Prediction of Ultimate Bearing Capacity of Eccentrically Inclined Loaded Strip Footing Resting Over Dense and Medium Dense Sand Using Generalized Regression Neural Network

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1 Introduction

The evaluation of bearing capacity is the major criterion in the construction of any infrastructure like buildings, dams, bridges, etc. There are several empirical, semi-empirical formulas or methods to estimate the ultimate bearing capacity of footing like Terzhaghi, Meyerhoffs, plate load method, etc. But these are conventional methods that are both time-consuming and less accurate and even sometimes the field methods are less applicable in a remote area as well. These laboratory and field methods are also limited to simpler problems and could not easily manipulate complexities that are generated mostly during performing these conventional methods. There are several geotechnical calculations that require complexities to attain a final result like settlement, slope failure, estimation of bearing capacity, etc. The estimation of bearing capacity of eccentrically inclined loaded strip footing by field methods is itself a challenging task. Therefore, the neural network technique is used to eliminate these complications. With the help of this technique, a well-trained and tested software model can be prepared.

The neural network works on the learning of the experimental or theoretical data. It relates the data with the output in the form of "activation functions". It provides the approximate result or output as compared to the desired output by minimizing errors through iterations. The objective of the current study is to develop a GRNN-General regression neural network prediction model using experimental datasets from laboratory model tests performed by Patra [1] over dense sand and medium dense sand. Three input parameters (Df/B, e/B, and α/ϕ) are used to predict a single output in the form of reduction factor (RF). The results found by GRNN are then

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compared with the empirical as well as ANN results [2]. The software used to apply GRNN in the present study is DTREG, the results of which are further compared with the results of ANN prediction.

2 Literature Review

2.1 Laboratory Model Test

Laboratory model test was conducted by Patra [1] to determine the ultimate bearing capacity of shallow strip footing subjected to eccentrically inclined load resting over dense and medium dense sand. Reduction factor (RF) value is defined as a ratio of ultimate bearing capacity considering eccentrically inclined load to the bearing capacity centrally loaded with no inclination. The poorly graded dense sand having a coefficient of curvature (Cc), coefficient of uniformity (Cu), and effective size of 1.15, 1.45, and 0.325 mm, respectively, was used in the investigation. The embedment ratio (Df/B), eccentricity ratio (e/B), and inclination ratio were varied from 0 to 1, 0 to 0.15, and 0 to 20°, respectively. Empirical equations were also used to calculate the value of reduction factor and treated as calculated RF which was further compared with the experimental values of RF. A variation of around 15% or less was seen and in some cases deviation was about 30% or less. The experimental value of RF is given by

$$\mathrm{RF} = \left[q_{u(Df/B, e/B, \alpha/\phi)}\right] / \left[q_{u(Df/B, e/B=0, \alpha/\phi=0)}\right] \tag{1}$$

2.2 ANN Modeling

The experimental datasets were utilized for training and testing ANN. A total of 120 datasets were used for model preparation out of which 70% was utilized for training data and 30% was utilized as testing or validation data. Embedment ratio, Eccentricity ratio, and inclination ratio were used as predictor variables, and a single output as reduction factor (RF) which was further utilized to get the bearing capacity. MATLAB software was used for ANN modeling, the training function utilized in this model building was TRAINLM, the adaptation learning function was LEARNGDM, and the performance function was MSE. The number of hidden layers used in the model was one. Figure 1 shows the connection strength of several inputs in the neural network diagram.

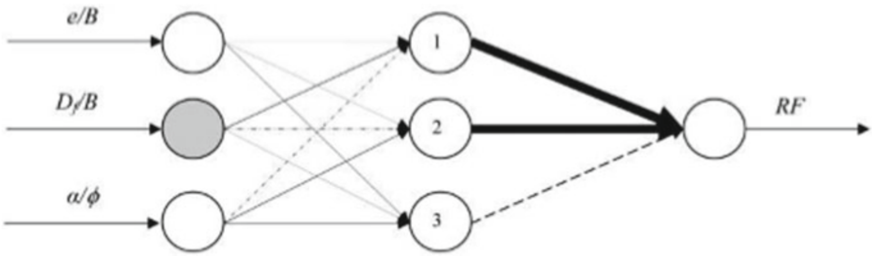

Fig. 1 Neural network diagram showing connection strength of several inputs [2]

3 Methodology

Probability density function used in GRNN is a normal distribution function with each training sample. In the GRNN modeling network, the output is calculated on the basis of weight adjustment mechanism with the help of "Euclidean distance" which is approximately the square of the difference between the training data sample and the testing data sample. If the Euclidean distance of a certain variable is large, then it means that the weight will be less and connection strength will be less for that variable. But if the Euclidean distance is small for certain variables, it will have a large amount of weight and connection strength. The equation used in GRNN is

$$Y(x) = \frac{\sum Y_i e^{-(\frac{d_i^2}{2\sigma^2})}}{\sum e^{-(\frac{d_i^2}{2\sigma^2})}} \tag{2}$$

Input sample is denoted as "X" and input sample in training as "Xi". Yi is the output sample regarding the input sample of Xi. Euclidean distance is denoted as di^2 which is the distance between X and Xi. The activation function which actually denotes the weight of that input sample is given by $e^{-(\frac{d_i^2}{2\sigma^2})}$.

The activation function utilized is the Gaussian type which comes under the type of radial basis function. The normal distribution is widely described by the Gaussian function. It is the best kernel function whose equation is given by

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \tag{3}$$

3.1 Architecture of GRNN Model

The network model is divided into four layers starting from the input layer and ending at the output layer.

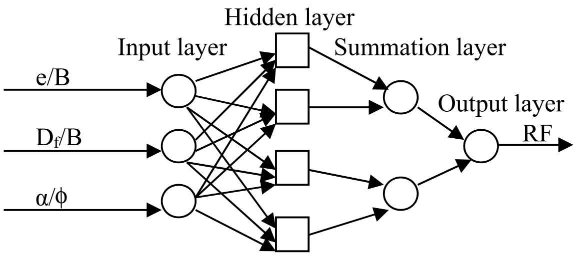

Fig. 2 GRNN architecture model

Input Layer: Each input layer is provided with one neuron in the input layer. The range of input neurons is standardized by the subtraction of median and division of interquartile range. At the end of the layer, the value of each neuron is then provided to the next hidden layer neurons for further processing.

Hidden Layer: The hidden layer neurons are fed values by the input layer neurons. Then comes the hidden layer which is provided with a single neuron for each case in the training data set. The values of predictor and target variables for similar case are stored by the neuron. The input values are presented over the X-axis, the distance from the neurons center point known as Euclidean distance, and is computed by the hidden layer after which using sigma values radial basis activation function is applied. This layer is mainly provided to compute the Euclidean distance which helps in adjusting the weightage of certain predictor variables and in the application of suitable activation function. The output of the hidden layer is then fed to the next layer known as the Pattern layer.

Pattern Layer or Summation Layer: This layer takes values from the hidden layer as input. It contains only two neurons, one is the Numerator neuron and the other is the Denominator neuron. The value of the denominator is computed by the summation of all values of activation function. The value for numerator neuron is computed by summation of multiplicative values of activation function and output data set values. The output values of both numerator and denominator are fed to the next layer known as the decision layer.

Output or Decision Layer: This layer contains only a single neuron. This layer ultimately predicts the target variable by simply computing its value from the division of the Numerator neuron and Denominator neuron values which are fed from the Pattern layer. Figure 2 shows the GRNN architecture model used.

3.2 Training Principle

The primary work in training with generalized regression neural network technique is to select the optimum value of sigma (σ) which helps to control the spread of radial basis function (RBF). The conjugate gradient algorithm is used by the DTREG software for the computation of optimum sigma values. Separate sigma (σ) values for each predictor variable are used. The software uses the leave one out method for evaluation of σ values during optimization.

4 Database and Preprocessing

The laboratory model test conducted by Patra [1] over shallow strip footing subjected to eccentrically inclined load resting over dense and medium dense sand was used. Data sets are divided into two categories: training and testing, around 70% of data sets are used as training data, whereas 30% of data sets are utilized as testing data. Total 120 data sets are available, so the first 90 sets are taken as training sets, whereas the last 30 sets are used as testing sets.

Table 1 shows the soil characteristics and parameters used by Patra [1] in the investigation. Eccentricity ratio was varied from 0 to 0.15, inclination ratio was varied from 0 to 20° and embedment ratio was varied from 0 to 1 for dense and medium dense type sand. Experimental datasets are used to model the training and testing process given in Table 2.

In this study, a trial version of DTREG software was used which helps in providing results but is unable to generate an equation for the required output. After the selection of the type of model to be built, values of sigma are selected or decided for the model. In this model preparation, sigma for each variable is used. The minimum and maximum sigma value is kept as 0.0001 and 10, respectively and 20 search steps are set for the model. Leave one out method is used in model testing and validation. The target variable is the reduction factor and three input variables are used for prediction. The validation method applied in the modeling is "Leave one out method" in which cross validation is performed by leaving one row out for each model built.

Table 1 Soil parameters and its characteristics [1]

Sand type	Unit weight of compaction (Kg/m^3)	Relative density of sand (%)	Friction angle (ϕ) degree	D_f/B	e/B	Load inclination (α) degree
Dense	14.36	69	40.8	0 0.5 1.0	0 0.05 0.1 0.15	0 5 10 15 20
Medium dense	13.97	51	37.5	0 0.5 1.0	0 0.05 0.1 0.15	0 5 10 15 20

Table 2 Experimental model datasets [1]

Data type	Expt. No.	e/B	Df/B	α/ϕ	Experimental q_u (kN/m^2)	Experimental RF	Calculated RF
Training	1	0.05	0	0	133.42	0.8	0.9
	2	0.1	0	0	109.87	0.659	0.8
	3	0.15	0	0	86.33	0.518	0.7
	4	0	0	0.123	128.51	0.771	0.77
	5	0.05	0	0.123	103.01	0.618	0.693
	6	0.1	0	0.123	86.33	0.518	0.616
	7	0	0	0.245	96.14	0.576	0.57
	8	0.05	0	0.245	76.52	0.459	0.513
	9	0.15	0	0.245	51.99	0.312	0.399
	10	0	0	0.368	66.71	0.4	0.4
	11	0.1	0	0.368	44.15	0.265	0.32
	12	0.15	0	0.368	35.12	0.211	0.28
	13	0.05	0	0.49	34.83	0.209	0.234
	14	0.1	0	0.49	29.43	0.176	0.208
	15	0.15	0	0.49	23.54	0.141	0.182
	16	0	0.5	0	264.87	1	1
	17	0.05	0.5	0	226.61	0.856	0.9
	18	0.1	0.5	0	195.22	0.737	0.8
	19	0	0.5	0.123	223.67	0.844	0.822
	20	0.05	0.5	0.123	193.26	0.73	0.74
	21	0.15	0.5	0.123	140.28	0.53	0.575
	22	0	0.5	0.245	186.39	0.704	0.656
	23	0.1	0.5	0.245	137.34	0.519	0.525
	24	0.15	0.5	0.245	116.74	0.441	0.459
	25	0.05	0.5	0.368	129.49	0.489	0.453
	26	0.1	0.5	0.368	111.83	0.422	0.402
	27	0.15	0.5	0.368	94.18	0.356	0.352
	28	0	0.5	0.49	115.76	0.437	0.364
	29	0.05	0.5	0.49	98.1	0.37	0.328
	30	0.15	0.5	0.49	72.59	0.274	0.255
	31	0	1	0	353.16	1	1
	32	0.1	1	0	278.6	0.789	0.8
	33	0.15	1	0	245.25	0.694	0.7
	34	0.05	1	0.123	277.62	0.786	0.79
	35	0.1	1	0.123	241.33	0.683	0.702

(continued)

Table 2 (continued)

Data type	Expt. No.	e/B	Df/B	α/ϕ	Experimental q_u (kN/m^2)	Experimental RF	Calculated RF
	36	0.15	1	0.123	215.82	0.611	0.614
	37	0	1	0.245	264.87	0.75	0.755
	38	0.05	1	0.245	239.36	0.678	0.679
	39	0.1	1	0.245	212.88	0.603	0.604
	40	0	1	0.368	225.63	0.639	0.632
	41	0.1	1	0.368	179.52	0.508	0.506
	42	0.15	1	0.368	155.98	0.442	0.443
	43	0.05	1	0.49	166.77	0.472	0.459
	44	0.1	1	0.49	143.23	0.406	0.408
	45	0.15	1	0.49	126.55	0.358	0.357
	46	0	0	0	101.04	1	1
	47	0.05	0	0	84.37	0.835	0.9
	48	0.15	0	0	54.94	0.544	0.7
	49	0	0	0.133	79.46	0.786	0.751
	50	0.1	0	0.133	52.97	0.524	0.601
	51	0.15	0	0.133	42.18	0.417	0.526
	52	0.05	0	0.267	47.09	0.466	0.484
	53	0.1	0	0.267	38.46	0.381	0.43
	54	0.15	0	0.267	31.39	0.311	0.376
	55	0	0	0.4	38.26	0.379	0.36
	56	0.05	0	0.4	32.37	0.32	0.324
	57	0.1	0	0.4	26.98	0.267	0.288
	58	0	0	0.533	24.03	0.238	0.218
	59	0.05	0	0.533	19.62	0.194	0.196
	60	0.15	0	0.533	13.34	0.132	0.152
	61	0	0.5	0	143.23	1	1
	62	0.1	0.5	0	103.99	0.726	0.8
	63	0.15	0.5	0	87.31	0.61	0.7
	64	0.05	0.5	0.133	103.99	0.726	0.726
	65	0.1	0.5	0.133	90.25	0.63	0.645
	66	0.15	0.5	0.133	72.59	0.507	0.565
	67	0	0.5	0.267	98.1	0.685	0.628
	68	0.05	0.5	0.267	84.86	0.592	0.565
	69	0.1	0.5	0.267	72.59	0.507	0.502
	70	0	0.5	0.4	79.46	0.555	0.465

(continued)

Table 2 (continued)

Data type	Expt. No.	e/B	Df/B	α/ϕ	Experimental q_u (kN/m^2)	Experimental RF	Calculated RF
	71	0.05	0.5	0.4	67.89	0.474	0.418
	72	0.15	0.5	0.4	48.07	0.336	0.325
	73	0	0.5	0.533	58.27	0.407	0.319
	74	0.1	0.5	0.533	43.16	0.301	0.255
	75	0.15	0.5	0.533	36.3	0.253	0.223
	76	0.05	1	0	193.26	0.925	0.9
	77	0.1	1	0	175.6	0.84	0.8
	78	0.15	1	0	156.96	0.751	0.7
	79	0	1	0.133	186.39	0.892	0.867
	80	0.05	1	0.133	168.73	0.808	0.78
	81	0.1	1	0.133	153.04	0.732	0.693
	82	0	1	0.267	160.88	0.77	0.733
	83	0.05	1	0.267	144.21	0.69	0.66
	84	0.15	1	0.267	112.82	0.54	0.513
	85	0	1	0.4	133.42	0.638	0.6
	86	0.1	1	0.4	106.93	0.512	0.48
	87	0.15	1	0.4	94.18	0.451	0.42
	88	0.05	1	0.533	92.21	0.441	0.42
	89	0.1	1	0.533	84.37	0.404	0.373
	90	0.15	1	0.533	75.54	0.362	0.327
Testing	1	0	0	0	166.77	1	1
	2	0.15	0	0.123	65.73	0.394	0.539
	3	0.1	0	0.245	62.78	0.376	0.456
	4	0.05	0	0.368	53.96	0.324	0.36
	5	0	0	0.49	43.16	0.259	0.26
	6	0.15	0.5	0	164.81	0.622	0.7
	7	0.1	0.5	0.123	165.79	0.626	0.658
	8	0.05	0.5	0.245	160.88	0.607	0.59
	9	0	0.5	0.368	151.07	0.57	0.503
	10	0.1	0.5	0.49	85.35	0.322	0.291
	11	0.05	1	0	313.92	0.889	0.9
	12	0	1	0.123	313.92	0.889	0.877
	13	0.15	1	0.245	188.35	0.533	0.528
	14	0.05	1	0.368	206.01	0.583	0.569
	15	0	1	0.49	183.45	0.519	0.51

(continued)

Table 2 (continued)

Data type	Expt. No.	e/B	Df/B	α/ϕ	Experimental q_u (kN/m^2)	Experimental RF	Calculated RF
	16	0.1	0	0	68.67	0.68	0.8
	17	0.05	0	0.133	63.77	0.631	0.676
	18	0	0	0.267	55.92	0.553	0.538
	19	0.15	0	0.4	20.6	0.204	0.252
	20	0.1	0	0.533	16.68	0.165	0.174
	21	0.05	0.5	0	123.61	0.863	0.9
	22	0	0.5	0.133	120.66	0.842	0.807
	23	0.15	0.5	0.267	60.82	0.425	0.44
	24	0.1	0.5	0.4	56.9	0.397	0.372
	25	0.05	0.5	0.533	50.03	0.349	0.287
	26	0	1	0	208.95	1	1
	27	0.15	1	0.133	137.34	0.657	0.607
	28	0.1	1	0.267	129.49	0.62	0.587
	29	0.05	1	0.4	118.7	0.568	0.54
	30	0	1	0.533	98.1	0.469	0.467

5 Results and Discussions

Best performance analysis is done by using correlation coefficient (Cr), determination coefficient (R^2), mean square error (MSE), root mean square error (RMSE), and mean absolute error (MAE).

Table 3 reveals the training and testing parameters of the experimental model test conducted by Patra [1]. 70% of data sets are used as training data and a GRNN model is built. 30% of datasets are used in the testing data. The coefficient of correlation for training and testing data was 0.998 and 0.994, respectively. It shows the linearity between the value predicted and the actual output with greater precision. More close the value to 1 shows higher linearity. The values of statistical parameters for GNN for all continuous variables are shown in Table 4. As RF is the target variable and the other three are predictor variables. The maximum value for RF was 1, whereas its minimum value was 0.132, and the mean value was 0.65229. The value of standard deviation was 0. 19,833. The statistical parameters from ANN are shown in Table 5. In the statistical analysis, the maximum value of output was 1 and its minimum value was 0.132. Whereas as mean or the average value was set as 0.555. The standard deviation of the output variable was 0.217.

Figure 2 shows the relative importance of the variable in the prediction model. It can be easily seen in Fig. 2 that the inclination variable shows 100% importance or impact over the target variable while the eccentricity ratio variable shows 33.946% impact and embedment ratio makes the least impact or least important as compared

Table 3 Experimental model datasets [1]

Parameters	Training	Testing
Mean target value for input data	0.652293	0.652293
Mean target value for predicted values	0.6530206	0.6544816
Variance in input data	0.0393364	0.0393364
Residual (unexplained) variance after model fit	0.000158	0.0005374
Proportion of variance explained by model (R^2)	99.598%	98.634%
Coefficient of variation (C_v)	0.019267	0.035538
Normalized mean square error (NMSE)	0.004015	0.013661
Correlation between actual and predicted	0.998156	0.994194
Maximum error	0.492199	0.089428
Root mean square error (RMSE)	0.012568	0.0231815
Mean squared error (MSE)	0.000158	0.0005374
Mean absolute error (MAE)	0.0090726	0.0177511
Mean absolute percentage error (MAPE)	0.0179782	0.0323818

Table 4 Statistical parameters from GRNN

Variable	Rows	Minimum	Maximum	Mean	Standard deviation
Df/B	120	0	1	0.66956	0.37450
A	120	0	20	7.98676	6.80215
e/B	120	0	0.15	0.06603	0.05531
RF	120	0.132	1	0.65229	0.19833

Table 5 Statistical parameters from ANN [2]

Parameter	Maximum	Minimum	Average	Standard deviation
e/B	0.15	0	0.075	0.056
Df/b	1	0	0.5	0.408
α/ϕ	0.533	0	0.256	0.181
RF	1	0.132	0.555	0.217

to the other two variables with around 22.112% importance over the target variable. Figures 3 and 4 show the variation of experimental RF (Actual) versus predicted RF value from GRNN and experimental RF versus empirically calculated RF(CRF), respectively. Higher variation can be seen, but still, the graph proceeds in the linear direction but less linearity is shown as compared to the GRNN prediction model graph which is shown in Fig. 3.

Figure 5 shows the variation of actual target variable and AN N predicted output variable for training and testing data. It can be seen from Fig. 5 that the model built was providing good results, which were analyzed with the help of coefficient of

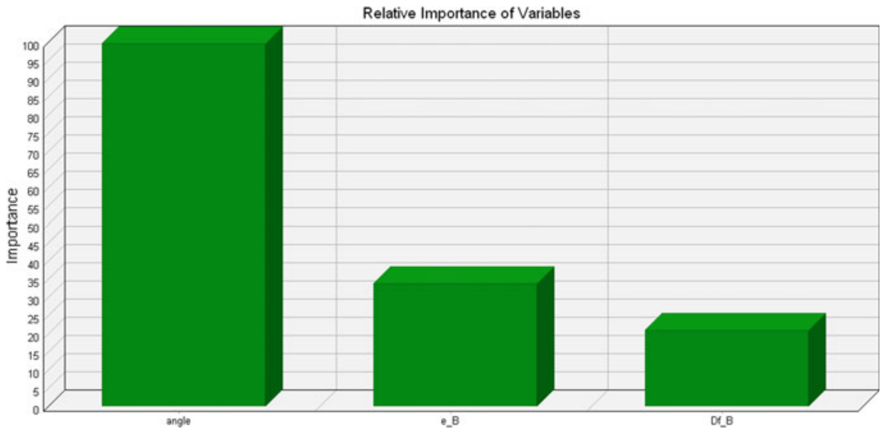

Fig. 3 Relative importance of input variables on output variable RF

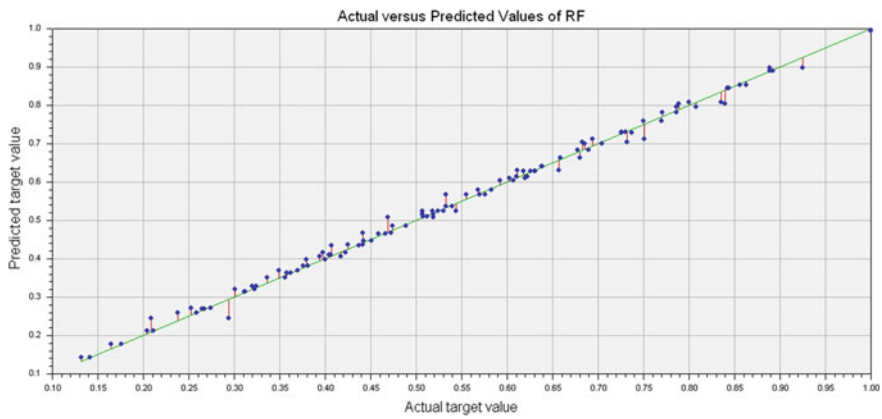

Fig. 4 Plot between experimental RF (Actual) and predicted RF value from GRNN

correlation. The value of Cr for training was 0.997 which is much closer to the value 1, whereas the Cr value for testing was 0.996 which represents a good prediction according to this model.

The comparison between ANN and GNN with different indices is shown in Table 6. No such variation in results was seen between the GRNN network model and the ANN model. The mean square error was less in GRNN prediction work. There was a slight difference between the correlation coefficient like for training work was 0.998 for GRNN model, whereas 0.997 for ANN model, and for testing it was 0.994 for GRNN and 0.996 for ANN. Results of both GRNN and ANN show higher accuracy than empirically calculated results, which are also shown in Figs. 3, 4, 5.

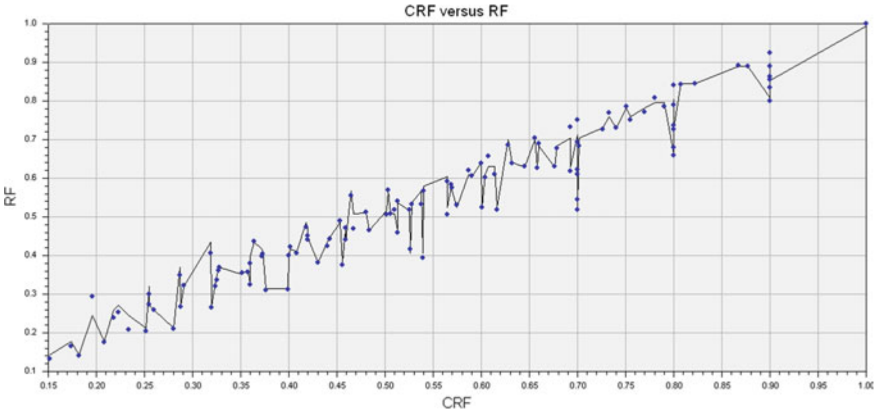

Fig. 5 Plot between Experimental RF and Empirically calculated RF(CRF)

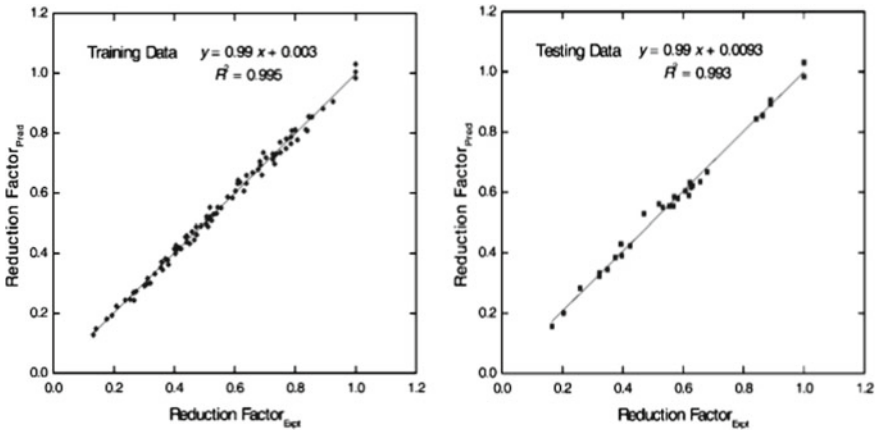

Fig. 6 Plot of experimental RF and predicted RF from ANN [2]

Table 6 Comparison between ANN and GRNN with Mathematical indices

Type of model	GRNN (present study)		ANN (results from [2])	
	Testing	Training	Testing	Training
MSE	0.0005	0.00015	0.0019	0.001
RMSE	0.0231	0.0125	0.043	0.032
R^2	0.986	0.996	0.992	0.994
Cr	0.994	0.998	0.996	0.997

6 Conclusions

Highlights of the present study are shown below:

1. No such variations in results between ANN and GRNN were spotted, both models were equally accurate.
2. The data available was never enough for backpropagation neural network, this GRNN neural network technique founds to be advantageous because of the ability of this technique in utilizing fewer data samples efficiently to converge the function.
3. The standard deviation found in the output reduction factor (RF) was lesser in the GRNN model as compared to the ANN model.
4. In the GRNN model, the inclination ratio was provided higher importance as compared to other two input variables like embedment ratio and eccentricity ratio (α/ϕ as 100%, e/B as 33.946% and Df/B as 21.112%), whereas in the ANN network model, as per Garson's algorithm, the inclination ratio (α/ϕ) was given more importance as compared to other two followed by embedment ratio (Df/B) and then eccentricity ratio (e/B).

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