

An Analytic and Genetic Algorithm Approach to Optimize Integrated Production-Inventory Model Under Time-Varying Demand



Isha Talati, Poonam Mishra, and Azharuddin Shaikh

Abstract This paper addresses the cost minimization problem of an integrated production-inventory model which has optimized by analytical method and evolutionary algorithm. We have formulated our model for items that deteriorate with respect to time and follow Weibull distribution. For controlling deterioration rate, we have used preservation technology. Further, we assumed that ordering cost is lot size dependent. Classical optimization methods demonstrate a number of difficulties when faced with complex problems. Moreover, most of the classical optimization methods do not have the global perspective and often get converged to a locally optimum solution. Genetic algorithm (GA) is an adaptive heuristic search algorithm based on the evolutionary ideas of natural selection and genetics. In this model, we optimized our model by gradient-based analytical method and GA in integrated as well as independent scenario. Numerical example is carried out. Sensitivity of different inventory parameters is carried out. The results of the proposed model help researchers to think about optimizing their complex problems using different evolutionary search algorithm.

Keywords Integrated inventory · Weibull distribution · Time-dependent demand · Genetic algorithm · Lot size-dependent ordering cost · Preservation technology

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1 Introduction

For survival and growth of business, proper coordination and communication among supply chain players place an important role in this competitive atmosphere. Firstly, Goyal (1976) formulated an integrated model for single supplier and single customers. Banerjee (1986) made an appropriate price adjustment and obtained joint order so that it is beneficial to both parties. Chung and Cárdenas-Barrón (2013) generated model for deteriorating items for stock dependent demand with two-level trade credits. Chung et al. (2014) extended previous model for exponentially deteriorating items. Shah (2015) derived model with two-level trade credits for items deteriorate constantly. Further, Shah et al. (2015) extended this model by taking price sensitive and time-dependent demand.

Most of the inventory researchers have used constant rate demand. But in the real world, demand is not always constant. It may vary with time. Donaldson (1977) obtained the fundamental result in EOQ model with time-varying linear demand over a known and finite time horizon. Dave and Patel (1981) extended model for deteriorating items. Further, Wee and Wang (1999) considered time varying demand and developed a variable production policy. Mishra and Singh (2011) had taken into account time-dependent holding cost and formulated an inventory model under shortages. Mishra (2013) extended model for time-varying deterioration.

Deterioration is the process in which the items loses its utility and become useless. In classical EOQ model, researcher considered inventory depletes due to demand only. But in the real world, inventory is not only reduce due to demand but also reduced due to deterioration. In earlier literature, Ghare and Schrader (1963) developed model for items those deteriorates exponential. Firstly, Philip and Covert (1973) formulated model for time-dependent deteriorating items which follow Weibull distribution. Further, Philip (1974) generalized this model. Manna and Chaudhuri (2001) derived inventory model under shortages for time-dependent deteriorating items. Bakker et al. (2012) gave up to date review of inventory models for deteriorating items. To reduce deterioration rate, different researcher used preservation technology. Mishra (2013) used preservation technology for time-dependent deteriorating items that follow Weibull Distribution. Chang (2013) used preservation technology for non-instantaneous deteriorating items. Singh and Rathore (2015) extended that model under shortages. Mishra and Talati (2018) derived integrated inventory model and used preservation technology under quantity discount scenario. Mahapatra et al. (2019) formulated inventory model for deteriorating items under fuzzy environment.

In last decades, to optimize the inventory models, researchers used different heuristic search algorithms like ant colony, swarm intelligence and genetic algorithm. Genetic algorithm describes a set of techniques inspired by natural selection like inheritance, mutation, selection and crossover. This technique requires fitness function and genetic representation of solution domain. In each generation, it uses fitness function to select global optimum. This process terminates when the satisfactory fitness level has been reached. Goldberg (1989) used GA for optimization. Then, different researchers like Murata et al. (1996), Goren et al. (2008), Radhakrishnan

et al. (2009, 2010), Narmadha et al. (2010), Woarawichai et al. (2012), Mishra and Talati (2015), Talati and Mishra (2019), Alejo-Reyes et al. (2021) used heuristic search algorithm for optimized their models.

2 Notations and Assumptions

2.1 Notations

2.1.1 Inventory Parameters for Manufacturer

- $D(t)$ Time-dependent demand
- P Production rate
- a Fix fraction of demand
- γ Salvage cost/unit (\$)
- h_m Holding cost/unit/annum
- A_m Set-up costs (\$)
- TC_m Total cost for manufacturer
- T The length of cycle time (Decision variable)
- b_1 Deteriorating cost/unit (\$)
- Q_m Inventory level for manufacturer
- ξ_1 Preservation technology cost for manufacturer that reduce deterioration rate in order to preserve the product $\xi_1 > 0$
- $\theta(t)$ Deterioration rate at t , where $\theta(t) = \alpha\beta t^\beta$
- m Reduce deteriorating rate
- τ_p Resultant deterioration rate $\tau_p = \theta(t) - m$

2.1.2 Inventory Parameters for Retailer

- Q_r Retailer's order
- $C_0 Q_r^\eta$ Ordering Cost/cycle ($0 < \eta < 1$)
- C_0 Fixed ordering cost, η (Decision variable)
- ξ_2 Preservation technology cost for manufacturer that reduce deterioration rate in order to preserve the product $\xi_2 > 0$
- γ Salvage value associated with deteriorated items
- TC_r Total cost for retailer

2.2 Assumptions

1. In present model, we have considered two-echelon supply chain model (single manufacturer and single retailer) for single item.
2. Demand is time dependent $D(t) = a + bt; a, b > 0$.
3. Replenishment rate is infinite.
4. Lead time is zero.
5. Shortages are not allowed.
6. Constant production rate is considered. $P > D(t)$.
7. Ordering cost is lot size dependent.
8. The inventory deteriorate with respect to time and follow Weibull distribution $\theta(t) = \alpha\beta t^\beta$ where α is shape parameter $0 < \alpha < 1$, and β is scale parameter $\beta \geq 1$.
9. Preservation technologies are used for reducing the deterioration rate.
10. The salvage value $\gamma, 0 \leq \gamma \leq 1$ is associated to deteriorated units.

3 Model Formulation

3.1 Manufacturer's Total Cost

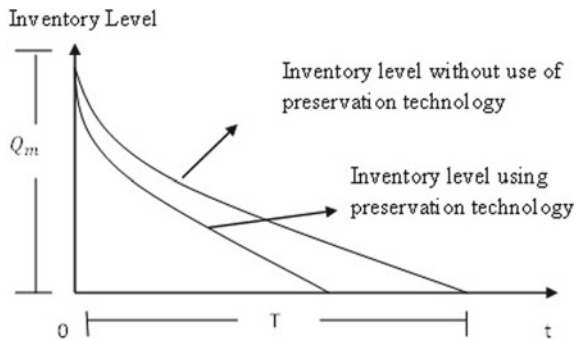
Here, we considered production dominates demand. Due to preservation technology, the rate of change of inventory during period $[0, T]$ is shown in Fig. 1.

Thus, the on-hand inventory for manufacturer is generated by the following differential equation

$$\frac{dQ_m}{dt} + \tau_p Q_m = P - D(t); \quad 0 \leq t \leq T \tag{1}$$

Solving Eq. (1) using boundary condition $Q_m(0) = 0$ and $Q_m(T) = Q_m$

Fig. 1 Inventory level for manufacturer. *Source own*



$$Q_m(t) = \left((P - a)t \left(1 + \frac{\alpha t^\beta}{\beta + 1} \right) - (Pm + b - ma)t^2 \left(\frac{1}{2} + \frac{\alpha t^\beta}{\beta + 2} \right) - mbt^3 \left(\frac{1}{3} + \frac{\alpha t^\beta}{\beta + 3} \right) \right) (1 - \alpha t^\beta + mt - m\alpha t^{\beta+1})$$

So total quantity by manufacturer per cycle is $Q_m(T) = Q_m$.

Basic Costs

1. Set-up cost

$$SC_m = A_m \quad (2)$$

2. Inventory holding cost per unit is given by

$$HC_m = h_m \int_0^T Q_m(t) dt \quad (3)$$

3. Now number of deteriorating units during cycle time T

$$DE_1(T) = Q_m - aT - \frac{(bT^2)}{2} \quad (4)$$

4. Deteriorating cost is given by

$$DC_m = b_1 DE_1(T) \quad (5)$$

5. Salvage value is given by

$$SV_m = \gamma DE_1(T) \quad (6)$$

6. Preservation cost is given by

$$PC_m = \xi_1 \quad (7)$$

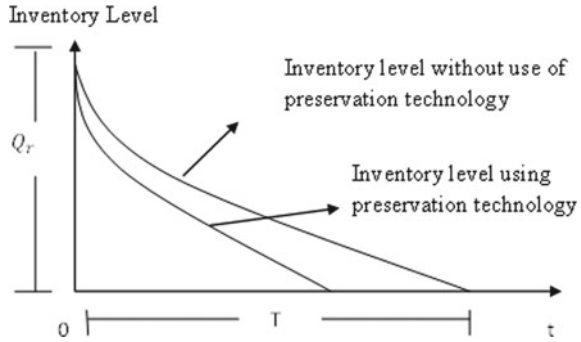
Thus, the total cost of manufacturer is

$$TC_m(T) = SC_m + HC_m + DC_m - SV_m + PC_m \quad (8)$$

3.2 Retailer's Total Cost

Retailer's on-hand inventory depletes with time-dependent demand and deterioration under preservation technology. The rate of change of inventory level due to preservation technology is shown in Fig. 2. So the governing differential equation describes the inventory level at instantaneous time t which is given by

Fig. 2 Inventory level for retailer. *Source* own



$$\frac{dQ_r}{dt} + \tau_p Q_r = -D(t); \quad 0 \leq t \leq T \tag{9}$$

Solving Eq. (9) using boundary condition $Q_r(T) = 0$ and $Q_m(0) = Q_r$, we get

$$Q_r(t) = \left[a(T-t) + \frac{b}{2}(T^2 - t^2) + \frac{\alpha a}{\beta + 1}(T^{\beta+2} - t^{\beta+2}) - \frac{ma}{2}(T^2 - t^2) - \frac{mb}{3}(T^3 - t^3) - \frac{\alpha a}{\beta + 2}(T^{\beta+2} - t^{\beta+2}) - \frac{m\alpha b}{\beta + 3}(T^{\beta+3} - t^{\beta+3}) \right] [1 - \alpha t^\beta + mt - m\alpha t^{\beta+1}] \tag{10}$$

∴ Total quantity purchase by retailer per cycle is

$$Q_r = Q_r(0) = \left[a(T) + \frac{b}{2}(T^2) + \frac{\alpha a}{\beta + 1}(T^{\beta+2}) - \frac{ma}{2}(T^2) - \frac{mb}{3}(T^3) - \frac{\alpha a}{\beta + 2}(T^{\beta+2}) - \frac{m\alpha b}{\beta + 3}(T^{\beta+3}) \right] \tag{11}$$

Basic costs associated with retailer total cost are

1. Ordering cost is lot size dependent

$$OC_r = C_0 Q_r^\eta \tag{12}$$

2. Holding cost per unit is given by

$$HC_r = h_r \int_0^T Q_r(t) dt \tag{13}$$

3. Total number of deteriorating units during cycle time T

$$DE_2(T) = Q_r - aT - \frac{(bT^2)}{2} \quad (14)$$

4. The deteriorating cost per time unit is

$$DC_r = b_1 DE_2(T) \quad (15)$$

5. Salvage value per time unit is

$$SV_r = \gamma DE_2(T) \quad (16)$$

6. Preservation cost is given by

$$PC_r = \xi_2 \quad (17)$$

The total cost for retailer is by

$$TC_r = OC_r + HC_r + DC_r + PC_r - SV_r \quad (18)$$

3.3 Joint Total Cost

Total cost for the inventory system is

$$TC = TC_m + TC_r \quad (19)$$

4 Computational Algorithm

4.1 Analytical Approach

- Set all parameters value in the mathematical model except decision variables.
- Find optimum T using TC_m .
- Used optimal T and Q_m to find total cost for manufacturer.
- Optimized T and η simultaneously from TC_r .
- Used optimal T , η and Q_r and obtain total cost for retailer.
- Find optimal T and η from system total cost.
- Used optimal T , η and optimal quantity and calculate total system cost.

4.2 Genetic Algorithm Approach

- Set all parameters value in the fitness function except decision variables.
- Start G.A. with an initial population of 20 chromosomes.
- On the basis of their fitness score rank the chromosomes.
- Chromosomes with good fitness score will enter in mating pool.
- Perform stochastic uniform crossover for reproduction. We have considered crossover fraction is 0.8 and each generation is 2-Elites.
- On the basis of their fitness value, rank all members and select members for new generation.
- Perform step (iii) and step (iv) till absolute difference between two successive members is 10^{-5} .

5 Numerical Example and Sensitivity Analysis

5.1 Numerical Example

Consider one integrated production-inventory system with $P = 500$, $a = 400$, $m = 0.5$, $b = 2$, $\alpha = 0.5$, $\beta = 2$, $h_m = 0.2$, $\xi_1 = 500\$$, $\xi_2 = 500\$$, $h_r = 0.2$, $C_0 = 2000$, $A_m = 2000$.

We have optimized this using analytical method by MAPLE18; we get some computational results those are shown in Table 1.

Here, in independent decision, the convexity of the function is given below
For manufacturer

$$\frac{d^2TC_m}{dT^2} |_{(T=T^*)} = 2808.527238 \geq 0$$

Table 1 Computational results obtained by analytical approach

Optimal	Independent scenario	Integrated scenario
Cycle time (year)	0.02501172258	0.02501546196
η	0.08506027179	0.03756340374
Lot size	25	25
Total cost	Independent scenario	Integrated scenario
Manufacturer (\$)	1972.520628	1499.249742
Retailer (\$)	2499.997751	2400.999625
System (\$)	4471.518379	3900.24567

Source own

Table 2 Computational results obtained by using genetic algorithm

	Independent scenario	Integrated scenario
Iterations	51 and 190	84
Optimal	Independent scenario	Integrated scenario
Cycle time (year)	0.02	0.02
η	0.05	0.05
Lot size	32	34
Total cost	Independent scenario	Integrated scenario
Manufacturer (\$)	1963.6	1452.21
Retailer (\$)	2288.41	1241.23
System (\$)	3952.01	2893.44

Source own

For retailer

$$\left| \begin{matrix} \frac{\partial^2 TC_r}{\partial \eta^2} & \frac{\partial^2 TC_r}{\partial \eta \partial T} \\ \frac{\partial^2 TC_r}{\partial \eta \partial T} & \frac{\partial^2 TC_r}{\partial T^2} \end{matrix} \right| = 5.545177044479552 \times 10^2 > 0$$

and

$$\frac{\partial^2 TC_r}{\partial T^2} = 8.256 \times 10^2 \geq 0$$

For integrated

$$\left| \begin{matrix} \frac{\partial^2 TC_r}{\partial \eta^2} & \frac{\partial^2 TC_r}{\partial \eta \partial T} \\ \frac{\partial^2 TC_r}{\partial \eta \partial T} & \frac{\partial^2 TC_r}{\partial T^2} \end{matrix} \right| = 5.26352 \times 10^2 > 0$$

and

$$\frac{\partial^2 TC_r}{\partial T^2} = 0.8039621 \times 10^3 \geq 0$$

Above example is also optimized by genetic algorithm using MATLAB16a. Computational results obtain by genetic algorithm are shown in Table 2. For independent decision, genetic algorithm took 51 for manufacturer, 190 for retailer and 84 for integrated system. Best fitness plot of manufacturer, retailer and the system is shown in Figs. 3, 4 and 5, respectively.

The sensitivity analysis for the above example is carried out to check the behaviour of inventory and supply chain parameters related to total cost in joint decision by varying inventory parameters as $-20, -10, 10$ and 20% . The computational results is shown in Table 3.

The results obtained in Table 3 can be summarized as follows:

- As inventory parameters a, b, α, h_m increase, integrated total cost decreases.
- As inventory parameters m, β, h_r increase, integrated total cost increases.

Table 3 Sensitivity analysis for inventory and supply chain parameters

Parameters	Change	Generations	Integrated total cost
a	-20%	80	2893.4422220173033
	-10%	80	2893.440313935822
	0	51	2893.4402274468635
	10%	75	2893.44014557451607
	20%	51	2893.4400680390536
m	-20%	108	2893.3983955247295
	-10%	149	2893.4191946532046
	0	51	2893.4401874468635
	10%	51	2893.4623950272157
	20%	89	2893.4846156768504
b	-20%	86	2893.449783693166
	-10%	76	2893.4425591065487
	0	51	2893.4401874468635
	10%	138	2893.439330496022
	20%	135	2893.4398683311897
α	-20%	120	2893.4448207696682
	-10%	51	2893.442330496022
	0	51	2893.4401874468635
	10%	58	2893.44006788676355
	20%	167	2893.44002539623709
β	-20%	75	2893.4341786444675
	-10%	74	2893.4388061031786
	0	51	2893.4401874468635
	10%	90	2893.440616458528
	20%	79	2893.4407451316824
h_m	-20%	117	2893.4431508922
	-10%	51	2893.4407278538433
	0	51	2893.4401874468635
	10%	72	2893.44002016096947
	20%	74	2893.4402023774837
h_r	-20%	51	2893.4401554573037
	-10%	114	2893.44016537294647
	0	51	2893.4401874468635
	10%	112	2893.4402036655782
	20%	51	2893.4402194364234

Source own

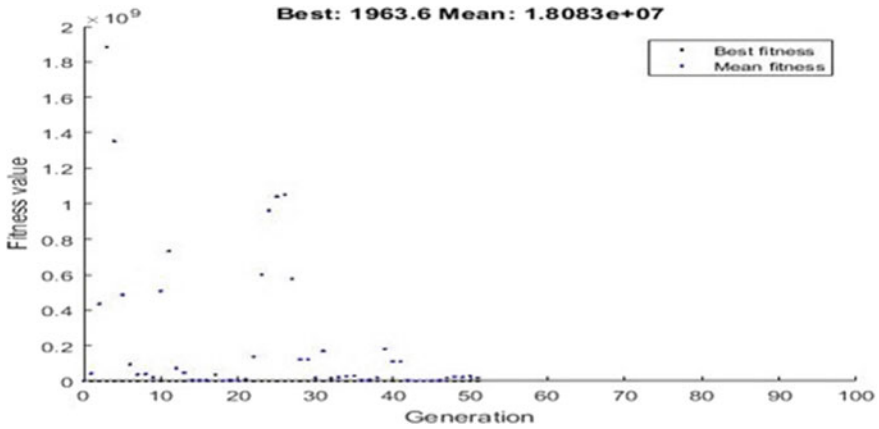


Fig. 3 Best fitness solution for manufacturer total cost. Source own

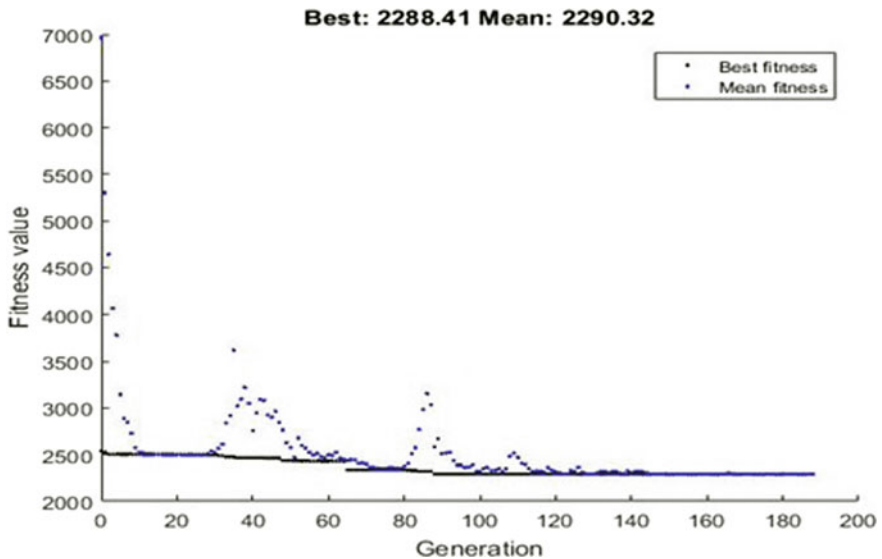


Fig. 4 Best fitness solution for retailer total cost. Source own

6 Conclusion

Supply chain management has required models and processes which can find a solution in a fast and efficient way. For comparison purposes, we have found a solution for the same numerical example using gradient-based analytical method and genetic algorithm. Complexity is explained mathematically for analytical techniques and graphically for genetic algorithms. It is shown that the decision taken in an integrated scenario reduces the cost compared to the decision in an isolated scenario

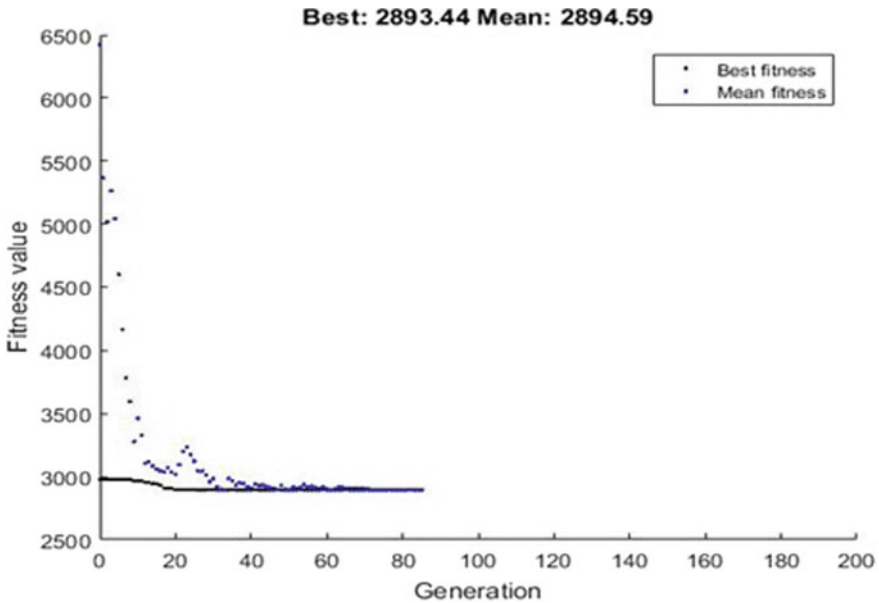


Fig. 5 Best fitness solution for system. *Source own*

in both techniques. Results clearly show that in our model, evolutionary algorithm provides global minimum while the analytical method fails. Future research may be extended into more realistic situations like shortages, random demand and inflation. Additionally, genetic algorithms can be modified to find solutions in a very efficient manner.

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