Bearing Capacity of Strip Footing Over Rectangular Tunnel in Soft Clay

Puja Dutta, Penke Sriharsha, and Paramita Bhattacharya

Abstract This research work examines the bearing capacity of strip footing over rectangular tunnel in soft clay. The bearing capacity under undrained condition is being obtained for a various conjunctions of *B/D*, *H/D*, $c_{uo}/\gamma D$ and *m* where *D*, *B* and *H* are height, width and soil cover depth of the tunnel, respectively, c_{uo} is the undrained shear strength of the soil at the ground surface, γ is the unit weight of the soil and *m* is a dimensionless parameter which depicts the linear variation of undrained shear strength of soil with depth. The analysis is being performed using lower bound limit analysis, finite elements and nonlinear programming. The present results indicate that normalized bearing pressure decreases with a decrease in normalized vertical distance between tunnel and footing and it becomes constant at a critical normalized distance or say critical cover depth of the tunnel. The values of the critical depth depends upon the size of underlying tunnel and *m*. In case the tunnel is placed above this critical depth, the bearing capacity of footing varies with size and cover depth of tunnel and also on non-homogeneity of undrained clay.

Keywords Footing · Bearing capacity · Tunnel · Limit analysis · Nonlinear programming

1 Introduction

The usage of underground space through tunnelling has been proved to be one of the best solutions for rapid urbanization. The presence of tunnel under any foundation might create risk of failure; hence it becomes an important task to study the stability of foundation above an underground opening. There has been quite a few literatures reported. Baus and Wang [2] carried out studies regarding the bearing capacity of strip footing on clay for singular continuous voids of circular and rectangular shape embedded. Wang and Badie [9] studied the effects on the capacity of strip and square footings of different size and embedment depths due to void location, size and shape

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(circular and cubical). Wang and Hsieh [10] suggested failure mechanisms for strip footing above circular void in cohesive-frictional soil. Azam et al. [1] performed the finite element analysis to estimate the bearing capacity of strip footing placed on a homogeneous soil and stratified deposit consist of different soil layers with and without an inclusion of void of rectangular/square/circular shape. Singh and Basudhar [8] studied the lower bound bearing capacity of smooth strip footing over a rectangular opening in cohesive-frictional soil. Kiyosumi et al. [5] studied the effects on the yield pressure of strip footing resting on calcareous soil due to multiple voids embedded below. Kiyosumi et al. [6] executed model tests of strip footing on stiff ground with square voids embedded under it. Lee et al. [7] studied stability of surface strip footing above rectangular voids by finite element analysis. Chakraborty and Sawant [3] determined the bearing capacity of strip above a circular void in presence of seismic forces. In this analysis they assumed a weightless soil condition.

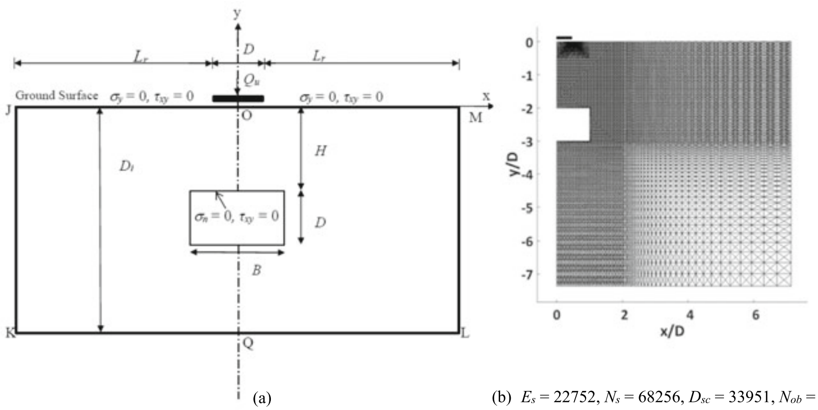
From the literature review, it is understood that no research work is available to determine the lower bound of the bearing pressure of strip footing in homogeneous and non-homogeneous clay soil under undrained condition with $\phi_u = 0°$ in the presence of rectangular and square openings placed centrally below the footing. The goal of this study is to determine the effect on bearing capacity of strip footing due to the presence of rectangular and square openings in undrained clay.

2 Problem Definition

The problem definition consist of a strip footing located over an existing rectangular tunnel of height (D) and width (B) present at a cover depth (H). In the present analysis, the width of footing has been taken equal to the height of the rectangular tunnel. The tunnel is located in soft clay with γ as unit weight of the soil and c_{uo} as undrained soil cohesion at ground surface; m is the normalized rate which implies a linear variation of increase in shear strength of soil with depth. The soil mass follows Mohr–Coulomb yield criteria and an associative flow rule. The soil cohesion (c_u) at any depth h_v from ground surface is defined by Eq. 1:

$$C_u = C_{uo}[1 + mh_{\rm v}/d] \tag{1}$$

The normalized bearing capacity p/c_{uo} has been determined for various H/D, B/D, $c_{uo}/\gamma D$ and m.



(a) (b) $E_s = 22752$, $N_s = 68256$, $D_{sc} = 33951$, $N_{ob} = 80$

Fig. 1 **a** Schematic diagram of strip footing over rectangular tunnel, **b** finite element mesh for $B/D = 2$ and $H/D = 2$

3 Finite Element Mesh, Domain and Stress Boundary Conditions

A strip footing of width D has been kept resting on the surface of a soil domain JKLM, as mentioned in Fig. 1a, where a tunnel with height D and width B is at a depth H. The problem definition is similar along the y-axis about the centre of the footing. Hence, only half of the domain, i.e. OQLM has been used for the analysis. The extent in the horizontal direction (L_r) varies from 4D to 10D and in the vertical direction (D_i) from 5D to 11D for $H/D = 1$ and $H/D = 5$, respectively. The stress boundary conditions are mentioned in Fig. 1a. The problem domain size is selected such that the yielded elements are well inside the boundaries and there is no change in the collapse load with further increase in domain size. Finite element mesh used for $B/D = 2$ and $H/D = 2$ is shown in Fig. 1b where the notations E_s, N_s, D_{sc} and N_{ob} expresses the total number of elements, nodes, discontinuities and objective nodes, respectively.

4 Analysis

The analysis is being carried out in plane strain condition using lower bound limit analysis with finite elements and second order conic programming technique. The domain is discretized into 3-noded triangular elements. The stress is varied linearly throughout each element and the nodal stresses σ_x, σ_y and τ_{xy} are the unknown basic variables . The equilibrium conditions along with stress discontinuities are satisfied throughout the problem domain. The stress conditions are enforced along the nodes of the boundary and tunnel periphery. Mohr–Coulomb yield criteria under plane strain condition is given as:

$$\sqrt{(\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2} \leq 2c\cos\phi - 0.5(\sigma_x + \sigma_y)\sin\phi \tag{2}$$

The above criteria can be exhibited as a 3-dimensional second order cone for each node i, which leads as $x^i{}_{SOCP} = \{s_{xx}, s_{xy}, s_{aux}\}^{\mathrm{T}}$ where the relation between basic stress variables and conic variables are as follows:

$$s_{xx} = 0.5(\sigma_x - \sigma_y);\ s_{xy} = \tau_{xy} \text{ and } s_{aux} = c\ \cos\phi - 0.5(\sigma_x + \sigma_y)\sin\phi \tag{3}$$

The inequality constraint given in Eq. (2) will be represented according to second order constraint,

$$\sqrt{s_{xx}^2 + s_{xy}^2} \leq s_{aux} \tag{4}$$

The value of $p/c_{u\mathrm{o}}$ is obtained by maximizing the compressive stress around the footing surface subjected to element equilibrium, discontinuity condition, stress boundary condition and second order inequality constraint satisfying Mohr–Coulomb yield criteria at all nodes. The bearing pressure (p) is calculated as: $p = (Q_{\mathrm{u}}/(D \times L_x))$, where Q_{u} is collapse load, L_x is the out of plane length of the footing. For plane strain problem, $L_x = 1$ unit.

5 Comparison and Results

5.1 Comparison

The present analysis has been compared with solutions of the elasto-plastic finite element analysis reported by Lee et al. [7] for square tunnel under strip footing in homogenous soil for $c_{u\mathrm{o}}/\gamma D = 3$ with size of the footing being equal to height of the tunnel and the comparison is presented in Fig. 2. The measure of $p/c_{u\mathrm{o}}$ of the present analysis is smaller than that obtained by Lee et al. [7]. The difference tends to reduce with an increase in *H/D* and becomes negligible at $H/D = 4$ for square tunnel and homogenous soil condition. Lower bound theorem always provides a safer collapse load which is lower or equal to the actual ultimate collapse load; hence the solution obtained from present analysis is lower than solution reported by Lee et al. [7].

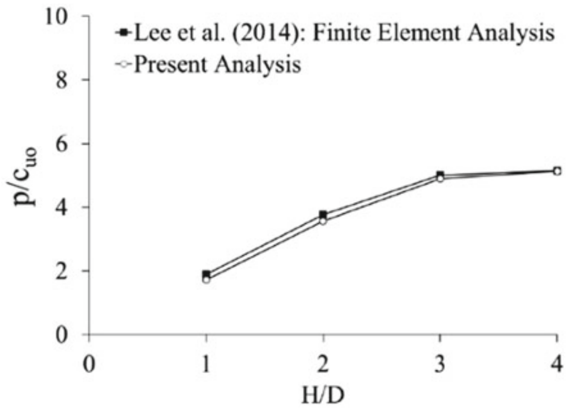


Fig. 2 Comparison of present analysis with Lee et al. [7] for $c_{uo}/\gamma D = 3$, $m = 0$ and $B/D = 1$

5.2 Results

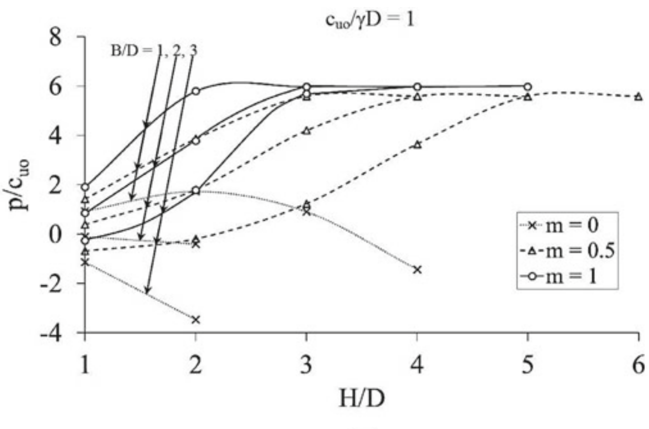
The plane strain analysis is being carried out for rectangular tunnel in soft clay under strip footing of width D for various values of (i) $B/D = 1$–3 with an interval gap of 1, (ii) $c_{uo}/\gamma D$ equal to 1, 2, 3 and 4 and (iii) $m = 0$, 0.25, 0.5, 0.75 and 1. The results of analysis for (i) $B/D = 1$, 2 and 3, (ii) $m = 0$, 0.5 and 1 and (iii) $c_{uo}/\gamma D =$ 1, 2, 3, and 4 are plotted in terms of variation of p/c_{uo} with H/D as shown in Figs. 3 and 4. It has been observed from the numerical analysis that for $m = 0$ and $c_{uo}/\gamma D = 1$ and 2 the value of p/c_{uo} decreases with increase in cover depth ratio for every B/D. This suggests that in these cases the existence of tunnel under strip footing reduces its capacity substantially. Apart from the above mentioned cases, the value of normalized bearing pressure (p/c_{uo}) has been found to decrease with decrease in depth of tunnel for all combinations of m, $c_{uo}/\gamma D$ and B/D. The increase in p/c_{uo} continues till it reaches to a maximum value which is equal to the value of p/c_{uo} obtained without underground opening and thereafter becomes constant. The charts also show the effect of B/D on the value of p/c_{uo}. Wider rectangular tunnel tends to decrease the capacity of the footing.

5.3 Proximity of Stress State at a Point to Failure

The stress state at a point with respect to the available shear strength of soil is determined by a ratio a/d where:

$$a = (\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2 \text{ and } d = (2c\cos\phi - (\sigma_x + \sigma_y)\sin\phi)^2.$$

The ratio of a/d remains smaller than 1 for non-yielding points and becomes unity where shear failure has occurred. The failure plots are shown in Fig. 5a–c for $c_{uo}/\gamma D = 1$ and $m = 0.5$. Figure 5a, b illustrates the failure plot for $H/D = 2$ and



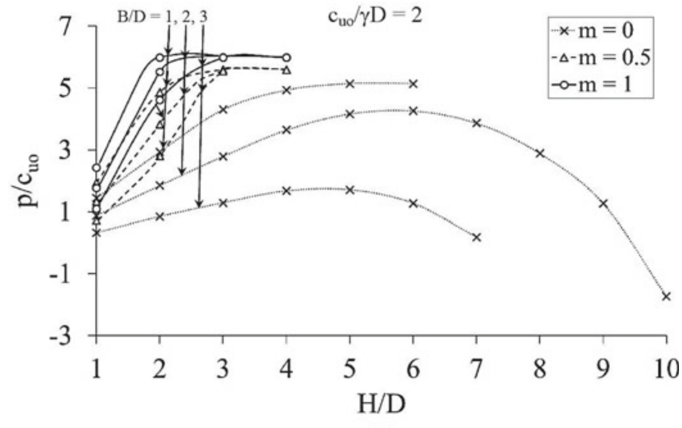
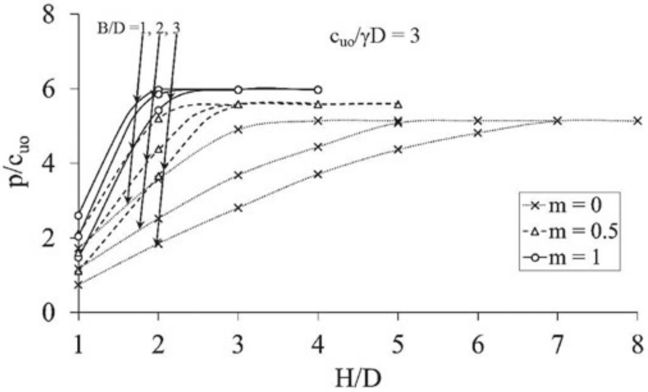
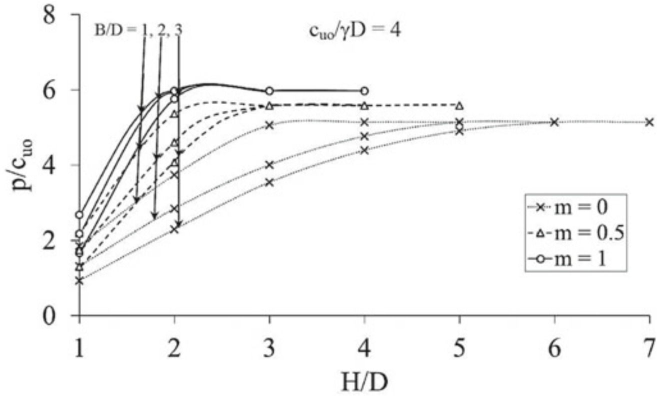


Fig. 3 Charts showing variation of p/c_{uo} with *H/D* for **a** $c_{uo}/\gamma D = 1$ and **b** $c_{uo}/\gamma D = 2$

4, respectively with $B/D = 1.5$ and Fig. 5c shows the failure plot for $H/D = 2$ with $B/D = 3$. It can be noted from the plots that the failure surface extends towards the tunnel until it reaches the critical normalized depth and thereafter the existence of tunnel does not pose any hindrance to the workability of the footing. It has also been found that the size of failure surface increases when there is an increase in size of the underlying tunnel (Table 1).



(a)



(b)

Fig. 4 Charts showing variation of p/c_{uo} with H/D for **a** $c_{uo}/\gamma D = 3$ and **b** $c_{uo}/\gamma D = 4$

6 Conclusion

By implementing the use of lower bound limit analysis, finite elements and SOCP technique, the normalized bearing capacity of strip footing above a rectangular tunnel has been evaluated. It has been found that in most cases except a few, the value of normalized bearing pressure increases with increase in normalized depth of tunnel till it reaches critical normalized depth and thereafter becomes constant attaining the maximum normalized bearing pressure. As the strength of soil increases with increase in non-homogeneity, the value of maximum normalized bearing pressure also increases. The value of $(H/D)_{cr}$ for non-homogenous soil condition has been observed to decrease with increase in strength of the soil. The influence of aspect ratio of the tunnel has also been observed to affect the capacity of the footing.

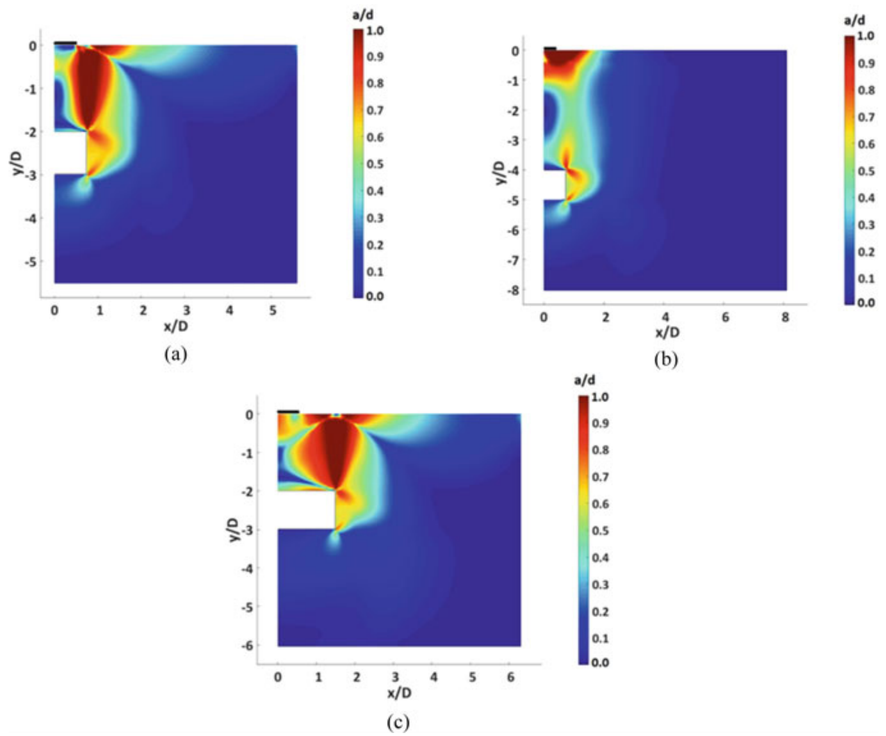


Fig. 5 Stress contour plots for $m = 0.5$; **a** $H/D = 2$, $B/D = 1.5$, **b** $H/D = 4$, $B/D = 1.5$ and **c** $H/D = 2$, $B/D = 3$

Constructing the tunnel at depths higher than the critical depth will prevent the interference of tunnel with the footing, thus preventing the reduction of bearing capacity. The present work provides a few charts on variation of p/c_{uo} with H/D for various values of (i) B/D, (ii) $c_{uo}/\gamma D$ and (iii) m which are expected to be useful in evaluating the performance of a strip footing over a rectangular tunnel.

Table 1 Critical normalized cover depth $(H/D)_{cr}$

m	$c_{uo}/\gamma D$	(B/D)					(p/c_{uo}) at $(H/D)_{cr}$
		1	1.5	2	2.5	3	
0	1						5.14 (5.14)*
	2	5	8				
	3	4	5	6	6	7	
	4	4	4	5	6	6	
0.25	1	4	5	6	7	8	5.37 (5.4)*
	2	3	4	4	4	5	
	3	3	3	4	4	4	
	4	3	3	3	4	4	
0.5	1	3	4	4	5	5	5.59 (5.58)*
	2	3	3	3	3	4	
	3	3	3	3	3	3	
	4	3	3	3	3	3	
0.75	1	3	3	4	4	4	5.79 (5.79)
	2	2	3	3	3	3	
	3	2	3	3	3	3	
	4	2	3	3	3	3	
1	1	3	3	3	3	4	5.97 (5.96)*
	2	2	3	3	3	3	
	3	2	2	3	3	3	
	4	2	2	3	3	3	

* Bearing capacity factors given by Davis and Booker [4]

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