The Dispersion Effect of Pseudo-noise Ranging and Time Delay Measurement for Ka Inter-satellite Link

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Abstract. BD-3 navigation satellite system uses the Ka Inter-satellite Link (ISL) technology. The accuracy of the distances between satellites is one of important parameters for autonomous navigation, which depends on the accuracy of the time delay of devices at both ends of ISL. However, because of the dispersion effect of RF channel and antenna, it is difficult to accurately give the time delay by traditional group delay measurement. In this paper, the Phase Spectrum Integration (PSI) method for pseudorange delay testing adapted for dispersive network is given. Then, taking the Ka reflector ISL antenna as an example, the pseudorange delay of this antenna is obtained by the PSI and other methods. The result shows the PSI method improves the accuracy of pseudorange delay measurement and meets the requirements of the in orbit application directly for ISL ranging.

Keywords: Inter-satellite link · ISL · Pseudo-noise ranging · Dispersion effect · Transmission function · Group delay · Pseudorange delay · Phase Spectrum Integration · PSI

1 Introduction

In the realization of constellation autonomous navigation of BD-3 global navigation satellite system, one of the key points is to use Ka band Inter-satellite Link (ISL) to measure inter satellite distance [1–7], then the clock error and the pseudorange of the phase center can be solved [1, 2, 5, 7] regarding the time delay of devices (antenna, RF channel, etc.) at both ends of ISL as constants in a short time (such as 3 days [6]), which could become the main error source in ISL range.

In order to improve the accuracy of ISL ranging, the popular method is the satellite and ground joint solution method [3–6, 17]. TANG Cheng Pan et al. (2017) [6] of Shanghai Astronomical Observatory (SHAO) analyzed the in orbit data of BD-3 satellite, and the delay of satellite end calculated is better than 0.4 ns within 4 months, but the variation of delay of ground anchor station is up to 4.13 ns. RUAN Rengui et al. (2020) [4, 5, 17] of Xi'an Institute of Surveying and Mapping gave the stability of the delay calculated is more than 0.5 ns when 8 BD-3 satellites, 2 anchor stations and 7 domestic iGMAS monitoring stations included. Therefore, although the satellite and ground joint solution of ISL ranging can solve the delay of each end of the ISL as an unknown value,

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it needs an accurate delay (satellite end or anchor station end) as a benchmark, so it cannot avoid the problem of determining the delay of devices accurately.

The delay of devices are generally considered as group delay and measured by vector network analyzer [8, 9]. However, there are many discussions that group delay cannot represent the delay of Pseudo-Noise (PN) code ranging because of the dispersion effect of the RF devices [12–14]. A solution is to calculate the pseudorange delay by numerical processing. Among them, Mike Brookes of Imperial College of Technology (2006) [14] proposed several calculation methods for group delay in speech signal processing, including DC component, average (AV), energy weighted (EW) and so on, but they are all approximate methods. ZHU Xiangwei et al. (2008) [12, 13] proposed Taylor expansion for group delay curve and took its zero order expansion term as pseudorange delay but the error is about 0.3 ns. TANG Dezhi et al. (2019) [11] of Chery Automobile Co., Ltd. analyzed the influence of dispersive network on time-domain pulse signal according to the transmission function theory, but the approximation of transmission function of this method is not suitable for the analysis of PN code signal.

Based on the research of the transmission function of dispersive networks, this paper proposes the Phase Spectrum Integration (PSI) method for pseudorange delay determination. And then, taking the Ka reflector ISL antenna of BD-3 M1S satellite as an example, the pseudorange delay is calculated by PSI method. The results show its accuracy is better than EW and AV methods mentioned above.

2 Pseudorange Delay of Dispersive Networks

2.1 Group Delay and Pseudorange Delay

In the propagation theory of electromagnetic waves, the concepts of phase velocity and group velocity are widely used. Phase velocity v_p is defined as the moving velocity of phase plane. Group velocity v_g is defined as the moving velocity of equal amplitude points on the time domain waveform which occupies a certain bandwidth. It is well known that the group velocity is related to a specific frequency. Accordingly, the time of electromagnetic wave signal passing through a dual port network is group delay. When the signal bandwidth and dispersion effect increase, the significance of group velocity and group delay will decrease.

For PN code ranging signal, the delay should be defined as pseudorange delay. The general PN code ranging signal is direct sequence spread spectrum (DSSS) signal in baseband which meets the assumption of ideal code sequence correlation [15]:

$$s(t) = \sum_{n=-\infty}^{+\infty} (-1)^{c_n} p(t - nT_c) \tag{1}$$

Where c_n is a PN code with bipolar definition, $p(t)$ is the time domain waveform of a single chip, T_c is the time width of the chip, and the code rate is $f_c = 1/T_c$. Generally, the correlation peak method was performed for pseudorange time delay determination.

2.2 Transmission Function of Networks

Because of the dispersion effects of microwave networks are universal, it is necessary to research their delay characteristics. Now consider a general lossless dual port network as shown in Fig. 1. The input and output voltage are U_1, U_2, and the characteristic impedance is Z_0, and the transfer matrix (ABCD) and scattering matrix [s] describe the characteristics of the network.

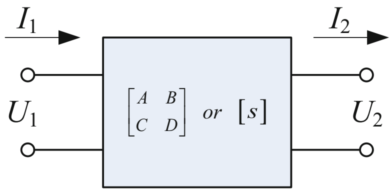

Fig. 1. The principle of dual-port networks

According to the theory of microwave network, the transmission function $H(\omega)$ of the network is:

$$H(\omega) = \frac{U_2}{U_1} = \frac{1}{A + B/Z_0} = \frac{s_{21}}{1 + s_{11}} \tag{2}$$

When the input port matches, $s_{11} = 0$, and $|s_{21}| = 1$, and we have $H(\omega) = \exp(\mathrm{j}\phi(\omega))$ (where $\phi(\omega)$ is the phase of s_{21}). Generally, the group delay is determined by $\tau_g = \mathrm{d}\phi(\omega)/\mathrm{d}\omega$, so the transmission function and group delay are related.

2.3 Frequency Conversion on Transmission Function

Because the dispersive network is located in the RF channel and the PN cross-correlation operation is located in the baseband, the transmission function of the dispersive network cannot be directly used to analyze the pseudorange delay. Let $P(\omega)$ is the spectrum of the signal $p(t)$, and let the ω is angular frequency for either baseband or RF, and let ω_0 is the angular frequency of RF carrier.

We define the frequency-domain process of up conversion, down conversion and transmission as functions UP(), DW() and TR(), and the corresponding time-domain process as functions up(), dw() and tr(), shown as follows:

$$\begin{aligned}
\mathrm{DW}(P(\omega)) &= P(\omega + \omega_0) \\
\mathrm{UP}(P(\omega)) &= P(\omega - \omega_0) \\
\mathrm{TR}(P(\omega)) &= P(\omega)e^{\mathrm{j}\phi(\omega)} \\
\mathrm{dw}(p(t)) &= p(t)e^{-\mathrm{j}\omega_0 t} \\
\mathrm{up}(p(t)) &= p(t)e^{\mathrm{j}\omega_0 t} \\
\mathrm{tr}(p(t)) &= \mathbb{F}^{-1}(\mathrm{TR}(P(\omega)))
\end{aligned} \tag{3}$$

Where $\mathbb{F}^{-1}(\cdot)$ is the inverse Fourier Transform. Therefore, the process of a single chip signal $p(t)$ going through up conversion, through dispersion network, and then down conversion to dispersive chip signal $p'(t)$ in time domain can be described as $p'(t) = \mathrm{dw}(\mathrm{tr}(\mathrm{up}(p(t))))$. After a series of deduction, we can get that:

$$\mathrm{dw}(\mathrm{tr}(\mathrm{up}(p(t)))) = \mathbb{F}^{-1}\left(P(\omega - \omega_0)e^{\mathrm{j}\phi(\omega)}\right)e^{-\mathrm{j}\omega_0 t} \tag{4}$$

Let $\omega' = \omega - \omega_0$, the equation above yields:

$$p'(t) = \mathbb{F}^{-1}\left(P\left(\omega'\right)e^{\mathrm{j}\phi(\omega'+\omega_0)}\right) \tag{5}$$

Therefore, if ω is the frequency of the baseband and $\phi(\omega)$ is the transmission function shifted to the baseband, then the spectrum directly acting on the baseband time domain waveform is:

$$P'(\omega) = P(\omega)e^{\mathrm{j}\phi(\omega)} \tag{6}$$

This conclusion has important guiding significance for studying the influence of dispersive network on pseudorange delay.

2.4 The Numerical Calculation Methods of Pseudorange Delay

On the RF channel testing conditions it is difficult to perform the correlation peak method for pseudorange delay determination, for the RF testing bandwidth BW is so narrow for sufficient time resolution $\Delta t = 1/BW$.

According to the theory of integral transformation, the cross-correlation curve $R'(t)$ is obtained by time-domain waveform $p(t)$ and $p'(t)$, which can be converted into the following operation of each spectrum, namely:

$$R'(t) = \mathbb{F}^{-1}\left(P(\omega)P'^{*}(\omega)\right) \tag{7}$$

Where * is conjugate operator. It is assumed that the receiver takes the peak value of the autocorrelation curve as the time reference, and the self-correlation curve was as follows:

$$R(t) = \mathbb{F}^{-1}\left(|P(\omega)|^2\right) \tag{8}$$

Assuming that two curves have only time difference it could be derived by the Law of Delay that:

$$R'(t) = R(t - \tau_0) = \mathbb{F}^{-1}\left(|P(\omega)|^2 e^{-\mathrm{j}\omega\tau_0}\right) \tag{9}$$

Where τ_0 is the pseudorange delay to be determined. Furthermore, according to Eq. (6) and Eq. (7), the ranging cross-correlation curve at the receiving end of the transceiver is as follows:

$$R'(t) = \mathbb{F}^{-1}\left(|P(\omega)|^2 e^{\mathrm{j}\phi(\omega)}\right) \tag{10}$$

It is obtained by Eq. (9):

$$\mathbb{F}^{-1}\left(|P(\omega)|^2 e^{\mathrm{j}\phi(\omega)}\right) \approx \mathbb{F}^{-1}\left(|P(\omega)|^2 e^{-\mathrm{j}\omega\tau_0}\right) \tag{11}$$

And it is equivalents to:

$$\int |P(\omega)|^2 e^{\mathrm{j}\phi(\omega)} \mathrm{d}\omega = \int |P(\omega)|^2 e^{-\mathrm{j}\omega\tau_0} \mathrm{d}\omega \tag{12}$$

Let:

$$C = \int |P(\omega)|^2 e^{\mathrm{j}\phi(\omega)} \mathrm{d}\omega \tag{13}$$

Therefore, the problem of pseudorange delay is transformed into finding $\tau = \tau_0$ to make the formula hold:

$$g(\tau) = \left| \int |P(\omega)|^2 e^{-\mathrm{j}\omega\tau} \mathrm{d}\omega - C \right| = 0 \tag{14}$$

The above equation can be further transformed into summation form and solved by numerical method. Because of the use of the phase frequency characteristics of the transmission function and the signal spectrum, this method is called Phase Spectrum Integration (PSI) in this paper.

It is worth noting that the EW method [14] can also be obtained from PSI method. By transforming Eq. (12) into summation and took the derivative of this with respect to ω from both sides, it is concluded that:

$$\begin{aligned} &\sum \left(2|P(\omega)||P(\omega)|' e^{\mathrm{j}\phi(\omega)} - \mathrm{j}|P(\omega)|^2 e^{\mathrm{j}\phi(\omega)} \tau(\omega)\right) \\ &= \sum \left(2|P(\omega)||P(\omega)|' e^{-\mathrm{j}\omega\tau_0} - \mathrm{j}|P(\omega)|^2 e^{-\mathrm{j}\omega\tau_0} \tau_0\right) \end{aligned} \tag{15}$$

Let all of the $\exp(\mathrm{j}\phi(\omega))$ in the left side of the above equation approximate to $\exp(-\mathrm{j}\omega\tau_0)$, it is obtained so called EW method that:

$$\tau_0 \approx \frac{\sum |P(\omega)|^2 \tau(\omega)}{\sum |P(\omega)|^2} \tag{16}$$

So, it can be seen that the EW method is an approximate method of PSI method, and is only suitable for the case of small dispersion effect. The AV method [14] could be obtained from Eq. (16) by further approximating $|P(\omega)| \approx 1$. Therefore, PSI method has better accuracy than EW and AV method.

3 ISL Antenna Pseudorange Delay Measurement

An antenna with single feed port is generally regarded as a single port network, but can also be equivalent to a dual port network considering its phase center [10, 16] as the 2$^{\mathrm{nd}}$ port.

This paper takes the Ka reflector ISL antenna of BD-3 M1S satellite as an example to illustrate the measurement of pseudorange delay by PSI method. The code rate of Ka reflector ISL is $f_c = 10.23$ Mcps, and the center frequency of the Ka reflector ISL antenna is f_0 and $BW = 100$ MHz (~$\pm 5 f_c$).

3.1 ISL Antenna Group Delay Measurement

The aperture of the Ka reflector ISL antenna is $\Phi 0.4$ m, and the far-field distance is $2D^2/\lambda \approx 27$ m. Therefore, this paper adopts a group delay measurement method [16]. It will get the phase frequency $\phi(f)$ (unit: degree). The group delay can be obtained by differential processing as follows:

$$\tau_g(f) = -\frac{1}{360}\frac{\Delta\phi(f)}{\Delta f} \tag{17}$$

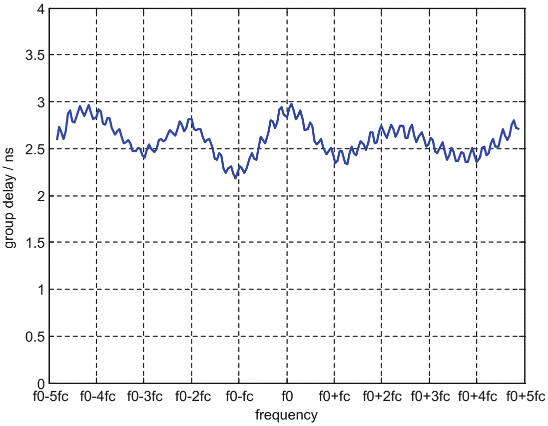

Fig. 2. The group delay of the Ka reflector ISL antenna

After processing the measured data, the group delay curve of the antenna can be obtained as shown in Fig. 2. It can be seen that within the bandwidth of $\pm 5\, f_c$, the fluctuation range of group delay is about 0.8 ns, and shows obvious dispersion effect. Therefore, group delay is not suitable for pseudorange delay of ISL ranging, and must be converted into pseudorange delay by post processing.

3.2 Determination of Pseudorange Delay by PSI Method

Because the test bandwidth is 100 MHz and the corresponding time-domain resolution is only 10 ns, the accuracy of pseudorange delay is difficult to be less than 0.1 ns when used in correlation peak analysis method. If the pseudorange delay is directly calculated according to the far-field phase frequency curve $\phi(f)$ or group delay $\tau_g(f)$ obtained in Sect. 3.1, there are three numerical methods, one is the PSI method proposed in this paper, the other two are EW and AV method, and different to the correlation peak analysis method, these three methods do not need to calculate the time-domain waveform, so the requirement of spectrum bandwidth is greatly reduced. The spectrum of a chip signal shows in Fig. 3.

After calculating the pseudorange delay in different bandwidth, the following data is obtained as Table 1.

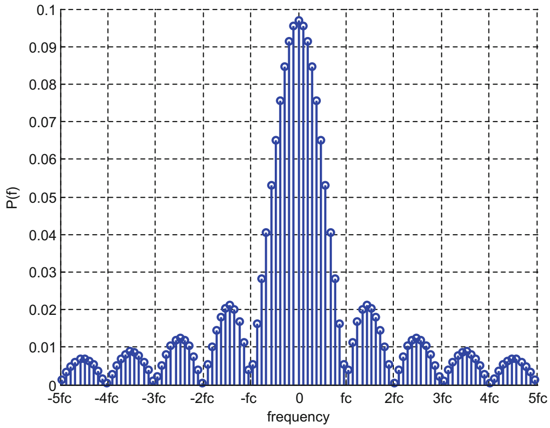

Fig. 3. The spectrum of a chip of PN code

Table 1. The calculated pseudorange delay

BW	PSI (this paper)	EW [14]	AV [14]
$\pm f_c$	2.82	2.77	2.60
$\pm 2f_c$	2.69	2.75	2.56
$\pm 3f_c$	2.65	2.75	2.59
$\pm 4f_c$	2.64	2.75	2.58
$\pm 5f_c$	2.64	2.75	2.60

It can be seen that the PSI method converges to the result of 2.64 ns within $\pm 5f_c$, while the result of EW method is more than 0.11 ns bigger, and the result of AV method fluctuates and the result is 0.04 ns smaller. According to the conclusion in Sect. 2.4, the EW method is the approximation of the PSI method under the condition of low dispersion, and the AV method is the further approximation of the EW method under the condition of time-domain impulse waveform. Therefore, the results of the PSI method have obvious better accuracy, and eliminate the ambiguity of about 0.15 ns caused by using the EW method and the AV method, so that the measurement accuracy of the pseudorange delay is improved. However, the existing third party calibration methods are not enough to verify such high accuracy results. The error of PSI method verified by current third party calibration method is no more than 0.25 ns.

In principle, PSI method is more accurate than EW and AV method. EW and AV method are different degrees of approximation of PSI method. Therefore, this paper takes PSI method as the recommended pseudorange measurement method for dispersive networks.

4 Conclusion

In this paper, a general PSI method for calculating pseudorange delay by transmission function and signal spectrum is given. Through the work of this paper, we can draw the following conclusions:

1) The pseudorange delay of dispersive network cannot be simply considered as the group delay of the center frequency point or the median value of the fluctuation range of the group delay, but should consider the influence of the transmission function of the dispersive network;
2) The influence of the transmission function (mainly the phase-frequency function) of the dispersive network working in the carrier RF band on the baseband signal can be equivalent to that the transmission function in the RF domain acts directly on the baseband signal after it is shifted to the baseband;
3) The correlation peak analysis method needs to calculate the time domain waveform. Because of the narrow RF bandwidth and low resolution in time domain when RF measure data used, it is difficult to determine the pseudorange delay accurately;
4) The PSI method proposed in this paper has better accuracy than that of the EW and AV methods.

Especially for the ISL antennas with dispersion effect, using PSI method to determine its pseudorange delay can improve the accuracy of pseudorange delay, and meet the requirements of constellation ISL ranging in orbit. The PSI method can also be used to determine the pseudorange delay of the ground station antenna with dispersion effect.

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