Estimate of Initial Installation Angle of INS in Vehicle MEMS-INS/GNSS Integrated Navigation System

Hongsong Zhao[1,2](✉), Juhong Xing[1,2], Mobo Qiu[1,2], and Yongsong Wang[1,2]

[1] Qianxun Spatial Intelligence Inc., Shanghai, China
hongsong.zhao@wz-inc.com
[2] Qianxun Spatial Intelligence (Zhejiang) Inc., Deqing, Zhejiang, China

Abstract. As a result of the low precision of MEMS inertial devices in vehicle MEMS-INS/GNSS integrated navigation systems, Nonholonomic constraints (NHC) or odometers are generally used to maintain high-precision navigation, especially when GNSS signal is blocked or interfered. The application of NHC or odometer generally requires that MEMS-INS is accurately installed in the vehicle body frame. But this requirement is sometimes very harsh, such as the limitation of vehicle installation conditions, the non-professionalism of operators, and so on. Therefore, a method for estimating the initial installation angle of MEMS-INS is proposed, which can quickly estimate the initial installation angle. This method requires the vehicle to stop on a relatively horizontal road for estimating the horizontal installation angle. Then the heading installation angle is calculated by using the trajectory information of MEMS-INS during the process from stationary to acceleration. Since this method will be affected by the accuracy of MEMS inertial devices, the factors affecting the accuracy of installation angle estimate are analyzed in detail. Finally, the effectiveness of the method is verified by the actual vehicle test. The method has good engineering practicability for the application of MEMS-INS/GNSS integrated navigation system in the field of vehicle.

Keywords: MEMS-INS · GNSS integrated navigation system · Nonholonomic constraint · Odometer · Installation angle estimate

1 Introduction

In recent years, with the development of Beidou industry, more and more integrated navigation systems based on Beidou satellite navigation are applied in various fields. Among them, MEMS-INS/GNSS integrated navigation system has the advantages of low cost, small size and good dynamic performance, which is widely used in small unmanned vehicle control, civil vehicle navigation, automatic driving precision positioning, precision agriculture and other fields [1–7]. Due to the low precision of MEMS inertial devices, it is difficult to maintain high precision inertial navigation capability for a long time. In the vehicle application, when GNSS signal is blocked or interfered,

C. Yang and J. Xie (Eds.): *China Satellite Navigation Conference (CSNC 2021) Proceedings*, LNEE 773, pp. 606–615, 2021.
https://doi.org/10.1007/978-981-16-3142-9_58

the Nonholonomic constraint (NHC) or odometer is generally used to maintain high-precision navigation [2, 6, 7]. Both NHC and odometer make use of the characteristic that the velocity in the plane perpendicular to the longitudinal direction is almost zero to restrict the inertial navigation errors [2]. Therefore, the installation of MEMS-INS must be completely coincident with the vehicle body frame, or only has a small angle installation error [8].

In many cases, due to the limitation of vehicle installation conditions, the lack of professional installation and calibration conditions, it is difficult to meet the above installation conditions of MEMS-INS. Therefore, it is necessary to obtain the actual installation angle information of MEMS-INS, which can be used to transform the IMU output to the vehicle frame. But it is difficult to obtain the installation angle in many cases. Most of the existing literature focuses on the estimate method when the installation angle is small. For example, Chen [8] and Zhao [9] model the installation angle error and estimate it online. Yan [10] and Fu [11] use the trajectory similarity principle to estimate the installation angle. The acceleration information is used to estimate the installation angle, which has a better estimation effect under the condition of large acceleration and small vibration noise [12]. But when the acceleration is small and the vibration noise is large, the accuracy of estimation cannot be guaranteed. FIR filter and algebraic observer are used to estimate the longitudinal and lateral accelerometer biases as well as the yaw misalignment [13]. Nevertheless, due to the influence of noise and parameter identifiability, it is difficult to ensure the stability of the estimate. To help the popularization and application of MEMS-INS/GNSS integrated navigation system, a fast estimate method of initial installation angle is proposed. This method has no restrictions on installation conditions, low requirements on vehicle acceleration and vibration environment, and it only need short time to get the stable and reliable estimate accuracy. Generally speaking, the estimate accuracy of the initial installation angle is no need to be high. As long as the residual installation angle error is small, the installation angle estimate model can be used in the integrated navigation system to accurately estimate the residual installation angle error through vehicle movement [8, 9].

2 Principle of the Installation Angle Estimate

2.1 Axis Transformation

Firstly, the coordinate system is defined as follows:

IMU frame (s-frame): x-axis refers to the right, y-axis refers to the front, and z-axis refers to the upper, which form the right-hand coordinate system.

Vehicle frame (b-frame): y-axis points to the forward direction of the vehicle body, x-axis points to the right side of transverse axis, z-axis points to the top of the vehicle body, which constitute a right-hand coordinate system.

Navigation frame (n-frame): the reference frame for navigation solution generally selects the east-north-up coordinate.

The direction cosine matrix is defined as

$$C_{\mathrm{m}}^{\mathrm{p}} = \begin{bmatrix} \mathrm{c}\gamma\mathrm{c}\psi - \mathrm{s}\gamma\mathrm{s}\psi\mathrm{s}\theta & \mathrm{c}\gamma\mathrm{s}\psi + \mathrm{s}\gamma\mathrm{c}\psi\mathrm{s}\theta & -\mathrm{s}\gamma\mathrm{c}\theta \\ -\mathrm{s}\psi\mathrm{c}\theta & \mathrm{c}\psi\mathrm{c}\theta & \mathrm{s}\theta \\ \mathrm{s}\gamma\mathrm{c}\psi + \mathrm{c}\gamma\mathrm{s}\psi\mathrm{s}\theta & \mathrm{s}\gamma\mathrm{s}\psi - \mathrm{c}\gamma\mathrm{c}\psi\mathrm{s}\theta & \mathrm{c}\gamma\mathrm{c}\theta \end{bmatrix} \tag{1}$$

where m and p denote any two coordinate, ψ 、θ 、γ represents the azimuth angle, pitch angle, and roll angle from m-frame to p-frame, the abbreviations of sθ and cθ represent sinθ and cosθ respectively, others are similar.

Due to the limitation of installation conditions, the IMU frame is not completely coincident with the vehicle frame, as shown in Fig. 1. Under some installation conditions, the IMU frame may be that any axis points above the vehicle. In this case, the IMU frame can be transformed by the static axis transformation. As the static axis transformation relationship is relatively simple, it will not be introduced in detail. The IMU frame (s-frame) described later has been transformed into the installation relationship as shown in Fig. 1 through the above static axis transformation.

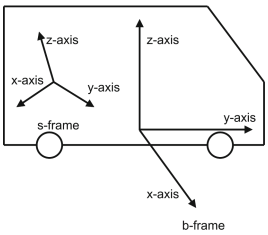

Fig. 1. Relative installation relationship between IMU frame and vehicle frame

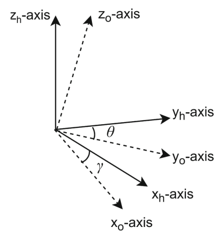

Fig. 2. Relative relationship between original IMU frame (x_o-y_o-z_o) and leveling IMU frame (x_h-y_h-z_h)

2.2 Estimate of the Horizontal Installation Angle

Due to the low precision of MEMS-IMU, it cannot be used for heading alignment [5]. In the inertial navigation, the heading error will lead to the platform command angular velocity error. However, when the working time of SINS based on MEMS-IMU is short, the motion velocity is small, and the motion trajectory is relatively short, the platform command angular velocity error is almost negligible relative to the device error of MEMS-IMU. In addition, when only the relative motion information, such as displacement information, is needed, the error caused by heading misalignment can be ignored. The SINS based on MEMS-IMU relies on the output of the accelerometer to easily complete the horizontal coarse alignment. The heading angle is directly initialized to 0°, so that the SINS works in a virtual navigation coordinate. As the navigation frame involved in this paper is the virtual navigation frame, for the convenience of expression, the navigation frame is n-frame.

First, the vehicle is required to stop on a basically level ground and be stationary. When the alignment is completed in the virtual navigation frame as above, the zero-velocity updates (ZUPT) mode is entered [14]. Due to the low precision of low-cost MEMS-IMU, it is necessary to add the heading keeping technology in ZUPT mode to ensure that the heading angle does not diverge.

The error state of SINS is selected as speed error $\delta \boldsymbol{v}^{\mathrm{n}}$, attitude error $\boldsymbol{\phi}^{\mathrm{n}}$, position error $\delta \boldsymbol{P}$, accelerometer constant bias ∇^{b}, gyro constant drift $\boldsymbol{\varepsilon}^{\mathrm{b}}$, totally 15 dimensions. Set the state variable as

$$X_{\mathrm{SINS}} = \left[(\delta \boldsymbol{v}^{\mathrm{n}})^{\mathrm{T}} \ (\boldsymbol{\phi}^{\mathrm{n}})^{\mathrm{T}} \ (\delta \boldsymbol{P})^{\mathrm{T}} \ (\nabla^{\mathrm{b}})^{\mathrm{T}} \ (\boldsymbol{\varepsilon}^{\mathrm{b}})^{\mathrm{T}} \right]^{\mathrm{T}} \tag{2}$$

where T denotes matrix transpose.

According to the error model of SINS, the system state equation is expressed as

$$\dot{\boldsymbol{X}}_{\mathrm{SINS}} = \boldsymbol{F}_{\mathrm{SINS}} \boldsymbol{X}_{\mathrm{SINS}} + \boldsymbol{G}_{\mathrm{SINS}} \boldsymbol{W} \tag{3}$$

where $\boldsymbol{F}_{\mathrm{SINS}}$ is the system state matrix, $\boldsymbol{G}_{\mathrm{SINS}}$ is the noise input matrix, $\boldsymbol{W}$ is the system state noise. The detailed definition can be referred to [15].

In ZUPT mode, the real velocity of the system is 0, so the velocity measurement equation is expressed as

$$\boldsymbol{v}^{\mathrm{n}} - 0 = \delta \boldsymbol{v}^{\mathrm{n}} \tag{4}$$

The attitude matrix of SINS is defined as $C_{\mathrm{b}}^{\mathrm{n}} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}$.

The heading angle of the first static frame is recorded as ψ_0, and the heading angle calculated by SINS in real time is recorded as ψ_{SINS}. According to the reference [16], the heading measurement equation for heading keeping technology is obtained as

$$\delta\psi = \psi_{\mathrm{SINS}} - \psi_0 = \phi_{\mathrm{U}} - \frac{T_{22}T_{32}}{T_{12}^2 + T_{22}^2}\phi_{\mathrm{N}} - \frac{T_{12}T_{32}}{T_{12}^2 + T_{22}^2}\phi_{\mathrm{E}} \tag{5}$$

From Eq. (4) and Eq. (5), the measurement equation is expressed as

$$\boldsymbol{Z} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{V} \tag{6}$$

where $\boldsymbol{Z} = \left[(\boldsymbol{v}^{\mathrm{n}})^{\mathrm{T}} \ \delta\psi \right]^{\mathrm{T}}$, $\boldsymbol{V}$ is the measurement noise. The measurement matrix is denoted as $\boldsymbol{H} = \begin{bmatrix} \boldsymbol{I} & 0_{3\times3} & 0_{9\times3} \\ 0_{1\times3} & \boldsymbol{M} & 0_{9\times3} \end{bmatrix}$, where $\boldsymbol{I}$ is the 3-dimensional identity matrix, $\boldsymbol{M} = \left[-\frac{T_{12}T_{32}}{T_{12}^2+T_{22}^2} \ -\frac{T_{22}T_{32}}{T_{12}^2+T_{22}^2} \ 1 \right]$.

The above state Eq. (3) and measurement Eq. (6) are discretized to obtain as $\boldsymbol{X}_k = \boldsymbol{\Phi}_{k/k-1}\boldsymbol{X}_{k-1} + \boldsymbol{\Gamma}_{k-1}\boldsymbol{W}_{k-1}$ and $\boldsymbol{Z}_k = \boldsymbol{H}_k\boldsymbol{X}_k + \boldsymbol{V}_k$.

The classical Kalman filtering algorithm is used for state estimate. The detailed algorithm can be referred to the references [10] and [15], which will not be repeated here. In ZUPT mode, the accurate horizontal attitude angle can be obtained, which is theoretically only affected by the biases of the horizontal accelerometers as stated in the reference [10] and [11]. At the same time, when the ZUPT lasts for a long time, the biases of the three-axis gyroscope and the biases of the vertical accelerometer can be estimated, and the bias errors of the MEMS-IMU can be reduced, which is beneficial to the pure inertial navigation when the vehicle is moving.

As described above, the horizontal attitude angle is the horizontal installation angle of the IMU frame relative to the vehicle frame. Through the horizontal installation angle, the IMU output can be converted to a new IMU frame (denoted as s_h-frame), as shown in Fig. 2, the plane determined by the x-axis and y-axis of the s_h-frame is parallel to the plane determined by the x-axis and y-axis of the vehicle frame, and the difference between the s_h-frame and the vehicle frame is only one heading installation angle ψ, as shown in Fig. 3. Thus, the heading angle in Eq. (1) can be set to 0, and after substituting the horizontal attitude angle and transposing, the transformation matrix from s-frame to s_h-frame is obtained as

$$C_s^{s_h} = \begin{bmatrix} c\gamma & 0 & s\gamma \\ s\gamma s\theta & c\theta & -c\gamma s\theta \\ -s\gamma c\theta & s\theta & c\gamma c\theta \end{bmatrix} \quad (7)$$

The IMU output is transferred to the s_h-frame by $C_s^{s_h}$, and then the navigation calculation is carried out in the s_h-frame.

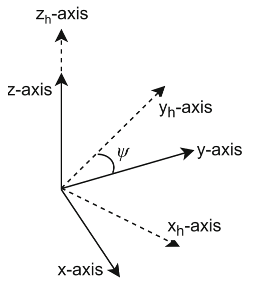

Fig. 3. Relative relationship between Leveling IMU frame (x_h-y_h-z_h) and vehicle frame (x-y-z)

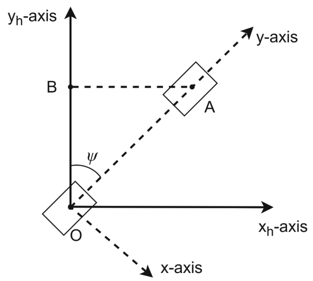

Fig. 4. Estimate principle of heading installation angle

2.3 Estimate of the Heading Installation Angle

After completing the estimate of the horizontal installation angle, the vehicle starts moving in a straight line from standstill, and the forward acceleration will be sensitive by the horizontal x_h-axis and y_h-axis accelerometers in the s_h-frame. Theoretically, the heading installation angle ψ can be calculated by the output of the horizontal x_h-axis and y_h-axis accelerometer. However, when the acceleration is small or the vibration noise is large, the accuracy of estimate is not guaranteed. In addition, low-cost MEMS inertial devices usually have large errors, which have a greater impact on the estimate of the heading installation angle. To meet the needs of most application scenarios, according to the short-term high-precision characteristics of inertial navigation in the virtual navigation frame (n-frame), the displacement information calculated by inertial navigation is used to estimate the heading installation angle, as shown in Fig. 4. At first, the vehicle is stationary at point O, and then the vehicle begins to move forward in a straight line. After moving to point A, the displacement components BA and OB calculated in the virtual navigation frame are used to estimate the heading installation angle. The displacement component BA and OB is defined respectively as P_E and P_N. Then the heading installation angle from the $\mathrm{s_h}$-frame to the vehicle frame (b-frame) is calculated as

$$\psi = \begin{cases} -\tan^{-1}(P_E/P_N), P_N > 0 \\ -\tan^{-1}(P_E/P_N) - \pi, P_E \geq 0, P_N < 0 \\ -\tan^{-1}(P_E/P_N) + \pi, P_E < 0, P_N < 0 \\ -\pi/2, P_E > 0, P_N = 0 \\ +\pi/2, P_E < 0, P_N = 0 \end{cases} \tag{8}$$

The transformation matrix from $\mathrm{s_h}$-frame to the b-frame is obtained as

$$C_{\mathrm{s_h}}^{\mathrm{b}} = \begin{bmatrix} \mathrm{c}\psi & \mathrm{s}\psi & 0 \\ -\mathrm{s}\psi & \mathrm{c}\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

According to Eq. (7) and Eq. (9), the transformation matrix from the s-frame to the b-frame can be obtained as

$$C_{\mathrm{s}}^{\mathrm{b}} = C_{\mathrm{s_h}}^{\mathrm{b}} C_{\mathrm{s}}^{\mathrm{s_h}} = \begin{bmatrix} \mathrm{c}\psi\mathrm{c}\gamma + \mathrm{s}\psi\mathrm{s}\gamma\mathrm{s}\theta & \mathrm{s}\psi\mathrm{c}\theta & \mathrm{c}\psi\mathrm{s}\gamma - \mathrm{s}\psi\mathrm{c}\gamma\mathrm{s}\theta \\ \mathrm{c}\psi\mathrm{s}\gamma\mathrm{s}\theta - \mathrm{s}\psi\mathrm{c}\gamma & \mathrm{c}\psi\mathrm{c}\theta & -\mathrm{s}\psi\mathrm{s}\gamma - \mathrm{c}\psi\mathrm{c}\gamma\mathrm{s}\theta \\ -\mathrm{s}\gamma\mathrm{c}\theta & \mathrm{s}\theta & \mathrm{c}\gamma\mathrm{c}\theta \end{bmatrix} \tag{10}$$

The installation angle calculated above still has a certain error, which is generally a small angle error. According to Eq. (10), the original IMU output is converted to the b-frame for navigation solution. Then NHC or odometer can be used for the integrated navigation system. At the same time, the linear error model can be used to estimate the remaining installation angle errors [8, 9].

3 Error Analysis

Firstly, the error of horizontal installation angle is estimated in ZUPT mode. As the ground is assumed to be horizontal, the error caused by uneven ground is not considered. In ZUPT mode, the estimate accuracy of the horizontal installation angle is mainly affected by the bias of the horizontal accelerometer. Generally, the bias of low-cost MEMS accelerometers is about 10 ~ 70 mg, and the estimate error of the horizontal installation angle is about 4°according to the maximum 70 mg. This error satisfies the assumption of small angle error.

The estimate accuracy of the heading installation angle is mainly affected by the displacement error. As stated in the reference [11], it is known that the initial alignment error, gyro output error, and accelerometer output error will affect the position accuracy of SINS. Since the estimate process is relatively short, it can generally be completed within 10 s, so only the gyro constant drift and the accelerometer constant bias can be considered. In the ZUPT mode, if the time is sufficient, the constant drift of the three-axis gyro and the constant bias of the vertical accelerometer can generally be estimated well.

Therefore, for the short time estimate process, only the initial horizontal attitude error and the constant bias of the horizontal accelerometer should be considered.

As stated in the reference [16], the position error is generated by the accumulation of velocity error over time, so the velocity error can be directly analyzed. In addition, considering the low precision of low-cost MEMS-IMU, the short duration of the estimate process, and the small velocity as generally less than 2 m/s, the errors related to the earth rotation rate and the traveling velocity in the velocity error model of SINS can be ignored, so the simplified velocity error model is expressed as $\delta\dot{V}_E = -\phi_N f_U + \nabla_E$ and $\delta\dot{V}_N = \phi_E f_U + \nabla_N$. According to the reference [11], the error of horizontal attitude angle in ZUPT mode is expressed as $\phi_E = -\nabla_N/f_U$ and $\phi_N = \nabla_E/f_U$. Substituting into the velocity error model, the velocity error model is obtained as $\delta\dot{V}_E = 0$ and $\delta\dot{V}_N = 0$.

It can be seen that the influence of the horizontal attitude error and the constant bias of the horizontal accelerometer can be ignored in the short straight-line driving. Based on the above error analysis, it can be seen that the influence of horizontal attitude error, the gyro constant drift and the accelerometer constant bias on the estimate accuracy of heading installation angle can be ignored.

4 Field Tests

To evaluate the estimate accuracy of the proposed method, the field test is carried out by using the Qianxun magic cube of Beidou integrated navigation system. The main technical specifications of MEMS-IMU in the test equipment are shown in Table 1.

Table 1. Main specifications of MEMS-IMU

Angular rate zero-rate level ($\circ$/s)	Rate noise density in high-performance mode ($\circ/\sqrt{Hz}$)	Linear acceleration zero-g level offset accuracy (mg)	Acceleration noise density in high-performance mode ($\mu g/\sqrt{Hz}$)
±1	5×10^{-3}	±10	60

The number of the tested Qianxun magic cube is 4 in the field test. The test route is shown in Fig. 5 and the test equipment is shown in Fig. 6. The initial installation angle is estimated by the method proposed in this paper. After the estimate of the initial installation angle is completed, the navigation mode is changed to normal mode. The method in the reference [9] is applied to estimate the residual installation angle. As it is difficult to directly measure the real installation angle of the test setup, the estimated value of the residual installation angle can be used as the estimate error of the initial installation angle. In addition, it can be seen from the references [8] and [9] that the residual roll installation angle cannot be estimated, so only the estimate errors of the pitch installation angle and heading installation angle are listed in the test results, as shown in Table 2.

Fig. 5. Field test route

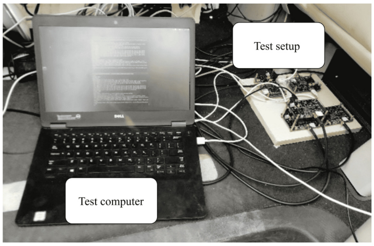

Fig. 6. Field test equipment

Table 2. Results of field test

Number	Estimate error of pitch installation angle (°)	Estimate error of heading installation angle (°)	True heading installation angle (°)	Static duration (s)	Maneuvering duration (s)	Vehicle velocity at completion of installation angle estimate (m/s)
1	0.81	0.07	2.54	42	2	0.98
2	0.43	0.33	−88.38	42	2	0.98
3	0.56	0.27	−88.25	42	2	0.98
4	1.55	0.10	2.90	42	2	0.98

From the above test results, it can be seen that the pitch installation angle error estimated by the proposed method is within 2° and the heading installation angle error is within 0.5° which can ensure that the estimated installation angle error is small. In addition, the whole estimate process takes a short time, the static duration is 42 s, the maneuvering duration is only 2 s, and the requirement of the vehicle maneuvering velocity is not very high.

In order to verify the influence of IMU bias error on the estimate accuracy of the installation angle, different gyro constant drifts and accelerometer constant biases are added to IMU data of the No. 3 test setup. The test results are shown in Table 3.

The test results in Table 3 fully show that the gyro constant drift has little influence on the pitch installation angle and heading installation angle, while the accelerometer constant bias has great influence on the pitch installation angle but has little influence on the heading installation angle. The above results basically verify the conclusion of error analysis in Sect. 3.

In summary, the experimental results verify that the proposed method has the characteristics of stable estimate accuracy and short time-consuming. It does not need to have higher requirement for vehicle maneuvering velocity. It only needs to be static for a period of time and then drive in a straight line to complete the estimate of the initial installation angle. As the practical operation is convenient, it is very easy to be popularized.

Table 3. Test results of increasing IMU bias error

Increased gyro constant drift (°/s)	Increased accelerometer constant bias (mg)	Estimate error of pitch installation angle (°)	Estimate error of heading installation angle (°)	Static duration (s)	Maneuvering duration (s)	Vehicle speed at completion of installation angle estimate (m/s)
0.2	0	0.53	0.07	42	2	0.98
0.4	0	0.51	0.14	42	2	0.98
0.6	0	0.49	0.36	42	2	0.98
0	20	0.62	0.24	42	2	0.98
0	40	1.71	0.29	42	2	0.98
0	60	2.81	0.37	42	2	0.98

5 Conclusion

Since MEMS-INS/GNSS integrated navigation system may be installed arbitrarily in vehicle environment, a convenient and fast estimate method of the initial installation angle is proposed. The estimated installation angle is used to convert the IMU frame, so that the converted IMU frame approximately coincides with the vehicle frame, which can facilitate the use of odometer or NHC. This method uses the navigation information calculated in virtual navigation coordinate system to estimate the installation angle. It only needs a short static and short-term straight-line driving to complete the initial installation angle estimate. It has the characteristics of simple operation, short time consumption, stable estimate accuracy and so on. As the displacement information of navigation solution is used to estimate the heading installation angle, this method does not have high requirements for dynamic maneuvering and vibration environment. Therefore, this method is applicable to civil passenger cars, small-unmanned vehicles or agricultural vehicles, which has obvious engineering significance. Finally, the effectiveness of the proposed method is verified by the field test.

References

1. El-Sheimy, N., Youssef, A.: Inertial sensors technologies for navigation applications: state of the art and future trends. Satell. Navig. **1**(1), 1–21 (2020)

2. Niu, X., Nassar, S., El-Sheimy, N.: An accurate land-vehicle MEMS IMU/GPS navigation system using 3D auxiliary velocity updates. Navig. **54**(3), 177–188 (2007)
3. Hong, S., Lee, M.H., Kwon, S.H., Chun, H.H.: A car test for the estimation of GPS/INS alignment errors. IEEE Trans. Intell. Transp. Syst. **5**(3), 208–218 (2004)
4. Noureldin, A., Karamat, T.B., Eberts, M.D., El-Shafie, A.: Performance enhancement of MEMS-based INS/GPS integration for low-cost navigation applications. IEEE Trans. Veh. Technol. **58**(3), 1077–1096 (2008)
5. Zhang, Q., Li, S., Xu, Z., Niu, X.: Velocity-based optimization-based alignment (VBOBA) of low-end MEMS IMU/GNSS for low dynamic applications. IEEE Sens. J. **20**(10), 5527–5539 (2020)
6. Abdel-Hafez, M.F., Saadeddin, K., Jarrah, M.A.: Constrained low-cost GPS/INS filter with encoder bias estimation for ground vehicles' applications. Mech. Syst. Signal Process. **58**, 285–297 (2015)
7. Dissanayake, G., Sukkarieh, S., Nebot, E., Durrant-Whyte, H.: The aiding of a low-cost strapdown inertial measurement unit using vehicle model constraints for land vehicle applications. IEEE Trans. Robot. Autom. **17**(5), 731–747 (2001)
8. Chen, Q., Zhang, Q., Niu, X.: Estimate the pitch and heading mounting angles of the IMU for land vehicular GNSS/INS integrated system. IEEE Trans. Intell. Transp. Syst. (2020). https://doi.org/10.1109/TITS.2020.2993052
9. Zhao, H., Miao, L., Shen, J.: High accuracy algorithm for SINS/odometer integrated navigation system. Acta Armamentarii **35**(4), 433–440 (2014)
10. Yan, G.: Research on vehicle position and azimuth determining system. Doctoral Dissertation, Northwestern Polytechnic University, Xi'an, China (2006)
11. Fu, Q.: Key technologies for vehicular positioning and orientation system. Doctoral Dissertation, Northwestern Polytechnical University, Xi'an, China (2015)
12. Vinande, E., Axelrad, P., Akos, D.: Mounting-angle estimation for personal navigation devices. IEEE Trans. Veh. Technol. **59**(3), 1129–1138 (2009)
13. Huttner, F., Kalkkuhl, J., Reger, J.: Offset and misalignment estimation for the online calibration of an MEMS-IMU using FIR-filter modulating functions. In: 2018 IEEE Conference on Control Technology and Applications (CCTA), pp. 1427–1433 (2018)
14. Liu, C.Y.:The performance evaluation of a real-time low-cost MEMS INS/GPS integrated navigator with aiding from ZUPT/ZIHR and non-holonomic constraint for land applications. In: Proceedings of the 25th International Technical Meeting of the Satellite Division of the Institute of Navigation (ION GNSS 2012), pp. 1500–1520 (2012)
15. Qin, Y., Zhang, H., Wang, S.: Kalman filtering and integrated navigation principle, pp. 312–314. Northwestern Polytechnical University Press, Xi'an (2004)
16. Qin, Y.: Inertial Navigation (2nd ed.), pp. 334–336. Science Press, Beijing (2014)