# The Scale Effects on the Shear Strength Behavior of Silty Sand Soil in Direct Shear Tests

**Omar H. Al-Emami and Ammar A. Al-Sultan**

**Abstract** The direct shear test is widely used in geomechanics investigation to obtain the shear strength properties of soils. The simplicity and repeatability of the results presented by this apparatus have kept the direct shear tests in use for more than 60 years. An effort has been made to study the effect of two various sizes of the shear box (60 × 60 and 300 × 300 mm) on the shear strength characteristics of silty sand soil used in this research and to evaluate the effect of test condition (saturated and wet conditions) on the experimental results of this test. The disturbed samples are almost prepared at an initial water content of (8 ± 1%) and two different initial dry unit weights (1.662 and 1.330 Mg/m$^3$). The results from two apparatuses appear that the internal angle of friction obtained from small shear box is higher (1–2°) compared to the big shear box. Also, the samples sheared under unsaturated conditions exhibited higher shear strength corresponding to those tested under saturated conditions. The results indicate that an increasing the specimen density caused an increase in the maximum shear strength in each of two shear box sizes.

**Keywords** Scale effects · Direct shear · Peak shear strength · Silty sand · Density · Test condition

## 1 Introduction

In geotechnical engineering, it is well known that the values of shear strength parameters, angle of friction, and cohesion are adopted to solve many tasks in the construction field, such as retaining walls, pile foundations, and shallow footings. There are two kinds of tests to obtain those parameters laboratory tests and in-situ testing. One of the common tests used in the study of shear strength characteristics is the direct shear experiments, as it is simple and gives reliable results. In the middle of

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the last century, the shear strength behavior's sample size effects have been investigated experimentally by Parsons [1]. The author performed a series of different sizes of shear boxes (60 × 60 mm, 120 × 100 mm, and 120 × 200 mm) direct shear tests on crushed quartz and Ottawa clean uniform sandy soil. The laboratory results revealed that the friction angle slightly decreased with increasing the shear cell dimensions. It was shown that the friction angle of Ottawa sand ranged from 28.5° to 31.0° and the friction angle resulted from testing the crushed quartz ranged from 30.7° to 31.5°. Cerato et al. [2] examined the scale effects of three-square shear boxes (60 mm, 101.6 mm, and 304.8 mm) on the shearing behavior of five sands with different relative densities (loose, medium, and dense). The authors indicated a noticeable dependency of friction angle on the sample size, and the effect of sample dimensions is also a function of the sand type and relative density. A similar trend of behavior was observed by other researchers, such as [3–5]. Recently, Shakri et al. [6] performed a series of direct shear tests of two shear box sizes (60 × 60 mm and 300 × 300 mm) on modified sand-column (PFA-sand mixture) and soft soil.

Their test results revealed that as the shear cell's dimensions increased, a decrease in the shear strength was observed. Conversely, Palmeira and Milligan [5] reported no significant difference in frictional angle with the increasing the size of the shear box. The authors obtained their results by performing several laboratory tests on dense Leighton Buzzard Sand using three different shear boxes (small, medium, and large). In the current study, in continuation of previous research, soil specimens with two densities were adopted and tested under saturated and unsaturated conditions to examine the effect of two various shear box sizes (60 × 60 mm and 300 × 300 mm) on the shear strength of silty sand soil. It is noteworthy to mention here that the term "unsaturated condition" used in this study indicates that the soil samples tested in the direct shear apparatus under two stages consolidation and shearing (i.e. without saturation stage).

## 2 Experimental Program

**Direct shear apparatus.** Consolidated-drained direct shear experiments were performed on silty sand soil samples based on the ASTM D3080 (2011) [7]. According to this specification, several requirements related to the ratio of particle dimensions to the box dimensions should be considered when preparing samples for testing. It is recommended that the minimum specimen width should not be less than 10 times the maximum particle-size diameter, and the minimum initial specimen thickness should not be less than 6 times the maximum particle diameter. In addition, the ratio of minimum specimen width to thickness is required to be 2 [2, 8]. Two types of direct shear apparatus were taken for this research, as shown in Fig. 1. The first apparatus has a square shear box having dimensions 60 × 60 × 20 mm. The normal load is applied on the tested samples using a lever arm frame. The small-sized shear box apparatus is equipped with two displacement transducers to gauge the horizontal displacement and vertical deformation and load cell for horizontal shear

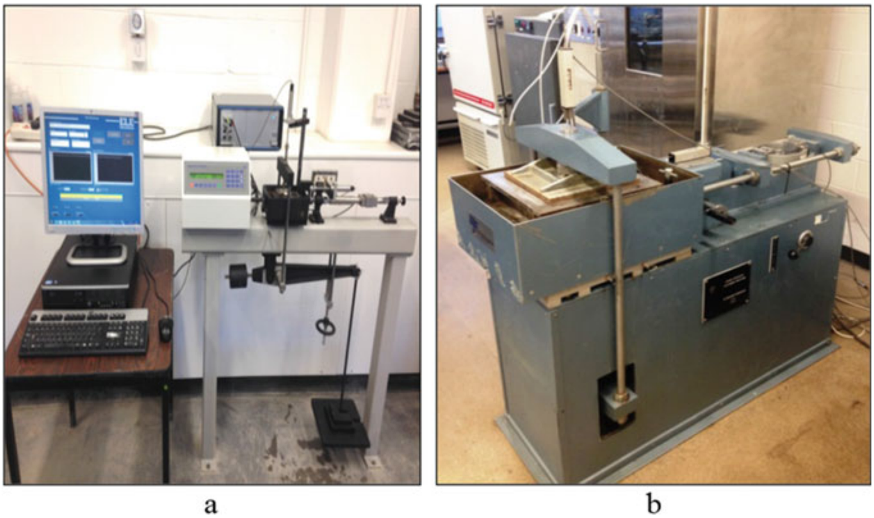

**Fig. 1** Direct shear test apparatus: **a** small shear box, **b** large-shear box

force measurement. The second apparatus has a square cross-section of 300 mm by 300 mm and a thickness of 140 mm. In this apparatus, the vertical pressure for consolidating the sample is applied and controlled by an automatically closed-loop hydraulic system. The soil samples were tested under four vertical stress values (50, 100, 200, and 400 kPa). Whereas, in the large-dimensioned shear box, tests were done under three different normal stresses (100, 200, and 400 kPa). For each shear box size, the samples were subjected to saturated and unsaturated conditions for the selected dry density.

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**Properties of the Tested Material.**

The soil used for the experimental investigations in this research is classified as Silty sand (SM) based on the Unified Soil Classification System (USCS). The soil was collected from a depth ranging from 16.5–25 m below the natural ground level. The particle size distribution of the tested material is presented in Fig. 2. The soil consists of 0.6% gravel, 77.3% sand, 20.1% silt, and 2% clay. The mean particle size ($D_{50} = 0.23$ mm) and specific gravity ($Gs = 2.67$) were determined from the sieve analysis and particle density tests. In addition, the used material was found non-plastic. The laboratory compaction characteristics of the silty sand samples, maximum dry unit weight (1.629 $Mg/m^3$), and optimum moisture content (11.25%) were measured following ASTM standard procedure D1557-09 [9].

**Sample Preparation and Testing Procedure.** All direct shear samples were prepared at an initial water content of 8 ± 1%. In small and large sizes of shear boxes, the required amount of soil mixture to achieve the targeted dry unit weight (i.e., 1.662 and 1.330 $Mg/m^3$) is compacted inside the shear box in four layers. After completing the compaction, the samples were allowed to soak water for 24 h for

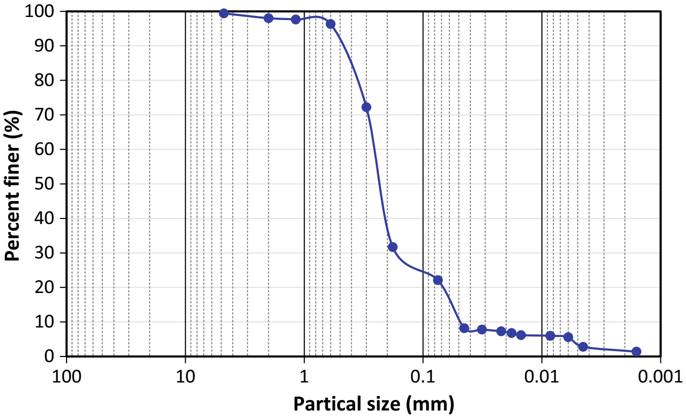


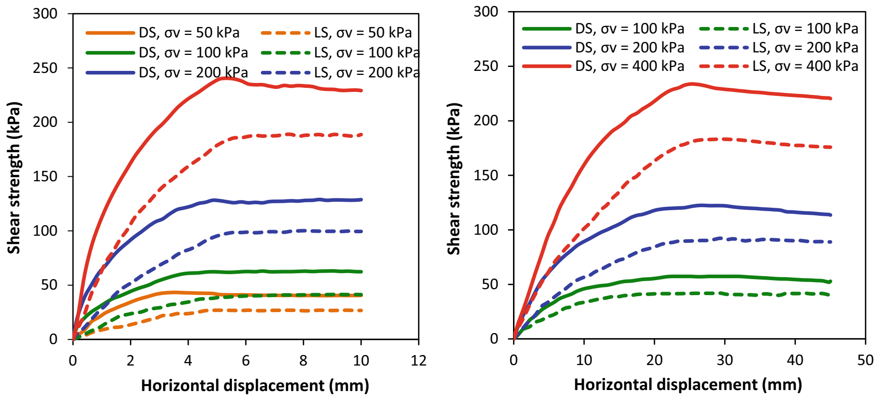
**Fig. 2** Particle size distribution curve for the tested soil

saturated tests. A pre-calculated vertical load corresponding to the targeted vertical stress was applied during the consolidation stage. After attaining the consolidation, the tested specimen was subjected to constant vertical stress and constant horizontal shear rate. Considering the previous studies, the saturated samples were sheared at a constant rate of shear displacement of 0.04 mm/min. In contrast, the samples conducted under unsaturated conditions were directly consolidated after completing the compaction process and then sheared at 0.0095 mm/min shear rate displacement.

## 3 Results and Discussion

**Effect of the shear cell dimensions on the shearing behavior.** Typical direct shear exam results are best showed through the plots of shear strength versus horizontal displacement. Figures 3 and 4 show these relationships for silty sand samples having an initial dry unit weight of 1.662 and 1.330 Mg/m$^3$ and tested in two different shear box sizes under saturated conditions. Generally, at any given value of soil density, the shear strength increased with increasing the level of applied vertical stress. However, at any stress level, the denser samples showed higher shear strength than the looser samples. A similar trend of behavior has been reported by previous researchers [e.g., 1, 10, 11]. The shear strength plots presented in Figs. 3 and 4 revealed two differentiated patterns: strain-softening pattern (peak pattern) and strain-hardening pattern (non-peak pattern). Denser samples ($\rho_{dmax} = 1.662$ Mg/m$^3$) exhibited peak patterns, whereas looser samples ($\rho_{dmax} = 1.330$ Mg/m$^3$) exhibited non-peak patterns. Also, the horizontal displacement corresponding to the peak and/or maximum shear strength increased with increasing applied vertical stress, as shown in Table 1.

Similar to saturated tests, the results of soil samples tested under unsaturated conditions showed a similar trend of behavior, in the sense that the shear strength



a

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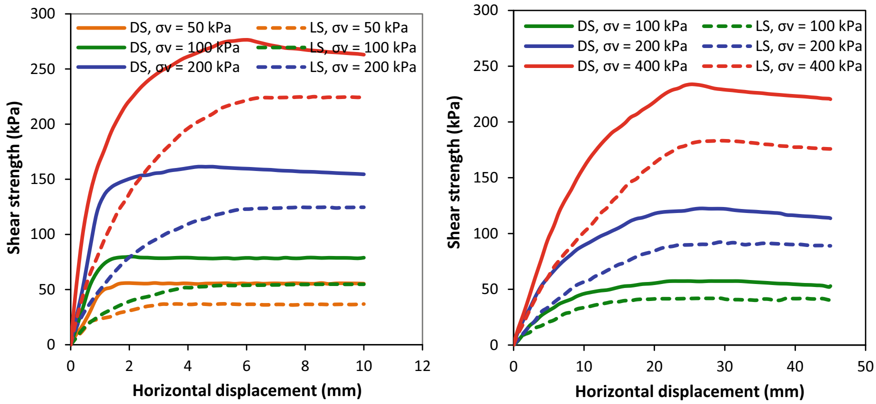
**Fig. 3** Shear strength versus horizontal displacement for dense and loose samples tested in **a** small-sized box and **b** large-sized box under saturated condition



a

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**Fig. 4** Shear strength versus horizontal displacement for dense and loose specimens tested in **a** small-sized box and **b** large-sized box under unsaturated condition

increased with increasing the level of applied stresses. From the observation of Figs. 3 and 4, the obtained shear strength of samples tested under constant water content is distinctly more significant than those obtained from saturated samples and all levels of vertical stress. The latter phenomenon is attributed to the fact that the meniscus around particle contact points tends to attract the particle together, which causes the increase of soil skeleton stiffness resulting in greater resistance during shearing, and consequently, the shear strength is increased [6, 11, 12]. For dense samples, a noticeable strain-softening behavior was observed after the peak shear strength (peak pattern). In contrast, looser samples exhibited non-peak patterns (little to no strain

**Table 1** Values of horizontal displacement corresponding to the peak/maximum shear strength for samples with different densities tested under (saturated and unsaturated) conditions

| Test condition | Shear box sizes, (mm) | Vertical stress, $\sigma$ (kPa) | $\rho_{dry} = 1.662$ (Mg/m$^3$) | $\rho_{dry} = 1.330$ (Mg/m$^3$) | Horizontal dis. at peak shear strength, $\delta_h$ (mm) | |
|---|---|---|---|---|---|---|
| | | | $\tau_{peak/maximum}$ (kPa) | $\tau_{peak/maximum}$ (kPa) | $\rho_{dry} =$ 1.662 (Mg/m$^3$) | $\rho_{dry} =$ 1.330 (Mg/m$^3$) |
| Saturated | 60 × 60 | 50 | 43.18 | 27.23 | 3.36 | 3.11 |
| | | 100 | 61.31 | 38.86 | 3.61 | 4.35 |
| | | 200 | 128.17 | 98.70 | 4.84 | 5.09 |
| | | 400 | 240.48 | 187.58 | 5.58 | 5.58 |
| | 300 × 300 | 100 | 59.07 | 41.05 | 20.04 | 14.04 |
| | | 200 | 122.22 | 91.83 | 23.43 | 21.76 |
| | | 400 | 233.73 | 182.65 | 25.98 | 25.15 |
| Unsaturated | 60 × 60 | 50 | 52.81 | 38.24 | 1.63 | 2.98 |
| | | 100 | 77.49 | 52.48 | 1.91 | 3.61 |
| | | 200 | 160.60 | 117.02 | 4.10 | 4.84 |
| | | 400 | 276.29 | 220.36 | 6.08 | 5.83 |
| | 300 × 300 | 100 | 74.31 | 52.05 | 21.76 | 13.26 |
| | | 200 | 156.40 | 118.16 | 25.15 | 20.92 |
| | | 400 | 277.92 | 217.29 | 28.53 | 23.43 |

hardening–softening behavior). Figure 3 that the different patterns of behavior in the post-peak shear strength region indicate that the soil suction is less significant, whereas the soil suction contributes clearly to the peak shear strength. This behavior agrees with that found by [10, 13, 14].

Examining Table 1 closely, it can be seen that the peak and/or maximum shear strength of the tested samples slightly decreased (2–8 kPa) as the size of the shear box increase. This behavior is consistent with many previous researchers' observations [e.g., 5, 11, 15]. Wu et al. [15] attributed the decrease in the peak shear strength of the dense Toyoura sandy soil with an increase in the ratio L/$D_{50}$ to two reasons: (i) a decrease in the influence of mechanical boundary restraint on the free development of the shear band, and (b) increasing in the effect of the gradual failure with an increase in length specimen of relative to the size of the sand particles. Figures 3 and 4, in conjunction with Table 1 revealed that the horizontal displacement corresponding to the peak and/or maximum shear strength is not consistent for the two shear box sizes. Hence, the comparison between the results is difficult. The shear strength is plotted against relative lateral strain (horizontal displacement/shear box length) instead of horizontal displacement to overcome this problem.

Figures 5 and 6 show that the same results shown in Figs. 3 and 4 are redrawn but plotting the x-axis as a relative lateral strain. The results showed that the peak and/or

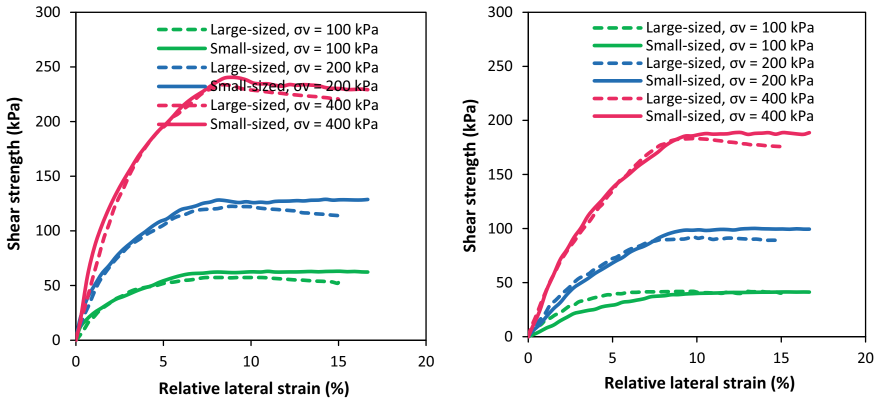

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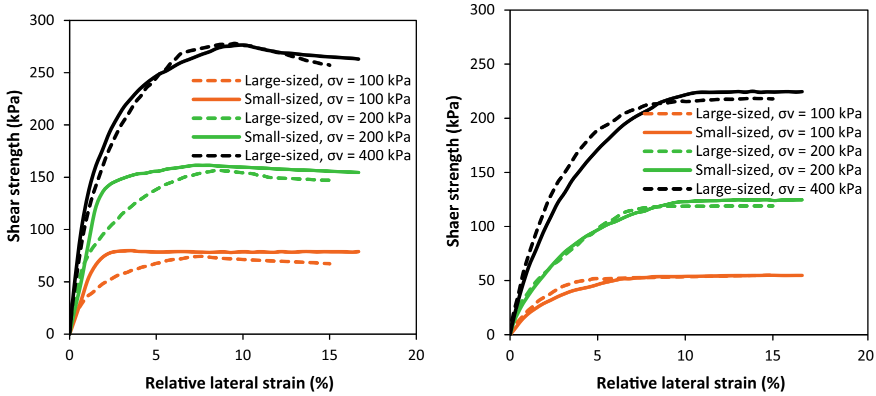
**Fig. 5** Comparison of the shear strength curves using two different shear box sizes under saturated condition **a** dense samples and **b** loose samples



a

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**Fig. 6** Comparing the shear strength curves using two different shear box sizes under unsaturated conditions **a** dense samples and **b** loose samples

maximum shear strength achieved approximately similar relative lateral strain. In addition, the peak/maximum shear strength slightly decreased as the size of the shear box increased (Table 1). This trend was mostly observed for all the tested samples performed under saturated and unsaturated conditions. The researchers [e.g., 3, 8] attributed this behavior to the different heights of samples that influence the vertical stress distribution of the sample shear plane, caused by the moment of shear force applied to the upper half of the shear box, which is transferred to the specimen. The researchers have found that the thickness of the specimen and the shear box's adequate length is necessary to consider to allow fully generation of the shear zone.

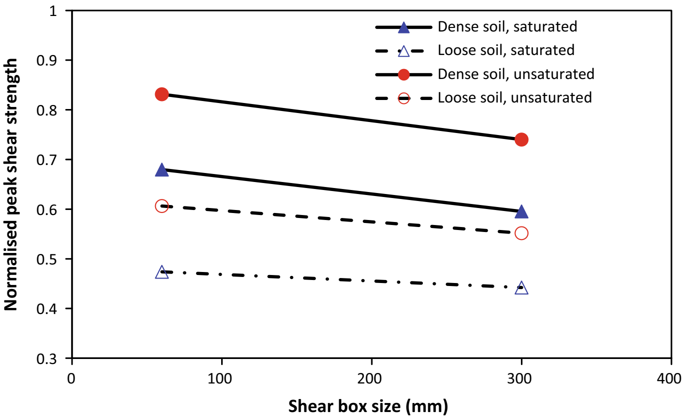

**Fig. 7** Influence of shear box size on the average of normalized shear strength for saturated and unsaturated silty sand samples

The average values of the normalized peak and/or maximum shear strength ($\tau_{peak/maximum}/\sigma_v$) of the tested samples are plotted against the shear box's width, as shown in Fig. 7. Test results of saturated and unsaturated samples presented in this figure showed that the shear box's size influences the values of ($\tau_{peak/maximum}/\sigma_v$), which is generally decreased with increasing the specimen's length. This agrees with the observations obtained by other researchers [5, 15, 16]. The authors attributed this behavior to the matter that the shear region in a small-sized shear box may not be fully developed, leading to a higher angle of friction. Moreover, Fig. 6 reveals a noticeable correlation between ($\tau_{peak/maximum}/\sigma_v$) and the initial dry unit weight of the tested material. For both saturated and unsaturated test conditions, denser samples appeared greater ($\tau_{peak/maximum}/\sigma_v$) than looser samples.

**Effect of the Shear Cell Dimensions on Shear Strength Parameters**. Plots of peak and/or maximum shear strength versus vertical stress corresponding to failure for dense and loose samples performed under saturated and unsaturated conditions are shown in Figs. 8 and 9, respectively. These figures showed clearly that the shear strength envelopes showed good linearity over the vertical stress range of 50 to 400 kPa. It can be seen from these figures, as expected, that the shear strength envelopes shifted upward with increasing the initial dry unit weight of the tested samples. The soil shear strength parameters are calculated from Figs. 8 and 9 and tabulated in Table 2. Also, the shear envelopes' R-square values are indicated in this table to verify the linearity of the relations. A very little scattering was noticed regarding the R-square values of the first order failure envelope lines of different void ratios samples for soil and interfaces, ranging from 0.9937 to 0.9992. This can be attributed to many factors, such as the accuracy adopted during the tested samples' preparation method and performing the laboratory tests under controlled conditions to some extent.

**Fig. 8** Failure envelopes corresponding to different unit weights under saturated conditions

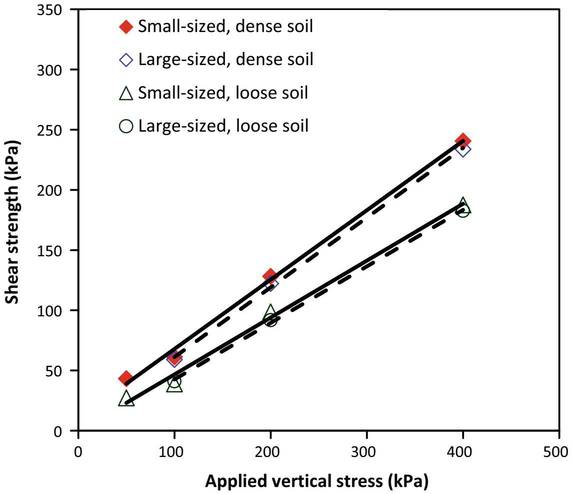

**Fig. 9** Failure envelopes corresponding to different unit weights under unsaturated conditions

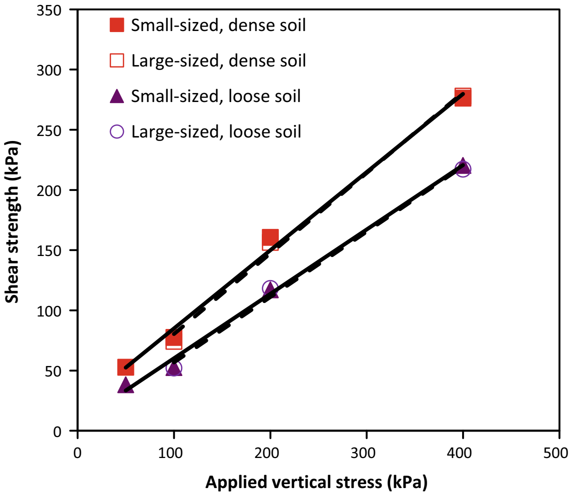

Figure 8 and Table 2 indicate that the friction angle, ϕ′ slightly decreases (1.5°) with the increase of the shear box size. The test results were found to agree with other researchers' findings [e.g., 1–3]. Also, it can be noted that only the failure envelopes of the dense samples tested under saturated conditions passed a little above the point of origin on the abscissa. Similar trends have been reported by Yokoi [17] after studying the relationship between shear strength and soil cohesion of Kibushi and Kashima soils. The author attributed this behavior to the matter that the absolute value of soil cohesion cannot be evaluated owing to the effective angle of extreme edge; soil cohesion measured by the metal-wedge method is closely related to the applied initial stress. Hence, it is probable that soil cohesion acts as compressive stress to

**Table 2** Shear strength parameters of samples with different densities tested under (saturated and unsaturated) conditions

| Shear box sizes (mm) | Initial density, $\gamma_{dry}$ (kN/m$^3$) | Test condition | $\phi'$ (Degree) | c$'$ (kPa) | R$^2$ |
|---|---|---|---|---|---|
| 60 × 60 | 16.62 | Saturated | 32.5 | 7.5 | 0.9972 |
| | 13.30 | | 28.0 | 0.0 | 0.9937 |
| 300 × 300 | 16.62 | Saturated | 31.0 | 5.5 | 0.9990 |
| | 13.30 | | 26.5 | 0.0 | 0.9992 |
| 60 × 60 | 16.62 | Unsaturated | 36.5 | 21.0 | 0.9940 |
| | 13.30 | | 31.0 | 4.5 | 0.9955 |
| 300 × 300 | 16.62 | Unsaturated | 36.0 | 16.0 | 0.9938 |
| | 13.30 | | 29.0 | 3.0 | 0.9943 |

the corresponding shear cohesion. In addition, it is found that the cohesionless soils under saturated conditions have little soil cohesion.

Similar to saturated tests, at any dry unit weight used in this study, the value of $\phi'$ obtained from unsaturated samples exhibits decrease (0.5° to 2°) with increasing the size of the shear box. The test results presented in Fig. 9 and Table 2 showed that the soil suction mainly influences the shear strength parameters. More specifically, the values of soil cohesion, c$'$ presented in Table 2 of the dense samples increased by twofold when performed at the unsaturated condition compared to those tested under saturated condition. Likewise, a slight increase in the value of effective cohesion was observed for loose samples. It can also be noticed from Table 2 that there is a noticeable dependency of the effective friction angle on the test conditions (saturated or unsaturated). Similar to c$'$, the friction angle values obtained from unsaturated samples are higher than saturated samples, as expected. These results match those observed in studies [e.g., 5, 10, 13, 14].

## 4 Conclusions

A laboratory testing program was designed to examine the sample size effect on the shearing behavior of silty sand soil with different initial dry unit weights and test conditions. The samples were tested in a large-sized (300 × 300 mm) and small-sized (60 × 60 mm) direct shear apparatus. Tests results presented in this research revealed the followings:

1. Based on the results, soil suction plays a considerable influencing role in increasing the tested samples' shear strength. However, this role is less significant beyond the peak and/or maximum shear strength.
2. The test results obtained from two different shear box sizes appeared that there is no remarkable difference in shear strength's measured values (2–8 kPa). Also,

the effect of samples size and the dry unit weight on the residual shear strength of the samples can be considered insignificant.

3. The angle friction angle values obtained from the small-sized shear box are slightly higher (1° to 2°) than those obtained from the large-sized shear box. The decrease in the friction angle increased with decreasing the initial dry unity weight of the tested samples.
4. The soil cohesion of dense samples performed under saturated conditions slightly reduces with increasing the shear box size. Simultaneously, an increase in sample size had a negligible effect on the soil cohesion of loose samples. Similarly, all large samples tested under unsaturated conditions showed a lower value of apparent cohesion than small samples.
5. Test results of saturated and unsaturated samples exhibited that the shear box's size influences the average values of the normalized peak and/or maximum shear strength ($\tau_{\text{peak/maximum}}/\sigma_{\text{v}}$), which is generally decreased with increasing the length of the samples. Also, denser samples appeared greater values of ($\tau_{\text{peak/maximum}}/\sigma_{\text{v}}$) compared with looser samples.

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