Improved BDS RAIM Algorithm Based on M-Estimation

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Abstract. Aiming at the robustness for M-estimation, the improved RAIM algorithm based on M-estimation is proposed. The algorithm introduces the estimated value of the weighted least absolute value algorithm as the improved initial value of M-estimation, constructs an equivalent weighting matrix, improves its robustness, restrains the influence of outliers between fault detection and identification rate, and improves the availability of RAIM. And BDS raw data from IGS are used to compare the improved M-estimation algorithm with the M-estimation algorithm and the least squares algorithm. The results show that the improved M-estimation algorithm can obviously improve the rate both of fault detection and identification. Therefore, the performance of the improved M-estimation algorithm is better than the M-estimation algorithm and the least square algorithm.

Keywords: BeiDou Navigation Satellite System · Receiver Autonomous Integrity Monitoring (RAIM) · M-estimation algorithm · Fault detection

1 Introduction

The Beidou-3 system is networked successfully. It will provide global positioning, navigation, and timing services [1]. The BeiDou Navigation Satellite System will become important in the future. Based on the Beidou RAIM (Receiver Autonomous Integrity Monitoring) algorithm, it will play a more important role in aviation fields [2].

The RAIM is used to monitor the consistency of the residual vector and determine which satellite is most likely to occur fault [3].Typical RAIM algorithms include snapshot RAIM algorithm and filtering RAIM algorithm [4]. The current filtering algorithm is mainly based on the Kalman filter algorithm, and snapshot algorithms are mainly included the range-comparison method, the least squares-residuals method and the parity method [5]. The parity method is used to the projection of the error in the vector to construct the test statistics [6]. The least squares residual method is based on the pseudo-range residual and false alarm rate to construct test statistics and detection threshold [7]. The classic RAIM algorithm generally is constructed test statistics by residuals. Due to the test statistics are estimated functions, it will lead to decline fault detection and fault identification rate by outliers in the fault [8]. To solve this problem, some domestic scholars have proposed the M-estimation algorithm [9]. The algorithm uses the LS

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(Least Square) estimation value as the initial value, and process the abnormal value by constructing an equivalent weight matrix to reduce the influence of the outliers on fault detection and fault identification. The M-estimation algorithm uses the LS algorithm to estimate the initial value, and the initial value will affect the M-estimation reliability under the influence of outliers.

Based on M-estimation to improve the initial value algorithm is proposed in this paper. It is proposed that the WLAV (Weighted Least Absolute Value) estimation value is used as the initial value of the improved M-estimation [10], and iterative operations are carried out to construct an equivalent weight matrix. It can improve the robustness of the M-estimation algorithm at the initial value, improve the fault detection and fault identification. The results show the performance of the improve M-estimation in RAIM technology for fault detection and fault identification, and compare with the LS and M-estimation algorithms.

2 Improved RAIM Algorithm for M-Estimation

2.1 RAIM Algorithm Based on Least Square Approach

RAIM algorithm mainly includes two parts: fault detection and fault identification [11]. Firstly, judging whether the number of visible satellites reaches 4 and reaches 4 satellites for positioning; then, when the number of visible satellites reaches more than 5, performing fault detection to determine whether the test statistics exceed the test threshold. If it exceeds the test threshold, it will be proved that the satellite is fault satellite in this epoch. Therefore, it is necessary to identify the fault satellite under this epoch, and eliminate the fault satellite [12, 13].

2.2 Basic Principles of M-Estimation

M-estimation has robust function. When outliers are unavoidable, it can be reduced the influence of outliers on the final result by choosing appropriate weights. The greater weight is pointed with small distance residuals, and the smaller weight is pointed with larger distance residuals. And the solution closest to the normal value can be solved. Therefore, the mathematical expression of M-estimation can be expressed as follows.

$$\sum_{i=1}^{n} \rho(P_i^{\frac{1}{2}}(\boldsymbol{H_i}\hat{\boldsymbol{X}} - \mathbf{Y_i})) = \min \sum_{i=1}^{n} \rho(P_i^{\frac{1}{2}}(\boldsymbol{H_i}\hat{\boldsymbol{X}} - \boldsymbol{Y}_{\mathbf{i}})) \tag{1}$$

Where, $\boldsymbol{H_i}$ is the design matrix, and $\hat{\boldsymbol{X}}$ is the estimated extreme value. M-estimation is essentially an iterative weighted least squares algorithm by the standardized size of the distance residual assigning weights. Therefore, the algorithm expression for converting M-estimation into RAIM can be as follows.

$$\boldsymbol{X = (H^T PH)^{-1} HPY} \tag{2}$$

In Eq. (2), H is the observation matrix vector; Y is the difference between the predicted value and the true value; $\boldsymbol{\varepsilon}$ is the mean square error, and obeys the Gaussian distribution of $(0, \sigma^2)$. P is the equivalent weight matrix, and the influence of outliers is suppressed by selecting an appropriate weight function.

In the selection of the weight function, the analysis of different weight functions based on robust estimation as described in [14] is proposed. It fully analysed that when the data contains gross errors, the Huber weight function can be better eliminated than other weight functions gross error.

The weight function is Eq. (3) by proposing Huber.

$$\rho(u_i) = \begin{cases} 1 & |u_i| \leq c \\ \frac{c}{|u_i|} & |u_i| > c \end{cases} \tag{3}$$

In Eq. (3), c is set up 1.345 [15]. If the standard residual error is greater than 1.345, the weight will gradually become smaller as the residual error increases. If the standard residual error is less than 1.345, the weight will remain within a certain range.

Where, u_i is the standardized residual parameter, and $med()$ is the median. It can be expressed as follows.

$$u_i = \frac{v_i}{s} = \frac{0.6745 \times v_i}{med|v_i - med(v_i)|} \tag{4}$$

The steps of the M-estimation algorithm are as follows:

Steps 1: Calculating the initial value of M-estimation, and using the least squares estimation value as the initial iteration value.

$$\boldsymbol{X_0 = (H^T H)^{-1} H^T Y} \tag{5}$$

Steps 2: Calculating pseudo-range residuals v_i and standardized residuals u_i.

$$u_i = \frac{v_i}{s} = \frac{0.6745 \times v_i}{med|v_i - med(v_i)|} \tag{6}$$

Steps 3: Constructing an equivalent weight moment P and calculating the M-estimation result.

$$\boldsymbol{X = (H^T P H)^{-1} H^T P Y} \tag{7}$$

Steps 4: Making judgment, if $|X_{i+1} - X_i| < \varepsilon$, it will end the loop. Otherwise the loop continues to step 1, the initial value is recalculated and started iterative.

M-estimation is calculated based on the LS estimated value as the initial value. The position of (x, y, z) and the receiver clock bias directly affect the initial position of the next iteration. If the satellite occurs fault, the LS algorithm will be not robust and weaken the robustness of M-estimation in a certain extent. It will cause the initial calculation result to error too far from the real position solution, and causing a certain degree of error in the positioning solution, so reducing the RAIM availability. In order to solve the above problem, the WLAV algorithm as the initial value of improved M-estimation is proposed.

2.3 Improved M-Estimation Algorithm

The objective function estimated by the WLAV algorithm can be written as follows.

$$J(x) = \sum_{i=1}^{m} W_i |z - h_i(x)| = \sum_{i=1}^{m} W_i |v_i| \tag{8}$$

Where, W_i is the i^{th} observation weight coefficient; z is the i^{th} measured value observed distance;h_i is the estimated observation and measurement distance; x is the coefficient to be estimated, and $|v_i|$ is the calculated distance residual.

Relative to the objective function of WLAV algorithm, the expression of WLS (Weighted Least Squares) algorithm can be written as follows.

$$J(x) = \sum_{i=1}^{m} |z - h(x)|^2 W_i = \sum_{i=1}^{m} v_i^2 W_i \tag{9}$$

Therefore, the WLAV algorithm expression can be rewritten in the following equation.

$$J(x) = \sum_{i=1}^{n} \frac{W_i |v_i|^2}{|v_i|} \tag{10}$$

Where

$$W_i^* = \frac{W_i}{|v_i|} \tag{11}$$

So, WLAV algorithm expression can be written as follows.

$$J(x) = \sum_{i=1}^{m} W_i^* v_i^2 \tag{12}$$

Where, W_i^* is the weighting factor, which can be obtained by comparing the weighting function with the pseudo-range residual. And v_i is the pseudo-range residual. The WLAV algorithm estimation value is used as the initial value, and introduced into the M-estimation algorithm. The initial value of the M-estimation algorithm is solved.

$$\boldsymbol{X_0 = (H^T\ W^* H)^{-1} H^T W^* Y} \tag{13}$$

W^* is the weighting factor, the ionosphere, the troposphere, the multipath, receiver thermal noise variance, etc. And it is part of the weighting factor. Therefore, the expression for the variance of observation noise can be written as follows [16].

$$\sigma^2 = \sigma_{URA}^2 + \sigma_{IONO}^2 + \sigma_{MP}^2 + \sigma_{TCVR}^2 + \sigma_{TROPO}^2 \tag{14}$$

σ_{URA}^2 is the clock mean squared error of the satellite and the ephemeris mean squared error of the satellite and is set up 2.4; σ_{IONO}^2 is the ionosphere mean squared error and is Calculated by Klobuchar; σ_{MP}^2 is the multipath mean squared error and is set up $(0.13 + 0.53e^{\frac{-E_i}{10}})^2$, E_i is the satellite elevation; σ_{TCVR}^2 is the thermal noise mean squared error of the receiver and is set 0.001; σ_{TROPO}^2 is the tropospheric mean squared error and is set up $(\frac{0.12\times 1.001}{\sqrt{0.002001+\sin^2(E_i)}})^2$.

Therefore

$$\boldsymbol{W} = \begin{bmatrix} \frac{1}{\sigma_1^2} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_2^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{\sigma_n^2} \end{bmatrix} \tag{15}$$

The weighting factor is established as follows.

$$\boldsymbol{W}^* = \frac{\boldsymbol{W}}{\boldsymbol{v}} = \begin{bmatrix} \frac{1}{\sigma_1^2 v_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_2^2 v_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{\sigma_n^2 v_n} \end{bmatrix} \tag{16}$$

Where, $(v_1, v_2, v_3, \cdots v_n)$ is the pseudo-range residual distance. According to Eq. (13), the initial iterative value of M-estimation is calculated, and the initial value is used as the initial value of M-estimation. The improved M-estimation fault detection and identification is similar to the M-estimation fault detection and identification algorithm. Firstly, the detection statistics need to be constructed.

$$T_{WMS} == \sqrt{\frac{SSE}{n-4}} = \sqrt{\frac{\boldsymbol{v}^T \boldsymbol{P} \boldsymbol{v}}{n-4}} \tag{17}$$

If there is no fault, $\boldsymbol{H_0}$ will be obey $\frac{SSE}{\sigma_0^2} \sim \chi^2(n-4)$.

If there is fault, $\mathbf{H_1}$ will be obey $\frac{SSE}{\sigma_0^2} \sim \chi^2(n-4, \lambda)$.

Where, λ is a decentralized parameter. When the false alarm rate and probability density function are known, the test threshold T_{WMD} is constructed to perform fault detection and fault identification.

If $T_{WMS} > T_{WMD}$, it will be proved that there is a faulty satellite at that moment, and the faulty satellite needs to be identified and excluded.

If $T_{WMS} < T_{WMD}$, it will be proved that there is no fault satellite at that moment.

3 Experimental Verification and Result Analysis

In order to compare and analyze the improved M-estimation algorithm with the M-estimation algorithm and the least squares algorithm. The BDS data is collected on June 6, 2020. The observation station is JFNG. The time starts from 2020.6.6.00:00:00, which lasts 27000 s and 900 simulations epochs are taken. The receiver coordinates are [−2279828.6768, 5004705.5635, 3219777.3655], and the false alarm rate is set up 0.002/h.

In order to verify that the improved M-estimation algorithm can detect satellite fault, we collect 900 simulation epochs. The number of visible satellites and the GDOP (Geometric Dilution of Precision) are shown in Fig. 1.

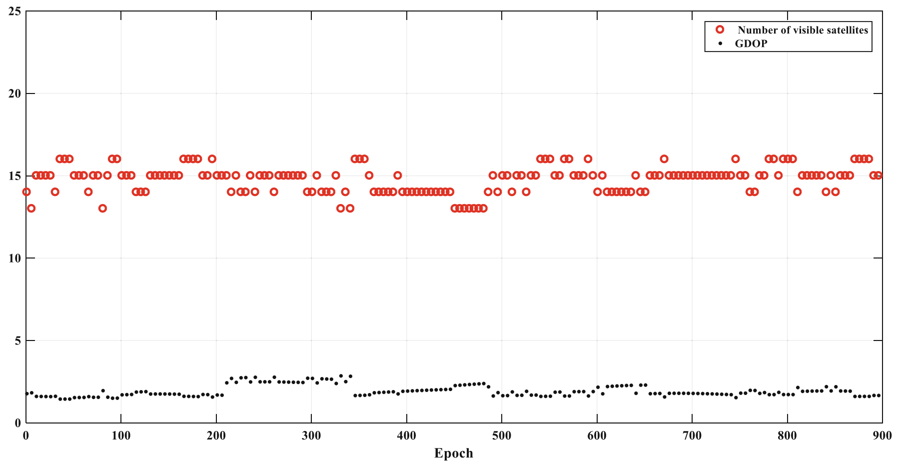

Fig. 1. Number of visible satellites and GDOP

It can be seen from Fig. 1 that the number of visible satellites is relatively stable. The number satellites remain between 13 and 16. The GDOP decreases as the number of visible satellites increasing.

Figure 2 shows the fault detection effects of the three algorithms with no pseudo-range error between 0 and 900 epochs.

In Fig. 2, the three algorithms are all less than the detection threshold with no fault. It can be indicated that the detection effects of the three algorithms are effective with no fault satellites.

Figure 3 is the fault detection results with pseudo-range error on C03 satellite. Adding 2 m/epoch pseudo-range error between 200 epochs and 400 epochs; the step length is 5 epochs. The fault detection results are shown in Fig. 3.

In Fig. 3, the three algorithms are all more than the detection threshold between 200 epochs and 400 epochs. It means that all faults can be detected within these epochs. However, the detection statistics are all lower than the detection threshold in other time periods. It means that there is no fault satellite in this period. In Fig. 2 and Fig. 3, the three detection algorithms are all effective both no fault and fault satellites.

In addition, the M-estimation algorithm has a good impact on the resisting outliers. Therefore, a random satellite is selected and a pseudo-range of 20 m is added between 0 and 900 epochs. The position calculations of the three algorithms are shown in Fig. 4, Fig. 5, and Fig. 6.

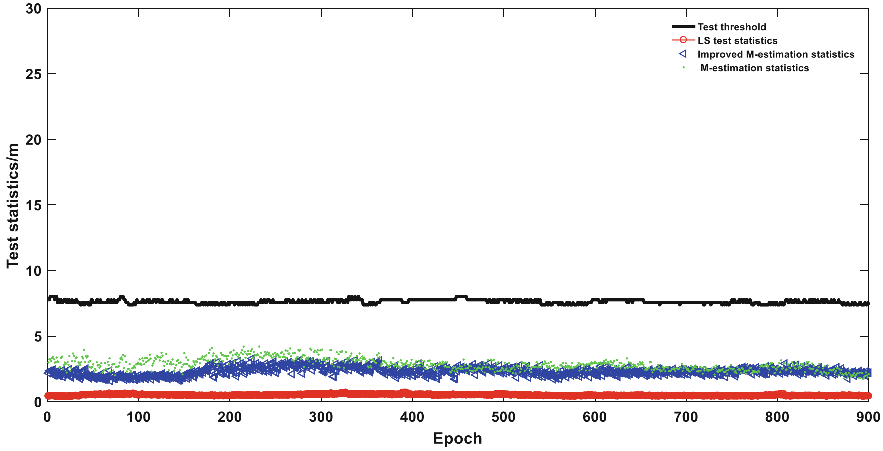

Fig. 2. Detection results with no fault for three algorithms

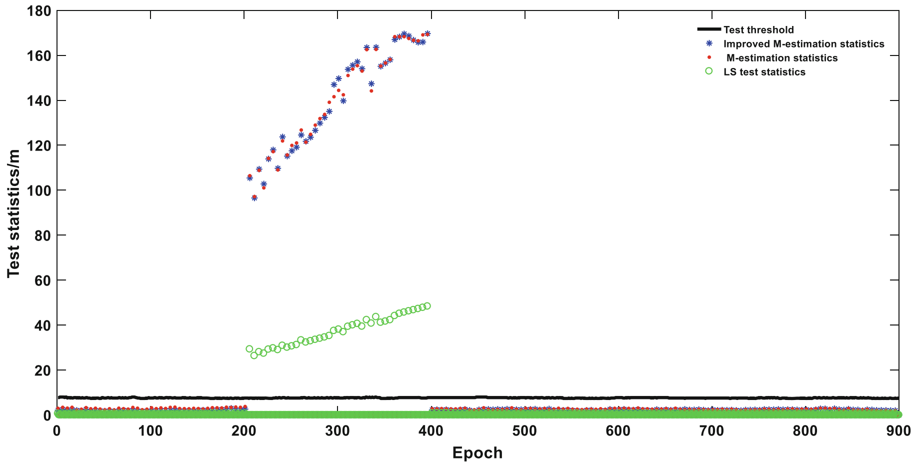

Fig. 3. Detection results with fault for three algorithms

It can be seen from Fig. 4, Fig. 5 and Fig. 6 that when 20 m error is added to the satellite at a certain moment and the position is directly calculated with fault satellites. M-estimation and the improved M-estimation algorithm are good at resisting outliers. It can be seen from the above three figures, the position error value calculation with the pseudo-rang error is better than the position error of the LS algorithm.

In order to test the performance of the improved algorithm in terms of fault detection rate and fault identification rate, a fault error of 1–120 m is added to the satellite C03, the simulation time is 900 epochs. Figure 7 shows the fault detection rate comparison of three algorithms.

It can be seen from Fig. 7 that the improved M-estimation algorithm can quickly detect satellite faults. The improved M-algorithm can detect the faulty satellites at a

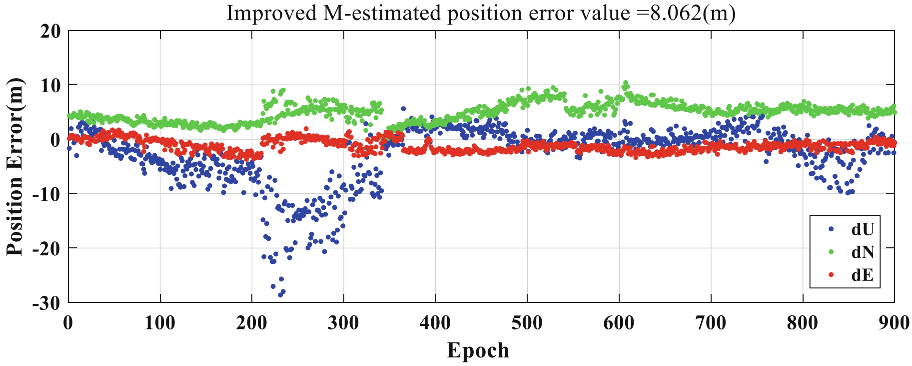

Fig. 4. Position error of the improved M-estimation algorithm

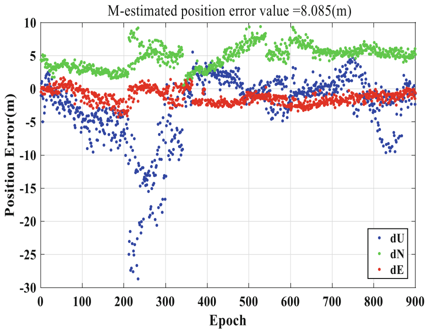

Fig. 5. M-estimation position error

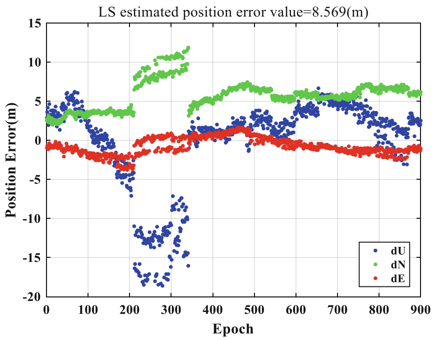

Fig. 6. LS estimation position error

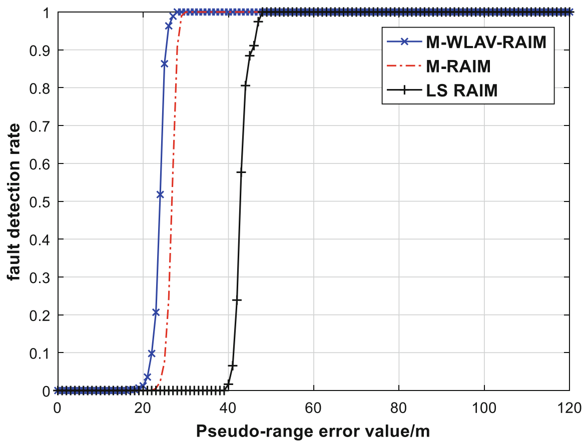

Fig. 7. Fault detection rate comparison of three different algorithms

pseudo-range error of 19 m. Meanwhile, the fault detection rate is 0.00697. The fault detection rate of M-estimation and the LS algorithm are 0, too. The M-estimation algorithm fault detection rate is 0.0056 and the improved M-estimation fault is 0.2067 at 23 m. When the pseudo-range error exceeds 31 m, the fault detection rate of the M-estimation algorithm and the improved M-estimation algorithm reach 100%. When the pseudo-range error exceeds 48 m, the detection rates of the three algorithms all reach 100%. Under the same error, it can be seen that the improved M-estimation detection rate is better than the M-estimation and is far better than the LS algorithm. The improved M-estimation algorithm is more robust in the initial value than the LS algorithm and is less affected by outliers. The initial value is not easy to deviate from the true position, improves the RAIM availability, and improves the fault detection rate of the algorithm.

As Fig. 7 shows that it is necessary to perform fault identification on all satellites when a fault satellite is detected under the epoch. Figure 8 shows the fault identification rate comparison of three algorithms.

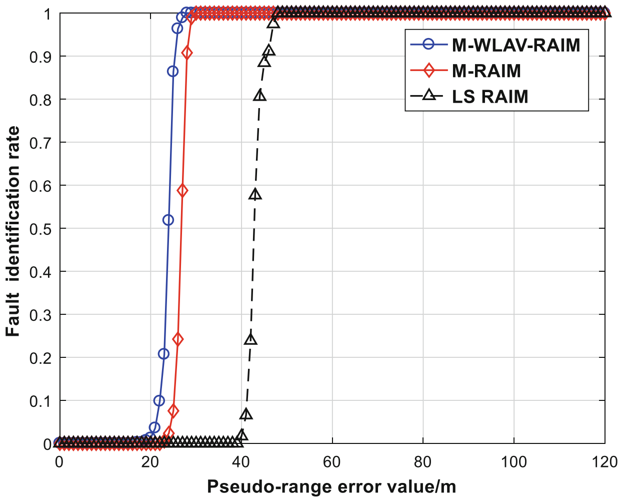

Fig. 8. Fault identification rate comparison of three different algorithms

Figure 8 shows the improved M-estimation algorithm, M-estimation algorithm, and the LS algorithm fault identification rate. It can be seen that the fault identification rate of the improved M-estimation algorithm is bigger than that of the M-estimation, which is much bigger than that of the LS algorithm. Under the pseudo-range error of 19 m, the improved M-estimation fault identification rate is 0.00697, while the M-estimation and the LS algorithm fault identification rate are 0. When the pseudo-range error is 23 m, M-estimation can identify the faulty satellite, the identification rate is 0.0056, the identification rate of the improved M-estimation algorithm is 0.2067, and the identification rate of the least square algorithm is 0. When the pseudo-range error is 26 m, the fault identification rate of the improved M-estimation algorithm is 0.9633, which is much higher than that of the M-estimation algorithm; and the fault identification rate of the least squares algorithm is 0. When the pseudo-range error is 31 m, the M-estimation algorithm is improved, and the fault identification rate of the M-estimation algorithm is 100%. When the pseudo-range error is greater than 48 m, the three detection algorithms can identify the faulty satellite, and the fault identification rate reaches 100%. It can be seen that the improved M-estimation RAIM algorithm is more sensitive to faulty satellites and has better fault identification ability than the M-estimation algorithm and the LS algorithm.

4 Conclusion

In the paper, the improved RAIM algorithm based on M-estimation is proposed. The improved M-estimation algorithm is derived.Through BDS raw data, the algorithm is verified and compared with the LS algorithm and M-estimation algorithm. The results show that the performance of the improved M-estimation algorithm is better than the M-estimation algorithm and the LS algorithm under resisting fault. In addition, the improved M-estimation algorithm can improve the fault detection and identification rate by comparing M-estimation and LS algorithm.

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