Event-Based H_∞ Control for Networked Control System with Saturation Constraints

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Abstract. The event-based H_∞ control problem is researched for networked control systems subject to saturation constraints. In consideration of event-triggered mechanism and saturation constraints concurrently, the networked control system is modeled as a nonlinear system. By using Lyapunov-functional method and the linear matrix inequality techniques, the mean square stable criteria and H_∞ controller has been got for the controlled system with suitable event-driven matrix. A numerical example is given to illustrate the feasibility of the control approach in the paper.

Keywords: Event-driven mechanism · Sensor saturation
Networked control system

1 Introduction

Event-based mechanism has been widely studied recently for the advantages of reducing transmission frequency and saving limited communication resources. The event-driven communication mechanism does not transmit information according to a fixed time period, but decides whether to transmit sampled signals based on predefined trigger conditions. Different from the traditional time-triggered scheme, the experimental results in reference [1] show that the event-triggered scheme can greatly reduce the information transmission frequency on the premise of ensuring the control performance of the system. Event-triggered filtering and control problems have been widely studied in many types of networked control systems, such as linear control systems [2,3], Markov jumping systems [4], descriptor systems [5], T-S fuzzy systems [6], and multi-agent systems [8]. In the literature mentioned above, event-triggering mechanism can be divided into two types: one is

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the continuous event-triggering mechanism [2,3,7,9], and the other is the discrete event-triggering mechanism [4–6]. The continuous event trigger is accomplished by detecting whether the current state exceeds the pre-defined trigger threshold via additional hardware continuously supervising state of the system. Based on discrete event-triggering mechanism, the event generator only needs to use the monitoring and sampling signals at discrete moments. For most practical systems, it is easy to realize the joint design of controller and trigger parameters.

Because of technical, physical, or safety constraints, the output of the sensor or actuator may be saturated. Saturation will introduce nonlinear characteristics into the control system, which may seriously affect the performance of the control system. If the saturation constraints are not properly handled, it not only might degrade the filter or control performances but also results in undesirable oscillatory behavior and even instability [11]. Considering the theoretical and practical significance of actuator/sensor saturation, the control and filter problem of control systems subject to saturation constraints has been extensively studied in recent years [12,13]. For example, in [12], the robust sampled-data control problem has been studied for an automotive seat-suspension system with actuator saturations. It should be emphasized that in most relevant literatures, few existing results involve the control systems subject to both actuator and sensor saturations [10,14], although this phenomenon is very common in engineering applications.

In this paper, the main contributions can be summed up as follows: (1) a new model of nonlinear system is developed for the use of robust H_∞ control synthesis with simultaneous consideration of event-based mechanism and both sensor and random actuator saturations; (2) by way of Lyapunov-functional method and the LMI technique, the mean square stability criteria and H_∞ controller are obtained for the controlled system with suitable trigger parameter matrices.

2 System Description

Consider the linear networked control system as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + G\omega(t) \\ z(t) = Cx(t) + Eu(t) + F\omega(t) \\ x(t_0) = x_0 \end{cases} \tag{1}$$

in which $u(t) \in \mathbb{R}^m$, $x(t) \in \mathbb{R}^n$, $z(t) \in \mathbb{R}^p$ and $\omega(t) \in \mathcal{L}_2[0,\infty)$ are the control input, state vector, control output and disturbance input, respectively. A, B, C, E, G and F are system parameter matrices, and $x_0 \in \mathbb{R}^n$ is the initial condition at initial time t_0.

In order to save the valuable bandwidth resources and reduce the signal transmission frequency in the control system, an event-triggering mechanism is introduced between the controller and the sensor. Considering the control system which has sensor saturation, we employ the trigger condition similar to [1]:

$$\begin{aligned} &[\sigma_s(x(t_kh+jh)) - \sigma_s(x(t_kh))]^T \,\Omega\, [\sigma_s(x(t_kh+jh)) - \sigma_s(x(t_kh))] \\ &\qquad \leq \rho\sigma_s^T(x(t_kh+jh)\Omega\sigma_s(x(t_kh+jh)) \end{aligned} \tag{2}$$

where $\Omega > 0$ is a trigger matrix, $\rho \in (0,1)$ is a constant and $\sigma_s(\bullet)$ is the sensor saturation function. $jh, t_k h$ are the sampling instant and the trigger instant, respectively. The latest sampled signal $x(t_k h + jh)$ will be sent to the controller on condition that the condition (2) is violated. Denote $e_k(t) = \sigma_s(x(t_k h)) - \sigma_s(x(t_k h + jh)), t \in [t_k, t_{k+1})_{k=0}^{\infty}$, and $\eta(t) = t - t_k h - jh, l = 0, 1, 2, \cdots$. Suppose $\eta(t) \in [0, \eta_M)$, where η_M is a positive real number. According to the definition of $e_k(t)$, we can rewrite the trigger condition (2) as follows:

$$e_k^T(t)\Omega e_k(t) \leq \rho\sigma_s^T(x(t-\eta(t)))\Omega\sigma_s(x(t-\eta(t))), \tag{3}$$

We can express the controller as follows:

$$u(t) = K(\sigma_s(x(t-\eta(t))) + e_k(t)) \tag{4}$$

If we consider random actuator saturation, the controller (4) can be rewritten as

$$\bar{u}(t) = \beta(t)u(t) + (1-\beta(t))\sigma_a(u(t)) \tag{5}$$

where $\sigma_a(\bullet)$ denotes the function of actuator saturation. $\beta(t)$ is a white sequence which obeys Bernoulli distribution, and its value is 0 or 1, and satisfies the statistical characteristics as follows:

$$Prob\{\beta(t) = 1\} = \beta_0,\ Prob\{\beta(t) = 0\} = 1 - \beta_0 \tag{6}$$

$$\mathbb{E}\{\beta(t)\} = \beta_0, \quad \mathbb{E}\{(\beta(t) - \beta_0)^2\} = \beta_0(1-\beta_0) \tag{7}$$

In which $\beta_0 \in [0,1]$ is a known constant. The stochastic variable $\beta(t)$ can be applied to describe the random actuator saturation. According to the formula (5), when $\beta(t)$= 1, the real actuator output is $\bar{u}(t) = u(t)$, which means that there is no actuator saturation. When $\beta(t) = 0$, the real actuator output is $\bar{u}(t) = \sigma_a(u(t))$, which shows that the actuator saturation comes up.

Referring to the method proposed in [15], we can the decompose the saturation functions $\sigma_s(x(t-\eta(t)))$ and $\sigma_a(u(t))$ as follows:

$$\sigma_s(x(t-\eta(t))) = x(t-\eta(t)) - \zeta(x(t-\eta(t))), \tag{8}$$

$$\sigma_a(u(t)) = u(t) - \xi(u(t)), \tag{9}$$

in which $\xi(u(t))$ and $\zeta(x(t-\eta(t)))$ are two nonlinearity functions. By using $\eta(t)$, $e_k(t)$, (8) and (9), the control system (1) can be rewritten as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t-\eta(t)) + BKe_k(t) - BK\zeta(x(t-\eta(t)) \\ \qquad\quad -(1-\beta(t))B\xi(u(t)) + G\omega(t) \\ z(t) = Cx(t) + EKx(t-\eta(t)) + EKe_k(t) - EK\zeta(x(t-\eta(t)) \\ \qquad\quad -(1-\beta(t))E\xi(u(t)) + F\omega(t) \end{cases} \tag{10}$$

3 Main Results

In the following, we firstly obtain a stable criterion for the control system (10).

Theorem 1. *For prescribed scalars $\rho \in [0,1)$, $\varepsilon_1, \varepsilon_2, \beta_0 \in (0,1)$, $\gamma > 0$, $\eta_M > 0$ and the controller K, the system (10) is mean square stable with the event-driven mechanism (2), if there exist positive definite matrices P, Q, R, Ω, and matrices S, U satisfying the following matrix inequalities:*

$$\Sigma(s) \triangleq \begin{bmatrix} \Sigma_{11}+\Gamma+\Gamma^T & * & * & * & * & * & * & * & * & * \\ \Sigma_{21} & -\gamma^2 I & * & * & * & * & * & * & * & * \\ \mathcal{G}^s & 0 & -R & * & * & * & * & * & * & * \\ \sqrt{\eta_M}\mathcal{A} & \sqrt{\eta_M}G & 0 & -R^{-1} & * & * & * & * & * & * \\ \Sigma_{51} & 0 & 0 & 0 & -I & * & * & * & * & * \\ \Sigma_{61} & 0 & 0 & 0 & 0 & -I & * & * & * & * \\ \Sigma_{71} & 0 & 0 & 0 & 0 & 0 & -\Omega & * & * & * \\ \mathcal{B} & F & 0 & 0 & 0 & 0 & 0 & -I & * & * \\ \mathcal{C} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R^{-1} & * \\ \mathcal{D} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \qquad s = 1, 2. \quad (11)$$

where

$$\Sigma_{11} = \begin{bmatrix} PA + A^T P + Q & * & * & * & * & * \\ K^T B^T P & 0 & * & * & * & * \\ 0 & 0 & -Q & * & * & * \\ K^T B^T P & 0 & 0 & -\Omega & * & * \\ -K^T B^T P & 0 & 0 & 0 & -I & * \\ -B^T P & 0 & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\begin{aligned} \Sigma_{21} &= \begin{bmatrix} G^T P \; 0 \; 0 \; 0 \; 0 \; 0 \end{bmatrix} \\ \mathcal{G}^1 &= \sqrt{\eta_M} S^T, \quad \mathcal{G}^2 = \sqrt{\eta_M} U^T \\ \Gamma &= \begin{bmatrix} S \; U - S \; -U \; 0 \; 0 \; 0 \end{bmatrix} \\ \mathcal{A} &= \begin{bmatrix} A \; BK \; 0 \; BK \; -BK \; -(1-\beta_0)B \end{bmatrix} \\ \Sigma_{51} &= \begin{bmatrix} 0 \; \sqrt{\varepsilon_1} I_n \; 0 \; 0 \; 0 \; 0 \end{bmatrix} \\ \Sigma_{61} &= \begin{bmatrix} 0 \; \sqrt{\varepsilon_2} K \; 0 \; \sqrt{\varepsilon_2} K \; -\sqrt{\varepsilon_2} K \; 0 \end{bmatrix} \\ \Sigma_{71} &= \begin{bmatrix} 0 \; \sqrt{\rho}\Omega \; 0 \; 0 \; -\sqrt{\rho}\Omega \; 0 \end{bmatrix} \\ \mathcal{B} &= \begin{bmatrix} C \; EK \; 0 \; EK \; -EK \; -(1-\beta_0)E \end{bmatrix} \\ \mathcal{C} &= \sqrt{\eta_M \beta_0 (1-\beta_0)} \begin{bmatrix} 0 \; 0 \; 0 \; 0 \; 0 \; B \end{bmatrix} \end{aligned}$$

$$\mathcal{D} = \sqrt{\beta_0(1-\beta_0)}\, 0 \; 0 \; 0 \; 0 \; 0 \; E$$

Proof. Define the Lyapunov-Krasovskii functional as follows:

$$V(x_t) = x^T(t)Px(t) + \int_{t-\eta_M}^{t} x^T(s)Qx(s)ds + \int_{t-\eta_M}^{t}\int_{s}^{t} \dot{x}^T(v)R\dot{x}(v)dvds \quad (12)$$

in which P, Q and R are positive definite matrices. Calculating the infinitesimal operator of $V(x_t)$ and using free matrices approach, by using a similar proof method in [1], we can prove that the control system (10) is mean square stable. Because of page limitation, the proof procedure is omitted.

Based on analysis results in Theorem 1, A robust design method of the controller K and trigger parameter matrices is proposed.

Theorem 2. *For prescribed scalars $\rho \in [0,1)$, $\varepsilon_1, \varepsilon_2, \beta_0 \in (0,1)$, $\varepsilon_3, \varepsilon_4 > 0$, $\gamma > 0$, $\eta_M > 0$, the control system (10) is mean square stable, if there exist positive definite matrices X, $\tilde{Q}$, $\tilde{R}$, $\tilde{\Omega}$, and matrices $\tilde{U}$, $\tilde{S}$, Y, satisfying the following LMIs:*

$$\tilde{\Sigma}(s) \triangleq \begin{bmatrix} \tilde{\Sigma}_{11}+\tilde{\Gamma}+\tilde{\Gamma}^T & * & * & * & * & * & * & * & * & * \\ \tilde{\Sigma}_{21} & -\gamma^2 I & * & * & * & * & * & * & * & * \\ \tilde{\mathcal{G}}^s & 0 & -\tilde{R} & * & * & * & * & * & * & * \\ \sqrt{\eta_M}\tilde{\mathcal{A}} & \sqrt{\eta_M}G & 0 & -2\varepsilon_3 X+\varepsilon_3^2\tilde{R} & * & * & * & * & * & * \\ \tilde{\Sigma}_{51} & 0 & 0 & 0 & -I & * & * & * & * & * \\ \tilde{\Sigma}_{61} & 0 & 0 & 0 & 0 & -I & * & * & * & * \\ \tilde{\Sigma}_{71} & 0 & 0 & 0 & 0 & 0 & -\tilde{\Omega} & * & * & * \\ \tilde{\mathcal{B}} & H & 0 & 0 & 0 & 0 & 0 & -I & * & * \\ \mathcal{C} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\varepsilon_4 X+\varepsilon_4^2\tilde{R} & * \\ \mathcal{E} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \quad (13)$$

where

$$s = 1, 2.$$

$$\tilde{\Sigma}_{11} = \begin{bmatrix} AX+XA^T+\tilde{Q} & * & * & * & * & * \\ Y^TB^T & 0 & * & * & * & * \\ 0 & 0 & -\tilde{Q} & * & * & * \\ Y^TB^T & 0 & 0 & -\tilde{\Omega} & * & * \\ -Y^TB^T & 0 & 0 & 0 & -2\varepsilon X+\varepsilon^2 I & * \\ -B^T & 0 & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\tilde{\Sigma}_{21} = \begin{bmatrix} G^T \; 0 \; 0 \; 0 \; 0 \; 0 \end{bmatrix}$$
$$\tilde{\mathcal{G}}^1 = \sqrt{\eta_M}\tilde{S}^T, \quad \tilde{\mathcal{G}}^2 = \sqrt{\eta_M}\tilde{U}^T$$
$$\tilde{\Gamma} = \begin{bmatrix} \tilde{S} \; \tilde{U}-\tilde{S} \; -\tilde{U} \; 0 \; 0 \; 0 \end{bmatrix}$$
$$\tilde{\mathcal{A}} = \begin{bmatrix} AX \; BY \; 0 \; BY \; -BY \; -B \end{bmatrix}$$
$$\tilde{\Sigma}_{51} = \begin{bmatrix} 0 \; \sqrt{\varepsilon_1}I \; 0 \; 0 \; 0 \; 0 \end{bmatrix}$$
$$\tilde{\Sigma}_{61} = \begin{bmatrix} 0 \; \sqrt{\varepsilon_2}Y \; 0 \; \sqrt{\varepsilon_2}Y \; -\sqrt{\varepsilon_2}Y \; 0 \end{bmatrix}$$
$$\tilde{\Sigma}_{71} = \begin{bmatrix} 0 \; \sqrt{\rho}\tilde{\Omega} \; 0 \; 0 \; -\sqrt{\rho}\tilde{\Omega} \; 0 \end{bmatrix}$$
$$\tilde{\mathcal{B}} = \begin{bmatrix} CX \; EY \; 0 \; EY \; -EY \; -E \end{bmatrix}$$
$$\mathcal{C} = \sqrt{\eta_M\beta_0(1-\beta_0)}\begin{bmatrix} 0 \; 0 \; 0 \; 0 \; 0 \; BX \end{bmatrix}$$
$$\mathcal{D} = \sqrt{\beta_0(1-\beta_0)}\begin{bmatrix} 0 \; 0 \; 0 \; 0 \; 0 \; E \end{bmatrix}$$

The controller of the system is $K = YX^{-1}$.

4 Illustrative Examples

Example 1. Consider the control system (1) with parameter matrices as follows:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -2 & -0.1 \\ -0.1 & 0.01 \end{bmatrix} x(t) + \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix} \sigma_a(u(t)) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \omega(t) \\ z(t) = \begin{bmatrix} 0.5 & -0.1 \\ -0.8 & 0.1 \end{bmatrix} x(t) + \begin{bmatrix} -0.7 \\ 0.3 \end{bmatrix} \sigma_a(\bar{u}(t)) + \begin{bmatrix} -1 \\ 1 \end{bmatrix} \omega(t) \end{cases} \tag{14}$$

Suppose the external disturbance as

$$\omega(t) = \begin{cases} 0.4\text{sgn}(\sin(t)), & if\ t \in [0, 15] \\ 0, & otherwise \end{cases}$$

Set $\varepsilon = 1$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.2$, $\rho = 0.03$, $\varepsilon_3 = \varepsilon_4 = 1$, $\beta_0 = 0.7$, $\gamma = 20$, and the sampling period $h = 0.05$. According to Theorem 2, the upper bound of η_M is 0.8523. Choosing $\eta_M = 0.7$, we can derive the trigger matrix Ω as

$$\Omega = \begin{bmatrix} 11.1874 & 0.1434 \\ 0.1434 & 0.9024 \end{bmatrix} \tag{15}$$

and the controller K of the system is

$$K = \begin{bmatrix} 0.1649 & -0.3794 \end{bmatrix} \tag{16}$$

When we choose the initial condition of the system as $x(0) = \begin{bmatrix} 1.2 & -0.7 \end{bmatrix}^T$, the state responses of the system, the output of the sensor with saturation constraints and the input of the system with random saturation are illustrated

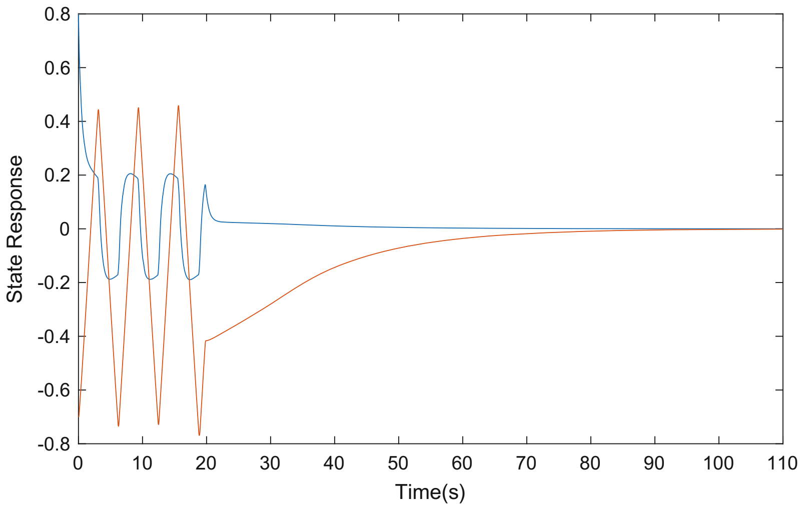

Fig. 1. The state of the system with the controller (16)

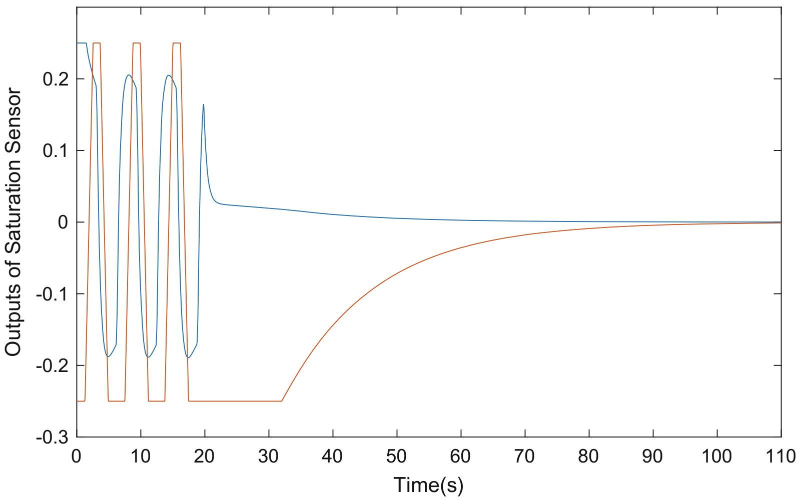

Fig. 2. The sensor outputs with the controller (16)

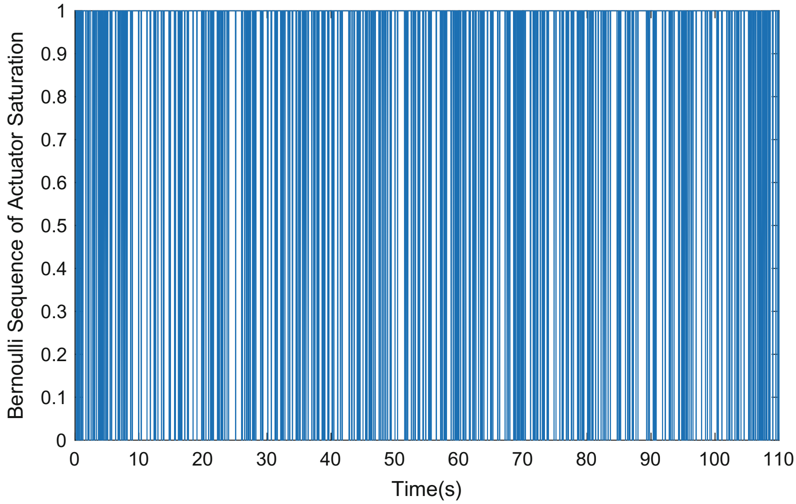

Fig. 3. The probability distribution of the randomly occurring actuator saturation

in Figs. 1, 2, and 4, respectively. The probability distribution of the randomly occurring actuator saturation is illustrated in Fig. 3. Figure 5 shows the signal release instants and event intervals. The transmitted signals only take 4.8% of all sampled signals.

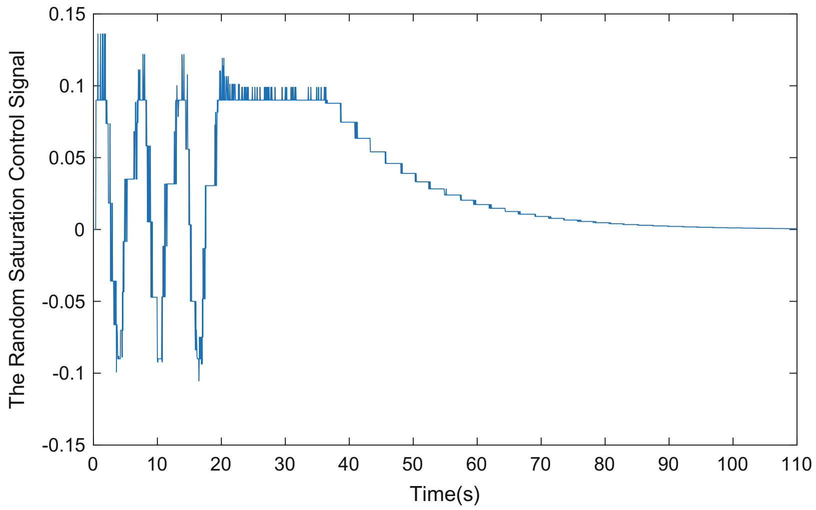

Fig. 4. The outputs of the random saturation actuator with the controller (16)

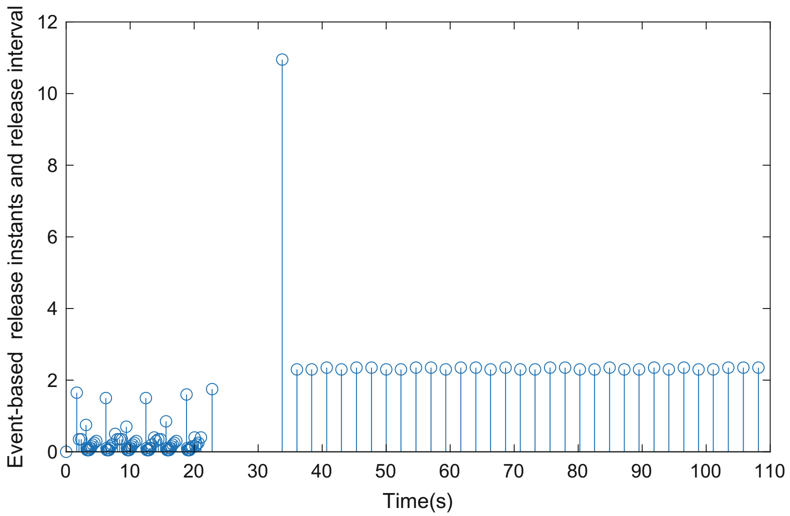

Fig. 5. The event-triggered instants and event-triggered interval with the controller (16)

5 Conclusion

This paper mainly researches the H_∞ control problem of networked control systems with event-driven mechanism and sensor and random actuator saturation. In order to overcome the adverse effects of sensor and random actuator saturations, a novel networked nonlinear control system model is established based on event-triggered mechanism. By using Lyapunov function method, stochastic analysis theory and linear matrix inequality technique, the stability criterion of the system is obtained and the co-design method of the controller and trigger parameter matrices is implemented simultaneously. A numerical example has been given to illustrate the feasibility of the proposed control technology.

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