Experimentally Investigating the Resonance of the Vibration of Two Masses One Spring System Under Different Friction Conditions

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Abstract. This report presents some experimental results of the effect of the Coulomb friction on the resonant frequency of the two-mass system connected by a nonlinear leaf spring. The experimental apparatus designed and built has an ability to vary the excitation frequency, the excitation force and friction force. Experimental data show that the resonance frequency of the system tends to decrease when increasing the friction force. The resonant frequency of the system can be expressed as the functions depending both on the amplitude of excitation force and on the Coulomb friction force. The experimental results serve as the basic premise for the development of studies applied in self-movement structure operating under different resistant environments.

Keywords: Resonance frequency · 2DOF · Vibration-driven locomotion · Friction force · Excitation force

1 Introduction

The nonlinear oscillator consists of two masses connected by one spring is the basic model used to theoretically study and applied to a variety of research fields, such as vibrations, multi-body systems, structural dynamics and transportation… especially in the operability of a self-moving mechanism (new locomotion system) [1–6]. Generally, the mathematical model of the system contains two ordinary conjugate differential equations with cubic non-linearity. Some researchers have presented several techniques for solving analytically a second order differential equation with various strong non-linear characteristic. S.K. Lai and C.W. Lim [5] used an analytical approach developed for non-linear free vibration of a conservative, two degrees of freedom mass-spring system having linear and non-linear stiffness (with model in Fig. 1). Max–Min Approach (MMA) is applied to obtain an approximate solution of three practical cases in terms of a non-linear oscillation system [3]. Recently, many researchers based on two-mass one-spring model developed the mobile devices employing vibration for motion, also

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called vibration-driven locomotion systems; such as designing, modeling and experimental validation; dynamical analysis; optimal progression and motion control [7–13]. Figure 2 is the physical model of the system employing vibration for motion.

However, most of these studies have not yet evaluated the effect of friction on the mechanical movement. In previously theoretical studies, the friction usually is not fully considered. In fact, the self-moving models, such as biomedical applications of capsule and rehabilitation robots in medical, pipe capsule robots… have to work in different environments, where the environmental resistance is different. Therefore, the study of self-movement under different resistance conditions is very important. Recently, Christoph Kossack et al. [14] reported frequency response function (FRF) determined by either experiment or simulation for a dynamic oscillator with a sliding friction contact (Coulomb friction). The results of this study have been just used for single degree of freedom (SDOF) system, thus haven't applied to capsule robots.

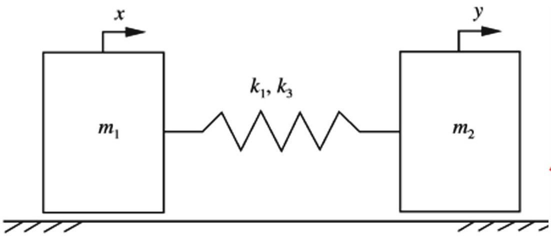


Fig. 1. Basic system of the two masses connected by linear or non-linear spring [2, 3, 5]

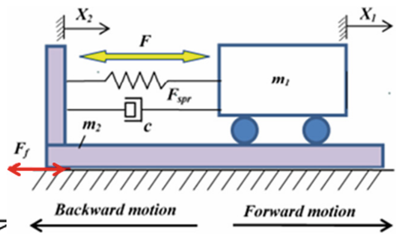


Fig. 2. Physical model of the system

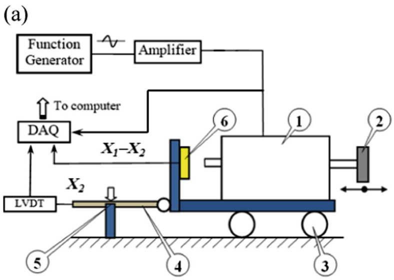
With vibration-driven locomotion systems based on two masses one spring, phenomenon resonance plays an important role. When an oscillating force is applied at a resonant frequency of a dynamical system, the system will oscillate at a higher amplitude than when the same force is applied at the others, i.e. non-resonant frequencies. Mathematically, when a system operates at resonant frequency, there is a 90° phase between the exciting force and the system's response. Therefore, it's possible to say that the force is in phase with the speed response of the system. It means that the sense of the force is the same as the sense of the motion [15, 16]. Consequently, the exciting force applies permanently in the direction and does positive work-done on the system body. This can enhance the effectively changing energy of the system.

This study uses the model of two masses connected by a nonlinear spring to evaluate oscillation of mechanical system, specifically in terms of vibration resonance. The research results are very significant in assessing the ability of vibration-driven system under the different resistance conditions.

2 Design and Setup the Model of Apparatus

2.1 Design and Setup Experimental Apparatus

Design and prototype are two cornerstone aspects in studying vibration-driven locomotion systems. Generally, the design of vibration-driven locomotion systems is based on the mechanics of the interaction between the inertial mass and the system body. In such system, the inertial mass is excited and controlled to periodically move inside the system body, creating the inertial force when changing either its direction of motion, or acceleration, or velocity. The locomotion thus can be generated with the presence of inertial force inside and resistant force from surrounding environment.



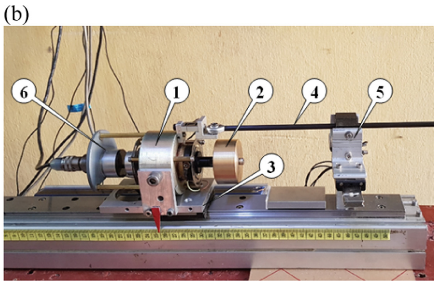


Fig. 3. (a) The experimential diagram and (b) experimential apparatus

This experimental study uses an apparatus based on the model of two masses and a non-linear leaf spring system, as shown in Fig. 2 and Fig. 3. A mini electro-dynamical shaker (1) is placed on a slider of a commercial linear bearing guide (3), providing a tiny friction force. An additional mass (2) was clamped on the shaker shaft. Generally, a sinusoidal current applying to the shaker leads to relative linear oscillation of the shaker shaft with the inertial mass added on, creates the excitation force F_e. This force depends on the current supplied and can be adjusted the sinusoidal voltage supplying to the amplifier. The moveable mass, combined by the addition mass and the shaker shaft, is assigned as inertial mass m_1, playing the role of the internal mass of the capsule robot. A non-contact position sensor (6), model Kaman KD-2306, was used to measure the relative motion of the inertial mass and the shaker body, i.e. measuring X_1–X_2. The movement of the shaker body was recognized by a linear variable displacement transformer (LVDT), i.e. absolute distance X_2. A carbon tube (4) is connected with the shaker body by means of a flexible joint, avoiding any misalignment when moving. As shown on Fig. 4(a), the carbon tube is able to slide between two aluminum pieces in the form of a V-block (5). The two V-blocks are fixed on two electromagnets (7). The body shaker, including the sensors LVDT and the carbon tube, was referred as the mass m_2 of the mass-spring model. The total weight of additional mass and the shaker shaft was considered as the inertial mass m_1.

A set of tests was carried out to determine the stiffness of leaf spring connecting the movable shaft with the shaker body. It is noted that the shaker body was fixed on the slider during these tests. A string was attached to the shaker shaft and rode over a pulley while a series of certain masses were hung on the other end of the string. The gravitational force of the masses pulled the shaft moving forward. The displacement of the shaker shaft, corresponding to each level of masses, was measured by using the non-contact position sensor. A nonlinear curve fitted with the cubic function was then applied to determine the relationship between the gravitational force and the movement. The experiment tests revealed that the spring is a nonlinear spring with hard characteristics (the cubic term is positive). In addition, the damping coefficient, c was determined by logarithmic decrement method.

2.2 Experimental Operation

Supplying a certain value of electrical current to the electromagnets provides a desired clamping force on the tube and thus receiving a corresponding value of sliding friction. The friction force was measured by pushing or pulling the body to move at a steady speed V_s. Experimental data revealed that the friction force F_f depends on the applied voltage V by fitted curve in the following relationship:

$$F_f = 1.52889 + 0.13591 * e^{1.25642*V} \tag{1}$$

This is configuration allows varying the friction force without changing the body mass and thus can experimentally check with the locomotion capacity in different friction levels. The signals from the sensors were captured by a data acquisition system (DAQ) and then stored and analyzed.

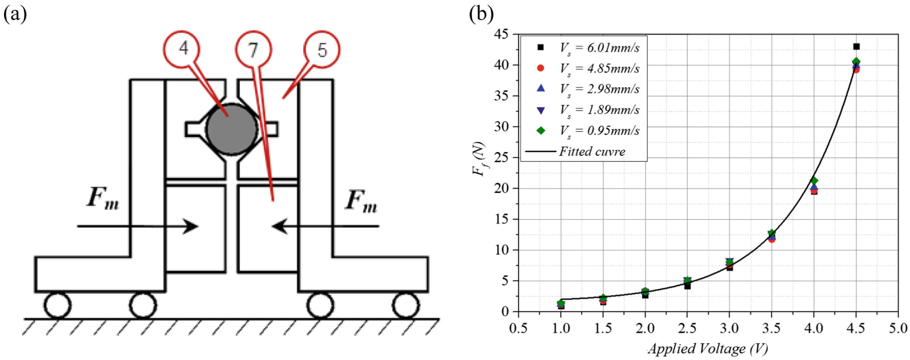


Fig. 4. Varying the friction force: (a) apparatus structure and (b) the dependency of friction force on supplied voltage

A supplementary experiment was implemented to determine the relationship of the magnetic force and the supplied current. A load-cell was used as an obstacle resisting the shaker movement and thus to measure the magnetic force induced. A DC voltage was supplied to the shaker to generate the magnetic force. Varying the voltage, several pairs of the current passing the shaker and the force were collected. Experimental data revealed that the force is proportional to the current supplied to the shaker (see Nguyen et al. 2017 [7] for detailed information of how to determine this relationship). In this study, the amplitude of supplied current was setup with three values, as 0.5 A, 0.75 A and 1.0 A, providing the magnitude of the exciting force, as 5.4 N, 8.1 N and 10.8 N, respectively. During each experiment, these values were kept without changing. All of the experimental parameters are summarized on Table 1.

Table 1. Parameters of experiments.

Parameter	Notation	Value	Unit
Internal mass	m_1	0.518	Kg
Body mass	m_2	1.818	Kg
Linear stiffness	k_1	2988.388	N/m
Cubic stiffness	k_2	52158700	N/m^3
Damping	c	8.893542	Ns/m
Friction force	F_f	0; 3.56; 8.31; 16.63; and 28.5	N
Excitation force	F_e	5.4; 8.1; and 10.8	N
Excitation frequency	f_exc	[2–30]; swept by 1 Hz step in 2 s	Hz

3 Results and Discussion

Theoretically, resonance occurs when the phase difference between the excitation force F_e and the X_1–X_2 shift is 90 degrees. At that time, the excitation current is minimum and the swept amplitude of oscillation X_1–X_2 is maximum. Figure 5 shows an experimental data received by sweeping from 2 Hz to 30 Hz with 3.56 N of friction force and 5.4 N of excitation force. The parameters included sweeping step as 1 Hz and sweeping time as 3 s each step. The experimental data was firstly conducted in time regime (right figure) then changed into frequency regime (right figure). The maximum amplitude of oscillation X_1–X_2 was revealed at 14 Hz. At this frequency, the excitation force is the smallest. Others experimental data were carried out similarly by changing the excitation forces and friction forces.

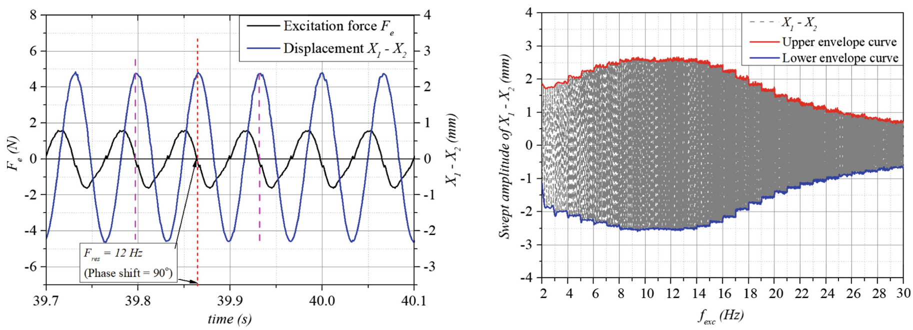


Fig. 5. Determining the experimental frequency resonance

Figure 6 is the experimental data showing the relationship of resonance frequency and friction force at three levels of excitation force 5.4 N, 8.1 N and 10.8 N, respectively. Firstly, as can be seen in all sub-plots, the amplitude relative displacement of the internal mass and the shaker body increases when raising the excitation frequency to maximum, then decreases if continuously raising. The excitation frequency, where the relative displacement reaches maximum value, is the resonant frequency. Secondly, at each level of excitation force, when increasing the friction force, the resonance frequency tends to decrease. Thirdly, without changing the level of resistant force, the resonance frequency increases as the amplitude of excitation force F_e increases from 5.4 N to 10.8 N.

Theoretical studies of the dynamics of the mechanical system with linear springs show that the natural frequency f_n of this system can be determined by following:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_1 * (m_1 + m_2)}{m_1 * m_2}} = \frac{1}{2\pi}\sqrt{\frac{2988.388 * (0.518 + 1.818)}{0.518 * 1.818}} = 13.703Hz$$

The results showed that the resonance frequency of the system was different from the natural frequency f_n. The causes of this difference may be from the nonlinearity of the system, the friction force and damping applied to the system.

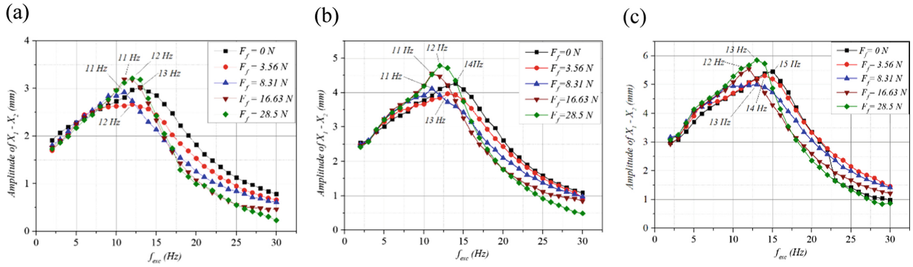


Fig. 6. The variation of resonance frequency under different friction force without changing the exciting force, respectively: (a) $F_e = 5.4\ N$; (b) $F_e = 8.1\ N$; and (c) $F_e = 10.8\ N$

Another view of relationship of resonance frequency and friction force is shown on Fig. 7 to support the above mentioned ideas. The amplitude of oscillation of the mechanical system at resonance increases with increasing the amplitude of excitation force at all investigated resistance levels F_f. Besides, with the same value of the excitation force, the magnitude of oscillation (i.e. amplitude of X_1–X_2) at resonance decreases when increasing the resistant force F_f.

As shown on Fig. 8(a), the resonant frequency f_{res} of the system at each amplitude of excitation force can be represented by a quadratic polynomial function depending on the Coulomb friction force as following:

$$f_{res} = A + B * F_f + C * F_f^2 \tag{2}$$

By fitting the plot data (Fig. 8(a)), the detailed expressions describe the relationship between resonant frequency and friction force as following:

$$f_{res1} = 12.98197 - 0.38692 * F_f + 0.01123 * F_f^2$$

$$f_{res2} = 13.98197 - 0.38692 * F_f + 0.01123 * F_f^2 \tag{3}$$

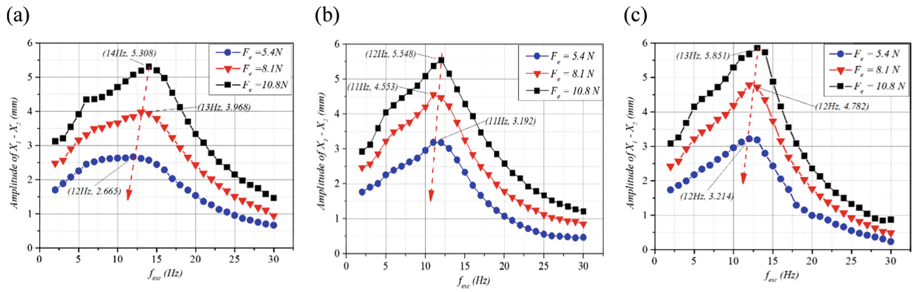


Fig. 7. The variation of resonant frequency under the same Coulomb friction when varying the excitation force, respectively: (a) $F_f = 3.56\ N$; (b) $F_f = 16.63\ N$; and (c) $F_f = 28.5\ N$

$$f_{res3} = 15.03625 - 0.32791 * F_f + 0.00898 * F_f^2$$

Where $f_{res1}, f_{res2}, f_{res3}$ were the resonant frequency of the system depending on friction force at exctitation force as 5.4 N, 8.1 N and 10.8 N, respectively.

The experimental data also show that the coefficients of Eq. (2) depend on excitation force, as shown on the Fig. 8(b). This relation can be expressed as following equations:

$$y = a + b * F_e$$

$$A = 10.91864 + 0.38042 * F_e$$

$$B = -0.45577 + 0.01093 * F_e \tag{4}$$

$$C = 0.01386 - 4.167 * 10^{-4} * F_e$$

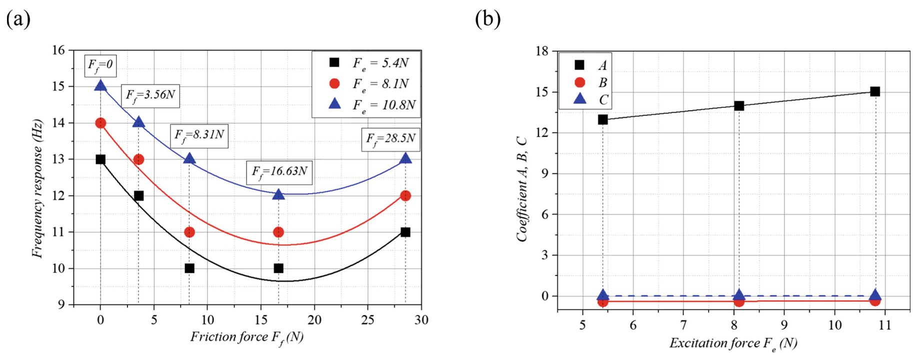


Fig. 8. Fitted curves of the Frequency Response Function with the Coulomb friction force and excitation force.

4 Conclusion

The effects of the Coulomb friction on the resonant frequency of the two-mass system connected by a nonlinear leaf spring were experimentally investigated by sweeping in the range from 2 Hz to 30 Hz. Experimental data show that the resonant frequency of the system seemed to decrease when increasing the resistant force. Under the different friction, the resonant frequencies have been changed quite in comparing with the natural frequency of the system. The resonant frequency of the system can be expressed as the functions depending both on the amplitude of excitation force and on the Coulomb friction force. The experimental results serve as the basic premise for the development of studies applied in self-movement structure operating under different resistant environments.

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References

1. Cveticanin, L.: Vibrations of a coupled two-degree-of-freedom system. J. Sound Vib. **247**(2), 279–292 (2001)
2. Cveticanin, L.: The motion of a two-mass system with non-linear connection. J. Sound Vib. **252**(2), 361–369 (2002)
3. Ganji, S.S., Barari, A., Ganji, D.D.: Approximate analysis of two-mass–spring systems and buckling of a column. Comput. Math. Appl. **61**(4), 1088–1095 (2011)
4. Guo, S., et al.: Development of Multiple Capsule Robots in Pipe. Micromachines (Basel) **9**(6) (2018)
5. Lai, S.K., Lim, C.W.: Nonlinear vibration of a two-mass system with nonlinear stiffnesses. Nonlinear Dyn. **49**(1–2), 233–249 (2006)
6. Xu, J., Fang, H.: Improving performance: recent progress on vibration-driven locomotion systems. Nonlinear Dyn. **98**(4), 2651–2669 (2019). https://doi.org/10.1007/s11071-019-04982-y
7. Nguyen, V.-D., et al.: The effect of inertial mass and excitation frequency on a Duffing vibro-impact drifting system. Int. J. Mech. Sci. **124–125**, 9–21 (2017)
8. Liu, L., et al.: Reliability of elastic impact system with Coulomb friction excited by Gaussian white noise. Chaos Solitons Fractals **131**, 109513 (2020)
9. Liu, Y., et al.: Bifurcation analysis of a vibro-impact experimental rig with two-sided constraint. Meccanica **55**(12), 2505–2521 (2020). https://doi.org/10.1007/s11012-020-01168-4
10. Nguyen, V.-D., et al., A new design of horizontal electro-vibro-impact devices. J. Comput. Nonlin. Dyn. **12**(6), 061002 (2017)
11. Safaeifar, H., Farshidianfar, A.: A new model of the contact force for the collision between two solid bodies. Multibody Syst. Dyn. **50**(3), 233–257 (2020). https://doi.org/10.1007/s11044-020-09732-2

12. Yan, Y., Liu, Y., Manfredi, L., Prasad, S.: Modelling of a vibro-impact self-propelled capsule in the small intestine. Nonlinear Dyn. **96**(1), 123–144 (2019). https://doi.org/10.1007/s11071-019-04779-z
13. Duong, T.-H., et al.: A new design for bidirectional autogenous mobile systems with two-side drifting impact oscillator. Int. J. Mech. Sci. **140**, 325–338 (2018)
14. Christoph Kossack, J.Z., Schmitz, T.L.: The sliding friction contact frequency response function. Procedia Manufacturing **34**, 73–82 (2019)
15. Polesel-Maris, J.R.M., et al.: Experimental investigation of resonance curves in dynamic force microscopy. Nanotechnology **14**(9), 1036–1042 (2003)
16. Bleck-Neuhaus, J.: Mechanical resonance 300 years from discovery to the full understanding of its importance (2018)