Fuzzy Inventory Model for Deteriorating Items in a Supply Chain System with Time Dependent Demand Rate

Srabani Shee and Tripti Chakrabarti

Abstract An inventory model for a single deteriorating item under fuzzy environment has been presented in this paper. Here demand rate is considered to be constant for some time period, post which the same is a linear function of time. This situation is common during the time of a new product launch in the market. As the product becomes popular, its demand increases with time although it remains constant during the initial days. Cycle time is considered to be constant in most of the models. However, practically it has been observed that it is difficult to pro-actively predict the cycle time. Because of this problem, cycle time has been considered as uncertain and has been further described as Symmetric Triangular Fuzzy number. The Signed Distance method has been used for defuzzification of the total cost function. For illustration of the process for finding the total optimal cost and the cycle time, numerical examples have been considered. The effects of changing parameter values on the optimal solution of the system have been demonstrated through Sensitivity Analysis.

Keywords Supply chain management · Constant and time dependent demand rate · Deterioration · Symmetric triangular fuzzy number · Signed distance method

1 Introduction

The most important and difficult role that inventory plays in supply chain is that of facilitating the balancing of demand and supply. To effectively manage the forward and reverse flows in the supply chain, firms have to deal with upstream supplier exchanges and downstream customer demands. Uncertainty is another key issue to deal with in order to define effective Supply Chain inventory policies. Demand, supply (e.g., lead time), various relevant cost, backorder costs, deterioration rate, etc.

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are usually uncertain. To solve these types of practical problems, we use the Fuzzy Set Theory. Bellman and Zadeh (1970) first studied fuzzy set theory to solve decision making problem. Then, Dubois and Prade (1978) introduced some operations on fuzzy number. Thereafter, Park (1987) developed fuzzy set theoretical interpretation of EOQ. Several researchers like Wu and Yao (2003), Wang et al. (2007), Hu et al. (2010), Jaggi et al. (2013), Yao and Chiang (2003), Wang et al. (2007), Kao and Hsu (2002), Dutta et al. (2005), Roy and Samanta (2009) have developed different types of inventory model under Fuzzy environment. In this area, a lot of research papers have been published by several researchers, viz. Bera et al. (2013), He et al. (2013), Dutta and Kumar (2015), Mishra et al. (2015), etc. Priyan and Manivannan (2017) developed an optimal inventory modeling of supply chain system involving quality inspection errors in fuzzy situation.

Lin et al. (2000) and Mishra et al. (2015) developed an economic order quantity model that focused on time varying demand and deteriorating items. After that, Ghosh and Chaudhuri (2004) proposed an inventory model with Weibull distribution rate of deterioration, time quadratic demand and shortages. A lot of research papers have been published by several researchers, viz. Wang and Chen (2001), Pal et al. (2006), Bera et al. (2013), He et al. (2013), Dutta and Kumar (2015), etc.

This paper has presented a Fuzzy supply chain inventory model in which the demand rate is constant for some time and then it increases or decreases according to the popularity of the product. This type of situation occurs when a new product is launched in the market. When the product becomes popular the demand of the product increases with time. It is also assumed that the cycle time is taken as Symmetric Triangular Fuzzy number. In addition, expressions for order quantity, cycle time and the total average cost (for both the models) are obtained. The convexity of the total cost function is established to ensure the existence of a unique optimal solution. The problem is solved by using LINGO 17.0 software.

2 Assumptions and Notations

The proposed model is developed under the following notations and assumptions:

Notations

1. $I(t)$ is the inventory level at time $t(\geq 0)$.
2. Demand $R(t) = \begin{cases} a, & \text{for } 0 \leq t \leq \mu \\ a + b(t - \mu), & \text{for } \mu \leq t \leq T \end{cases}$.
3. θ is the rate of deterioration.
4. q is the number of items received at the beginning of the period.
5. C is the deterioration cost per unit.
6. C_1 is the inventory holding cost per unit per-unit-time.
7. C_2 is the setup cost per cycle.
8. μ is the time point at which deterioration starts and also demand increases with time.

9. T is the cycle length.
10. $\tilde{T}$ is the fuzzy cycle length.
11. $K(t)$ is the total inventory cost of the system per unit time.
12. $\widetilde{K(t)}$ is the fuzzy total inventory cost of the system.

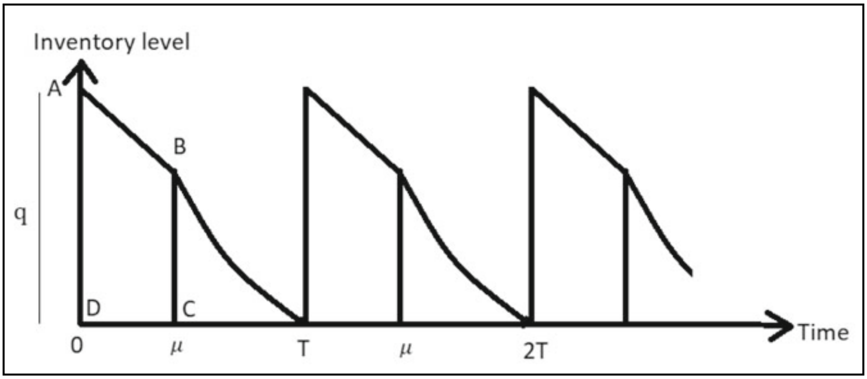
Assumptions

1. The deterioration cost, holding cost and ordering cost remain constant over time.
2. There is no deterioration for the period $[0, \mu]$. The deterioration rate is constant, say θ, for the period $[\mu, T]$, which is practically very small.
3. A single item is considered over a prescribed period of T units of time.
4. The cycle time is uncertain and we assume it as symmetric triangular fuzzy number.
5. The replenishment is instantaneous.
6. Lead time is zero.
7. There is no replacement or repair of deteriorated items.
8. Shortage is not allowed.

3 Mathematical Model

The inventory cycle starts at time $t = 0$ with the inventory level q. During the time interval $[0, \mu]$, the inventory level decreases due to the constant demand a units per unit time. After time $t = \mu$, the inventory level gradually decreases mainly to meet demands and partly for deterioration and falls to zero at time $t = T$. The cycle then repeats itself after time T.

This model is represented by the following diagram:



Now, the total demand for the time period $[0, \mu]$, is $= a\mu$.

Therefore, the inventory level is decreased by the factor $a\mu$ and $(q - a\mu)$ inventory is left for the time period $[\mu, T]$.

The holding cost for the period $[0, \mu]$ is

$$= C_1(\text{Area of trapezium } ABCD)$$
$$= C_1 \cdot \frac{1}{2}[q + (q - a\mu)]\mu$$
$$= C_1\mu\left[(q - a\mu) + \frac{a\mu}{2}\right]$$

Then, the differential equation governing the instantaneous state of $I(t)$ during the time interval $\mu \le t \le t_1$ is,

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -\theta I(t) - [a + b(t - \mu)], \quad 0 \le t \le t_1 \tag{1}$$

where $t_1 = (T - \mu)$, the origin has been shifted just for the sake of mathematical simplicity.

With the boundary conditions, $t = 0,\ I(t) = (q - a\mu)$ and $t = t_1,\ I(t) = 0$.

Solving the differential equation we get,

$$\mathrm{e}^{\theta t} I(t) - (q - a\mu) = -\int_0^t [a + b(t - \mu)]\mathrm{e}^{\theta t}\mathrm{d}t$$

At $t = t_1,\ I(t) = 0$

$$\therefore (q - a\mu) = \int_0^{t_1} [a + b(t - \mu)]\mathrm{e}^{\theta t}\mathrm{d}t \tag{2}$$

We know that $\mathrm{e}^{\theta t} = \sum_{n=0}^{\infty} \frac{(\theta t)^n}{n!}$. Using this exponential expansion in Eq. (2) and then integrating term bay term we have,

$$(q - a\mu) = (a - b\mu)\sum_{n=0}^{\infty} \frac{\theta^n}{n!}\frac{t_1^{n+1}}{n+1} + b\sum_{n=0}^{\infty} \frac{\theta^n}{n!}\frac{t_1^{n+2}}{n+2} \tag{3}$$

Now, the holding cost for the time period $(0, t_1)$ is

$$= C_1\frac{1}{2}(q - a\mu)t_1$$

Total amount of inventory that has deteriorated during this cycle is

$$= (q - a\mu) - \int_0^{t_1} [a + b(t - \mu)]\mathrm{e}^{\theta t}\mathrm{d}t$$

$$= (q - a\mu) - (a - b\mu)t_1 - \frac{1}{2}bt_1^2 \tag{4}$$

Therefore, the total inventory cost per unit time is,

$$\begin{aligned}
K(T) &= \text{inventory carrying cost} + \text{deterioration cost} + \text{set up cost} \\
&= \frac{1}{T}\Bigg[C_1\mu(q - a\mu) + C_1\frac{a\mu^2}{2} + \frac{1}{2}C_1(q - a\mu)t_1 \\
&\quad + C\left\{(q - a\mu) - (a - b\mu)t_1 - \frac{1}{2}bt_1^2\right\} + C_2\Bigg] \\
&= \frac{1}{T}\left[(q - a\mu)\left\{C_1\mu + \frac{1}{2}C_1t_1 + C\right\} + C_1\frac{a\mu^2}{2} - C(a - b\mu)t_1 - \frac{C}{2}bt_1^2 + C_2\right] \\
&= \frac{1}{T}\Bigg[(C_1\mu + C)\left\{(a - b\mu)\sum_{n=0}^{\infty}\frac{\theta^n}{n!}\frac{(T-\mu)^{n+1}}{n+1} + b\sum_{n=0}^{\infty}\frac{\theta^n}{n!}\frac{(T-\mu)^{n+2}}{n+2}\right\} \\
&\quad + \frac{C_1}{2}\left\{(a - b\mu)\sum_{n=0}^{\infty}\frac{\theta^n}{n!}\frac{(T-\mu)^{n+2}}{n+1} + b\sum_{n=0}^{\infty}\frac{\theta^n}{n!}\frac{(T-\mu)^{n+3}}{n+2}\right\} \\
&\quad + C_1\frac{a\mu^2}{2} - C(a - b\mu)(T - \mu) - \frac{C}{2}b(T - \mu)^2 + C_2\Bigg]
\end{aligned}$$

Since θ is very small, the terms involving θ^n with $n(> 1)$ can be neglected. Hence, retaining the terms in the summation for $n = 0$ and $n = 1$ only, we have,

$$\begin{aligned}
K(T) &= \frac{1}{T}\Bigg[P\left\{A(T-\mu) + \frac{A\theta}{2}(T-\mu)^2 + \frac{b}{2}(T-\mu)^2 + \frac{b\theta}{3}(T-\mu)^3\right\} \\
&\quad + \frac{C_1}{2}\left\{A(T-\mu)^2 + \frac{A\theta}{2}(T-\mu)^3 + \frac{b}{2}(T-\mu)^3 + \frac{b\theta}{3}(T-\mu)^4\right\} \\
&\quad + C_1\frac{a\mu^2}{2} - CAT + CA\mu - \frac{C}{2}b(T-\mu)^2 + C_2\Bigg] \\
&= \frac{C_1b\theta}{6}T^3 + \left(-\frac{2}{3}\mu C_1b\theta + \frac{Pb\theta}{3} + \frac{C_1A\theta}{4} + \frac{C_1b}{4}\right)T^2 \\
&\quad + \left(C_1b\theta\mu^2 - Pb\theta\mu - \frac{3C_1A\theta\mu}{4} - \frac{3C_1b\mu}{4} + \frac{PA\theta}{2} + \frac{Pb}{2} + \frac{C_1A}{2} - \frac{Cb}{2}\right)T \\
&\quad + \left(-\frac{2}{3}C_1b\theta\mu^3 + Pb\theta\mu^2 + \frac{3}{4}C_1A\theta\mu^2 + \frac{3}{4}C_1b\mu^2 - PA\theta\mu - Pb\mu\right. \\
&\quad \left. - C_1A\mu + Cb\mu + PA - CA\right) + \left(\frac{C_1b\theta\mu^4}{6} - \frac{Pb\theta\mu^3}{3} - \frac{C_1A\theta\mu^3}{4} - \frac{C_1b\mu^3}{4}\right. \\
&\quad \left. + \frac{PA\theta\mu^2}{2} + \frac{Pb\mu^2}{2} + \frac{C_1A\mu^2}{2} - \frac{Cb\mu^2}{2} - PA\mu + CA\mu + C_1\frac{a\mu^2}{2} + C_2\right)\frac{1}{T} \\
&= U_1T^3 + V_1T^2 + W_1T + X_1 + Y_1\frac{1}{T}
\end{aligned} \tag{5}$$

where, $P = (C_1\mu + C)$ and $A = (a - b\mu)$

$U_1 = \frac{C_1b\theta}{6}$

$V_1 = \left(-\frac{2}{3}\mu C_1b\theta + \frac{Pb\theta}{3} + \frac{C_1A\theta}{4} + \frac{C_1b}{4}\right)$

$$W_1 = \left(C_1 b\theta\mu^2 - Pb\theta\mu - \tfrac{3C_1 A\theta\mu}{4} - \tfrac{3C_1 b\mu}{4} + \tfrac{PA\theta}{2} + \tfrac{Pb}{2} + \tfrac{C_1 A}{2} - \tfrac{Cb}{2}\right)$$

$$X_1 = \left(-\frac{2}{3}C_1 b\theta\mu^3 + Pb\theta\mu^2 + \frac{3}{4}C_1 A\theta\mu^2 + \frac{3}{4}C_1 b\mu^2 - PA\theta\mu - Pb\mu\right.$$
$$\left.-C_1 A\mu + Cb\mu + PA - CA\right)$$

$$Y_1 = \left(\frac{C_1 b\theta\mu^4}{6} - \frac{Pb\theta\mu^3}{3} - \frac{C_1 A\theta\mu^3}{4} - \frac{C_1 b\mu^3}{4} + \frac{PA\theta\mu^2}{2} + \frac{Pb\mu^2}{2} + \frac{C_1 A\mu^2}{2}\right.$$
$$\left.-\frac{Cb\mu^2}{2} - PA\mu + CA\mu + C_1\frac{a\mu^2}{2} + C_2\right)$$

Now, let us describe the cycle time T as triangular fuzzy number $\tilde{T} = (T - \Delta, T, T + \Delta)$.

So, from Eq. (5) the total Fuzzy cost function is

$$\widetilde{K(T)} = U_1\tilde{T}^3 + V_1\tilde{T}^2 + W_1\tilde{T} + X_1 + Y_1\frac{1}{\tilde{T}} \tag{6}$$

From the definition of the signed distance method, we have,

$$\mathrm{d}\left(\tilde{A}, 0\right) = \frac{1}{2}\int_0^1 [A_\mathrm{L}(\alpha) + A_\mathrm{U}(\alpha)]\mathrm{d}\alpha$$

where, $\tilde{A} = (a, b, c),\ A_\mathrm{L}(\alpha) = a + (b - a)\alpha,\ A_\mathrm{U}(\alpha) = c - (c - b)\alpha.$

Now, $T_\mathrm{L}(\alpha) = (T - \Delta) + \Delta\alpha,\ T_\mathrm{U}(\alpha) = (T + \Delta) - \Delta\alpha.$

Therefore,

$$\begin{aligned}\mathrm{d}\left(\tilde{T}, 0\right) &= \frac{1}{2}\int_0^1 [T_\mathrm{L}(\alpha) + T_\mathrm{U}(\alpha)]\mathrm{d}\alpha \\ &= \frac{1}{2}\int_0^1 [(T - \Delta) + \Delta\alpha + (T + \Delta) - \Delta\alpha]\mathrm{d}\alpha \\ &= \frac{1}{2}\int_0^1 2T\,\mathrm{d}\alpha = T \end{aligned} \tag{7}$$

And

$$\mathrm{d}\left(\frac{1}{\tilde{T}}, 0\right) = \frac{1}{2}\int_0^1 \left[\left(\frac{1}{\tilde{T}}\right)_\mathrm{L}(\alpha) + \left(\frac{1}{\tilde{T}}\right)_\mathrm{U}(\alpha)\right]\mathrm{d}\alpha$$

$$= \frac{1}{2}\int_0^1 \left[\frac{1}{T + \Delta - \Delta\alpha} + \frac{1}{T - \Delta + \Delta\alpha}\right] \mathrm{d}\alpha$$

$$= \frac{1}{2\Delta} \ln\left(\frac{T + \Delta}{T - \Delta}\right) \tag{8}$$

From (6), (7) and (8) we have

$$\widetilde{K(T)} = U_1 T^3 + V_1 T^2 + W_1 T + X_1 + \frac{1}{2\Delta} Y_1 \ln\left(\frac{T + \Delta}{T - \Delta}\right) \tag{9}$$

To minimize $K(T)$ the necessary condition is

$$\frac{\mathrm{d}K(T)}{\mathrm{d}T} = 0$$

By simplifying $\frac{\mathrm{d}K(T)}{\mathrm{d}T} = 0$ we get a bi-quadratic equation in T, which is,

$$3U_1 T^4 + 2V_1 T^3 + W_1 T^2 - Y_1 = 0 \tag{10}$$

We can solve Eq. (5) by Newton–Raphson's method for a positive T (T^* say).
If $\frac{\mathrm{d}^2 K(T)}{\mathrm{d}T^2} > 0$ for $T = T^*$, then T^* will be an optimal solution.
Hence, $K(T)$ is strictly convex.
Substituting the value of $T = T^*$ in (5), the optimum average cost $K(T^*)$ can also be determined.

4 Numerical Example

To illustrate the results obtained for the suggested model, a numerical example with the following parameter values is considered.

$$a = 20\,\text{units},\ b = 0.2,\ \mu = 0.4\,\text{days},\ \theta = 0.02,$$

$$C = \text{Rs. } 18\,\text{per unit},$$

$$C_1 = \text{Rs. } 0.50\,\text{per unit per day},\ C_2 = \text{Rs. } 80.$$

We obtain for crisp model optimum total cost is $K(T^*) = 50.4065$ per day.
And cycle time is $T^* = 2.975$ days.
For fuzzy model total cost $\widetilde{K(T^*)} = 53.5294$ and cycle time $\widetilde{T^*} = 3.016$.
The convexity of the total cost function is shown in Fig. 1.

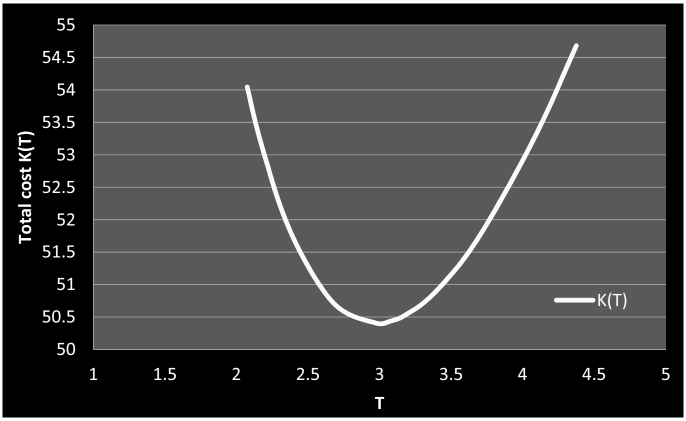


Fig. 1 Convexity of cost function w. r. t. T

5 Sensitivity Analysis

Sensitivities of the parameters are shown in Tables 1, 2, 3, 4 and 5 and graphically illustrated in Figs. 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11.

Table 1 Sensitivity on μ

Change value		Crisp model		Fuzzy model	
		$K(T^*)$	T^*	$\widetilde{K(T^*)}$	$\widetilde{T}^*$
μ	0.1	52.5818	2.954	53.5589	2.995
	0.2	51.8308	2.960	53.5247	3.001
	0.3	51.1057	2.967	53.5148	3.008
	0.4	50.4065	2.975	53.5294	3.016
	0.5	49.7332	2.985	53.5684	3.025
	0.6	49.0858	2.996	53.6318	3.036
	0.7	48.4643	3.009	53.7195	3.049

Table 2 Sensitivity on C_2

Change value		Crisp model		Fuzzy model	
		$K(T^*)$	T^*	$\widetilde{K(T^*)}$	$\widetilde{T}^*$
C_2	60	43.2216	2.591	46.3794	2.637
	70	46.9379	2.790	50.0764	2.834
	80	50.4065	2.975	53.5294	3.016
	90	53.6722	3.148	56.7822	3.187
	100	56.7680	3.312	59.8670	3.348

Table 3 Sensitivity on C_1

Change value		Crisp model		Fuzzy model	
		$K(T^*)$	T^*	$\widetilde{K(T^*)}$	$\widetilde{T^*}$
C_1	0.10	36.0927	4.069	39.0516	4.098
	0.30	43.8291	3.393	46.8647	3.428
	0.50	50.4065	2.975	53.5294	3.016
	0.70	56.2313	2.684	59.4507	2.729
	0.90	61.5158	2.465	64.8401	2.514

Table 4 Sensitivity on C

Change value		Crisp model		Fuzzy model	
		$K(T^*)$	T^*	$\widetilde{K(T^*)}$	$\widetilde{T^*}$
C	14	48.5454	3.112	50.0011	3.151
	16	49.4898	3.041	52.2789	3.081
	18	50.4065	2.975	53.5294	3.016
	20	51.2974	2.614	54.7544	2.955
	22	52.1641	2.855	55.9556	2.898

Table 5 Sensitivity on θ

Change value		Crisp model		Fuzzy model	
		$K(T^*)$	T^*	$\widetilde{K(T^*)}$	$\widetilde{T^*}$
θ	0.01	45.8050	3.349	47.4191	3.385
	0.015	48.1976	3.154	50.5652	3.183
	0.02	50.4065	2.975	53.5294	3.016
	0.025	52.4629	2.832	56.3430	2.875
	0.03	54.3902	2.709	59.0291	2.753

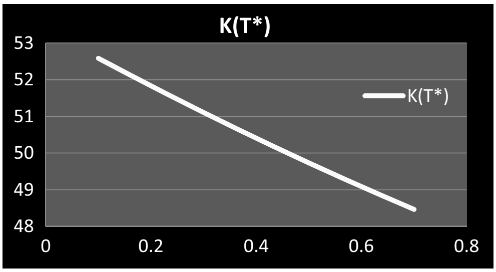
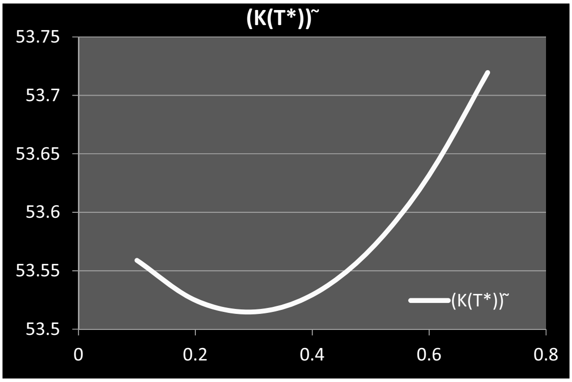


Fig. 2 Impact of μ on $K(T^*)$: crisp model (from Table: 1)

Fig. 3 Impact of μ on $\widetilde{K(T^*)}$: fuzzy model (from Table: 1)



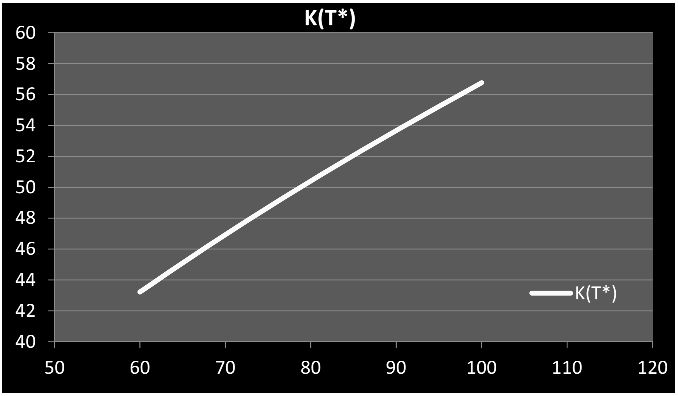
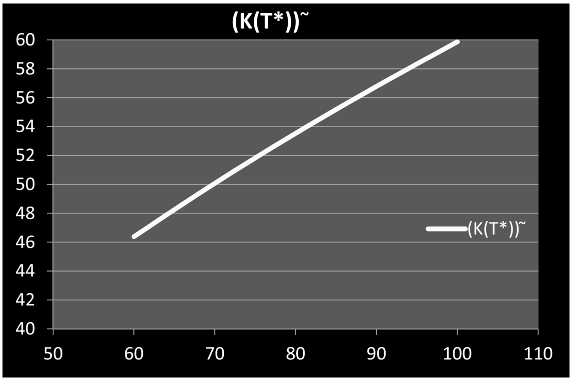


Fig. 4 Impact of C_2 on $K(T^*)$: crisp model (from Table: 2)

Fig. 5 Impact of C_2 on $\widetilde{K(T^*)}$: fuzzy model (from Table: 2)



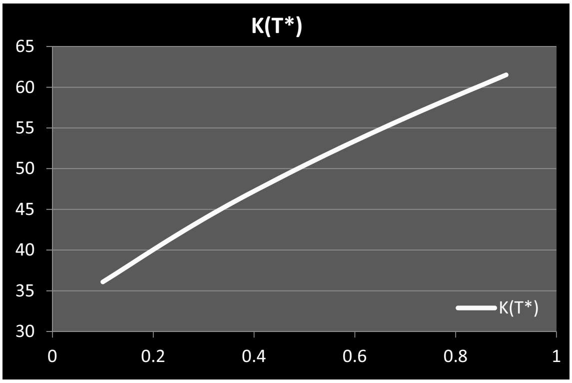


Fig. 6 Impact of C_1 on $K(T^*)$: crisp model (from Table: 3)

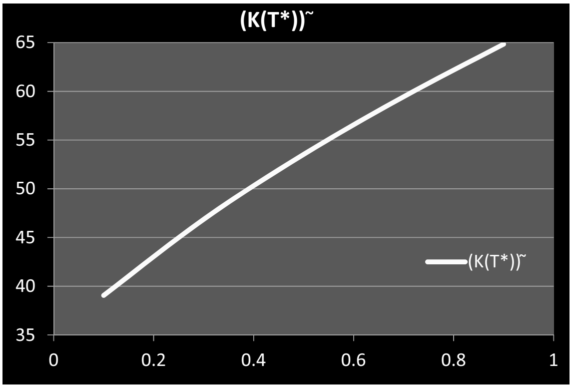


Fig. 7 Impact of C_1 on $\widetilde{K(T^*)}$: fuzzy model (from Table: 3)

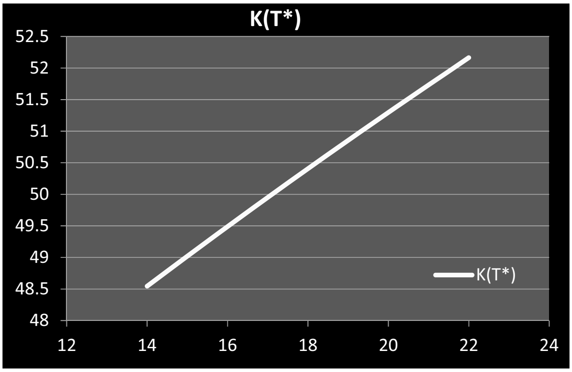


Fig. 8 Impact of C on $K(T^*)$: crisp model (from Table: 4)

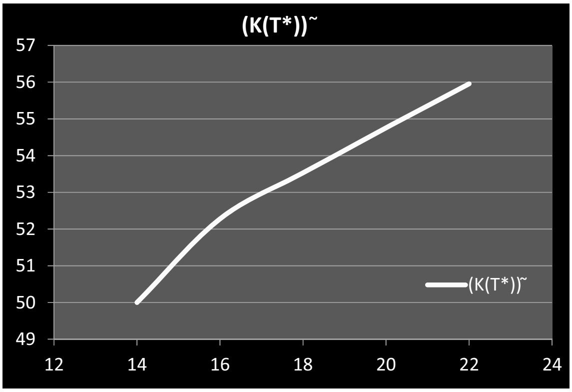


Fig. 9 Impact of C on $\widetilde{K(T^*)}$: fuzzy model (from Table: 4)

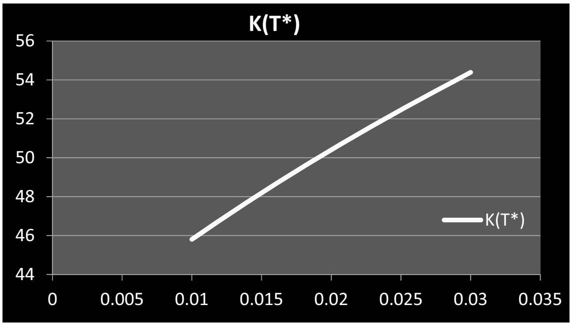


Fig. 10 Impact of θ on $K(T^*)$: crisp model (from Table: 5)

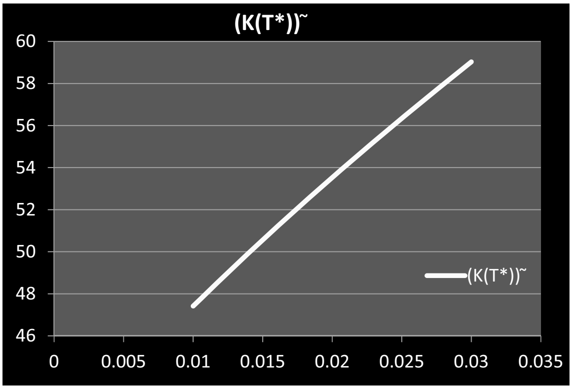


Fig. 11 Impact of θ on $\widetilde{K(T^*)}$: fuzzy model (from Table: 5)

Observations

It is observed from the tables that:

(i) In crisp model, if the parameter μ is increased (or decreased), the value of optimum cycle time increases (or decreases) while the optimal total cost decreases (or increases). Further, in fuzzy model, if the parameter μ

is increased (or decreased) the value of optimum cycle time increases (or decreases) while the optimal cost increases.

(ii) The increases (or decrease) in setup cost C_2 increases (or decreases) the total inventory cost for both the models.

(iii) The total cost (for both the models) increases (or decreases) as the holding cost C_1 per unit time increases (or decreases).

(iv) With the increase (or decrease) of the rate of deterioration θ, the total inventory cost (for the two models) also increase (or decrease).

(v) As the deterioration cost C per unit increase (or decrease), the total costs for the two models also increase (or decrease).

6 Conclusion

In the present chapter, we have dealt with a fuzzy inventory model where we have introduced the cycle time T as a Triangular Symmetric Fuzzy number. It is assumed the demand rate is constant for some time and then as a linear function of time. In our real life, we generally find that the cycle time is uncertain. So keeping this situation in mind we have tried to compare crisp model with the fuzzy model and have observed that the cycle time and the total cost obtained by fuzzy model is greater than those obtained by crisp model. The sensitivity analysis shows that the total cost of both the model increases as the cost associated with the model increases. In future, researchers can do more work about several types of demand, variable cost, etc.

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