Empirical Formulation of (n, p) Nuclear Cross-Section at 14–15 meV

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Abstract The empirical equation of (n, p) nuclear cross-section is formulated at 14–15 meV incident neutron energy. The data used for the construction of the empirical formula is obtained from EXFOR with a mass range of $9 < A < 111$. From the data, we fit empirical formula with exponential dependency of $\left(\frac{N-Z+A}{A}\right)^n$ where n is determined from the non-linear fitting. The empirical formulation is then compared using the chi-square coefficient as a quality criterion. The derived empirical formulation is compared with five previously published formulation. We show that the new formulation derived in this work gives good agreement with the experimental nuclear cross-section value.

Keywords Nuclear cross-section data · Empirical formulation · (n · p) nuclear reaction

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1 Introduction

Neutron-induced nuclear cross-section data is important in the study of nuclear structure, radiation damage, nuclear heating and structural transmutation of nuclear reactors [1]. Nuclear cross-section data is also valuable in determining the possible nuclear reaction channels which are important in the production of radionuclide [2]. Experimental nuclear cross-sections can be obtained from the EXFOR database, obtained from the measurement of various authors [3–5]. However, there are problems of the discrepancy between measurements done by different authors. This discrepancy is especially large at neutron incident energy around 14 meV. Some nuclear reaction is very hard to experiments on, the experimental cross-section data is scarce.

The empirical and semi-empirical formulation is crucial in theoretical calculations of nuclear cross-sections. For (n, p) nuclear reaction, the various empirical and semi-empirical formulation has been developed at neutron incident energy around 14 meV. In all of the empirical formulation for (n, p) nuclear cross-section, the nuclear cross-section is assumed to have exponential dependence on the number of nucleons in the target nucleus [6]. In this work, we proposed a new empirical formulation of (n, p) nuclear reaction by the non-linear fitting of EXFOR nuclear cross-section data for 61 nuclear isotopes with atomic mass range $9 < A < 111$ at neutron incident energy around 14–15 meV.

2 Methodology and Data

Generally, the statistical model defined the (n, p) nuclear reaction cross-section as

$$\sigma_{(n,p)} = \sigma_C(n) * G(p) \tag{1}$$

where $\sigma_C(n)$ is the cross-sections for the compound nucleus form during the collisions of the nucleus with incident neutron [7]. $\sigma_C(n)$ is generally known as the non-elastic cross sections. $G(p)$ is the probability of the compound nucleus to decays, where a proton is ejected from the nucleus. $G(p)$ can be defined as

$$G(p) = \frac{\Gamma_{\mathrm{p}}}{\Gamma_{\mathrm{n,p}}} \tag{2}$$

where Γ_{p} is the partial decay width of emitted proton and $\Gamma_{\mathrm{n,p}}$ is the total decay width. Due to $\Gamma_{\mathrm{n,p}}$ to be the total decay width, it is given as the sum of partial decay width of emitted proton (Γ_p), neutron (Γ_n), deuteron (Γ_d), triton (Γ_t), hellion (Γ_{He}) and alpha particles (Γ_α). However, all of the modes of decays is assumed as negligible excepts for Γ_n as shown below.

$$\Gamma_{n,p} = \Gamma_p + \Gamma_n + \Gamma_d + \Gamma_t + \Gamma_{He} + \Gamma_\alpha$$

$$\Gamma_{n,p} \approx \Gamma_n$$

The $\Gamma_{\rm p}$ can be defined as follows

$$\Gamma_{\rm p} = h \int \frac{d^2W}{d\varepsilon_p * dt} * d\varepsilon_p$$

where according to Weisskopf-Ewing theory, the emission rate of the proton, $\frac{d^2W}{d\varepsilon_p * dt}$ can be defined as

$$\frac{d^2W}{d\varepsilon_p * dt} \approx \frac{2I_p + 1}{2\pi^2 h} k_p^2 \sigma_C \exp - \frac{\varepsilon_p + S_p}{T}$$

Here, k_p and I_p are the wavenumber and the spin of the emitted proton while T and S_p is the nuclear temperature and separation energy of proton []. At around 14 meV (fast neutrons energy range), the nuclear cross-section for charged can be approximated as

$$\sigma(n, p) = C * \sigma_C * \exp a\left(\frac{N - Z}{A}\right) \quad (3)$$

This is the empirical nuclear cross-section proposed by Levkovski [6], where the nuclear cross-section has an exponential dependence on asymmetry parameter, $\frac{N-Z}{A}$. From this empirical formulation, many other authors have subsequently modified and developed other version of empirical formulation that has exponential dependent on $\frac{N-Z}{A}$. Other forms of exponential dependency have also been developed.

In Table 1, we tabulated previously proposed empirical nuclear cross sections by other authors. In this table, we tabulated the expression proposed, the mass region, the incident neutron energy and the chi-square value (χ^2) of the fitting. Here we defined the chi-square value (χ^2) as

$$\chi^2 = \frac{1}{N - n_0} \sum_{i=1}^{N} \frac{|\sigma_{exp} - \sigma_{emp}|}{\sigma_{exp}} \quad (4)$$

where N is the total number of cross-sections data points, n_0 is the number of free parameters, σ_{exp} is the experimental nuclear cross-sections and σ_{emp} is the empirical nuclear cross-sections. In this work, we proposed an empirical formulation in the form of 3 free parameters.

$$\sigma(n, p) = C_1 * \left(A^{\frac{1}{3}} + 1\right) * \exp C_2\left(\frac{N - Z + A}{A}\right)^n \quad (5)$$

Table 1 Empirical formulation of (n, p) nuclear cross-section at around 14 meV incident energy

Authors	Energy (meV)	Mass region	Formula (mb)	χ^2
Ait Tahar [8]	14	$40 \leq A \leq 239$	$\sigma = 107.98\left(A^{\frac{1}{3}}+1\right)^2 \exp\left(-36.749\frac{N-Z+1}{A}\right)$	7.08
Kasugai et al. [9]	14	$19 \leq A \leq 188$	$\sigma = 1830(N-Z+1)\exp\left(-50.7\frac{N-Z+1}{A}\right)$	8.23
Doczi et al. [10]	14.7	$31 \leq A \leq 181$	$\sigma = 23.659\left(A^{\frac{1}{3}}+1\right)^2 \exp\left(-23.041\left(\frac{N-Z}{A}+\left(\frac{N-Z}{A}\right)^2\right)\right)$	4.24
Luo et al. [11]	14.5	$46 \leq A \leq 196$	$\sigma = 62.98\left(A^{\frac{1}{3}}+1\right)^2 \exp\left(-34.45\frac{N-Z}{A}\right)$	2.64
Yigit [7]	14–15	$9 \leq A \leq 239$	$\sigma = 48.367\left(A^{\frac{1}{3}}+1\right)^2 \exp\left(-31.859\frac{N-Z}{A}\right)$	0.29

where C_1, C_2 and n is obtained from fittings. The fitting is done by using a dataset of nuclear isotopes obtained from the EXFOR database. The dataset contains the list of 61 nuclear isotopes with atomic mass range $9 < A < 111$ at neutron incident energy around 14–15 meV. Then, we compare the empirical formulation we obtained with previous formulations in terms of chi-square value (χ^2).

3 Results and Discussions

From Fig. 1, we plotted the experimental nuclear cross-sections (orange dot) and the fitted empirical formulation (blue dot) that we obtained as a function of $\left(\frac{N-Z+A}{A}\right)$. From the patterns of the plot in Fig. 1, there is an exponential dependence of $\sigma(n, p)$ with $\left(\frac{N-Z+A}{A}\right)$. From non-linear fitting, we obtained that the value of C_1, C_2 and n that minimized the chi-square value (χ^2) are 7, 4.3 and -19.6. We tabulated the fitting parameters in Table 2. The complete empirical formulation proposed in this work for (n, p) nuclear reaction with mass range $9 \leq A \leq 111$ is as follows:

$$\sigma(n, p) = 7 * \left(A^{\frac{1}{3}}+1\right) * \exp 4.3\left(\frac{N-Z+A}{A}\right)^{-19.6} \tag{6}$$

The minimum chi-square value (χ^2) obtained for our empirical formulation is 1.23, which is better than the results from Ait Tahar [8], Kasugai et al. [9], Doczi et al. [10] and Luo et al. [11]. Our results is higher compared to Yigit [7]. However, we find that our result is acceptable and the empirical formulation fit strong enough to the experimental nuclear cross-section data which can be observed in Fig. 2.

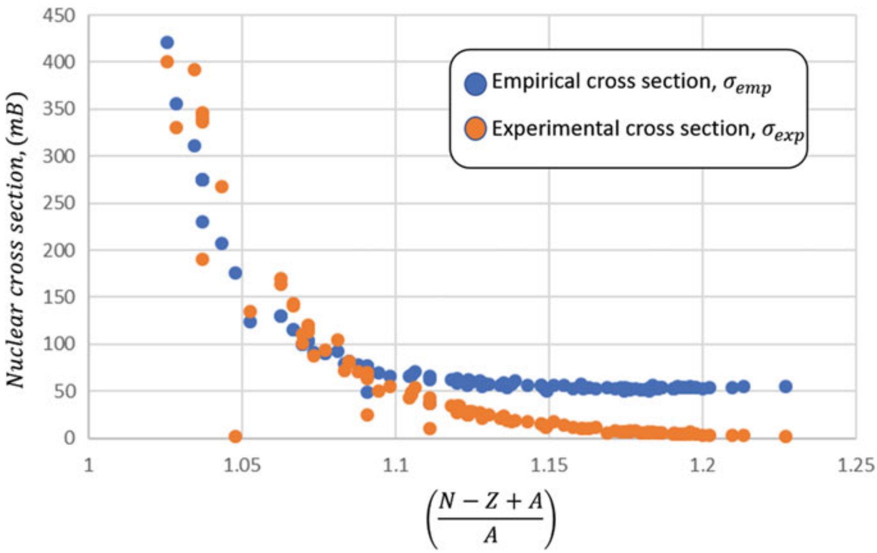

Fig. 1 Non-linear fitting of experimental cross-sections. The blue dots are the empirical nuclear cross-section while the orange dots are the experimental nuclear cross-section at incident neutron energy around 14 meV

Table 2 Fitting parameter obtained for the empirical formulation of (n, p) nuclear cross-section at around 14 meV incident energy

C_1	C_2	n	χ^2
7	4.3	− 19.6	1.230680986

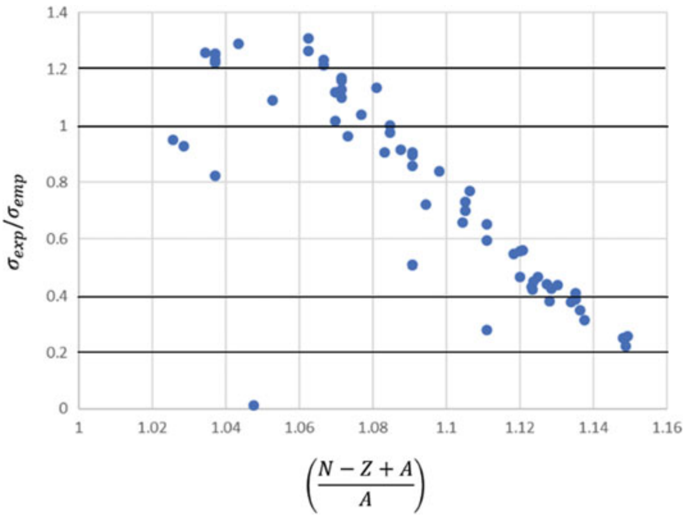

Fig. 2 Ratio of $\sigma_{exp}/\sigma_{emp}$ as a function of $\left(\frac{N-Z+A}{A}\right)$

4 Conclusions

In this work, we derived the empirical nuclear cross-section formulation of (n, p) nuclear reactions for neutron incident energy around 14 meV. This is done by non-linear fittings of experimental nuclear cross-section data obtained from the EXFOR database. The empirical formulation derived in this work shows an exponential dependency to $\left(\frac{N-Z+A}{A}\right)$. The empirical formula we derived have 3 free parameters which are $C_1 = 7$, $C_2 = 4.3$ and $n = -19.6$ that minimized $\chi^2 = 1.23$. Thus, we conclude that our empirical formulation shows an acceptable agreement with experimental data.

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