0.6	0.696	1154.13	297,506.30	20.87	67,387.58	$X_1 = 3550.35$ $X_2 = 895.45$ $X_3 = 3792.46$ $X_4 = 4129.57$ $X_5 = 15,485.91$ $X_6 = 12,903.36$ $X_7 = 0$ $X_8 = 34,413.13$ $X_9 = 6079.66$ $X_{10} = 0$	81,249.89 (57.36)
0.7	0.691	1083.26	271,381.50	19.43	63,080.88	$X_1 = 3183.67$ $X_2 = 959.80$ $X_3 = 3657.92$ $X_4 = 3894.76$ $X_5 = 14,856.34$ $X_6 = 12,378.40$ $X_7 = 0$ $X_8 = 32,456.41$ $X_9 = 5560.14$ $X_{10} = 0$	76,947.04 (54.32)

(continued)

Table 8.2 (continued)

Precision level (μ)	Level of satisfaction (λ)	NB (Million Rupees)	CP (Tons)	EG (Million Man Days)	MU (Tons)	Acreages for different crops (ha)	Total area for Irrigation (ha) and irrigation intensity (%)
0.8	0.689	1015.38	246,261.20	18.06	58,942.82	$X_1 =$ 2813.51 $X_2 =$ 1028.54 $X_3 =$ 3513.13 $X_4 =$ 3661.05 $X_5 =$ 14,195.92 $X_6 =$ 11,827.71 $X_7 = 0$ $X_8 =$ 30,508.77 $X_9 =$ 5100.05 $X_{10} = 0$	72,648.68 (51.29)
0.9	0.686	948.53	222,457.00	16.74	54,885.51	$X_1 =$ 2461.54 $X_2 =$ 1079.80 $X_3 =$ 3357.19 $X_4 =$ 3426.06 $X_5 =$ 13,496.61 $X_6 =$ 11,244.58 $X_7 = 0$ $X_8 =$ 28,550.52 $X_9 =$ 4687.88 $X_{10} = 0$	68,304.18 (48.22)

(continued)

Table 8.2 (continued)

Precision level (μ)	Level of satisfaction (λ)	NB (Million Rupees)	CP (Tons)	EG (Million Man Days)	MU (Tons)	Acreages for different crops (ha)	Total area for Irrigation (ha) and irrigation intensity (%)
1.0	0.682	881.26	202,565.30	15.43	50,838.54	$X_1 = 2211.14$ $X_2 = 1064.16$ $X_3 = 3186.90$ $X_4 = 3186.90$ $X_5 = 12,747.60$ $X_6 = 10,620.00$ $X_7 = 0$ $X_8 = 26,090.44$ $X_9 = 5058.57$ $X_{10} = 0$	64,165.71 (45.30)

Regulwar and Gurav (2012)

scenario helps the DM to take judicious decision with reference to particular crops in the specific crop season.

It is seen that for a minimum level of precision (i.e., zero), values of various objectives under consideration are maximum such as NB = 1875.27 Million Rupees, CP = 381,720.70 Tons, EG = 35.80 Million Man Days and MU = 109,306 Tons. These values are minimum for the maximum level of precision (i.e., one) such as NB = 881.26 Million Rupees, CP = 202,565.30 Tons, EG = 15.43 Million Man Days and MU = 50,838.54 Tons, respectively. Similarly, for a minimum level of precision (i.e., zero) the net cropped area = 116,893.09 ha and irrigation intensity = 82.52%, as well as these values, are minimum for the maximum level of precision (i.e., one) such as net cropped area = 64,165.71 ha and irrigation intensity = 45.30%, respectively. It is also seen that when the level of precision is minimum (i.e., zero), all the crops under existing cropping pattern suggests the maximum area to be irrigated and vice versa for the maximum value of the level of precision. Apart from these, Groundnut (HW) is having zero acreage irrespective of the any value of the level of precision (varying from zero to one). Also, when $\mu \geq 0.6$, there is crop such as Sorghum (R) having zero acreage.

8.4 Conclusion

The Multi Objective Fuzzy Linear Programming Model using Fuzzy Parametric Programming Approach has been developed, and its applicability has been demonstrated through its application for the case study of Jayakwadi Project Stage-I in Godavari River sub-basin of Maharashtra State, India.

The proposed methodology and models focus on how to tackle the uncertainty and vagueness involved in the form of fuzziness in the different decision parameters of the irrigation planning problem (i.e., technological coefficients, stipulations/resources, and coefficients of the objective function). The solution for the same has also presented.

The proposed model provides a series of the solution along with the level of precision required to the field situation with the view of irrigation planning based in sustainability concept, which allows the DM to take a judicious decision.

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