Chapter 10
Addressing the Skills Gap: What Schools Can Do to Cultivate Innovation and Problem Solving

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Abstract Singapore students have consistently demonstrated outstanding levels of performance in mathematics and problem solving captured in international assessments. However, these stellar results stand in contrast to Singapore's real-world problem-solving capacities, evidenced by her diffident innovation levels and a limited talent pool with problem-solving skills that are high in the value chain. This chapter seeks to address this "skills gap" between what schools develop in students and the high-value workforce skills needed for innovation and enterprise. Focusing on mathematics problem solving, we first examined the historical and socio-cultural development of Singapore mathematics education to identify the system's affordances in cultivating the performance in international assessments, and its trade-offs in developing students' skills in dealing with authentic, non-routine and complex real-world problems. We then examined the trajectories and the impact of pedagogical innovations that were designed for the Singapore mathematics classrooms and that sought to address the trade-offs. From a postulation of factors behind the challenges of implementing and sustaining these innovations in the classrooms, implications for policy, practice, and research are put forth to propose how the Singapore mathematics education can be enhanced to mould the value–creating talent that Singapore needs to stay competitive.

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10.1 Introduction

10.1.1 Singapore's Mathematics Performance in International Assessments: Status and Significance

Singapore students have consistently achieved high levels of performance in mathematics international assessments, securing top positions in the Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA). Considering that such assessments provide evidence about the comparative success of the Singapore education system in the teaching and learning of mathematics (Mullis et al., 2016) and in equipping her future workforce with the necessary competencies to deal with the authentic contexts deemed essential for life and work in the twenty-first century, this naturally begs the following questions: What factors could be behind Singapore mathematics education's success in driving the stellar performances in these international assessments? Through an insight into these factors, what steps can Singapore's mathematics education embark on next to ensure that the country stays ahead of the curve in terms of ensuring a high-quality workforce equipped with the necessary mathematical skills and problem-solving competencies? To answer these questions, we pursued a systemic perspective to identify the current affordances and trade-offs in the current Singapore education system, which would in turn allow us to examine the possible actions that the Singapore mathematics education can adopt to move forward.

An overview of the Singapore education system could be obtained from the surveys conducted by the TIMSS and PISA (Mullis et al., 2016; Organisation for Economic Co-operation and Development [OECD], 2013a, 2013b, 2013c, 2014) and other related secondary analyses that followed (e.g., Yi & Lee, 2017; Zhu, 2017). With regard to mathematics education, the surveys and the analyses revealed the following findings of the key players of the system. Singapore *students* were showed to be highly intrinsically and instrumentally motivated towards mathematics, were generally confident in the subject, but also had higher than average levels of anxiety towards the subject. Singapore mathematics *teachers* were also generally more qualified than their international counterparts in terms of educational certifications and trainings, and had greater opportunities in receiving professional development programmes that focus on mathematics. Mathematics lessons provided students with adequate exposure to the necessary pure and applied mathematical knowledge and were above international average in terms of their levels of support, classroom management, disciplinary climate, cognitive activation, exposure to pedagogical practices that are both student-oriented and teacher-directed and avenues for formative assessments. In terms of *school governance*, while principals have lower than average levels of autonomy for resource allocation and curriculum and assessment, they practised high levels of strong instructional leadership, and assessments and examinations were highly employed for purposes such as school effectiveness and progress, teacher effectiveness, and the design of instruction. Finally, the *school and external environment* were supportive of students' academic development in mathematics: students perceived schools as adequately resourced and safe, and their home environments provided them access to resources for learning. Students also engaged in mathematics homework and activities in informal, out of school hours. Collectively, these factors illustrated the amount of resources Singapore has invested in her students' mathematics education, which has a central focus in problem solving.

While Singapore's outstanding performance in TIMSS and PISA, as well as its emphasis on mathematics education and problem solving, may imply that Singaporeans are well prepared for problem-solving situations in real life, the validity of this inference is, however, questionable on two fronts. First, real-life problems are often complex, non-routine, ill-structured, admit multiple solutions, and require not only cognitive skills, but also a range of other competencies such as creativity, and social and emotional skills. This contrasts the pre-determined and well-articulated problems that are found in standardized assessments (e.g., Deng & Gopinathan, 2016). Second, Singapore's laudable performance in cognitive-based problem-solving assessments, stands in contrast to her diffident performance in other global indicators of real-world problem solving (e.g., innovation, entrepreneurship). For instance, she was lowly ranked in its "innovation efficiency ratio" (63rd place) in Global Innovation Index 2017 (Cornell University, INSEAD, & WIPO, 2017). Similarly, a recent report published jointly by Telstra and The Economist Intelligence Unit (2017) ranked Singapore 21st (out of 45 countries) for industries' confidence in "innovation and entrepreneurship". These indicators suggest that Singaporeans' exceptional lead in problem solving in international assessments bears little relation to the emergence of a critical pool of value-creators and high-value-chain skilled problem-solvers that can help propel the Singapore economy to greater heights. We refer to "skills gap" as the gap between what schools develop in students and the high-value workforce skills needed for innovation and enterprise. In the context of this chapter, the gap is confined to mathematics problem solving. We postulate that problem solving is one of the key enablers for filling this gap.

Given this paradox, it therefore warrants an examination on the underlying factors accounting for the misalignment between Singapore's demonstrated high levels of mathematics and problem-solving capacities captured in international assessments, and the actual demonstrated real-world problem-solving capacities that are measured by the innovation, entrepreneurship, and other drivers of economic growth. Clearly, the various findings about the current Singapore education system from the TIMSS and PISA cannot address the paradox; we will need to understand how it has evolved to its present state. An appreciation of the historical development of the mathematics education in Singapore could provide us with an insight into why certain strategies were pursued to upskill the numeracy and problem-solving competencies of her population. The affordances and trade-offs of these strategies would also foreground the challenges that Singapore faces in developing a sizeable indigenous talent pool with problem-solving skills that could stay high in the value chain, and maintain her competitiveness in the knowledge age.

Hence, in this chapter, we will first outline the historical and socio-cultural development of Singapore mathematics education, and from this analysis, identify the factors that the system has afforded in the high performance in mathematics international assessments, and the trade-offs that may impede the development of students' skills in dealing with the non-routine and complex real-world problems. We then examined the pedagogical innovations that were designed for the Singapore mathematics classrooms that sought to address the trade-offs. From a detailed understanding of the innovations trajectories and their impact in the local classrooms, we then reflect on and postulate the factors behind the challenges of implementing and sustaining these innovations in the classrooms. Implications for policy, practice, and research are put forth to propose how the Singapore mathematics education can be further enhanced to mould the high value-skilled, value–creating talent that Singapore needs to stay competitive in future.

10.1.2 Going to the Genesis of the Singapore's Mathematics Education: Explaining Singapore's Success in International Assessments and Its Trade-Offs

Prior to her independence under the British colonial rule, Singapore was a small, free entrepôt port and a flourishing hub for trade and commerce (Lee, 2008). In her post-independence years, Singapore's founding leaders leveraged education as an important driver to maximize the potential of Singapore's only resource—her people—in a land-scarce island and used education as a vehicle to level up her pluralistic, multi-ethnic populace that was largely illiterate and unskilled, and to restructure her economy. As Singapore constantly seeks to develop herself to become a major location for research, financial services, and high-end manufacturing (Tan & Bhaskaran, 2015), mathematics has been perceived and employed as a key subject of modernization necessary for economic growth. The mathematics education shifted in tandem with the nation's response to a global environment that is highly susceptible to change and nation building efforts (Lee, 2008). Guided by the principle of meritocracy, pragmatism, and accountability, the mathematics curriculum matured over the years, as it progressed through three major phases of Singapore's education history.

In the ***survival*** phase (1959–1978), which spans both the post-colonial and post-independence periods in Singapore's history, there was a need for the government to develop a common mathematics syllabus for her multi-ethnic citizens, given the vernacular nature of education offered then. Mathematics was to be instructed in English, the language of commerce, and had pedagogical recommendations that was progressive. Problem solving was included as one of the learning objectives in the 1970s. Despite the constant revisions of the syllabus during this period, the strategy did not level up the low numeracy rates of the population. Identifying teacher quality, misalignment between curriculum and assessments, and the perceived lack of coherence of what was learnt in school to what was required in the workforce (Lee, 2017) as possible contributory factors, this led to the next phase of the development in the Singapore education system. Dubbed as the ***efficiency*** phase (1979–1996), this phase aimed to address the high education wastage and the low literacy rates identified in the late 1970s, as well as the need for Singapore to evolve to a higher-skilled economy. The strategy was the employment of an ability-based streaming system at both primary and secondary levels of education that takes into consideration variations in learning capacities of children (Kaur, 2014). In tandem with the streaming initiative, assessments like Primary School Leaving Examinations (PSLE) and Singapore-Cambridge Ordinary and Advanced (O- and A-) levels examinations played major roles in providing information to the placement, selection, and certification of pupils at the key stages of education (Lim & Tan, 1999), and became more high stakes. The Curriculum Development Institute of Singapore (CDIS) was established to design a highly prescriptive mathematics curriculum (e.g., syllabus, textbooks, teacher guides) to allow for differentiated instruction and could be employed by less experienced and skilled teachers. More important, a pentagonal framework for the mathematics curriculum that has a centrally focused on problem solving was developed. Concerted efforts were made to develop and support students' mathematical problem-solving abilities; heuristics, like the model method, (Kho, 1987) were introduced. The policies and strategies pursued during this period resulted in a dramatic decline in dropout rates and an impressive rise in literacy and numeracy rates (OECD, 2011; Mourshed et al., 2010). By 1984, the performance for O-level English was a 90% pass rate, and in 1995, Singapore led the world in mathematics in TIMSS (OECD, 2011).

With the Asian financial crisis in 1997, there was a need to prepare students to be lifelong learners for them to survive the challenges that were brought about by the rapid economic, technological, and cultural changes. This was also necessary to cultivate an environment that breeds innovation, which has become the key driver of growth for advanced economies. These developments brought about the current ***ability*** (1997–2011) and ***student-centric, value-driven*** phases (2012 onwards) of the Singapore education system. A new educational vision, "Thinking Schools, Learning Nation", was mooted, with aspirations for the Singapore schools to develop creative thinking skills, the passion for lifelong learning, and nationalistic commitment in the young. There was a shift in focus to enabling students to reach the fullest of his or her potential, to encouraging student-centred learning, and to the development of ethics, character, and dispositions. The previous streaming system evolved into one where students could now cross over from one stream to another, with multiple bridges and ladders to move from one trajectory to another (Lee et al., 2016). Several support programmes, such as the Learning Support Programme for Mathematics (LSM) and the ICAN project (Improving Confidence and Achievement in Numeracy), are offered for the mathematically less endowed. For the mathematically more capable students, there are gifted education programmes, advanced mathematics options within the syllabus, and also specialist institutions, such as the National University of Singapore (NUS) High School of Mathematics and Science, devoted to the nurturing of mathematical and science talent. With the education policy's focus on developing students' potential, recommendations are made to ensure more "quality" (e.g., related to classroom interaction, opportunities for expression, and innovative and effective teaching approaches and strategies), rather than "quantity" (e.g., in terms of rote-learning, repetitive tests, and following prescribed answers) for instruction (Kaur, 2014). There was also explicit recommendations on teacher practices that would enhance students' process skills, with the curricular document detailing the kind of learning experiences that students should have in their mathematics lessons (MOE, 2012).

Corresponding to the policy changes, there was a reduction and re-organization in the mathematics syllabus content to facilitate innovation development (Kaur, 2014). Aspects of the pentagon framework were also appraised to reflect an increased emphasis on the thinking skills and processes that are necessary for effective mathematical problem solving. Recognizing that processes like creativity and critical thinking, and soft twenty-first century competencies like collaborating with others, perseverance, and initiation could not be driven solely from the top, the Ministry of Education (MOE) allowed for more decentralization, where schools were given much greater flexibility and responsibility for how they should teach and manage their students (Kaur, 2014). For example, funding was provided for ground up initiatives like the *Ignite! Programme*, which was introduced to help fund schools to engage in innovative practices that may help transform learning (see Lee, 2014).

Nonetheless, as much as there was increasing autonomy, there was also increased accountability for results. Decentralization reforms are initiated by the MOE, and while schools have the autonomy to decide on administrative procedures and tasks, such as setting up their own directions, vision and mission, and deciding the percentage of students via school based merit criteria, and the choice of pedagogy to deliver the national curriculum, all schools must conform to the rationale and intents of national policies to the MOE (Toh et al., 2016), and remain rooted to the system of central coordination to ensure that education ends are met (Ng, 2010). As such, despite the increased autonomy for instructional changes in mathematics classroom, the increased accountability for academic results that is part of centralized–decentralization system has led mathematics instruction to continue to be transmissionist, teacher-directed, and dominated by teaching *for* problem solving, so as to ensure that students achieve content mastery for high-stake examinations.

10.1.3 Observations from the Evolution of Singapore Mathematics Education that Explain Her Performance in Assessments Internationally

From the development of mathematics education in Singapore, which is guided by the principle of meritocracy, pragmatism, and accountability, the following four factors could have arisen to explain her stellar mathematics performance. First, in terms of the ***historical and cultural development***, Singapore's historical beginnings as a port for trade and commerce and her post-independence economic strategy to develop herself as a major location for high-end manufacturing pre-disposed the development of the necessary numeracy and problem-solving competencies required to develop the trading psyche, and the need to have good mathematics education. Coupled with a societal culture that upholds Confucian teachings that emphasize the respecting of authority and order and the importance of education in upgrading oneself, Singapore students are compliant to work hard for their studies. This gave rise to the competitive, high-performing, and high-stressed system that possibly propelled high performance in international assessments.

Second, there was a ***strong alignment of intended curriculum, assessment, and pedagogy***. As the Singapore mathematics education evolved over time, and with MOE maintaining a strong control on curriculum and assessment matters in the centralized–decentralized system, the intended curriculum, assessment, and pedagogical support to meet the educational needs of the population gets more and more aligned. The mathematics curriculum, which consults the curriculum and teaching approaches from both Eastern and Western countries, lays out a balanced asset of mathematical priorities centred for problem solving and build deep understanding of mathematical concepts (Ginsburg et al., 2005). Heuristics and the teaching for problem-solving approaches in the classrooms that geared towards the mastery of mathematics helped students rise to the demands of high-stakes assessments, which were described as of high standard and challenging (Ginsburg et al., 2005).

Third, Singapore's stellar mathematics could be attributed to its ***quality teachers***. Singapore mathematics teachers are generally more qualified than their international counterparts in terms of certifications and training, and were selected based on a stringent criteria prior to joining the service. They are well-compensated and have access to more opportunities of professional development (PD) in mathematics. Kaur (2009) also noted that mathematics teachers in Singapore have high standards of professionalism and work ethos.

Finally, the Singapore education system allows ***opportunities for levelling up***, and for crossing of pathways. For students who are ready to move to a more advanced level of learning, they can move from their current stream, and thereby allowing for levelling up. There are also availability of support programmes (e.g., LSM and ICAN) and setting of institutions (e.g., NUS High Schools) to cater to students of diverse mathematically skills and talents.

The Singapore mathematics education system, with its high standard of curriculum and assessments, quality teachers, and differentiated support for students, has undoubtedly aided students to excel in problem solving within test-taking situations. However, to prepare students to problem solve beyond the school context, Gravemeijer et al. (2017) argued that there is a need for mathematics taught in the classroom to not only be responsive to the increased digitalization and automatization of work processes, but also to be aligned to the characteristics of mathematics in the workplace. Specifically, mathematics education should develop students' ability to (i) *recognize where mathematics is applicable;* (ii) *translate practical problems into mathematical problems;* (iii) *solve the mathematical problem; and* (iv) *interpret and evaluate the outcomes.* Like most mathematical education systems around the world, the current Singapore mathematics education focused largely on the third area of solving mathematical problem solving, which Gravemeijer and colleagues (2017) noted are increasingly carried out by computers. This narrow focus on problem solving is also evident from the mainly teaching *for* problem-solving approaches adopted in the Singapore mathematics classrooms (Fan & Zhu, 2007; Hogan et al., 2013; Kaur, 2017). This teaching approach is reinforced by the centralized–decentralized system, where tight control is exerted on curriculum and assessment, assessments are high-stakes, and students' achievements are part of teachers' performative indicators. Although teaching *for* problem solving has been effective in helping students develop mastery of skills and content, and in preparing them to achieve in standardized examinations, it is at the expense of the less measurable but equally important development of soft competencies. To promote these mathematical problem-solving processes that would help students to deal with non-routine and complex real-world problems and where skills are increasingly automated by technology and machineries, there is a need to encourage the teaching *about* and via problem-solving strategies (Lester, 2013; Shroeder & Lester, 1989) to engender more meaning in problem solving, allowing for more deeper understanding of mathematics through inquiry-based environments (Lester, 2013; Shroeder & Lester, 1989), and could afford the development of the necessary twenty-first century competencies.

Two pedagogical innovations—the *Mathematical Problem Solving for Everyone* (*M-ProSE;* Toh et al., 2011) and the use of constructivist learning designs (e.g., *Productive Failure* [*PF*]; Kapur, 2008, 2010)—were introduced for this purpose. Against the backdrop of the current system, we describe the ways each innovation diffused into a centralized–decentralized system in their bids to help transform practice in the Singapore mathematics classroom.

10.2 Transforming Mathematical Practice to Get Singapore to Stay Ahead of the Curve: Pedagogical Innovations and Their Trajectories

M-ProSE relates to the teaching *about* mathematical problem solving, as it involves a 10-lesson problem-solving module that explicates the teaching of Pólya's four-stage problem-solving strategy through the use of appropriate non-routine problems, teacher instruction, and teacher modelling.

(i) ***How did the innovation travel?*** To help teachers understand how to implement teaching *about* problem solving, the researchers designed a prototype model with one school first, and in the process developed the module and built teacher capacity. Participating teachers were also provided with a comprehensive three-stage PD training (Leong et al., 2011). The "success story" in the high ability school paved the way to diffusing the innovation to four more secondary schools that were representative of the spectrum of schools in the Singapore education landscape (Toh et al., 2017).

(ii) ***How did the research practice nexus pan out?*** The research team operationalized Pólya (1954)'s and Schoenfeld (1985)'s problem-solving model into a lesson plan that encouraged the use of explicit instruction, scaffolding, and non-routine problem practices to help students understand the nature of a Math problem, Pólya's Four-Step Problem-Solving Process, the selection and functions of heuristics, and Schoenfeld's (1985) notion of control. Practical worksheets that explicate students' thinking processes were also constructed. During the implementation, researchers worked closely with teachers to find out teachers' concerns (e.g., task difficulty) to further refine the innovation.

(iii) ***How systematic was the research done?*** To demonstrate the tractability of M-ProSE, research on the project proceeded in three identifiable phases: (a) *exploration and pilot phase*, where the research team laid the groundwork for the problem-solving curriculum; (b) *development and implementation phase*, where M-ProSE team used a "design experiment" method to develop and implement the innovation in a high ability school; and (c) *infusion and diffusion* phase, where M-ProSE was diffused to four more secondary schools (Toh et al., 2017), and further infused in the original M-ProSE school's curriculum.

PF (Kapur, 2008), a constructivist learning design, promotes the teaching of mathematical concepts via problem solving. It includes a two-phase learning design: (a) *generation phase*, where students generate and explore solutions collaboratively to a novel problem that targets a mathematics concept they have yet to learn; and (b) *consolidation and assembly phase*, where the concept is taught and teachers compare and contrast the canonical solution to what students have produced in their problem-solving efforts.

(i) ***How did the innovation travel?*** Given that constructivist learning designs, such as PF, counter the conventional instruction problem-solving cycle adopted in most mathematics classrooms, a series of quasi-experiments were conducted to first establish a strong proof-of-concept (Kapur, 2008; Kapur et al., 2008), and it was later followed by an expansion of the evidence base for the innovation (Kapur, 2012). Following which, there was a PD research programme that helped to build teachers' design, content, and pedagogical knowledge in designing PF units. The empirical studies and teacher capacity building effort also enabled a collaboration between MOE and the research team to translate and scale the PF learning design across key concepts in the A-level statistics curriculum in 2014.

(ii) ***How did the research practice nexus pan out?*** The crux of PF research lay in the re-examination of the roles of structure and failure in problem solving. The PF learning design embodies four core interdependent mechanisms: (i) activation and differentiation of prior knowledge in relation to the targeted concepts, (ii) attention to critical conceptual features of the targeted concepts, (iii) explanation and elaboration of these features, and the (iv) organization and assembly of the critical conceptual features into the targeted concepts (Kapur & Bielaczyc, 2012). PF, however, would require support for the teachers in developing the necessary knowledge to design for, and enact the learning design, and also a change in classroom culture. Hence, from the comprehensive PD and in-situ support, the research team worked with the teachers to effect the paradigm shift in employing the teaching of concepts via problem solving and help them see the relevance and meaning of employing the pedagogy in the deep learning of mathematics.

(iii) ***How systematic was the research done?*** The PF research proceeded through four identifiable phases: (a) *proof-of-concept,* which sought to demonstrate the efficacy of PF as compared to DI; (b) the *expansion of evidence base,* which sought to examine the effectiveness of PF across curricular units, grade levels, and schools; (c) *building teacher capacity*, which sought to develop teachers' design, content, and pedagogical knowledge; and (d) *translation and scale*, which sought to translate and scale PF across key concepts in the A-level statistics curriculum. In its progress from its proof-of-concept to the translation phases over the years, the PF research conducted design experiments in establishing the pedagogical tractability of the design for learning, and the progression of the research; the evidence collected allowed the research to convince stakeholders to engender this diffusion into the system. Three important findings have emerged: (1) despite failing to discover the canonical solution in their problem-solving efforts, PF students significantly outperformed their counterparts in the traditional Direct Instruction (DI) classrooms in conceptual understanding and transfer problems without compromising procedural fluency, and this trend was consistent in schools with different academic profiles; (2) students with significantly different mathematical abilities were not as different in terms of their ability to generate multiple representations and solution methods to the complex problems; and (3) students' capacity to generate solution methods positively predicts how much they learnt from PF. Taken together, these findings suggest that the PF design not only combines the benefits of exploratory problem-solving and instruction, but is also a promising way of maximizing the learning potential in Singapore mathematics classrooms. The research also shed light on the importance of the role of the teacher in building upon students' ideas when instructing the canonical concept.

To date, all five schools in the M-ProSE research continue to implement the 10-lesson module. As for PF, the innovation impacted mathematics classrooms from 23 schools, 240 teachers and more than 8700 students. The translation project has impacted 16 out of the 20 JCs (80%) in Singapore, 8 of which expressed interest in the continuation of PF in the instruction of statistics in their school. A ground-up initiative of a Networked Learning Committee (NLC) comprising eight junior college teachers also emerged to advance the use of constructivist learning design like PF, in the design of mathematics instructional units.

10.3 Discussion and Conclusion

In the course of unpacking the paradox behind Singapore's stellar performance in mathematics international assessments and the under-developed pool of value-creating talent, we postulate that this could be due to the current mathematics classroom practices being predominantly transmissionist, and centres on teaching *for* problem solving. Although innovations like M-ProSE and PF were introduced to address this issue, their uptake remained with a selected few schools. Reflecting on the education eco-system, we postulate the following factors that may explain the general inertia in Singapore mathematics practice in embracing innovations:

(i) ***Teacher level***. Teacher capacity and practice may be impeding factors. This is related to (a) the nature of teachers' practice, which is time pressured to fulfil multiple instructional goals within an allocated time, resulting in further decreased sense of competencies in adopting a new instructional approach (Leong & Chick, 2011); (b) the innovations' demands on teachers' design (DK), content (CK), and pedagogical content knowledge (PCK); and (c) teachers' beliefs that acquiring knowledge is more important than how it is acquired, that certain conceptual strategies are more suited for their high achieving students than for the low-achieving ones, and that pedagogical innovations are less efficient than their usual practice.

(ii) ***Institutional level***. Teachers' lack of efficacy and unwillingness to implement constructivist learning designs in the classroom are also influenced by the type of trainings that they were exposed to prior to their incumbency. While most mathematics teachers are graduates, not all possessed the requisite mathematical disciplinary knowledge, given that most of them are non-mathematics majors or underwent training in more applied mathematics disciplines, such as engineering and business. While the National Institute of Education (NIE) provides comprehensive training in the mastery of mathematics content and does expose teachers to constructivist learning designs, the short duration of the pre-service training and the demands of implementing such designs possibly explain the low take up of pedagogical innovations. In addition, for in-service teachers, the general training for the use of constructivist training methods in practice is not extensive.

(iii) ***Policy level***. While there is a push at the policy to effect more constructivist ways of instruction in the classroom, the high-stake assessments system may be a disincentive for teachers to take up instructional methods that are less efficient in getting students to master the necessary content knowledge to tackle the assessments, or methods that allow exploration and failure.

(iv) ***Cultural level***. At the macro-level, two cultural forces that are inherent in the Singapore culture may affect teachers' and students' actions and motivations, which in turn impede the adoption of pedagogical innovations: (a) *fear of failure*, which inhibits students' creative problem-solving capacity and teachers' openness to instructional methods that take up more time, rely on failure mechanisms, and possibly might not have any comparative advantage to the tried and tested DI; and (b) high *power distance* (Hofstede, 1991), which is reinforced in mathematics classrooms where teachers play an authoritative role concerning knowledge, and where students are comfortable not be invited to voice themselves and participating in the knowledge construction process. Such forms of instruction will propagate an absolute form of epistemology about knowledge, i.e., knowledge provided by teacher or an authority is absolute and final.

Taken together, given the centralized system which demands that teachers meet standards, and the demands that new innovations place on teachers' capacity, and their beliefs and attitudes, it will take a leap of faith and lots of courage for teachers to make space to implement pedagogical innovations independently. However, the focus on just content and procedures will be ineffective in the long run, given the gradual obliteration of such technical knowledge with the increased automatization of the world. Hence, despite MOE's attempts to encourage teachers to complement their current strategies with pedagogies that are more student-centred and encourage higher-order thinking, the lack of a wider uptake of these pedagogical innovations in Singapore mathematics classrooms reflects a policy practice translation gap.

To address this gap, there is a need to enhance Singapore mathematics education, taking into account its position in a centralized education system, its current heavy emphasis on the mastery of content knowledge for the preparation of high-stake examinations, and the general culture of conformity and risk-aversion. We need a more concerted movement to develop and incentivize teachers to consider the *process* of learning mathematics, in order to ensure the kind of depth in learning and development of competencies that are necessary for the development of value-creating talent for the future. Constructivist pedagogies, especially those that afford students to tinker and explore ideas, elicit their intuitive conceptions prior to the formal instruction of targeted concepts, and persist in their failed problem-solving efforts, could be ways to engage students both in the deep learning of concepts, and creative problem solving. Considering that mathematics practice is couched in the unique Singapore education ecology system, implications of how this can be achieved are detailed below:

(i) ***Implications on taxonomy, mathematics curriculum, and assessments***. There is a clear emphasis from MOE for teachers to focus on how mathematics should be taught to allow students to experience the discipline of mathematics deeply. However, given the general pedagogical practices in the current mathematics classrooms, this possibly requires a stronger push from policy to transform the socio-mathematical culture in Singapore classrooms through (a) stipulating the use of such innovations nationwide; (b) developing a taxonomy that defines and operationalises features of effective mathematics lessons in which mathematics teachers could leverage; (c) providing directives on the use of the various assessment methods to assess mathematical competencies at the national level; and (d) freeing up more time for teachers to implement these new pedagogies in the classroom.

(ii) ***Implications on Practice***. The implementation of pedagogies for the teaching about and via problem solving demands teachers' CK, PCK, and DK. Considering the background of the majority of mathematics teachers, where most come from non-mathematics major background, there is a need for training institutions (i.e. NIE, Academy of Singapore Teachers, AST) to not only continue the development of CK and PCK, but also in specialized content knowledge (SCK), which is the mathematical knowledge and skills unique to the teaching (Ball et al., 2008). Master Teachers from AST could also help to form the necessary Professional Learning Communities (PLCs) and Networked Learning Communities (NLCs) to help support teachers, whereas NIE research teams could provide teachers with in-situ support of the pedagogy during implementation, and follow up with the teachers to identify sustainability issues. The other source of scaffolding for teachers could come from the teachers' immediate work environment, i.e., the school culture. With the push to ease the cultures of fear of failure and power distance, existing leadership in schools will need to find ways to lead the micro-cultures within each school in the realization of the change of culture in the classrooms. The collective efforts from NIE, AST, and schools will be instrumental in the development of teachers in leading the innovations in the mathematics and effecting ecological leadership (e.g., Toh et al., 2016). These might be help to overcome the cultural barriers such as power distance. Finally, formal learning environments would need to be redesigned to include pedagogies that support the teaching about and via problem solving, and schools could also collaborate with external agencies to enable students to participate in informal learning environments (e.g., learning of coding) for authentic learning.

(iii) ***Implications on Research***. With the slow uptake of pedagogical innovations in the classroom, as well as the lack of adequate expertise in NIE to support the development and training of all mathematics teachers in terms of implementing and designing resources to realize constructivist learning in the classrooms, there is a need for NIE research fraternity to (a) work with MOE and AST to develop the necessary resources in advancing these pedagogies; (b) develop effective PD models that could equip Singapore teachers with the necessary capacities; (c) play the role of the broker, understand the needs of the ground, and suggest the necessary ideas and avenues to get teachers to be the implementers of these strategies; (d) to embrace the essence of action research and teacher inquiry as measures of success of adaptation on the ground, and; (e) continue their roles in helping MOE and schools improve deep levels of mathematical learning in schools.

For the past 20 years, Singapore students have demonstrated high levels of mathematical competencies and problem-solving capabilities in TIMSS and PISA. However, Singapore's innovation levels, which are demonstrations of a country's comparative advantage in problem solving in a competitive, globalized world, have a weak correspondence with the results of these international assessments. To address this skills gap, we need to cultivate Singaporeans to achieve skills that are at the highest end of the value chain. To do so, we argue that mathematics instruction should emphasize more on the *processes* of problem solving and argue for the teaching *about* and via problem solving as the necessary approaches to afford deep and meaningful learning and the development mathematical habits and dispositions in students. We postulate that cultural factors and teacher capacity are the reasons behind the slow development of these practices and that these factors can be addressed through the collective efforts of MOE, NIE, AST, and schools, in investing in teacher development, and in pushing for a change in school and classroom culture (e.g., reducing power distance). We believe that a concerted effort for change from policy, research, and practice could slowly help to close this skills gap.

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