A Stackelberg Game Approach in Supply Chain for Imperfect Quality Items with Learning Effect in Fuzzy Environment

Rita Yadav, Mandeep Mittal, Navneet Kumar Lamba, and Mahesh Kumar Jayaswal

Abstract In recent decades, researchers have effectively worked on seeking optimal policies under supply chain management for attaining practical and powerful outcomes. This paper studies supply chain model for imperfect quality items in which demand depends upon the buyer's price and marketing cost. The buyer segregates the defective items from supplied lot by seller and sell them at discounted price. In today's scenario, learning effect methodology has become a promotional tool in supply chain management. It impacts profit or loss of the members of the supply chain. The rapid change in the life cycle of product makes the parameters of the supply chain models more and more uncertain. Fuzzy analysis becomes a powerful tool to deal such type of vague or uncertain parameters in computing form. It examines the better assessment and performance of imprecise parameters. Keeping in view, some supply chain models for imperfect quality items have been developed by considering learning effect under fuzzy environment. A non-cooperative Stackelberg game theoretic approach is used to find the optimal decision variables and optimum profit of the supply chain members in fuzzy environment. Various numerical results with sensitivity analysis have been explained to justify the model.

Keywords Learning curve · Fuzzy system · Imperfect quality items · Non-cooperative games · Supply chain · Game theory

1 Introduction

Supply chain management is primarily related to the integration of activities and process between and within the organization. To analyze the interaction between

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the members of the supply chain, the theory of non-cooperative Stackelberg game is preferred and used to study supply chain connected problems. There have been many researchers/academicians involve in working of the mathematical model implementing the learning curve. Some researches like Wright (1936), Baloff (1966), Cunningham (1980), and Argote et al. (1990) have contributed their work in the field of learning and forgetting curves by discussing the mathematical behavior of learning theory. Salameh et al. (1993) developed production inventory model (limited manufactured stock form) to optimize the cost with the outcome of human knowledge by taking the variable demand rate and learning with respect to time. Jaber and Bonney (1996) discovered and discussed a comparative study of the theory of learning and forgetting and also focused analytically different types of models.

Salameh and Jaber (2000) explored EOQ model for defective items with an inspection process at the buyer's end. These items may occur due to any reasons. Further, Eroglu and Ozdemir (2007) stretched the model of Salameh and Jaber (2000) by permitting shortages. Jaber and Bonney (2003) developed mathematical model which focused on reducing the setup time, eliminating rework and increasing production capacity with the help of learning curve. Jaber et al. (2008) deliberated the EOQ model in which they discussed that by using the concept of the learning curve, the percentage of defective items per batch decreased. Khan et al. (2010) minimized the production cost and maximized the production in their EOQ model for defective items by letting learning in screening process. Anzanello and Fogliatto (2011) advised the mathematical forms with their applications of learning curves models. Konstantaras et al. (2012) established a model to maximize production by allowing shortages for the imperfect items with an inspection as learning. Jaber et al. (2013) considered a manufacture stock model with "learning and forgetting" theory in manufacture. Game theory is a competent tool to balance the coordination among the players like seller and buyer in supply chain industry. Jayaswal et al. (2019) established an inventory model for imperfect quality items with permission delay under learning effect. Mittal et al. (2017) proposed an inventory model for price and demand are time depended under inflation.

Many times, there is ups and downs in the market. So it becomes necessary and useful for business to use fuzzy number to get best strategy. Wei and Zhao (2013) discussed three supply chain models in which expected profit is determined by fuzzy game theory. Soleimani (2016) analyzed manufacturer–leader Stackelberg game in which manufacturing cost and demand of customer are precise in nature. Optimum value of whole sale price and buyer's price are obtained by game theoretic approach. Patro et al. (2017) investigated two models crisp as well as fuzzy EOQ models with imperfect quality items (proportionate discount items) under learning effect in a finite time horizon. The optimal order lot size is determined to maximize the total profit where the defective items follow a learning curve and the demand rate assumed as triangle fuzzy number. Chavoshlou et al. (2019) developed three players (government, manufacture, and customer) green supply chain optimization model under fuzzy environment. Optimal strategies are obtained by Nash equilibrium game, and positive effects of fuzzy game model over non-fuzzy game model are discussed.

Some researchers like Abad and Jaggi (2003) have developed supply chain model in which the seller endorsed credit period to the buyer (payer) by cooperative and non-cooperative game theoretical structure. Esmaeili et al. (2009) also developed supply chain models by the game theoretical approach (cooperative and non-cooperative) in which demand is influenced by both the selling cost and marketing expenditure cost. Yadav et al. (2020) developed supply chain models for defective items with learning effect by Stackelberg non-cooperative approach. None of the researchers have developed such model under fuzzy environment, where demand is sensitive to selling rate and marketing expenditure charges of the buyer. To obtain their optimal policies, the fuzzy set theory is adopted to solve these fuzzy models. Meanwhile, fuzzy analysis is a commanding tool that deals with the information which arises from computational awareness and perception. Therefore, we consider the correlation between one buyer and one seller in a fuzzy decision-marking environment, where the parameters of the models can be forecasted and expressed as the triangular fuzzy variables.

Fuzzy theory basically comprises the process to find out the proper range for indistinct items in a vague/imprecise environment for smooth coordination between players of supply chain. In this paper, ordering cost of the buyer and setup cost of the seller are imprecise in nature. The total optimal profit in fuzzy environment is defuzzified with the help of the centroid method. In this paper, two-level supply chain models under fuzzy environment with the learning effect have been developed. The non-cooperative game theoretic approaches have been discussed in which demand is influenced by the marketing expenditure/promotional cost and selling price of the player, purchaser. Seller-Stackelberg and Buyer-Stackelberg, two different game approaches, have been discussed.

In this paper, impact of learning curve (LC) curve is shown on the different parameters of the supply chain. In this paper, learning curve is assumed to be in the form of $p(n) = a/\left(g + s^{bn}\right)$, where a, b and $g > 0$ are the active parameters, and $p(n)$ is the percentage defective per batch n, whereas n is the cumulative number of lots.

2 Notations

Seller's decision variables

c_b Seller's selling price ($/unit)

Buyer's decision variables

M Marketing cost (promotional price) ($/unit)
p_b Buyer's selling price ($/unit)
y_n Order quantity (in units) in nth batch, where $n \geq 1$

Parameters

A_b	Buyer's ordering cost ($/order)
A_s	Seller's ordering cost ($/order)
H_b	Inventory cost ($/unit/time)
I	Percent of inventory's carrying cost ($/unit)
$p(n)$	Defective percentage of items/products per batch (n) in y_n (units)
C	Seller's purchasing cost ($/unit)
c_s	Cost of defective value items per unit ($year) ($c_s < c_b$)
β	Marketing expenditure (promotional) elasticity of demand ($0 < \beta < 1,\ \beta + 1 < e$)
e	Price elasticity of the marketing demand ($e > 1$)
D	Annual demand rate (unit/year) = $kp_b^{-e}M^{\beta}$
λ	Screening rate decided by the buyer in units per unit of time ($D < \lambda$)
s_c	Cost to screen the product ($/units)
t_n	Time taken to screen a lot for imperfect items, $t_n = y_n/\lambda$ (years)
k	Scaling constant for the promoting demand ($k > 0$)
$\widetilde{A_b}$	Fuzzy ordering cost of the buyer ($/order)
$\widetilde{A_s}$	Fuzzy ordering cost of the seller ($/order)
$\mathrm{TP}_b^c(p_b, M, y_n)$	Buyer's profit function
$\mathrm{TP}_s^c(c_b)$	Seller's profit function
$\mathrm{TP}_{b^*}^c(p_b, M, y_n)$	Fuzzy buyer's profit
$\mathrm{TP}_{s^*}^c(c_b)$	Fuzzy seller's profit
T_n	Cycle length/span of the buyer (in years), $T_n = y_n(1 - p(n))/D$
T_n^*	Cycle length/span of the seller (in years), $T_n^* = y_n/D$
T_n^{**}	Cycle length/span of the Stackelberg models (in years), $T_n^{**} = \mathrm{Max}\left(T_n, T_n^*\right)$

2.1 Assumptions

1. Marketing demand is considered as a function of p_b and M.
2. Planning horizon is assumed as infinite.
3. No shortages acceptable (the demand is fulfilled).
4. Demand and screening follows at the same time and ($D < \lambda$).
5. Holding/inventory cost is not reflected for the seller as a lot-to-lot strategy rule have been considered.
6. The defective percentage items follow the Wright's curve (assumed) and the worth of the good product is assumed to be more than that of the imperfect quality items.
7. The number of imperfect items present in each batch is assumed by learning curve $p(n) = \frac{a}{g+s^{bn}}$, b is the learning rate, where a, b and $g > 0$ are the effective parameters, n is the cumulative number of lots or shipment, and $p(n)$ is the percentage defective per batch n.

8. The buyer's ordering cost and seller's setup cost are imprecise in nature.
9. Defuzzify the total profit function by the triangular method.

2.2 Some Definitions

Fuzzification

Fuzzification is a function which assigns input of a set to some degree of membership. The degree of membership may lie within the closed interval [0, 1]. If interval value is 1, then the value completely belongs to the fuzzy set. If its value is 0, then value does not belong to the given fuzzy set and if value lie between 0 and 1, that signifies the degree of vagueness or uncertainty that the given value belong in the set. Fuzzy environment process tries to solve the problems with a rough/imprecise data that makes it possible to obtain a group of exact conclusions.

Defuzzification: If $\overset{\vee}{A} = (a_1, a_2, a_3)$ is triangular fuzzy number then centroid method for defuzzification is defined as $C\left(\overset{\vee}{A}\right) = \frac{a_1+a_2+a_3}{3}$.

3 Mathematical Crisp Models

3.1 Buyer's Model

The objective of the present model is to optimize the buyer's price, marketing cost, and the ordered quantity with the corresponding profit for the retailer with the learning effect.

Buyer's profit = Sales income − purchasing cost − screening cost − marketing expenditure cost − ordering cost − holding cost

$$\mathrm{TP_b}(p_\mathrm{b}, M, y_n) = p_\mathrm{b}(1 - p(n))y_n + c_\mathrm{s}p(n)y_n - c_\mathrm{b}y_n - s_\mathrm{c}y_n - My_n - A_\mathrm{b} - \left(\frac{Q(1 - p(n))T_1}{2} + \frac{p(n)Q^2}{\lambda}\right)H_\mathrm{b}$$

Put $T_n = \frac{(1-p(n))y_n}{D}$, $t = \frac{y_n}{\lambda}$, $H_\mathrm{b} = Ic_\mathrm{b}$ then buyer's profit is given by

$$\mathrm{TP_b}(p_\mathrm{b}, M, y_n) = p_\mathrm{b}(1 - p(n))y_n + c_\mathrm{s}p(n)y_n - c_\mathrm{b}y_n - My_n - s_\mathrm{c}y_n - A_\mathrm{b} - \left(\frac{y_n^2(1 - p(n))^2}{2D} + \frac{p(n)y_n^2}{\lambda}\right)Ic_\mathrm{b}$$

We assumed that the demand function is $D = kp_\mathrm{b}^{-e}M^{\beta}$.

Buyer's profit per cycle is given by

$$\mathrm{TP}_{\mathrm{b}}^{c}(p_{\mathrm{b}}, M, y_n) = \left[\frac{\mathrm{TP}_{\mathrm{b}}(p_{\mathrm{b}}, M, y_n)}{T_n}\right]$$

$$\frac{D}{(1-p(n))y_n}[p_{\mathrm{b}}(1-p(n))y_n + c_{\mathrm{s}}p(n)y_n - c_{\mathrm{b}}y_n$$

$$-My_n - s_{\mathrm{c}}y_n - A_{\mathrm{b}} - \left(\frac{y_n^2(1-p(n))^2}{2D} + \frac{p(n)y_n^2}{\lambda}\right)Ic_{\mathrm{b}}\Big]$$

$$= p_{\mathrm{b}}D + \frac{1}{(1-p(n))}\left[c_{\mathrm{s}}p(n)D - c_{\mathrm{b}}D - MD - s_{\mathrm{c}}D - \frac{A_{\mathrm{b}}D}{y_n}\right.$$

$$\left.-\left(\frac{y_n(1-p(n))^2}{2} + \frac{p(n)y_nD}{\lambda}\right)Ic_{\mathrm{b}}\right]$$

$$\mathrm{TP}_{\mathrm{b}}^{c}(p_{\mathrm{b}}, M, y_n) = kp_{\mathrm{b}}^{-e+1}M^{\beta} + \frac{kp_{\mathrm{b}}^{-e}M^{\beta}}{(1-p(n))}\left[c_{\mathrm{s}}p(n) - c_{\mathrm{b}} - s_{\mathrm{c}} - M - \frac{A_{\mathrm{b}}}{y_n}\right.$$

$$\left.-\frac{p(n)y_n}{\lambda}Ic_{\mathrm{b}}\right] - \left(\frac{y_n(1-p(n))^2}{2(1-p(n))}Ic_{\mathrm{b}}\right) \tag{1}$$

The buyer's goal is to find optimal values for order quantity y_n, selling price, p_{b}, marketing expenditure cost, M, such that his profit becomes maximum.

For this, we equate first derivative of Eq. (1) with respect to p_{b} to zero.

$\frac{\partial[\mathrm{TP}_{\mathrm{b}}^{c}(p_{\mathrm{b}}, M, y_n)]}{\partial p_{\mathrm{b}}} = 0$, yields

$$p_{\mathrm{b}} = \frac{e}{(e-1)(1-p(n))}\left[M + c_{\mathrm{b}} + s_c - c_{\mathrm{s}}p(n) + \frac{A_{\mathrm{b}}}{y_n} + \frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda}\right] \tag{2}$$

The buyer's profit $[\mathrm{TP}_{\mathrm{b}}^{c}(p_{\mathrm{b}}, M, y_n)]$ is pseudoconcave with respect to p_{b} for constants M and y_n (Yadav et al. 2018).

Substituting the value of p_{b} into Eq. (1) and then subsequent equation is

$$[\mathrm{TP}_{\mathrm{b}}^{c}(p_{\mathrm{b}}(M), M, y_n(M))] = \frac{K}{e}\left[\frac{e}{(e-1)(1-p(n))}\left(M + c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{A_{\mathrm{b}}}{y_n}\right.\right.$$

$$\left.\left.+\frac{p(n)y_nIc_{\mathrm{b}}}{\lambda} - c_{\mathrm{s}}p(n)\right)\right]^{-e+1}M^{\beta} - \left(\frac{y_n(1-p(n))^2}{2(1-p(n))}Ic_{\mathrm{b}}\right) \tag{3}$$

Taking differentiation of Eq. (3) w.r.t. M, we get

$$M = \frac{\beta}{(e-\beta-1)}\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{A_{\mathrm{b}}}{y_n} + \frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda} - c_{\mathrm{s}}p(n)\right] \tag{4}$$

The buyer's profit, $[\mathrm{TP}_{\mathrm{b}}^{c}(p_{\mathrm{b}}(M), M, y_n(M))]$, is concave with respect to M for constant y_n (Yadav et al. 2018).

Substituting the value of Eq. (4) into Eq. (2), we get

$$p_{\mathrm{b}} = \frac{e}{(e-\beta-1)(1-p(n))}\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{A_{\mathrm{b}}}{y_n} + \frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda} - c_{\mathrm{s}}p(n)\right] \tag{5}$$

$$\begin{aligned}\left[\mathrm{TP}^c(y_n)\right] = {} & k\left(\frac{e}{(e-\beta-1)(1-p(n))}\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{A_{\mathrm{b}}}{y_n} + \frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda} - c_{\mathrm{s}}p(n)\right]\right)^{-e} \\ & \left(\frac{\beta}{(e-\beta-1)}\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{A_{\mathrm{b}}}{y_n} + \frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda} - c_{\mathrm{s}}p(n)\right]\right)^{\beta} \\ & \left\{\left(\frac{e}{(e-\beta-1)(1-p(n))}\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{A_{\mathrm{b}}}{y_n} + \frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda} - c_{\mathrm{s}}p(n)\right]\right)\right. \\ & + \frac{1}{(1-p(n))}\left[c_{\mathrm{s}}p(n) - s_{\mathrm{c}} - c_{\mathrm{b}} - \left(\frac{\beta}{(e-\beta-1)}\left[c_{\mathrm{b}} + s_c + \frac{A_{\mathrm{b}}}{y_n}\right.\right.\right. \\ & \left.\left.\left.\left. + \frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda} - c_{\mathrm{s}}p(n)\right]\right) - \frac{A_{\mathrm{b}}}{y_n} - \frac{p(n)y_n}{\lambda}Ic_{\mathrm{b}} - \frac{y_n\left[(1-p(n))^2\right]}{2}Ic_{\mathrm{b}}\right]\right\}\end{aligned} \tag{6}$$

The first-order condition of Eq. (6) w.r.t. y_n finds the constraints as follows:

$$\begin{aligned}& y_n^2 Ic_{\mathrm{b}}\left((1-p(n))^2\right)\lambda + 2Dp(n) = 2D\lambda, \quad \text{i.e.} \\ & y_n^2 Ic_{\mathrm{b}}\left((1-p(n))^2\right)\lambda = 2ke^{-e}\beta^{\beta}\left(\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{A_{\mathrm{b}}}{y_n} + \frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda} - c_{\mathrm{s}}p(n)\right]\right)^{\beta-e} \\ & \quad (e-\beta-1)^{e-\beta}(1-p(n))^e\left(\lambda A_{\mathrm{b}} - y_n^2 p(n)\right)\end{aligned} \tag{7}$$

It is quite difficult to prove the concavity of the above total profit function defined in Eq. (6) analytically.

Thus, buyer's total profit $\left[\mathrm{TP}_{\mathrm{b}}^c(y_n)\right]$ defined in Eq. (6) is concave function with respect to order quantity is shown with the help of the graph (Fig. 1).

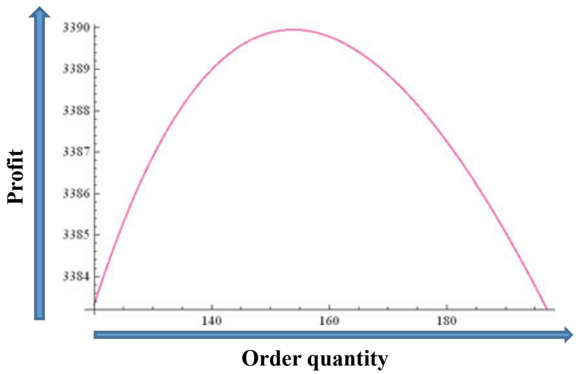

Fig. 1 Plot of buyer's profit function with respect to order quantity

3.2 Seller's Model

Seller's yield = Sales revenue − purchasing cost − ordering cost

$$\mathrm{TP_s}(c_\mathrm{b}) = c_\mathrm{b} y_n - C y_n - A_\mathrm{s}$$

Seller's cycle length, $T_n^* = \frac{y_n}{D}$.
Seller's profit per cycle is given by,

$$\begin{aligned}\mathrm{TP_s^c}(c_\mathrm{b}) &= \frac{D}{y_n}(c_\mathrm{b} y_n - C y_n - A_\mathrm{s}) \\ &= k p_\mathrm{b}^{-e} M^{\beta}\left(c_\mathrm{b} - C - \frac{A_\mathrm{s}}{y_n}\right) \end{aligned} \tag{8}$$

Seller's plan is to achieve his net profit, by finding the optimal value of selling price, c_b.

Seller's profit is zero at $c_{\mathrm{b}_0} = C + \frac{A_\mathrm{s}}{y_n}$.
Since the seller always would prefer to have positive profit,
$c_{\mathrm{b}_0} > C + \frac{A_\mathrm{s}}{y_n}$, let

$$\boldsymbol{c_b} = F c_{\mathrm{b}_0} = F\left(C + \frac{A_\mathrm{s}}{y_n}\right) \text{ for some, } F > 1 \tag{9}$$

i.e., the optimal value for c_b obtained through negotiation by seller and buyer.

3.3 The Non-cooperative Stackelberg Game Theory Approach

The Stackelberg non-cooperative game considers two players. Among them, one player is recognized as dominant player and takes the advantage of making the first move/travel and other player acts as follower, making their best probable move serially using preceding available information.

3.3.1 The Seller-Stackelberg Model

In this model, seller is treated as dominant player. The seller's objective is to find his yield on the basis of buyer's decision variables. The problem is,

$$\text{Max } \left(\mathrm{TP_s^c}(c_\mathrm{b})\right)$$

$$\mathrm{TP}_{\mathrm{s}}^{c}(c_{\mathrm{b}}) = \frac{D}{y_n}(c_{\mathrm{b}} y_n - C y_n - A_{\mathrm{s}}) = k p_{\mathrm{b}}^{-e} M^{\beta}\left(c_{\mathrm{b}} - C - \frac{A_{\mathrm{s}}}{y_n}\right) \tag{10}$$

Subject to

$$M = \frac{\beta}{(e-\beta-1)}\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{A_{\mathrm{b}}}{y_n} + \frac{I c_{\mathrm{b}} p(n) y_n}{\lambda} - c_{\mathrm{s}} p(n)\right] \tag{11}$$

$$p_{\mathrm{b}} = \frac{e}{(e-\beta-1)(1-p(n))}\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{A_{\mathrm{b}}}{y_n} + \frac{I c_{\mathrm{b}} p(n) y_n}{\lambda} - c_{\mathrm{s}} p(n)\right] \tag{12}$$

Constraints

$$y_n^2 I c_{\mathrm{b}}\left((1-p(n))^2\right)\lambda = 2ke^{-e}\beta^{\beta}\left(\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{A_{\mathrm{b}}}{y_n} + \frac{I c_{\mathrm{b}} p(n) y_n}{\lambda} - c_{\mathrm{s}} p(n)\right]\right)^{\beta-e} (e-\beta-1)^{e-\beta}(1-p(n))^{e}\left(\lambda A_{\mathrm{b}} - y_n^2 p(n)\right) \tag{13}$$

Cycle length, $T_n^{**} = \max\left(T_n, T_n^{*}\right)$.

By using Eqs. (11) and (12) and the constraints (13) in Eq. (10), the subsequent equation can be resolved using software Mathematica 9.0.

3.3.2 The Buyer-Stackelberg Model

$$\mathrm{Max}\left[\mathrm{TP}_{\mathrm{b}}^{c}(p_{\mathrm{b}}, M, y_n)\right] = k p_{\mathrm{b}}^{-e+1} M^{\beta} + \frac{k p_{\mathrm{b}}^{-e} M^{\beta}}{(1-p(n))}\left[c_{\mathrm{s}} p(n) - c_{\mathrm{b}} - s_{\mathrm{c}} - M - \frac{A_{\mathrm{b}}}{y_n} - \frac{p(n) y_n}{\lambda} I c_{\mathrm{b}}\right] - \left(\frac{y_n (1-p(n))^2}{2(1-p(n))} I c_{\mathrm{b}}\right) \tag{14}$$

Subject to

$$\text{At } c_{\mathrm{b}_0} = F\left(C + \frac{A_{\mathrm{s}}}{y_n}\right) \tag{15}$$

By using Eq. (15) on Eq. (14), the resultant nonlinear equation can be explained using software Mathematica 9.0.

4 Mathematical Fuzzy Model

In this section, different mathematical models such as buyer's fuzzy model, seller's fuzzy model, seller-Stackelberg fuzzy model, and buyer's Stackelberg fuzzy model with learning effect under fuzzy environment have been explained.

4.1 *Buyer's Fuzzy Model*

The objective of the present model is to optimize the buyer's price, marketing cost, and the ordered quantity with the corresponding profit for the retailer with learning effect under fuzzy environment.

Let us assume that due to uncertainty existing in parameters, the inventory model is in fuzzy environment. Also, we have assumed that the parameters $\widetilde{A_b} = (A_{b1}, A_{b2}, A_{b3})$, $\widetilde{A_s} = (A_{s1}, A_{s2}, A_{s3})$ are triangular fuzzy numbers, then the entire profit per unit time in fuzzy environment is in each model.

Buyer's total profit per cycle in fuzzy environment

$$\mathrm{TP}^c_{b^*}(p_b, M, y_n) = kp_b^{-e+1}M^\beta + \frac{kp_b^{-e}M^\beta}{(1-p(n))} \left[c_s p(n) - c_b - s_c - M - \frac{\widetilde{A_b}}{y_n} - \frac{p(n)y_n}{\lambda} I c_b\right] - \left(\frac{y_n(1-p(n))^2}{2(1-p(n))} I c_b\right) \quad (16)$$

$$\mathrm{TP}^c_{b1}(p_b, M, y_n) = kp_b^{-e+1}M^\beta + \frac{kp_b^{-e}M^\beta}{(1-p(n))} \left[c_s p(n) - c_b - s_c - M - \frac{A_{b1}}{y_n} - \frac{p(n)y_n}{\lambda} I c_b\right] - \left(\frac{y_n(1-p(n))^2}{2(1-p(n))} I c_b\right) \quad (17)$$

$$\mathrm{TP}^c_{b2}(p_b, M, y_n) = kp_b^{-e+1}M^\beta + \frac{kp_b^{-e}M^\beta}{(1-p(n))} \left[c_s p(n) - c_b - s_c - M - \frac{A_{b2}}{y_n} - \frac{p(n)y_n}{\lambda} I c_b\right] - \left(\frac{y_n(1-p(n))^2}{2(1-p(n))} I c_b\right) \quad (18)$$

$$\mathrm{TP}^c_{b3}(p_b, M, y_n) = kp_b^{-e+1}M^\beta + \frac{kp_b^{-e}M^\beta}{(1-p(n))} \left[c_s p(n) - c_b - s_c - M - \frac{A_{b3}}{y_n} - \frac{p(n)y_n}{\lambda} I c_b\right]$$

$$-\left(\frac{y_n(1-p(n))^2}{2(1-p(n))}Ic_{\mathrm{b}}\right) \tag{19}$$

Now we defuzzify the entire profit per unit time by centroid method

$$\mathrm{TP}^c_{\mathrm{b}^*}(p_{\mathrm{b}}, M, y_n) = \frac{\mathrm{TP}^c_{\mathrm{b}1}(p_{\mathrm{b}}, M, y_n) + \mathrm{TP}^c_{\mathrm{b}2}(p_{\mathrm{b}}, M, y_n) + \mathrm{TP}^c_{\mathrm{b}3}(p_{\mathrm{b}}, M, y_n)}{3} \tag{20}$$

Substituting the values from Eqs. (17), (18), and (19) in Eq. (20), we get

$$\begin{aligned}
\mathrm{TP}^c_{\mathrm{b}}\cdot(p_{\mathrm{b}}, M, y_n) = \frac{1}{3}\Bigg\{ & kp_{\mathrm{b}}^{-e+1}M^{\beta} + \frac{kp_{\mathrm{b}}^{-e}M^{\beta}}{(1-p(n))} \\
& \left[c_{\mathrm{s}}p(n) - c_{\mathrm{b}} - s_{\mathrm{c}} - M - \frac{A_{\mathrm{b}1}}{y_n} - \frac{p(n)y_n}{\lambda}Ic_{\mathrm{b}}\right] - \left(\frac{y_n(1-p(n))^2}{2(1-p(n))}Ic_{\mathrm{b}}\right) \\
& + kp_{\mathrm{b}}^{-e+1}M^{\beta} + \frac{kp_{\mathrm{b}}^{-e}M^{\beta}}{(1-p(n))}\left[c_{\mathrm{s}}p(n) - c_{\mathrm{b}} - s_{\mathrm{c}} - M - \frac{A_{\mathrm{b}2}}{y_n} - \frac{p(n)y_n}{\lambda}Ic_{\mathrm{b}}\right] \\
& - \left(\frac{y_n(1-p(n))^2}{2(1-p(n))}Ic_{\mathrm{b}}\right) + kp_{\mathrm{b}}^{-e+1}M^{\beta} + \frac{kp_{\mathrm{b}}^{-e}M^{\beta}}{(1-p(n))} \\
& \left[c_{\mathrm{s}}p(n) - c_{\mathrm{b}} - s_{\mathrm{c}} - M - \frac{A_{\mathrm{b}3}}{y_n} - \frac{p(n)y_n}{\lambda}Ic_{\mathrm{b}}\right] - \left(\frac{y_n(1-p(n))^2}{2(1-p(n))}Ic_{\mathrm{b}}\right)\Bigg\}
\end{aligned}$$

$$\begin{aligned}
\mathrm{TP}^c_{\mathrm{b}^*}(p_{\mathrm{b}}, M, y_n) = & \frac{(A_{\mathrm{b}1} + A_{\mathrm{b}2} + A_{\mathrm{b}3})}{3}\left(\frac{kp_{\mathrm{b}}^{-e}M^{\beta}}{(1-p(n))y_n}\right) \\
& + \Bigg[kp_{\mathrm{b}}^{-e+1}M^{\beta} + \frac{kp_{\mathrm{b}}^{-e}M^{\beta}}{(1-p(n))}[c_{\mathrm{s}}p(n) - c_{\mathrm{b}} \\
& - s_{\mathrm{c}} - M - \frac{p(n)y_n}{\lambda}Ic_{\mathrm{b}}\Big] - \left(\frac{y_n(1-p(n))^2}{2(1-p(n))}Ic_{\mathrm{b}}\right)\Bigg]
\end{aligned} \tag{21}$$

Now, our objective is to find the optimal values of three decision variables p_{b}, M, and y_n to optimize the profit function $\mathrm{TP}^c_{\mathrm{b}^*}(p_{\mathrm{b}}, M, y_n)$. The first-order condition of Eq. (21) w.r.t. p_{b} and M, we have

$$M = \frac{\beta}{(e-\beta-1)}\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{(A_{\mathrm{b}1} + A_{\mathrm{b}2} + A_{\mathrm{b}3})}{3y_n} + \frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda} - c_{\mathrm{s}}p(n)\right] \tag{22}$$

$$p_{\mathrm{b}} = \frac{e}{(e-\beta-1)(1-p(n))}\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{(A_{\mathrm{b}1} + A_{\mathrm{b}2} + A_{\mathrm{b}3})}{3y_n} + \frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda} - c_{\mathrm{s}}p(n)\right] \tag{23}$$

The total buyer's fuzzy profit is pseudoconcave with respect to p_{b} and M (Yadav et al., 2018).

Substituting the values of p_{b} and M in Eq. (21), we get

$$\left[\mathrm{TP}^c_{\mathrm{b}^*}(y_n)\right] = k\Bigg(\frac{e}{(e-\beta-1)(1-p(n))}\Bigg[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{(A_{\mathrm{b}1} + A_{\mathrm{b}2} + A_{\mathrm{b}3})}{3y_n}$$

$$+\frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda}-c_{\mathrm{s}}p(n)\Big]\Big)^{-e}\left(\frac{\beta}{(e-\beta-1)}\Big[c_{\mathrm{b}}+s_{\mathrm{c}}+\frac{(A_{\mathrm{b1}}+A_{\mathrm{b2}}+A_{\mathrm{b3}})}{3y_n}\right.$$
$$+\frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda}-c_{\mathrm{s}}p(n)\Big]\Big)^{\beta}\Big\{\Big(\frac{e}{(e-\beta-1)(1-p(n))}[c_{\mathrm{b}}+s_{\mathrm{c}}$$
$$+\frac{(A_{\mathrm{b1}}+A_{\mathrm{b2}}+A_{\mathrm{b3}})}{3y_n}+\frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda}-c_{\mathrm{s}}p(n)\Big]\Big)+\frac{1}{(1-p(n))}[c_{\mathrm{s}}p(n)-s_{\mathrm{c}}$$
$$-c_{\mathrm{b}}-\left(\frac{\beta}{(e-\beta-1)}\Big[c_{\mathrm{b}}+s_{\mathrm{c}}+\frac{(A_{\mathrm{b1}}+A_{\mathrm{b2}}+A_{\mathrm{b3}})}{3y_n}+\frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda}-c_{\mathrm{s}}p(n)\Big]\right)$$
$$-\frac{(A_{\mathrm{b1}}+A_{\mathrm{b2}}+A_{\mathrm{b3}})}{3y_n}-\frac{p(n)y_n}{\lambda}Ic_{\mathrm{b}}-\frac{y_n\big[(1-p(n))^2\big]}{2}Ic_{\mathrm{b}}\Big]\Big\} \tag{24}$$

The first-order condition of Eq. (24) w.r.t. y_n, we find the constraints as follows:

$$y_n^2Ic_{\mathrm{b}}\big((1-p(n))^2\big)\lambda+2Dp(n)=2D\lambda,\quad \text{i.e.}$$

$$y_n^2Ic_{\mathrm{b}}\big((1-p(n))^2\big)\lambda=2ke^{-e}\beta^{\beta}\Big(\Big[c_{\mathrm{b}}+s_{\mathrm{c}}+\frac{(A_{\mathrm{b1}}+A_{\mathrm{b2}}+A_{\mathrm{b3}})}{3y_n}$$
$$+\frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda}-c_{\mathrm{s}}p(n)\Big]\Big)^{\beta-e}(e-\beta-1)^{e-\beta}$$
$$(1-p(n))^{e}\left(\frac{\lambda}{3}(A_{\mathrm{b1}}+A_{\mathrm{b2}}+A_{\mathrm{b3}})-y_n^2p(n)\right)$$

Thus, total fuzzy profit $\big[\mathrm{TP}^{c}_{\mathrm{b}^*}(y_n)\big]$ defined in Eq. (24) is concave function with respect to order quantity which is shown analytically with the help of the graph (Fig. 2).

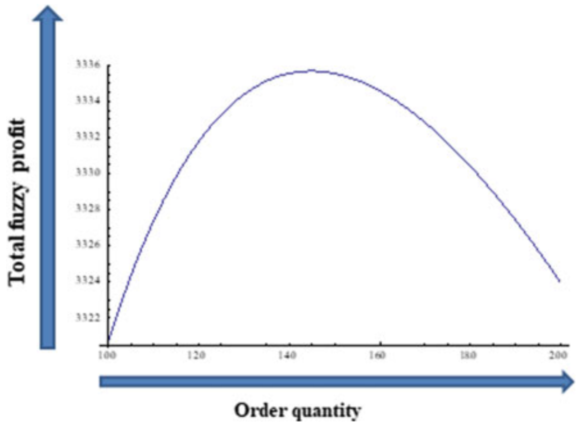

Fig. 2 Plot of fuzzy buyer's profit function with respect to order quantity

4.2 Seller's Fuzzy Model

The objective of the present model is to optimize the seller's price with the corresponding profit for the seller with learning effect under fuzzy environment.

Seller's total profit per cycle in fuzzy environment is given by

$$\mathrm{TP}^{c}_{\mathrm{s}*}(c_{\mathrm{b}}) = kp_{\mathrm{b}}^{-e}M^{\beta}\left(c_{\mathrm{b}} - C - \frac{\widetilde{A}_{\mathrm{s}}}{y_n}\right) \tag{25}$$

$$\mathrm{TP}^{c}_{\mathrm{s1}}(c_{\mathrm{b}}) = kp_{\mathrm{b}}^{-e}M^{\beta}\left(c_{\mathrm{b}} - C - \frac{A_{\mathrm{s1}}}{y_n}\right) \tag{26}$$

$$\mathrm{TP}^{c}_{\mathrm{s2}}(c_{\mathrm{b}}) = kp_{\mathrm{b}}^{-e}M^{\beta}\left(c_{\mathrm{b}} - C - \frac{A_{\mathrm{s2}}}{y_n}\right) \tag{27}$$

$$\mathrm{TP}^{c}_{\mathrm{s3}}(c_{\mathrm{b}}) = kp_{\mathrm{b}}^{-e}M^{\beta}\left(c_{\mathrm{b}} - C - \frac{A_{\mathrm{s3}}}{y_n}\right) \tag{28}$$

Now we defuzzify the entire profit per unit time by centroid method

$$\mathrm{TP}^{c}_{\mathrm{s}*}(c_{\mathrm{b}}) = \frac{\mathrm{TP}^{c}_{\mathrm{s1}}(c_{\mathrm{b}}) + \mathrm{TP}^{c}_{\mathrm{s2}}(c_{\mathrm{b}}) + \mathrm{TP}^{c}_{\mathrm{s3}}(c_{\mathrm{b}})}{3} \tag{29}$$

Substituting the values from Eqs. (26), (27), and (28) in Eq. (25), we get

$$\begin{aligned}\mathrm{TP}^{c}_{\mathrm{s}*}(c_{\mathrm{b}}) &= \left\{\frac{kp_{\mathrm{b}}^{-e}M^{\beta}\left(c_{\mathrm{b}} - C - \frac{A_{\mathrm{s1}}}{y_n}\right) + kp_{\mathrm{b}}^{-e}M^{\beta}\left(c_{\mathrm{b}} - C - \frac{A_{\mathrm{s2}}}{y_n}\right) + kp_{\mathrm{b}}^{-e}M^{\beta}\left(c_{\mathrm{b}} - C - \frac{A_{\mathrm{s3}}}{y_n}\right)}{3}\right\} \\ &= \frac{kp_{\mathrm{b}}^{-e}M^{\beta}}{3y_n}(A_{\mathrm{s1}} + A_{\mathrm{s2}} + A_{\mathrm{s3}}) + kp_{\mathrm{b}}^{-e}M^{\beta}(c_{\mathrm{b}} - C)\end{aligned} \tag{30}$$

4.3 Seller's Stackelberg Fuzzy Model

Seller is the dominant player. The seller's main aim is to find his profit on the basis of given buyer's decision variables. The problem is,

$$\text{Max } \mathrm{TP}^{c}_{\mathrm{s}*}(c_{\mathrm{b}}) = \frac{kp_{\mathrm{b}}^{-e}M^{\beta}}{3y_n}(A_{\mathrm{s1}} + A_{\mathrm{s2}} + A_{\mathrm{s3}}) + kp_{\mathrm{b}}^{-e}M^{\beta}(c_{\mathrm{b}} - C) \tag{31}$$

Subject to

$$M = \frac{\beta}{(e - \beta - 1)}\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{(A_{\mathrm{b1}} + A_{\mathrm{b2}} + A_{\mathrm{b3}})}{3y_n} + \frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda} - c_{\mathrm{s}}p(n)\right] \tag{32}$$

$$p_{\mathrm{b}} = \frac{e}{(e-\beta-1)(1-p(n))}\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{(A_{\mathrm{b}1}+A_{\mathrm{b}2}+A_{\mathrm{b}3})}{3y_n} + \frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda} - c_{\mathrm{s}}p(n)\right] \tag{33}$$

Constraints

$$y_n^2 Ic_{\mathrm{b}}\big((1-p(n))^2\big)\lambda = 2ke^{-e}\beta^{\beta}\left(\left[c_{\mathrm{b}} + s_{\mathrm{c}} + \frac{(A_{\mathrm{b}1}+A_{\mathrm{b}2}+A_{\mathrm{b}3})}{3y_n} + \frac{Ic_{\mathrm{b}}p(n)y_n}{\lambda} - c_{\mathrm{s}}p(n)\right]\right)^{\beta-e}(e-\beta-1)^{e-\beta}$$
$$(1-p(n))^{e}\left(\lambda\frac{(A_{\mathrm{b}1}+A_{\mathrm{b}2}+A_{\mathrm{b}3})}{3} - y_n^2 p(n)\right) \tag{34}$$

4.4 *The Buyer's Stackelberg Fuzzy Model*

The buyer is the dominant player. The buyer's main aim is to find his profit on the basis of given seller's decision variables. The problem is,

$$\text{Max } \mathrm{TP}^{c}_{\mathrm{b}^*}(p_{\mathrm{b}}, M, y_n) = \frac{(A_{\mathrm{b}1}+A_{\mathrm{b}2}+A_{\mathrm{b}3})}{3}\left(\frac{kp_{\mathrm{b}}^{-e}M^{\beta}}{(1-p(n))y_n}\right)$$
$$+\left[kp_{\mathrm{b}}^{-e+1}M^{\beta} + \frac{kp_{\mathrm{b}}^{-e}M^{\beta}}{(1-p(n))}\left[c_{\mathrm{s}}p(n) - c_{\mathrm{b}} - s_{\mathrm{c}} - M - \frac{p(n)y_n}{\lambda}Ic_{\mathrm{b}}\right]\right.$$
$$\left.-\left(\frac{y_n(1-p(n))^2}{2(1-p(n))}Ic_{\mathrm{b}}\right)\right] \tag{35}$$

Subject to

$$\text{At } c_{\mathrm{b}_0} = F\left(C + \frac{(A_{\mathrm{s}1}+A_{\mathrm{s}2}+A_{\mathrm{s}3})}{3y_n}\right) \tag{36}$$

5 Numerical Examples

Example 1

The seller-Stackelberg game model is shown in the given example which shows the effect of learning on the decision variables. Input parameters are taken from two papers Esmaeili et al. (2009) and Jaber et al. (2008), C = \$1.5 units, A_{b} = \$38, A_{s} = \$40, k = 36,080, F = 1.8, λ = 175,200 unit/year, c_{s} = \$3.5, $\beta = Le$, e = 1.7, L = 0.088, Sc = \$0.035, I = 0.38, F = 1.8, a = 40, b =

1.8, $n = 5$, $g = 999$, $s = 2.99$, $p(n) = 0.0019$. Equation (13) gives the results, $y_n = 138$ units and $c_b = \$4.291$. Equations (11) and (12) produce the results, $p_b = \$14.227$ and $M = \$1.252$. The seller's profit, $TP_s^c = \$1023.08$ and the buyer's profit, $TP_b^c = \$3309.80$.

Example 2

The buyer-Stackelberg game model is shown in the given example which shows the effect of learning on the decision variables. We consider the values of all parameters are same as defined in Example 1 except $c_s = 2.5$. Equation (14) gives the results, $p_b = \$9.280$, $M = \$0.817$, and $y_n = 413$ units. Equation (15) generates the results, $c_b = \$2.874$. Seller's profit, $TP_s^c = \$1013.11$ and buyer's profit, $TP_b^c = \$4103.86$.

Fuzzy Numerical Example 3

Effect of learning on the decision variables in fuzzy seller-Stackelberg game model is shown in the given example. Input parameters are taken from two papers Esmaeili et al. (2009) and Jaber et al. (2008), $C = \$1.5$ units, $\widetilde{A_b} = (35, 40, 45)$, $\widetilde{A_s} = (45, 50, 55)$, $k = 36{,}080$, $F = 1.8$, $\lambda = 175{,}200$ unit/year, $c_s = \$3.5$, $\beta = Le$, $e = 1.7$, $L = 0.088$, $Sc = \$0.035$, $I = 0.38$, $F = 1.8$, $a = 40$, $b = 1.8$, $n = 5$, $g = 999$, $s = 2.99$, $p(n) = 0.0019$. Equation (34) gives the results, $y_n = 141$ units and $c_b = \$4.320$. Equations (32) and (33) produce the results, $p_b = \$14.345$ and $M = \$1.263$. Fuzzy seller's profit, $TP_{s*}^c = \$999.50$ and fuzzy buyer's profit, $TP_{b*}^c = \$3291.34$.

Fuzzy Numerical Example 4

Effect of learning on the decision variables in fuzzy buyer-Stackelberg game model is shown in the given example. We consider the values of all parameters are same as defined in Example 1 except $c_s = 2.5$. Equation (35) gives the results, $p_b = \$9.357$, $M = \$0.824$, and $y_n = 446$ units and Eq. (36) generates the results, $c_b = \$2.902$. Fuzzy seller's profit, $TP_{s*}^c = \$ = \$1009.72$ and fuzzy buyer's profit, $TP_{b*}^c = \$4063.70$.

Results indicate that the high seller's selling price results the more gain in the profit to the seller in seller Stackelberg model. Result shows that seller got higher profit when he is leader and less when he is follower, whereas results also show that higher profit gained by the purchaser shows that he is better off in the second model. In both the cases, buyer got more profit as compared to the player seller due to the learning effect.

In case of fuzzy environment, result shows that buyer is more benefited when he is leader but seller got more in case of follower. When we compare crisp model example with fuzzy example, we conclude that both the players obtain less profit in fuzzy as compared to crisp model example.

6 Sensitivity Analysis

In this section, sensitivity analysis is carry out on the basis of key factors/parameters to estimate the strength of the model. This part shows the effect of learning rate on the different decision variables and profit of the players.

6.1 *Effect of Learning on the Player's Profit*

Seller-Stackelberg

See Table 1.

Buyer-Stackelberg

See Table 2.

Table 1 Effect of learning rate on the parameter p_b, M, Q, C_b, $[TP_s^c]$ and $[TP_b^c]$ with learning rate $b = 1.8$

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Buyer profit $[TP_b^c]$	Seller profit $[TP_s^c]$
1	0.0397	153	4.064	13.545	1.147	3382.30	1010.13
2	0.0381	152	4.074	13.576	1.152	3378.80	1010.66
3	0.0291	149	4.127	13.740	1.177	3360.79	1013.53
4	0.0107	142	4.237	14.069	1.228	3325.97	1019.83
5	0.0019	138	4.291	14.227	1.252	3309.80	1023.08

Table 2 Effect of learning rate on the parameter p_b, M, Q, C_b, $[TP_s^c]$ and $[TP_b^c]$ with learning rate $b = 1.8$

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Buyer profit $[TP_b^c]$	Seller profit $[TP_s^c]$
1	0.0397	427	2.868	9.3134	0.789	4072.04	999.648
2	0.0381	426	2.869	9.3118	0.790	4073.49	1000.24
3	0.0291	423	2.870	9.303	0.797	4081.05	1003.59
4	0.0107	416	2.873	9.287	0.810	4096.44	1010.08
5	0.0019	413	2.874	9.280	0.817	4103.86	1013.11

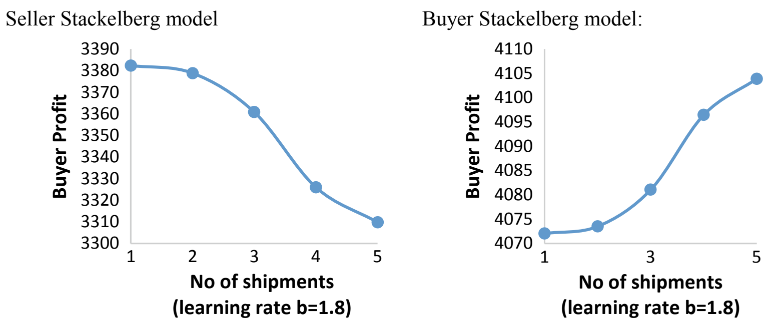

Fig. 3 Effect of shipments on buyer's profit

Effect of no. of shipments and learning rate on buyer's profit in both Stackelberg models (Fig. 3).

6.2 *Fuzzy Seller-Stackelberg*

See Table 3.

Fuzzy Buyer-Stackelberg

See Table 4.

Fuzzy Seller-Stackelberg

See Table 5.

Fuzzy Buyer-Stackelberg

See Table 6.

Table 3 Effect of learning rate on the parameter $p_b, M, Q, C_b, [TP_s^c]$ and $[TP_b^c]$ in fuzzy environment ($A_b = 40,\ A_s = 50,$ learning rate $b = 1.8$)

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Fuzzy buyer profit $[TP_b^c]$	Fuzzy seller profit $[TP_s^c]$
1	0.0397	155	4.092	13.690	1.1599	3359.38	982.49
2	0.0381	154	4.102	13.701	1.1629	3359.26	982.98
3	0.0291	151	4.156	13.862	1.187	3341.51	985.66
4	0.0107	144	4.266	14.189	1.238	3307.24	991.54
5	0.0019	141	4.320	14.345	1.263	3291.34	999.50

Table 4 Effect of learning rate on the parameter $p_b, M, Q, C_b, [TP_s^c]$ and $[TP_b^c]$ in fuzzy environment ($A_b = 40, A_s = 50$, learning rate $b = 1.8$)

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Fuzzy buyer profit $[TP_b^c]$	Fuzzy seller profit $[TP_s^c]$
1	0.0397	461	2.895	9.392	0.796	4032.22	995.93
2	0.0381	460	2.896	9.389	0.797	4033.65	996.61
3	0.0291	457	2.897	9.381	0.804	4041.14	999.90
4	0.0107	450	2.900	9.365	0.817	4056.36	1006.41
5	0.0019	446	2.902	9.357	0.824	4063.70	1009.72

Table 5 Effect of learning rate on the parameter $p_b, M, Q, C_b, [TP_s^c]$ and $[TP_b^c]$ in fuzzy environment ($A_b = 50, A_s = 60$, learning rate $b = 1.8$)

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Fuzzy buyer profit $[TP_b^c]$	Fuzzy seller profit $[TP_s^c]$
1	0.0397	173	4.092	13.768	1.166	3335.32	960.14
2	0.0381	172	4.102	13.801	1.171	3331.97	960.60
3	0.0291	168	4.156	13.965	1.196	3314.41	963.13
4	0.0107	160	4.265	14.295	1.247	3280.54	968.70
5	0.0019	156	4.319	14.455	1.273	4264.76	971.59

Table 6 Effect of learning rate on the parameter $p_b, M, Q, C_b, [TP_s^c]$ and $[TP_b^c]$ in fuzzy environment ($A_b = 50, A_s = 60$, learning rate $b = 1.8$)

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Fuzzy buyer profit $[TP_b^c]$	Fuzzy seller profit $[TP_s^c]$
1	0.0397	504	2.914	9.493	0.804	3982.57	985.96
2	0.0381	503	2.915	9.491	0.806	3983.98	986.62
3	0.0291	500	2.916	9.483	0.812	3991.38	989.74
4	0.0107	492	2.920	9.466	0.826	4006.41	996.46
5	0.0019	488	2.921	9.458	0.833	4013.67	999.69

Table 7 Effect of learning rate on the parameter $p_b, M, Q, C_b, \left[TP_s^c\right]$ and $\left[TP_b^c\right]$ in fuzzy environment ($A_b = 60,\ A_s = 70,$ learning rate $b = 1.8$)

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Fuzzy buyer profit $\left[TP_b^c\right]$	Fuzzy seller profit $\left[TP_s^c\right]$
1	0.0397	188	4.093	13.868	1.175	3310.39	939.63
2	0.0381	187	4.103	13.900	1.179	3307.05	940.07
3	0.0291	182	4.156	14.068	1.205	3289.78	942.46
4	0.0107	174	4.265	14.368	1.254	3260.09	947.75
5	0.0019	170	4.318	14.553	1.281	3240.75	950.49

Table 8 Effect of learning rate on the parameter $p_b, M, Q, C_b, \left[TP_s^c\right]$ and $\left[TP_b^c\right]$ in fuzzy environment ($A_b = 60, A_s = 70,$ learning rate $b = 1.8$)

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Fuzzy buyer profit $\left[TP_b^c\right]$	Fuzzy seller profit $\left[TP_s^c\right]$
1	0.0397	543	2.932	9.587	0.812	3937.22	976.85
2	0.0381	542	2.932	9.585	0.816	3938.61	977.61
3	0.0291	538	2.934	9.577	0.820	3945.92	980.74
4	0.0107	529	2.938	9.559	0.834	3960.78	987.73
5	0.0019	525	2.940	9.552	0.841	3967.95	990.81

Fuzzy Seller-Stackelberg

See Table 7.

Fuzzy Buyer-Stackelberg

See Table 8.

7 Observations

Following are the observations

(a) Results indicate from example 1 and example 2 that both the players are better off when they are leader and they got less profit when they are follower.

(b) Numerical example shows that seller profit and buyer profit obtained in seller-Stackelberg model and buyer-Stackelberg model are more as compared to obtained in fuzzy Stackelberg model.

(c) Figure 3 concludes that as number of shipments increases with a given learning rate at $b = 1.8$, buyer profit decreases in seller-Stackelberg model, whereas, Fig. 3 illustrates that buyer profit and seller profit increase as number of shipments increases with same learning rate in buyer-Stackelberg model.
(d) Data from Table 3 designate that seller-Stackelberg model under fuzzy environment as shipment increases in numbers, the buyer's profit decreases whereas seller's profit increases. This means that seller get benefitted in case of headship position.
(e) Data from Table 4 indicate that in buyer-Stackelberg model under fuzzy environment as shipment increases in numbers, the buyer's profit and seller's profit increases. This means both the players get benefitted in fuzzy environment.

8 Conclusions

Two-level supply chain models have been established for imperfect quality items under fuzzy environment with learning effect environment The effect of learning and fuzziness is shown on the players' optimal policies. Buyer's price, marketing expenditure cost, and order quantity and corresponding profit of players of supply chain are optimized. The learning impact on the calculation of gains or losses of the supply chain has been shown in the sensitivity analysis and numerical example. Results show that due to learning effect, buyer's gain is more than the seller in both the model. Both the players get benefitted in case of leadership position. It is shown from the result that seller's profit and buyer's profit obtained in mathematical crisp model is more that obtained in fuzzy model. A future extension to present model can be assume a stochastic learning curve instead of deterministic. This model can be extended by considering the idea of shortages and trade credit period.

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