A Novel Pythagorean Fuzzy MULTIMOORA Applied to the Evaluation of Energy Storage Technologies

Iman Mohamad Sharaf

1 Introduction

Nowadays, the decision-making process in organizations confronts many challenges due to the extensive changes and increasing complexity of issues faced in a rapidly developing business environment. In recent decades, researchers introduced several methods for multi-criteria decision-making (MCDM). These methods deal with the complexities faced in decision-making and facilitate this process. They also increase efficiency and improve the quality of the process [11].

Real-life decision-making problems are mainly composed of imprecise, and uncertain data together with the subjective evaluations of the decision-makers (DMs) which is based on their perceptions which is sure to differ from one person to another. To face the unrealistic supposition that exact numerical values are proper to model and handle real-life decision-making problems, Zadeh [65] introduced the notion of fuzzy sets, later named type-1 fuzzy sets (T1FSs).

Due to the deficiency of T1FSs to express the uncertainties and cognitive limitations in decision-making, different types of fuzzy sets were proposed by various researchers to model diverse situations of vagueness and ambiguity. For example, type-2 fuzzy sets (T2FSs) [66], neutrosophic sets (NFSs) [53], hesitant fuzzy sets (HFSs) [56, 57], and spherical fuzzy sets (SFSs) [26].

Atanassov [2] defined intuitionistic fuzzy sets (IFSs) as a more general form of fuzzy sets by adding the non-membership degree under the constraint that the sum of the membership degree and the non-membership degree is less than or equal to one. The indeterminacy degree is treated as a residual term such that the sum of the three degrees is equal to one. Although IFSs are proficient at imprecise treatment and inexact data, still, there are certain difficulties IFSs cannot handle. Atanassov [3]

I. M. Sharaf (✉)
Department of Basic Sciences, Higher Technological Institute, Ahmad Hamdy Street, First District, PO Box 228, Tenth of Ramadan City, Egypt

H. Garg (ed.), *Pythagorean Fuzzy Sets*,
https://doi.org/10.1007/978-981-16-1989-2_12

pointed out the possibility of changing the condition on the sum of the membership degree and the non-membership degree by increasing the power.

Yager and Abbasov [63] and Yager [62] discussed the model of Pythagorean fuzzy sets (PFSs) for a more human consistent reasoning under imperfect and imprecise data. In PFSs, the sum of the squares of the membership degree and the non-membership degree is less than or equal to one, and still, the indeterminacy degree is the square root of the residual term. The space of all PFSs includes IFSs. Thus, PFSs can be used more widely than IFSs in handling practical problems with imprecision and uncertainty.

Brauers and Zavadskas [5] proposed the MULTIMOORA (Multi-Objective Optimization by Ratio Analysis plus full multiplicative form) for MCDM. The MULTIMOORA method proved to be one of the most practical MCDM methods that have been applied in solving complex decision-making problems. It is a completely effective method for the evaluation and ranking of alternatives without subjective orientation in various phenomena [11]. It satisfies all the necessary robustness' conditions proposed by Brauers and Zavadskas [5] to become the most robust system of multi-objective optimization under the condition of support from the Ameliorated Nominal Group Technique and Delphi [5]. Due to its robustness and flexibility, MULTIMOORA was extended using different types of fuzzy sets and was applied to various practical fields. Throughout its application, it provided high efficiency and effectiveness in problem-solving [11].

The output of the MULTIMOORA results from ternary ranking techniques: ratio analysis, reference point theory, and full multiplicative form. Based on the scores of the three techniques, the alternatives are individually ranked in each technique. Then, using these individual rankings the rules of the dominance theory are applied to find the final ranking. However, the dominance theory has some limitations, e.g., multiple comparisons and circular reasoning [11].

As the ranking process using the dominance theory can be in some cases complex and challenging, it is not preferred in large-scale applications and the scores of the three techniques are aggregated instead [11, 68].

Up till now, when applying the reference point approach in the MULTIMOORA method, only the best solution is taken into consideration. Utilizing the two reference points, i.e., the best and worst solutions, was not previously studied.

When extended in the intuitionistic and Pythagorean fuzzy environment, the IF-MULTIMOORA and the PF-MULTIMOORA might have two main drawbacks. In the ratio analysis technique which exploits the additive utility, the weighted averaging operators play the main role. Most of these aggregation operators have a flaw that might result in a biased treatment and false ranking. For an alternative, a single criterion with the perfect rating (1, 0) will dominate regardless of its weight and abolish the effect of all the other criteria, which is not fair in the assessment process. Similarly, for the full multiplicative form in which the weighted geometric operators play the main role a single criterion with the worst rating (0, 1) will dominate regardless of its weight and abolish the effect of all the other criteria for an alternative, which is also not fair in an evaluation process.

In the last few decades, energy consumption increased and additional energy supplies are needed to balance the increasing demand. It was found that renewable energies are the best approach for the provision of energy due to their sustainable nature and broad utilization due to their diverse presence such as wind, solar, geothermal, bioenergy and hydropower. Yet, renewable sources usually cannot stand alone in power plants because of their intermittent nature and significant fluctuations, e.g., wind and solar energies. Energy storage technologies (ESTs) can solve this problem when coupled with renewable energy resources. ESTs improve the system's performance and increase the penetration of renewable energy sources. ESTs are continuously developed and different storage systems are established due to the multiple utilization of energy and the different types of applications [42]. The choice of a suitable EST is an MCDM problem since multiple technologies are defined for multiple conflicting criteria.

This chapter develops a new version MULTIMOORA in the Pythagorean fuzzy environment that eliminates the shortcomings of the previous versions. So far, when using the aggregation approach the result of each utility function is defuzzified using the score function before the aggregation for ranking. This can be mainly attributed to using the reference point approach which relies on the distance from the ideal situation that is always employed as a crisp value. In fact, being a distance between two fuzzy values it cannot be definitely determined. Hence, it is more appropriate to define distance on a fuzzy basis rather than a crisp basis. Consequently, this study will adopt the aggregation approach in which distances are utilized on a fuzzy basis. As a result, defuzzification is employed only in the final step for ranking. At this point, the accuracy function is also employed with the score function to make the comparison more discriminatory and to overcome any drawbacks that may be associated with the defuzzification using the score function solely. In addition, newly proposed aggregation operators are exploited. These operators guarantee fair treatment among the evaluation criteria since most of the aggregation operators have a flaw that might result in a biased treatment and false ranking in certain situations. A practical example that considers the evaluation of energy storage technologies is provided to illustrate the developed method and to make a comparative analysis.

From the previous discussion, the contribution of the study encompasses two main features. In the reference point approach, fuzzy distance is employed. This allows examining two reference points instead of one. The study also exploits aggregation operators that make the decision results more precise and exact.

The chapter is organized as follows. In Sect. 2, the literature is reviewed. Section 3 includes the basic concepts, definitions, and operators of PFSs together with the conventional MULTIMOORA. Section 4 explains the proposed PF-MULTIMOORA in detail. In Sect. 5, a practical example in the evaluation of energy storage technologies is provided to illustrate the newly developed method. Finally, the conclusion is given in Sect. 6.

2 Review of the Literature

2.1 The MULTIMOORA Method

Brauers and Zavadskas [4] proposed the MOORA method that combines two techniques, the ratio analysis, and the reference point methods. In the ratio analysis, the rating of each alternative to a criterion is compared to a denominator which is representative of all the alternatives concerning that criterion. While the reference point method measures the distance between the rating of an alternative for a criterion and a reference point. This reference point has the highest rating for maximization and the lowest for minimization. They applied the method to optimize privatization processes, especially for transition economies.

Brauers and Zavadskas [5] clarified that using two different methods of multi-objective optimization is more robust than using a single method. Also, using three methods is more robust than using two methods. Accordingly, they proposed the MULTIMOORA which is composed of the MOORA method and the full multiplicative form. In the full multiplicative form, the utility function is the multiplication the ratings of an alternative for the evaluation criteria. They applied the method for project management in a transition economy.

Since the introduction of MULTIMOORA, it has been applied in various practical sectors including industries, economics, civil services, and environmental policy-making, healthcare management, and information and communications technologies [28]. Several versions were also developed to handle uncertainty using different types of data. For a comprehensive review of the MULTIMOORA different versions applied to diverse practical problems until 2018, the reader is referred to Hafezalkotob et al. [28]. The most recent articles about MULTIMOORA are summarized by the type of data used as follows.

Using crisp data, Asante et al. [1] integrated MULTIMOORA with the Evaluation based on Distance from Average Solution (EDAS) method for the evaluation of renewable energy barriers in developing countries. Dizdar and Ünver [14] applied MULTIMOORA method for the assessment procedure of occupational safety and health based on the counts of occupational accidents and diseases. Fedajev et al. [17] used MULTIMOORA and the Shannon Entropy Index to rank and classify the European Union (EU) countries according to the progress achieved in the implementation of the "Europe 2020" strategy. Omrani et al. [45] proposed a new approach based on the Best–Worst Method (BWM) and MULTIMOORA methods to calculate semi-human development index (HDI). HDI is a useful tool for policymakers to understand the degree of development in their societies and establish new policies to improve it. Souzangarzadeh et al. [54] used the response surface methodology (RSM) D-optimal Design along with MULTIMOORA to find the optimum design of segmented tubes as energy absorbers in terms of various vehicles collision scenarios. Yörükoğlu and Aydın [64] applied MULTIMOORA for wind turbine selection problem according to qualitative and quantitative criteria.

As for type-1 fuzzy sets, Chen et al. [8] proposed an extended MULTIMOORA method using the ordered weighted geometric averaging (OWGA) operator and Choquet integral for failure mode and effects analysis (FMEA). Dai et al. [12] proposed a novel MULTIMOORA into the triangular fuzzy environment in which the period weights and attribute weights are completely unknown. Rahimi et al. [49] introduced a framework comprising Geographic Information System (GIS) techniques and fuzzy MCDM methods to select sustainable landfill site. The criteria weights were obtained using the fuzzy BWM. The suitability maps were generated based on the GIS analysis. The selected sites were then analyzed and ranked using the MULTIMOORA method. Tavana et al. [55] proposed an integrated approach for supply chain risk–benefit assessment and supplier selection that combines the fuzzy analytic hierarchy process (AHP) and the fuzzy MULTIMOORA. Dahooie et al. [11] applied an objective weight determination method called CCSD (Correlation Coefficient and Standard Deviation) to eliminate the limitations of the dominance theory and increase the robustness of the MULTIMOORA and enhance its performance by considering the importance level of the three different techniques (i.e., ratio system, reference point, and full multiplicative form).

In the context of probabilistic linguistic information (PLI), Chen et al. [7] proposed a MULTIMOORA and introduced an innovative two-step comparative method to evaluate cloud-based enterprise resource planning (ERP) systems. Liu and Li [39] established the prospect theory-based MULTIMOORA method. They used the probabilistic linguistic terms set (PLTS) to describe qualitative information not only to provide every possible evaluation value but also to give the weight of these values.

MULTIMOORA was also extended using the hesitant fuzzy linguistic term set (HFLTS). Liang et al. [34] designed a MULTIMOORA method using a dual hesitant fuzzy extended Bonferroni mean (DHFEBM) to select a renewable energy technology. Liao et al. [37] improved the MULTIMOORA method by integrating with the ORESTE (organísation, rangement et Synthèse de données relationnelles) method, and extended the method to the unbalanced hesitant fuzzy linguistic context based on an introduced score function to eliminate the defects of the subscript-based operations on HFLTSs.

Regarding picture fuzzy numbers (PFNs), Lin et al. [38] proposed a novel picture fuzzy MULTIMOORA to solve the site selection problem for car-sharing stations based on a novel score function and Borda rule.

In the intuitionistic fuzzy environment, Luo et al. [40] developed a distance-based IF-MULTIMOORA method integrating a novel weight-determining method to select medical equipment. Zhang et al. [68] proposed an IF-MULTIMOORA method for MCDM that involves information fusion to allow processing both crisp and fuzzy information.

On the subject of PFSs, Li et al. [33] proposed a new MULTIMOORA method to evaluate the passenger satisfaction level of the public transportation system under a large group environment. Xian et al. [60] developed a MULTIMOORA method using interval 2-tuple Pythagorean fuzzy linguistic sets to evaluate financial management performance in universities. Liang et al. [35] presented a MULTIMOORA method with interval-valued Pythagorean fuzzy sets (IVPFSs) to solve the selection problem of hospital open-source electronic health records (EHRs) systems for MedLab in Ghana.

Concerning the neutrosophic fuzzy environment, Liang et al. [36] proposed a MULTIMOORA approach based on linguistic neutrosophic numbers (LNNs) and applied this new approach to select the optimal mining method. Based on a neutrosophic MULTIMOORA technique, Siksnelyte et al. [52] presented an original framework for sustainable energy development indicators. They analyzed the trends of energy development across the Baltic Sea Region (BSR) countries from 2008 to 2015.

Lately, Gündoğdu [25] developed a MULTIMOORA method using spherical fuzzy sets (SFSs) to increase its efficiency at solving complex problems that require evaluation and estimation under unreliable data environment.

2.2 Pythagorean Fuzzy Operators

Various scholars have paid attention to MCDM problems under PF environment. To develop effective and efficient methods to solve these problems, PF operational laws and aggregation operators are crucial. Yager [61] developed the Pythagorean fuzzy weighted averaging (PFWA) operator and Pythagorean fuzzy weighted geometric (PFWG) operator to handle multiple attribute decision making (MADM) problems. Ma and Xu [41] defined some novel PFWG and PFWA operators for PF information which can treat the membership degree and the non-membership degree neutrally. Garg [19, 20] developed some generalized PF Einstein weighted and ordered weighted averaging operators. Zhang [69] presented a new PFWA operator and PF-ordered weighted averaging (PFOWA) operator to aggregate PFSs. Garg [18] presented the averaging and the geometric aggregation operators under the interval-valued PF environment. Peng and Yang [47] developed a PF Choquet integral operator for multiple attribute group decision-making (MAGDM) problems. Wei and Lu [59] proposed some PF Maclaurin symmetric mean operators for MADM. Garg [19] defined two new exponential operational laws for interval-valued Pythagorean fuzzy sets (IVPFSs) and their corresponding aggregation operators. Garg [22] developed some new logarithm operational laws (LOL) with a real number base for PFSs. Based on the properties of these LOL, various weighted averaging and geometric operators were developed and a decision-making method was introduced under PF information using the proposed operators. Garg [23] developed some new operational laws and their corresponding PFWGA operators by including the feature of the probability sum and the interaction coefficient into the analysis to get a neutral or a fair treatment to the membership and non-membership functions of PFSs. Later, Garg [24] defined some new PF weighted, ordered weighted, and hybrid neutral averaging aggregation operators for PF information. He utilized these operators that can neutrally treat the membership and non-membership degrees to present an algorithm to solve the MAGDM problems under the PF environment. Wang et al. [58] presented some PF interactive Hamacher power aggregation operators such as PF interactive

Hamacher power average, weighted average, ordered weighted average, PF interactive Hamacher power geometric, weighted geometric, and ordered geometric operators, respectively. In addition, they defined a PF entropy measure and established a method to determine the attribute weights. They explored a novel approach for MADM problems and assessed express service quality.

Shahzadi et al. [51] introduced six families of aggregation operators namely, PF Yager weighted averaging aggregation, PF Yager ordered weighted averaging aggregation, PF Yager hybrid weighted averaging aggregation, PF Yager weighted geometric aggregation, PF Yager ordered weighted geometric aggregation and PF Yager hybrid weighted geometric aggregation. These operators inherit the operational advantages of Yager parametric families.

2.3 Energy Storage Technologies

With the development of renewable energy, energy storage is becoming increasingly important; hence, finding and implementing cost-effective and sustainable energy storage and conversion systems is vital [13].

ESTs not only store the excess of energy but also increase renewable energy penetration and decrease its limitations as a power plant cannot depend solely on a renewable energy source without EST. This results in decreasing fuel consumption and CO_2 emissions. ESTs balance between the energy supply and demand while reducing renewable energy fluctuations due to its intermittent nature. They improve the overall efficiency of a power plant, thus reducing the operating cost in the long run. They also reduce the peak energy loads which will, in turn, decrease the risk of load shedding especially when the large capacity of storage is considered. The flexibility of ESTs makes it convenient and suitable to cover distant areas that suffer from the lack of electricity [42]. The major problem with ESTs is their investment cost and operational cost that should be within acceptable limits. Finding the possible low cost, efficient, and long term ESTs that don't harm the environment is a subject of extensive research [30].

The only way of storing electrical energy is by converting it to other forms of energy such as thermal energy, chemical energy, electrochemical energy, mechanical energy, and electromagnetic energy [31].

Thermal energy storage (TES): it is a technology that stores thermal energy by heating or cooling a storage medium and then utilizes this stored energy when needed. The stored energy can be used at a later time for heating and cooling applications and power generation [50]. Power is generated from this stored energy by applying a Rankine cycle turbine with the system. TES systems are applicable in diverse industrial and residential purposes, e.g., space heating or cooling, process heating and cooling, hot water production, and electricity generation. TES can be classified into three types: latent heat, sensible heat, and thermochemical heat storage. Sensible heat storage stores heat energy in any material depending on its heat capacity and the change of the material's temperature during the process of charging and discharging.

The main advantage of this type is that charging and discharging is completely reversible and have unlimited life cycles [30]. Latent heat storage is based on the amount of heat released or absorbed during the phase change of any material. Heat is stored in phase change materials which could be both organics and inorganics and can change their phase with varying heat [31]. The materials for storing the heat can be both liquid and solid. They are successfully integrated with solar energy systems [68].

Chemical energy storage (CES): Chemical energy is stored in the chemical bonds of atoms and molecules that can be only observed when released in a chemical reaction in the form of heat. After the release of chemical energy, the substance is often changed into a completely different substance. Chemical fuels are the dominant form of energy storage regarding electrical generation and energy transportation. Chemical energy storage is suitable for storing large amounts of energy and for longer durations [27]. CES includes hydrogen storage and biofuels [30]. Biofuels are produced by a biological process instead of geological processes. Biomass is an organic matter derived from the biodegradable fraction of energy crops, the waste matter of plants and animals. This biomass is used to produce biogas which can be converted through a generator to electricity. Biofuels include ethanol, biodiesel, bio-alcohols, bio-ether, green diesel, biofuel gasoline, vegetable oils, syngas, and solid biomass [30]. Hydrogen energy storage technology is one of the most prominent types. The process involves two steps: producing and storing hydrogen when there is excess power available, and then producing electricity from the stored hydrogen using fuel cells in case of power shortage [31].

Electrochemical energy storage (EcES): this storage technology converts electric energy into chemical energy and vice versa during energy storage and recovery. There are two main branches of EcES: electrochemical batteries and electrochemical capacitors. The type of the EcES differs according to the nature of the chemical reaction, structural features, and design. Electrochemical cells and batteries are classified according to three features. The first classification depends on the operation principle and contains 4 categories; primary cell or battery, secondary cell or battery, reserve cell, and fuel cell. In the primary batteries, the chemical once consumed cannot be recharged, while the secondary batteries can be charged and discharged many times. In power system applications, only secondary batteries are utilized [31]. The second classification is based on discharge depth, either shallow or deep cycle batteries. Deep cycle batteries are suitable for renewable applications. The third classification depends on the characteristic of the electrolyte in the battery, either flooded or wet and sealed. Flooded or wet batteries are vastly utilized in renewable applications [27]. EcES plays a vital role in our daily life since they are applied in small devices, e.g., laptops, tablets, and cell phones, and in larger devices, e.g., electric cars, to provide efficient and reliable use of energy. Battery energy storage is the most widespread storage method. It is available in different sizes ranging from tens of watts to megawatts [31]. Batteries have two main disadvantages. First, the long charging time since they have an intrinsically low power handling capability (<1 kW/kg, normalized by the device mass). Second, the short device cycle life. The specific power of modern batteries has been increased; yet, the cost of these advanced batteries is high, and they still do not fulfill the power demands of many applications, e.g., electric vehicles [32].

Electrical energy storage (EES): in this technology electrical energy is converted from a power network or source via an energy conversion module into another energy storage medium. This intermediate energy is stored for a limited time, then converted back into electrical energy when needed [43]. ESS is most suitable for any specific application in power systems. EES includes capacitors, supercapacitors, and superconducting magnetic energy storage (SMES). In supercapacitors the electric energy is stored in the form of the electrostatic field created in-between the two porous electrodes, separated by a separator. On the other hand, SMES stores electric energy in the electromagnetic field generated by a current following through a superconducting conductor [31]. The capacitors can be used for high currents, but for extremely short periods due to their relatively low capacitance generation. A Supercapacitor can replace a regular capacitor, but it offers very high capacitance in a small package. Superconducting magnetic energy storage systems are preferred on the outlet of power plants to stabilize the output or on industrial sites to accommodate peaks in energy consumption [27].

Mechanical energy storage (MES): this technology takes advantage of kinetic or gravitational forces to store energy. MES is easily adaptable to convert and store energy from water current, wave, and tidal sources [27]. Mechanical energy storage offers several advantages compared to other ESTs especially in terms of environmental impact, cost, and sustainability. In MES the energy is stored by doing some mechanical work, and then energy from mechanical work is exploited upon its requirement [30]. MES can be found in two forms according to the utilization of stored energy. The first form is pure mechanical if the system is directly used. The second form is mechanical–electrical when energy is transmitted via an electric motor-generator. The pure mechanical systems can provide mechanical work such as smoothing the rotation of a rotating mass; mechanical–electric systems are used to supply the grid with electricity. MES is classified by the working principle as follows: pressurized gas, forced springs, kinetic energy, and potential energy. The main types of MES are pumped hydroelectric storage (PHS), compressed air energy storage (CAES), and flywheel energy storage (FES) [42].

The wide range of ESTs, with each EST being different in terms of the scale of power, response time, energy/power density, discharge duration, and cost coupled with the complex characteristics matrices, makes it difficult to choose a particular EST for a specific application.

3 Preliminaries

3.1 Pythagorean Fuzzy Sets

In this section, the basic definitions, operations, and aggregation operators of PFSs are reviewed.

Definition 1 ([63]). A Pythagorean fuzzy set $\tilde{A}$ in a finite universe of discourse X is defined by

$$\tilde{A} = \left\{x, \mu_{\tilde{A}}(x), \upsilon_{\tilde{A}}(x) : x \in X\right\}, \tag{1}$$

where $\mu_{\tilde{A}}(x) : X \to [0, 1]$ denotes the membership degree,
$\upsilon_{\tilde{A}}(x) : X \to [0, 1]$ denotes the non-membership degree,
satisfying the constraint

$$0 \leq \mu^2_{\tilde{A}}(x) + \upsilon^2_{\tilde{A}}(x) \leq 1. \tag{2}$$

The hesitation margin, i.e., the degree of uncertainty, is represented by

$$\pi_{\tilde{A}}(x) = \sqrt{1 - \left(\mu^2_{\tilde{A}}(x) + \upsilon^2_{\tilde{A}}(x)\right)}. \tag{3}$$

Definition 2 ([62]). For the PFSs $\left\{\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n\right\}$ having weights $(w_1, w_2, \ldots, w_n)$, with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, the Pythagorean fuzzy weighted averaging operator (PFWA$_Y$) and the Pythagorean fuzzy weighted geometric operator (PFWG$_Y$) are defined as follows:

(i)

$$\text{PFWA}_Y = \left(\sum_{i=1}^{n} w_i \mu_{\tilde{A}_i}, \sum_{i=1}^{n} w_i \upsilon_{\tilde{A}_i}\right), \tag{4}$$

(ii)

$$\text{PFWG}_Y = \left(\prod_{i=1}^{n} \mu^{w_i}_{\tilde{A}_i}, \prod_{i=1}^{n} \upsilon^{w_i}_{\tilde{A}_i}\right). \tag{5}$$

Definition 3 ([41]). For any two PFSs $\tilde{A} = \left(\mu_{\tilde{A}}, \upsilon_{\tilde{A}}\right)$ and $\tilde{B} = \left(\mu_{\tilde{B}}, \upsilon_{\tilde{B}}\right)$ the operational laws are given by

(i)

$$\tilde{A} \oplus \tilde{B} = \left(\sqrt{\mu_{\tilde{A}}^2 + \mu_{\tilde{B}}^2 - \mu_{\tilde{A}}^2\mu_{\tilde{B}}^2}, \upsilon_{\tilde{A}}\upsilon_{\tilde{B}}\right), \tag{6}$$

(ii)

$$\tilde{A} \otimes \tilde{B} = \left(\mu_{\tilde{A}}\mu_{\tilde{B}}, \sqrt{\upsilon_{\tilde{A}}^2 + \upsilon_{\tilde{B}}^2 - \upsilon_{\tilde{A}}^2\upsilon_{\tilde{B}}^2}\right), \tag{7}$$

(iii)

$$\lambda \odot \tilde{A} = \left(\sqrt{1 - \left(1 - \mu_{\tilde{A}}^2\right)^{\lambda}}, \upsilon_{\tilde{A}}^{\lambda}\right), \tag{8}$$

(iv)

$$\tilde{A}^{\lambda} = \left(\mu_{\tilde{A}}^{\lambda}, \sqrt{1 - \left(1 - \upsilon_{\tilde{A}}^2\right)^{\lambda}}\right), \text{ where } \lambda > 0 \text{ is a scalar.} \tag{9}$$

Based on the operational laws given in Definition 3, the PFWA_{MX} and the PFWG_{MX} are defined as follows:

Definition 4 ([41]). Consider the PFSs $\left\{\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n\right\}$ with weights $(w_1, w_2, \ldots, w_n)$, where $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, the PFWA_{MX} and the PFWG_{MX} are defined as follows.

(i)

$$\begin{aligned}\text{PFWA}_{MX}\left(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n\right) &= \left(w_1 \odot \tilde{A}_1\right) \oplus \left(w_2 \odot \tilde{A}_2\right) \oplus \ldots \oplus \left(w_n \odot \tilde{A}_n\right) \\ &= \left(\left[1 - \prod_{i=1}^{n}\left(1 - \mu_{\tilde{A}_i}^2\right)^{w_i}\right]^{\frac{1}{2}}, \prod_{i=1}^{n} \upsilon_{\tilde{A}_i}^{w_i}\right),\end{aligned} \tag{10}$$

(ii)

$$\begin{aligned}\text{PFWG}_{MX}\left(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n\right) &= \tilde{A}_1^{w_1} \otimes \tilde{A}_2^{w_2} \otimes \ldots \otimes \tilde{A}_n^{w_n}. \\ &= \left(\prod_{i=1}^{n} \mu_{\tilde{A}_i}^{w_i}, \left[1 - \prod_{i=1}^{n}\left(1 - \upsilon_{\tilde{A}_i}^2\right)^{w_i}\right]^{\frac{1}{2}}\right)\end{aligned} \tag{11}$$

Definition 5 ([51]). For any two PFSs $\tilde{A} = \left(\mu_{\tilde{A}}, \upsilon_{\tilde{A}}\right)$ and $\tilde{B} = \left(\mu_{\tilde{B}}, \upsilon_{\tilde{B}}\right)$, $\theta > 0$ and $\lambda > 0$, Yager's t-norm and t-conorm operations are defined as follows:

(i)

$$\tilde{A} \boxplus \tilde{B} = \left(\sqrt{\min\left(1, \left(\mu_{\tilde{A}}^{2\theta} + \mu_{\tilde{B}}^{2\theta}\right)^{\frac{1}{\theta}}\right)}, \sqrt{1 - \min\left(1, \left(\left(1 - \upsilon_{\tilde{A}}^2\right)^{\theta} + \left(1 - \upsilon_{\tilde{B}}^2\right)^{\theta}\right)^{\frac{1}{\theta}}\right)} \right), \tag{12}$$

(ii)

$$\tilde{A} \boxtimes \tilde{B} = \left(\sqrt{1 - \min\left(1, \left(\left(1 - \mu_{\tilde{A}}^2\right)^{\theta} + \left(1 - \mu_{\tilde{B}}^2\right)^{\theta}\right)^{\frac{1}{\theta}}\right)}, \sqrt{\min\left(1, \left(\upsilon_{\tilde{A}}^{2\theta} + \upsilon_{\tilde{B}}^{2\theta}\right)^{\frac{1}{\theta}}\right)} \right), \tag{13}$$

(iii)

$$\lambda \boxdot \tilde{A} = \left(\sqrt{\min\left(1, \left(\lambda \mu_{\tilde{A}}^{2\theta}\right)^{\frac{1}{\theta}}\right)}, \sqrt{1 - \min\left(1, \left(\lambda\left(1 - \upsilon_{\tilde{A}}^2\right)^{\theta}\right)^{\frac{1}{\theta}}\right)} \right), \tag{14}$$

(iv)

$$\tilde{A}^{\lambda} = \left(\sqrt{1 - \min\left(1, \left(\lambda\left(1 - \mu_{\tilde{A}}^2\right)^{\theta}\right)^{\frac{1}{\theta}}\right)}, \sqrt{\min\left(1, \left(\lambda \upsilon_{\tilde{A}}^{2\theta}\right)^{\frac{1}{\theta}}\right)} \right). \tag{15}$$

Based on the operational laws given in Definition 5, the PFYWA and the PFYWG are defined as follows.

Definition 6 ([51]). For the PFSs $\left\{\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n\right\}$ having weights $(w_1, w_2, \ldots, w_n)$, with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, the PFYWA and the PFYWG are given as follows:

$$\begin{aligned} \text{PFYWA}\left(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n\right) &= \left(w_1 \boxdot \tilde{A}_1\right) \boxplus \left(w_2 \boxdot \tilde{A}_2\right) \boxplus \ldots \boxplus \left(w_n \boxdot \tilde{A}_n\right) \\ &= \left(\sqrt{\min\left(1, \left(\sum_{i=1}^{n}\left(w_i \mu_{\tilde{A}_i}^{2\theta}\right)\right)^{\frac{1}{\theta}}\right)}, \sqrt{1 - \min\left(1, \left(\sum_{i=1}^{n}\left(w_i\left(1 - \upsilon_{\tilde{A}_i}^2\right)^{\theta}\right)\right)^{\frac{1}{\theta}}\right)} \right), \end{aligned} \tag{16}$$

$$\begin{aligned} &\text{PFYWG}\left(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n\right) = \tilde{A}_1^{w_1} \boxtimes \tilde{A}_2^{w_2} \boxtimes \ldots \boxtimes \tilde{A}_n^{w_n} \\ &= \left(\sqrt{1 - \min\left(1, \left(\sum_{i=1}^{n}\left(w_i\left(1 - \mu_{\tilde{A}_i}^2\right)^{\theta}\right)\right)^{\frac{1}{\theta}}\right)}, \sqrt{\min\left(1, \left(\sum_{i=1}^{n}\left(w_i \upsilon_{\tilde{A}_i}^{2\theta}\right)\right)^{\frac{1}{\theta}}\right)} \right). \end{aligned} \tag{17}$$

For a PFS $\tilde{A} = \left(\mu_{\tilde{A}}, \upsilon_{\tilde{A}}\right)$, Zhang and Xu [70] proposed a score function to evaluate and compare PFSs and it is defined as follows:

$$\mathcal{S}\left(\tilde{A}\right) = \mu_{\tilde{A}}^2 - \upsilon_{\tilde{A}}^2, \text{ where } \mathcal{S}\left(\tilde{A}\right) \in [-1, 1]. \tag{18}$$

In addition, Peng and Yang [46] proposed an accuracy function to help in discrimination whenever a tie occurs. The accuracy function is defined as follows:

$$\mathcal{A}\left(\tilde{A}\right) = \mu_{\tilde{A}}^2 + \upsilon_{\tilde{A}}^2, \text{ where } \mathcal{A}\left(\tilde{A}\right) \in [0, 1]. \tag{19}$$

Definition 7 ([48]). The complement or the negation pf a PFS $\tilde{A} = \left(\mu_{\tilde{A}}, \upsilon_{\tilde{A}}\right)$ is denoted by

$$\tilde{A}^c = \left(\upsilon_{\tilde{A}}, \mu_{\tilde{A}}\right). \tag{20}$$

Definition 8 ([46]). Two PFSs $\tilde{A} = \left(\mu_{\tilde{A}}, \upsilon_{\tilde{A}}\right)$ and $\tilde{B} = \left(\mu_{\tilde{B}}, \upsilon_{\tilde{B}}\right)$ are compared as follows:

(a) If $\mathcal{S}\left(\tilde{A}\right) < \mathcal{S}\left(\tilde{B}\right)$, then $\tilde{A} < \tilde{B}$;

(b) If $\mathcal{S}\left(\tilde{A}\right) > \mathcal{S}\left(\tilde{B}\right)$, then $\tilde{A} > \tilde{B}$;

(c) If $\mathcal{S}\left(\tilde{A}\right) = \mathcal{S}\left(\tilde{B}\right)$, the accuracy function is employed

  (i) If $\mathcal{A}\left(\tilde{A}\right) < \mathcal{A}\left(\tilde{B}\right)$, then $\tilde{A} < \tilde{B}$;

  (ii) If $\mathcal{A}\left(\tilde{A}\right) > \mathcal{A}\left(\tilde{B}\right)$, then $\tilde{A} > \tilde{B}$;

  (iii) If $\mathcal{A}\left(\tilde{A}\right) = \mathcal{A}\left(\tilde{B}\right)$, then $\tilde{A} \approx \tilde{B}$.

Distance and similarity measures are important topics and have been extensively used in diverse fields such as pattern recognition, machine learning, and market prediction [68]. Some common metrics, e.g., Hamming distance and Euclidean distance, are widely used to find the distance between two PFSs. Initially, the membership degree and the non-membership degree were only considered in the distance formulas. Later, these formulas were modified to include the degree of hesitation as well. The distance formulas in a Pythagorean fuzzy environment are given as follows [29].

i. Hamming distance

$$d_{Hm}\left(\tilde{A}, \tilde{B}\right) = \frac{1}{2}\sum_{i=1}^{n}\left\{\left|\mu_{\tilde{A}}^2(x_i) - \mu_{\tilde{B}}^2(x_i)\right| + \left|\upsilon_{\tilde{A}}^2(x_i) - \upsilon_{\tilde{B}}^2(x_i)\right| + \left|\pi_{\tilde{A}}^2(x_i) - \pi_{\tilde{B}}^2(x_i)\right|\right\}. \tag{21}$$

ii. Normalized Hamming distance

$$d_{NHm}\left(\tilde{A},\tilde{B}\right)=\frac{1}{2n}\sum_{i=1}^{n}\left\{\left|\mu_{\tilde{A}}^{2}(x_i)-\mu_{\tilde{B}}^{2}(x_i)\right|+\left|\upsilon_{\tilde{A}}^{2}(x_i)-\upsilon_{\tilde{B}}^{2}(x_i)\right|+\left|\pi_{\tilde{A}}^{2}(x_i)-\pi_{\tilde{B}}^{2}(x_i)\right|\right\}. \tag{22}$$

iii. Euclidean distance

$$d_{E}\left(\tilde{A},\tilde{B}\right)=\sqrt{\frac{1}{2}\sum_{i=1}^{n}\left\{\left(\mu_{\tilde{A}}^{2}(x_i)-\mu_{\tilde{B}}^{2}(x_i)\right)^2+\left(\upsilon_{\tilde{A}}^{2}(x_i)-\upsilon_{\tilde{B}}^{2}(x_i)\right)^2+\left(\pi_{\tilde{A}}^{2}(x_i)-\pi_{\tilde{B}}^{2}(x_i)\right)^2\right\}}. \tag{23}$$

iv. Normalized Euclidean distance

$$d_{NE}\left(\tilde{A},\tilde{B}\right)=\sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left\{\left(\mu_{\tilde{A}}^{2}(x_i)-\mu_{\tilde{B}}^{2}(x_i)\right)^2+\left(\upsilon_{\tilde{A}}^{2}(x_i)-\upsilon_{\tilde{B}}^{2}(x_i)\right)^2+\left(\pi_{\tilde{A}}^{2}(x_i)-\pi_{\tilde{B}}^{2}(x_i)\right)^2\right\}}. \tag{24}$$

v. Hausdorff distance

$$d_{Hs}\left(\tilde{A},\tilde{B}\right)=\sum_{i=1}^{n}\max\left\{\left|\mu_{\tilde{A}}^{2}(x_i)-\mu_{\tilde{B}}^{2}(x_i)\right|,\left|\upsilon_{\tilde{A}}^{2}(x_i)-\upsilon_{\tilde{B}}^{2}(x_i)\right|\right\}. \tag{25}$$

vi. Normalized Hausdorff distance

$$d_{NHs}\left(\tilde{A},\tilde{B}\right)=\frac{1}{n}\sum_{i=1}^{n}\max\left\{\left|\mu_{\tilde{A}}^{2}(x_i)-\mu_{\tilde{B}}^{2}(x_i)\right|+\left|\upsilon_{\tilde{A}}^{2}(x_i)-\upsilon_{\tilde{B}}^{2}(x_i)\right|\right\}. \tag{26}$$

The concepts of distance measure and similarity measure are dual concepts. Hence, the distance between two PFSs is used to define the similarity between two PFSs.

Proposition 1 ([16]). Let $d\left(\tilde{A},\tilde{B}\right)$ be the distance between two PFSs $\tilde{A}$ and $\tilde{B}$, then the similarity measure between the two PFSs is given as

$$S\left(\tilde{A},\tilde{B}\right)=1-d\left(\tilde{A},\tilde{B}\right). \tag{27}$$

Hussian and Yang [29] defined other similarity measures based on the Hausdorff metric beside the simple linear function (27). These similarity measures use a rational function and an exponential function as follows:

$$S\left(\tilde{A}, \tilde{B}\right) = \frac{1 - d_{Hs}\left(\tilde{A}, \tilde{B}\right)}{1 + d_{Hs}\left(\tilde{A}, \tilde{B}\right)}, \tag{28}$$

and

$$S\left(\tilde{A}, \tilde{B}\right) = \frac{e^{d_{Hs}\left(\tilde{A}, \tilde{B}\right)} - e^{-1}}{1 - e^{-1}}. \tag{29}$$

3.2 The Classical MULTIMOORA Method

The MULTIMOORA method is an extension of the multi-objective optimization by ratio analysis (MOORA) method by incorporating the full multiplicative form of multiple objectives [5]. The MULTIMOORA method can be summarized as follows [5, 35]:

Suppose the general decision matrix of an MCDM problem is given by

$$\mathbf{D} = \left[X_{ij}\right] = \begin{array}{c} \\ X_1 \\ X_2 \\ \vdots \\ X_n \end{array} \begin{array}{c} \begin{array}{cccc} \mathrm{C}_1 & \mathrm{C}_2 & & \mathrm{C}_\mathrm{m} \end{array} \\ \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1m} \\ X_{21} & X_{22} & & X_{2m} \\ & \vdots & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & X_{nm} \end{bmatrix} \end{array},$$

where its element X_{ij} is the rating of the alternative X_i; $i = 1, 2, \ldots, n$ for the criterion C_j; $j = 1, 2, \ldots, m$. First, the data of the general decision matrix is normalized by dividing the rating of an alternative for a criterion by the square root of the sum of squares of the ratings of the entire alternatives for that criterion,

$$X_{ij}^N = \frac{X_{ij}}{\sqrt{\sum_{i=1}^n X_{ij}^2}}. \tag{30}$$

Hence, the normalized general decision matrix $\mathbf{D}_\mathrm{N} = \left[X_{ij}^N\right]$ is formed.

In the ratio system technique, the elements X_{ij}^N are added in case of maximization and subtracted in case of minimization. Let g be the number of benefit criteria to be maximized and $m - g$ be the number of cost criteria to be minimized, the overall index of each alternative is:

$$R_i = \sum_{j=1}^{g} X_{ij}^N - \sum_{j=g+1}^{m} X_{ij}^N. \quad (31)$$

Using (31), the alternatives are ranked. The higher the value of R_i, the higher the rank.

In the reference point technique, the Reference Point Theory is applied with the Min–Max Metric of Chebyshev. The jth criterion reference point is defined by

$$X_j^* = \begin{cases} \max_i X_{ij}^N, & \text{for benefit criteria,} \\ \min_i X_{ij}^N, & \text{for cost criteria.} \end{cases} \quad (32)$$

The deviation of the normalized rating of each alternative from the reference point is calculated by

$$d_i = \min_i \left\{ \max_j \left| X_j^* - X_{ij}^N \right| \right\}. \quad (33)$$

Using (33), the alternatives are ranked. The lower the value of d_i, the higher the rank.

The full multiplicative form combines both maximization and minimization of the multiplicative utility function. The overall utility of each alternative is given by the dimensionless number

$$U_i = \frac{U_i^b}{U_i^c}, \quad (34)$$

where $U_i^b = \prod_{j=1}^{g} X_{ij}$ denotes the product of an alternative's ratings of benefit criteria, and $U_i^b = \prod_{j=g+1}^{m} X_{ij}$ denotes the product of an alternative's ratings of cost criteria. Using (34), the alternatives are ranked. The higher the value of U_i, the higher the rank.

Utilizing the dominance theory, the alternatives are ranked based on the previous three ranking lists and the final decision is made, i.e., the alternative with the highest appearance in the first place on all the ranking lists is the best.

4 The Proposed PF-MULTIMOORA

In this section, the MULTIMOORA method is utilized in PF environment due to its appealing features. The MULTIMOORA is one of the most practical MCDM methods. It is an effective, efficient, flexible, and robust method. It was successfully applied to various practical fields.

Consider an MCDM problem with n alternatives $\{X_1, X_2, \ldots, X_n\}$ and m criteria $\{C_1, C_2, \ldots, C_m\}$, with weights $(w_1, w_2, \ldots, w_m)$ satisfying $\sum_{i=1}^{m} w_i = 1$. The Pythagorean fuzzy general decision matrix is represented as

$$\tilde{\mathbf{D}} = \left[\tilde{X}_{ij}\right] = \begin{matrix} & C_1 & C_2 & & C_m \\ X_1 & \tilde{X}_{11} & \tilde{X}_{12} & \cdots & \tilde{X}_{1m} \\ X_2 & \tilde{X}_{21} & \tilde{X}_{22} & & \tilde{X}_{2m} \\ \vdots & \vdots & & \ddots & \vdots \\ X_n & \tilde{X}_{n1} & \tilde{X}_{n2} & \cdots & \tilde{X}_{nm} \end{matrix},$$

where $\tilde{X}_{ij} = (\mu_{ij}, \upsilon_{ij})$ indicates the ratings of the alternatives for the assessment criteria expressed by PFSs. The value "μ_{ij}" indicates the degree to which an alternative X_i satisfies a criterion C_j, and the value "υ_{ij}" indicates the degree to which X_i fails to satisfy this criterion. In constructing the decision matrix, the complement of the PFS is used for the ratings of the cost criteria. Hence, the decision matrix needs no further processing and the three techniques are directly applied.

The three techniques are expressed in detail in the following subsections.

4.1 The Ratio System Technique

The ratio system is based on the additive utility function. As the ratings are already expressed by PFSs they do not need normalization. In addition, since the complement is used in case of cost criteria, all the criteria are treated as benefit ones and subtraction operation (31) is not required. A Pythagorean fuzzy weighted averaging aggregation operator is applied.

Most of the proposed weighted average aggregation operators cannot be applied in a certain situation that is illustrated by the following example. Consider a simple MCDM problem with two alternatives and three criteria. The criteria weights are 0.2, 0.3, and 0.5, respectively. The ratings of the alternatives for the criteria are given by the following decision matrix.

$$\tilde{\mathbf{D}} = \begin{matrix} & C_1 & C_2 & C_3 \\ A_1 & (1, 0) & (0.1, 0.9) & (0.1, 0.9) \\ A_2 & (0.9, 0.1) & (0.8, 0.2) & (0.9, 0.1) \end{matrix}$$

From the decision matrix, the performance of A_2 far exceeds that of A_1 for the second and third criteria that have larger weights, while the performance of A_1 is slightly better than that of A_2 for the first criterion that has the smallest weight. Therefore, it is obvious that A_2 is better than A_1 by intuition.

Using the PFWA_{MX} (10) and the score function (18), we get $\text{PFWA}_{MX}\left(A_1|C_j\right) = (1, 0)$ with $S(A_1) = 1$, while $\text{PFWA}_{MX}\left(A_2|C_j\right) = (0.8774, 0.1231)$ with $S(A_2) = 0.7547$. This result leads to the selection of A_1 despite being not the better choice by logic. Accordingly, it can be concluded that a single criterion with the perfect rating $(1, 0)$ will dominate regardless of its weight and abolish the effect of the rest of the evaluation criteria, which is not fair in the assessment process. In this case, the selection is biased to the alternative having a single perfect rating regardless of its ratings for the other criteria. Therefore, false ranking is obtained.

On the other hand, using the PFYWA (16) and the score function (18), we get $\text{PFYWA}\left(A_1|C_j\right) = (0.6688, 0.7222)$ with $S(A_1) = -0.0743$, while $\text{PFYWA}\left(A_2|C_j\right) = (0.8735, 0.1375)$ with $S(A_2) = 0.7441$. This result leads to the selection of A_2, which is the better alternative by intuition. Here, the obtained ranking is rational.

From the previous illustration, the PFYWA (16) is chosen for aggregation. Hence, the additive utility U_i^A of each alternative X_i is given by

$$\tilde{U}_i^A = \text{PFYWA}\left(\tilde{X}_{ij}|j = 1, 2, \ldots, m; w\right) = \left(w_1 \boxdot \tilde{X}_{i1}\right) \boxplus \left(w_2 \boxdot \tilde{X}_{i2}\right) \boxplus \ldots \boxplus \left(w_n \boxdot \tilde{X}_{im}\right)$$
$$= \left(\sqrt{\min\left(1, \left(\sum_{j=1}^{m}\left(w_i \mu_{\tilde{X}_{ij}}^{2\theta}\right)\right)^{\frac{1}{\theta}}\right)}, \sqrt{1 - \min\left(1, \left(\sum_{j=1}^{m}\left(w_j \left(1 - \upsilon_{\tilde{X}_{ij}}^2\right)^{\theta}\right)\right)^{\frac{1}{\theta}}\right)}\right).$$

4.2 The Reference Point Technique

So far, the reference point technique proceeds by identifying a reference point for each criterion; this reference point indicates the best rating obtained by an alternative for a criterion. Then, the distance between the rating of each alternative for a criterion and the reference point is calculated using a distance formula. Therefore, the reference point approach yields a crisp value.

The reference point can be the theoretical reference point defined by $(1, 0, 0)$. Otherwise, it can be an empirical reference point, i.e., defined from the data of the problem. In this case, it is given by

$$\tilde{R}_j = \left(\mu_j^{'}, \upsilon_j^{'}\right), \text{ where} \mu_j^{'} = \max_i \mu_{ij}, \upsilon_j^{'} = \min_i \upsilon_{ij}, j = 1, 2, \ldots, m. \quad (35)$$

Actually, a distance between two fuzzy values cannot be definitely and uniquely defined. It is closer to be a fuzzy value rather than a crisp value. Therefore, it is more proper to define the distance between two PFSs with a PFS.

In this proposed PF-MULTIMOORA two reference points are considered, the best rating and the worst rating. We are in favor of an alternative according to its degree of similarity to the best rating and oppose this alternative according to its degree of similarity to the worst rating, and the degree of indeterminacy can be calculated residually. The reference point utility value is estimated as follows:

The weighted general decision matrix is calculated first. It is given by

$$\tilde{\mathbf{D}}_w = \left[\tilde{\mathcal{X}}_{ij}\right] = \begin{matrix} & \begin{matrix} C_1 & C_2 & & C_m \end{matrix} \\ \begin{matrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{matrix} & \begin{bmatrix} \tilde{\mathcal{X}}_{11} & \tilde{\mathcal{X}}_{12} & \cdots & \tilde{\mathcal{X}}_{1m} \\ \tilde{\mathcal{X}}_{21} & \tilde{\mathcal{X}}_{22} & & \tilde{\mathcal{X}}_{2m} \\ \vdots & & \ddots & \vdots \\ \tilde{\mathcal{X}}_{n1} & \tilde{\mathcal{X}}_{n2} & \cdots & \tilde{\mathcal{X}}_{nm} \end{bmatrix} \end{matrix}, \quad \text{where } \tilde{\mathcal{X}}_{ij} = w_j \odot \tilde{\mathcal{X}}_{ij}.$$

The theoretical reference point is employed to guarantee that the resulting value is a PFS. Let $\tilde{R}_j^+$ be the best rating $(1, 0, 0)$ and $\tilde{R}_j^-$ be the worst rating $(0, 1, 0)$. Applying the normalized Euclidean distance (24) to find the distance between the ratings of an alternative for the criteria and the best rating

$$d_i^+\left(\tilde{\mathcal{X}}_{ij}, \tilde{R}_j^+\right) = \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left\{\left(\mu^2_{\tilde{\mathcal{X}}_{ij}} - 1\right)^2 + \upsilon^4_{\tilde{\mathcal{X}}_{ij}} + \pi^4_{\tilde{\mathcal{X}}_{ij}}\right\}}. \tag{36}$$

Similarly, the normalized Euclidean distance (24) is applied to find the distance between the ratings of an alternative for the criteria and the worst rating

$$d_i^-\left(\tilde{\mathcal{X}}_{ij}, \tilde{R}_j^-\right) = \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left\{\mu^4_{\tilde{\mathcal{X}}_{ij}} + \left(\upsilon^2_{\tilde{\mathcal{X}}_{ij}} - 1\right)^2 + \pi^4_{\tilde{\mathcal{X}}_{ij}}\right\}}. \tag{37}$$

Then, the utility value based on the reference point approach is expressed by

$$\tilde{U}_i^R = (\mu_i, \upsilon_i) \tag{38}$$

where

$$\mu_i = S_i^+\left(\tilde{\mathcal{X}}_{ij}, \tilde{R}_j^+\right) = 1 - d_i^+\left(\tilde{\mathcal{X}}_{ij}, \tilde{R}_j^+\right) \tag{39}$$

represents the degree of agreement on an alternative for the assessment criteria regarding its closeness to the best rating,

$$\upsilon_i = S_i^-\left(\tilde{\mathcal{X}}_{ij}, \tilde{R}_j^-\right) = 1 - d_i^-\left(\tilde{\mathcal{X}}_{ij}, \tilde{R}_j^-\right) \tag{40}$$

represents the degree of disagreement on an alternative for the assessment criteria regarding its closeness to the worst rating.

4.3 The Full Multiplicative Form Technique

The full multiplicative technique is based on the multiplicative utility function. Since all the criteria are treated as benefit criteria after using the complement of the ratings of the cost criteria, the division operation (34) is no longer required. A Pythagorean fuzzy weighted geometric aggregation operator is applied.

For an MCDM problem with two alternatives and three criteria with weights 0.2, 0.3, and 0.5, respectively, the ratings of the alternatives for the criteria are given by the following decision matrix.

$$\tilde{\mathbf{D}} = \begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} (0,1) & (0.9,0.1) & (0.9,0.1) \\ (0.1,0.9) & (0.2,0.8) & (0.1,0.9) \end{bmatrix} \end{array}.$$

From the decision matrix, the ratings of A_1 exceeds that of A_2 for the second and third criteria that have larger weights, and the rating of A_2 is slightly better than that of A_1 for the first criteria that has the smallest weight. Therefore, it is obvious that A_1 is better than A_2.

Using the PFWG_{MX} (11) and the score function (18) we get $\mathrm{PFWG}_{MX}(A_1|C_j) = (0,1)$ with $S(A_1) = -1$, while $\mathrm{PFWG}_{MX}(A_2|C_j) = (0.1231, 0.4637)$ with $S(A_2) = -0.1999$. This result leads to the selection of A_2 although it is not the better choice by intuition. Therefore, it can be concluded that a single criterion with the worst performance (0, 1) will dominate regardless of its weight and abolish the effect of the rest of the evaluation criteria, which is also not fair in evaluation. In this case, the selection is biased against the alternative having this worst performance regardless of its performance for the other criteria. This leads to false ranking.

On the other hand, using the PFYWG (17) and the score function (18) we get $\mathrm{PFYWG}(A_1|C_j) = (0.7222, 0.6688)$ with $S(A_1) = 0.0743$, while $\mathrm{PFYWG}(A_2|C_j) = (0.1375, 0.8735)$ with $S(A_2) = -0.7441$. This result leads to the selection of A_1, which is actually the better alternative by intuition.

From the previous illustration, the PFYWG (17) is chosen for aggregation. Hence, the multiplicative utility U_i^M of each alternative X_i is given by

$$\tilde{U}_i^M = \mathrm{PFYWG}\left(\tilde{X}_{ij} | j = 1, 2, \ldots, m; w\right) = \tilde{X}_{i1}^{w_1} \boxtimes \tilde{X}_{i2}^{w_2} \boxtimes \ldots \boxtimes \tilde{X}_{in}^{w_n}$$

$$= \left(\sqrt{1 - \min\left(1, \left(\sum_{j=1}^{m}\left(w_i\left(1 - \mu_{\tilde{X}_{ij}}^2\right)^{\theta}\right)\right)^{\frac{1}{\theta}}\right)}, \sqrt{\min\left(1, \left(\sum_{j=1}^{m}\left(w_i \upsilon_{\tilde{X}_{ij}}^{2\theta}\right)\right)^{\frac{1}{\theta}}\right)} \right).$$

4.4 The Overall Utility Score

Finally, the results of the three techniques are combined to get the overall utility value. In the early versions of the MULTIMOORA, the dominance theory was applied to rank the alternatives. When using the dominance theory several rules are utilized for discrimination: absolute dominance, general dominance, transitiveness, overall dominance, absolute equability, and partial equability [6]. In spite of all these rules, circular reasoning is possible. For example, consider the following alternatives in an MCDM problem with their ranking by the three techniques $X_1(11-20-14)$, $X_2(14-6-15)$, and $X_3(15-19-12)$. When applying the dominance rules we have: X_1 generally dominates X_2, X_2 generally dominates X_3, and X_3 generally dominates X_1. Accordingly, the same ranking is given to the three objects [6]. Therefore, the dominance theory in large scale applications has two main drawbacks: multiple comparisons and circular reasoning.

To overcome these drawbacks, recent researches proposed the aggregation of the three techniques to enhance the accuracy and efficiency of the MULTIMOORA method [11, 68].

Therefore, the three utility values are aggregated into the overall utility value. Here, the common trend is to defuzzify the utility values and then aggregate. This is accomplished by

$$U_i = \omega_A U_i^A + \omega_R U_i^R + \omega_M U_i^M, \tag{41}$$

where ω_A, ω_R, and ω_M are the coefficient of importance of the utility scores, and their sum is equal to one.

The main disadvantage of this trend is having equal scores for different PFSs which will surely affect the overall utility. For example, the PFSs $(0.6, 0.6)$ and $(0.3, 0.3)$ have the same score. Therefore, it is preferable to use a weighted average aggregation operator first, and then use the score function (18) for defuzzification. In this case, whenever we have equal scores the accuracy function (19) can be applied for discrimination.

The overall utility is computed by using the weighted average aggregation operator (4) for similar treatment of the membership and non-membership information for the three utility values:

$$\begin{aligned}\tilde{U}_i^T &= \mathrm{PFWA}_Y\left(\tilde{U}_i^A, \tilde{U}_i^R, \tilde{U}_i^M | \omega_A, \omega_R, \omega_M\right)\\ &= \left(\omega_A \mu_{\tilde{U}_i^A} + \omega_R \mu_{\tilde{U}_i^R} + \omega_M \mu_{\tilde{U}_i^M}, \omega_A \upsilon_{\tilde{U}_i^A} + \omega_R \upsilon_{\tilde{U}_i^R} + \omega_M \upsilon_{\tilde{U}_i^M}\right),\\ \omega_A &= \omega_R = \omega_M, \quad \text{and} \quad \omega_A + \omega_R + \omega_M = 1.\end{aligned} \tag{42}$$

Then, the score function (18) and the accuracy function (19) are used for ranking. The alternative with the highest overall utility score is the best.

The steps of the PF-MUTIMOORA are shown in Fig. 1.

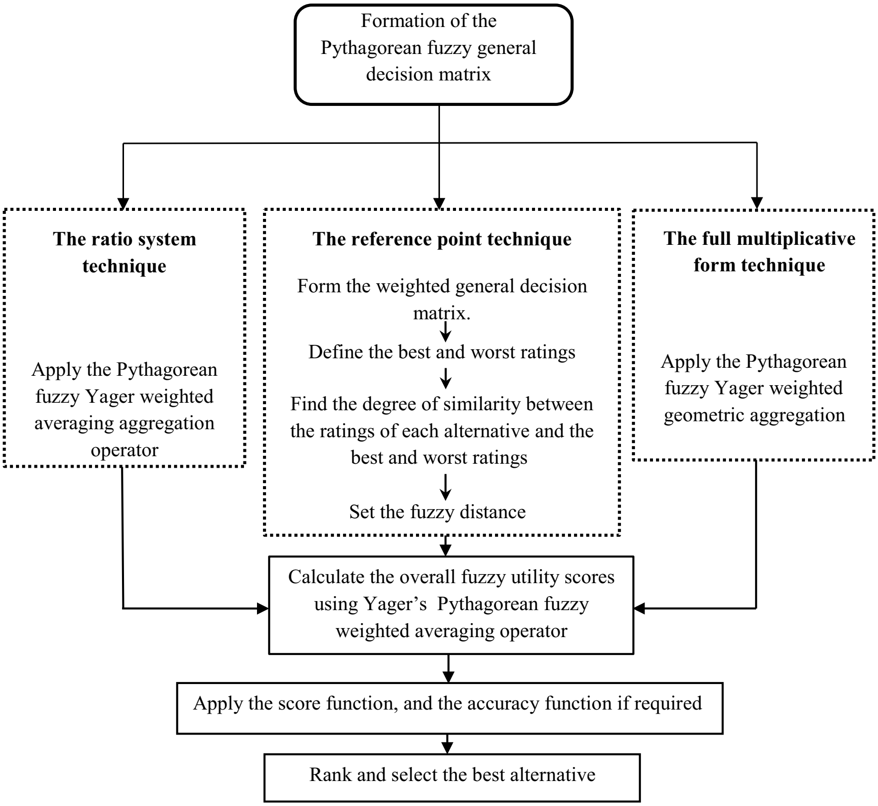

Fig. 1 The proposed PF-MULTIMOORA

5 Evaluation of Energy Storage Technologies

5.1 An Overview

As a result of industrialization and the growing population, energy demand has been increasing in the world. Renewable energy sources (RESs) are seen as effective alternatives to fulfill these increasing requirements. Since RESs are fluctuating and intermittent, energy storage technologies (ESTs) enable the storage of excess energy and utilize it when needed to secure energy supply [9]. ESTs provide a wide range of approaches to create a more resilient energy infrastructure and bring cost savings to utilities and consumers. Energy storage devices are charged when they absorb energy and discharged when they deliver the stored energy back into the grid. Charging and discharging processes normally require power conversion devices to transform electrical energy into a different energy form, e.g., chemical, electrochemical, electrical, mechanical, and thermal. In other words, energy storage enables supply and demand to be balanced even when the generation and consumption of energy do not happen at the same time [44].

Finding better ways to store energy is critical to becoming more energy efficient. Advances in energy storage can be achieved by finding new materials and understanding how current and new materials function. ESTs can be used in diverse applications. While some of them can be properly selected for specific applications, some others are applicable in wider frames. The key factor to success lies in matching the application to technology [27].

ESTs can be classified into five main categories thermal energy, chemical energy, electrochemical energy, mechanical energy, and electromagnetic energy storage as previously mentioned in Sect. 2. The form of converted energy determines the class of the EST. The power storing capacity, energy and power densities, response time, cost and economy scale, operating life, monitoring and control mechanisms, efficiency, and operating constraints are the critical parameters that govern the choice as to which type of technology.

The selection of an EST is an MCDM problem since the evaluation of ESTs is based on multiple conflicting criteria. The main criteria in the assessment of the performance of EST to achieve sustainability and energy security are technological, economic, and environmental criteria. Technological criteria allow assessing the reliability of the used technology and its ability to ensure safe energy supply. Economic criteria take into account competitiveness and affordability issues through the associated costs of installation and their impact on energy prices. Environmental criteria allow addressing environmental sustainability [68].

5.2 A Practical Example

The proposed PF-MULTIMOORA is utilized to rank a set of different ESTs. Fourteen alternatives are evaluated by using eleven criteria. The alternatives are given in Table 1. The criteria from one to eight are technological; the ninth and tenth criteria are economical, the eleventh criteria are environmental. The assessment criteria are defined as follows.

(C_1) **The power rating**: indicates the size of the power conversion subsystems resulting from the maximum power requirements of the electrical load on the discharging part (generation side) and the appearing excess power on the charging part (input side) [67]. The power rating is measured in megawatt (MW). High power rating indicates better EST [68].

(C_2) **The energy rating**: measured in hours, is the duration of discharge, i.e., the duration needed to empty the reservoir initially full at maximum outflow capacity [10]. It indicates how long a storage device can maintain output. Long discharge period is preferred since operating flexibility is required to manage variations in renewable energy generation and load to match demand. A Long duration EST refers to an EST with durations of 10 or more hours [15].

Table 1 The evaluated ESTs

Alternative	Name	Technology
X_1	Hydrogen	Chemical storage
X_2	Pumped hydroelectric storage (PHS)	Mechanical storage
X_3	Compressed air energy storage (CAES)	Mechanical storage
X_4	Flywheel	Mechanical storage
X_5	Superconducting magnetic energy storage (SMES)	Electrical storage
X_6	Supercapacitors (Supercap)	Electrical storage
X_7	Lead-acid (Pb-acid)	Electrochemical storage
X_8	Nickel–cadmium (NiCd)	Electrochemical storage
X_9	Lithium-ion (Li-ion)	Electrochemical storage
X_{10}	Sodium–Sulphur (NaS)	Electrochemical storage
X_{11}	Sodium–nickel chloride (NaNiCl)	Electrochemical storage
X_{12}	Vanadium redox (VRB)	Electrochemical storage
X_{13}	Zinc–bromine (ZnBr)	Electrochemical storage
X_{14}	Molten Salt	Thermal storage

(C_3) **The response time**: indicates the required time to activate the system, i.e., how quickly a storage technology can be brought into operation and discharge energy. ESTs with short response time provide electricity instantly, while ESTs with long response time provide electricity after a time interval [67]. It is measured on a linguistic scale. The lower this value the better the EST, since the rapid response is preferred [68].

(C_4) **The energy density**: the ratio of energy storage capacity to the system volume or mass [67]. It is measured in Wh/kg. High energy density indicates better EST [68].

(C_5) **The self-discharge time**: also known as idling losses, it is the losses occurring during the time in which energy remains stored [67]. It is measured in percentage per day, the lower the losses the better the EST [68].

(C_6) **The round-trip efficiency**: it is the ratio of input energy (in MWh) to the energy retrieved from storage (in MWh). It is measured in percentage. High round-trip efficiency is required [68].

(C_7) **The lifetime**: also known as the service period, it is expressed in years for a certain cycling rate, or in the total number of cycles, where a cycle is the time during which the system is fully charged and discharged. Long lifetime is required [68].

(C_8) **The number of cycles of operation**: the charge/discharge performance that represents the demands associated with a specific application placed on an EST.

(C_9) **The power cost**: it is the total costs of installation. It is measured in Eur/kW. Lower costs are desired [68].

(C_{10}) **The energy cost**: it is the costs of energy supply. It is measured in Eur/kWh. Lower costs are desired [68].

(C_{11}) **The environmental impact**: this encompasses the impacts of the construction, disposal/end of life, and usage of ESTs on the environment. For example, wastes from batteries manufacturing and recycling are a crucial and growing challenge for public health due to their toxicity, abundance and durability in the environment [13]. It is measured on a qualitative scale, and of course, the minimum impact must be attained.

From the previous illustration of the criteria, it is clear that they not only have quantitative and qualitative data but also the quantitative data have different units of measurement. Moreover, they differ in the objective. It is required to maximize $\{C_1, C_2, C_4, C_6, C_7, \text{and} C_8\}$, and minimize $\{C_3, C_5, C_9, C_{10}, \text{and} C_{11}\}$. Zhang et al. [68] transformed all the data into IFSs. In addition, the IFSs representing the criteria to be minimized was negated using (20). Therefore, the data given and used in this chapter is the data in its final form ready to be processed. The main difference is in the residual term, i.e., the hesitation margin, which is calculated under the PF condition. The problem data is given in Table 2. For detailed information about data transformation and fusion, the reader is referred to Zhang et al. [68].

Several weighting strategies were proposed by Zhang et al. [68] to test the effect of the different priorities on the selection of energy storage technologies. Each of the three strategies: technological, economic, and environmental is assigned a weight. Then, the weight of each strategy is equally distributed among its criteria.

First, they treated the main three strategies as equally important (balanced strategy). Hence, the weight of each dimension is (1/3). Therefore the weights of the criteria are

$$(0.042, 0.042, 0.042, 0.042, 0.042, 0.042, 0.042, 0.042, 0.167, 0.167, 0.333).$$

Second, they gave a high priority to the technological strategy (0.5) and (0.25) for the other two strategies. Then the weights of the criteria are given as follows:

$$(0.063, 0.063, 0.063, 0.063, 0.063, 0.063, 0.063, 0.063, 0.125, 0.125, 0.25).$$

Third, they gave a high priority to the economic strategy (0.5) and (0.25) for the other two strategies. Then the weights of the criteria are given as follows

$$(0.031, 0.031, 0.031, 0.031, 0.031, 0.031, 0.031, 0.031, 0.25, 0.25, 0.25).$$

Fourth, they gave a high priority to the environmental strategy (0.5) and (0.25) for the other two strategies. Then the weights of the criteria are given as follows:

$$(0.031, 0.031, 0.031, 0.031, 0.031, 0.031, 0.031, 0.031, 0.125, 0.125, 0.5).$$

The proposed PF-MULTIMOORA is applied to solve this problem for the differently proposed weights. The value of $\theta = 2$. The solution steps are demonstrated as follows:

Table 2 The PF ratings of the alternatives for the evaluation criteria

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
X_1	$\begin{pmatrix}0\\0.99\\0.1411\end{pmatrix}$	$\begin{pmatrix}0\\0.5223\\0.8528\end{pmatrix}$	$\begin{pmatrix}0.45\\0.5\\0.7399\end{pmatrix}$	$\begin{pmatrix}0.0796\\0.0046\\0.9968\end{pmatrix}$	$\begin{pmatrix}0.09829\\0.0043\\0.1841\end{pmatrix}$	$\begin{pmatrix}0.0477\\0.8808\\0.4711\end{pmatrix}$	$\begin{pmatrix}0.0354\\0.8937\\0.4473\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.9999\\0.0141\end{pmatrix}$	$\begin{pmatrix}0.7731\\0.078\\0.6295\end{pmatrix}$	$\begin{pmatrix}0.9984\\0.0001\\0.0565\end{pmatrix}$	$\begin{pmatrix}0.2\\0.75\\0.6305\end{pmatrix}$
X_2	$\begin{pmatrix}0.0199\\0.0028\\0.9998\end{pmatrix}$	$\begin{pmatrix}0.0199\\0.5223\\0.8525\end{pmatrix}$	$\begin{pmatrix}0.45\\0.5\\0.7399\end{pmatrix}$	$\begin{pmatrix}0\\0.9999\\0.0141\end{pmatrix}$	$\begin{pmatrix}1\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.1788\\0.7973\\0.5765\end{pmatrix}$	$\begin{pmatrix}0.3544\\0.2912\\0.8886\end{pmatrix}$	$\begin{pmatrix}0.0002\\0.9995\\0.0316\end{pmatrix}$	$\begin{pmatrix}0.4895\\0.0709\\0.8691\end{pmatrix}$	$\begin{pmatrix}0.9838\\0.0054\\0.1792\end{pmatrix}$	$\begin{pmatrix}0.1\\0.9\\0.4243\end{pmatrix}$
X_3	$\begin{pmatrix}0.0199\\0.9402\\0.3400\end{pmatrix}$	$\begin{pmatrix}0.0199\\0.5223\\0.8525\end{pmatrix}$	$\begin{pmatrix}0.2\\0.75\\0.6305\end{pmatrix}$	$\begin{pmatrix}0.003\\0.994\\0.1093\end{pmatrix}$	$\begin{pmatrix}1\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.1001\\0.8712\\0.4806\end{pmatrix}$	$\begin{pmatrix}0.1772\\0.7165\\0.6747\end{pmatrix}$	$\begin{pmatrix}0\\0.9995\\0.0316\end{pmatrix}$	$\begin{pmatrix}0.8369\\0.0567\\0.5444\end{pmatrix}$	$\begin{pmatrix}0.987\\0.0011\\0.1607\end{pmatrix}$	$\begin{pmatrix}0.1\\0.9\\0.4243\end{pmatrix}$
X_4	$\begin{pmatrix}0\\0.996\\0.0894\end{pmatrix}$	$\begin{pmatrix}0.00001\\0.995\\0.0999\end{pmatrix}$	$\begin{pmatrix}0.45\\0.5\\0.7399\end{pmatrix}$	$\begin{pmatrix}0.0005\\0.9871\\0.1601\end{pmatrix}$	$\begin{pmatrix}0.1452\\0.171\\0.9745\end{pmatrix}$	$\begin{pmatrix}0.2027\\0.7735\\0.6005\end{pmatrix}$	$\begin{pmatrix}0.1418\\0.8582\\0.4933\end{pmatrix}$	$\begin{pmatrix}0.001\\0.9005\\0.4349\end{pmatrix}$	$\begin{pmatrix}0.9575\\0.0142\\0.2881\end{pmatrix}$	$\begin{pmatrix}0.6219\\0.108\\0.7756\end{pmatrix}$	$\begin{pmatrix}0.9\\0.1\\0.4243\end{pmatrix}$
X_5	$\begin{pmatrix}0\\0.998\\0.0632\end{pmatrix}$	$\begin{pmatrix}0\\0.9983\\0.0583\end{pmatrix}$	$\begin{pmatrix}0.6\\0.35\\0.7194\end{pmatrix}$	$\begin{pmatrix}0\\0.9995\\0.0316\end{pmatrix}$	$\begin{pmatrix}0.8717\\0.0855\\0.4823\end{pmatrix}$	$\begin{pmatrix}0.2265\\0.7735\\0.5919\end{pmatrix}$	$\begin{pmatrix}0.1418\\0.8582\\0.4933\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.9999\\0.0141\end{pmatrix}$	$\begin{pmatrix}0.9433\\0.0142\\0.3316\end{pmatrix}$	$\begin{pmatrix}0.2437\\0.0756\\0.9669\end{pmatrix}$	$\begin{pmatrix}0.1\\0.9\\0.4243\end{pmatrix}$

(continued)

Table 2 (continued)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
X_6	$\begin{pmatrix}0\\0.9998\\0.0200\end{pmatrix}$	$\begin{pmatrix}0\\0.9801\\0.1985\end{pmatrix}$	$\begin{pmatrix}0.6\\0.35\\0.7194\end{pmatrix}$	$\begin{pmatrix}0\\0.9985\\0.0548\end{pmatrix}$	$\begin{pmatrix}0.6581\\0.0171\\0.7527\end{pmatrix}$	$\begin{pmatrix}0.2027\\0.7663\\0.6097\end{pmatrix}$	$\begin{pmatrix}0.1418\\0.8582\\0.4933\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.005\\1\end{pmatrix}$	$\begin{pmatrix}0.9433\\0.0142\\0.3316\end{pmatrix}$	$\begin{pmatrix}0.5678\\0.0324\\0.8225\end{pmatrix}$	$\begin{pmatrix}0.6\\0.35\\0.7194\end{pmatrix}$
X_7	$\begin{pmatrix}0\\0.99\\0.1411\end{pmatrix}$	$\begin{pmatrix}0\\0.9403\\0.3403\end{pmatrix}$	$\begin{pmatrix}0.9\\0.1\\0.4243\end{pmatrix}$	$\begin{pmatrix}0.003\\0.995\\0.0998\end{pmatrix}$	$\begin{pmatrix}0.9974\\0.0009\\0.0721\end{pmatrix}$	$\begin{pmatrix}0.1431\\0.7735\\0.6174\end{pmatrix}$	$\begin{pmatrix}0.0213\\0.8937\\0.4482\end{pmatrix}$	$\begin{pmatrix}0\\1\\0\end{pmatrix}$	$\begin{pmatrix}0.9078\\0.0284\\0.4184\end{pmatrix}$	$\begin{pmatrix}0.9676\\0.00054\\0.2524\end{pmatrix}$	$\begin{pmatrix}0.1\\0.9\\0.4243\end{pmatrix}$
X_8	$\begin{pmatrix}0\\0.992\\0.1262\end{pmatrix}$	$\begin{pmatrix}0\\0.9801\\0.1985\end{pmatrix}$	$\begin{pmatrix}0.9\\0.1\\0.4243\end{pmatrix}$	$\begin{pmatrix}0.004\\0.994\\0.1093\end{pmatrix}$	$\begin{pmatrix}0.9949\\0.0017\\0.1009\end{pmatrix}$	$\begin{pmatrix}0.1431\\0.783\\0.6053\end{pmatrix}$	$\begin{pmatrix}0.1063\\0.8582\\0.5022\end{pmatrix}$	$\begin{pmatrix}0\\1\\0\end{pmatrix}$	$\begin{pmatrix}0.8582\\0.0496\\0.5109\end{pmatrix}$	$\begin{pmatrix}0.892\\0.0216\\0.4515\end{pmatrix}$	$\begin{pmatrix}0.9\\0.1\\0.4243\end{pmatrix}$
X_9	$\begin{pmatrix}0\\1\\0\end{pmatrix}$	$\begin{pmatrix}0.003\\0.9801\\0.1985\end{pmatrix}$	$\begin{pmatrix}0.9\\0.1\\0.4243\end{pmatrix}$	$\begin{pmatrix}0.0075\\0.9751\\0.2216\end{pmatrix}$	$\begin{pmatrix}0.9974\\0.0009\\0.0721\end{pmatrix}$	$\begin{pmatrix}0.2027\\0.7616\\0.6155\end{pmatrix}$	$\begin{pmatrix}0.0354\\0.8937\\0.4473\end{pmatrix}$	$\begin{pmatrix}0\\0.9999\\0.0141\end{pmatrix}$	$\begin{pmatrix}0.5746\\0.0993\\0.8124\end{pmatrix}$	$\begin{pmatrix}0.8055\\0.0216\\0.5922\end{pmatrix}$	$\begin{pmatrix}0.6\\0.35\\0.7194\end{pmatrix}$
X_{10}	$\begin{pmatrix}0.0001\\0.99\\0.1411\end{pmatrix}$	$\begin{pmatrix}0\\0.9602\\0.2739\end{pmatrix}$	$\begin{pmatrix}0.9\\0.1\\0.4243\end{pmatrix}$	$\begin{pmatrix}0.0149\\0.9761\\0.2168\end{pmatrix}$	$\begin{pmatrix}0.829\\0.171\\0.5325\end{pmatrix}$	$\begin{pmatrix}0.2027\\0.7854\\0.5849\end{pmatrix}$	$\begin{pmatrix}0.0709\\0.8937\\0.4430\end{pmatrix}$	$\begin{pmatrix}0\\1\\0\end{pmatrix}$	$\begin{pmatrix}0.7164\\0.0993\\0.6909\end{pmatrix}$	$\begin{pmatrix}0.9028\\0.0216\\0.4295\end{pmatrix}$	$\begin{pmatrix}0.6\\0.35\\0.7194\end{pmatrix}$
X_{11}	$\begin{pmatrix}0\\0.9998\\0.0200\end{pmatrix}$	$\begin{pmatrix}0.0003\\0.9801\\0.1985\end{pmatrix}$	$\begin{pmatrix}0.9\\0.1\\0.4243\end{pmatrix}$	$\begin{pmatrix}0.0124\\0.9876\\0.1565\end{pmatrix}$	$\begin{pmatrix}0.8718\\0.1282\\0.4728\end{pmatrix}$	$\begin{pmatrix}0.2146\\0.7854\\0.5806\end{pmatrix}$	$\begin{pmatrix}0.0709\\0.9008\\0.4284\end{pmatrix}$	$\begin{pmatrix}0\\1\\0\end{pmatrix}$	$\begin{pmatrix}0.9716\\0.0142\\0.2362\end{pmatrix}$	$\begin{pmatrix}0.9838\\0.0076\\0.1791\end{pmatrix}$	$\begin{pmatrix}0.9\\0.1\\0.4243\end{pmatrix}$

(continued)

Table 2 (continued)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
X_{12}	$\begin{pmatrix} 0 \\ 0.9986 \\ 0.0529 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0.801 \\ 0.5987 \end{pmatrix}$	$\begin{pmatrix} 0.6 \\ 0.35 \\ 0.7194 \end{pmatrix}$	$\begin{pmatrix} 0.0075 \\ 0.9925 \\ 0.1220 \end{pmatrix}$	$\begin{pmatrix} 0.9145 \\ 0.1282 \\ 0.4046 \end{pmatrix}$	$\begin{pmatrix} 0.2027 \\ 0.7973 \\ 0.5685 \end{pmatrix}$	$\begin{pmatrix} 0.0354 \\ 0.8582 \\ 0.5121 \end{pmatrix}$	$\begin{pmatrix} 0.0001 \\ 0.9999 \\ 0.0141 \end{pmatrix}$	$\begin{pmatrix} 0.6455 \\ 0.3545 \\ 0.6765 \end{pmatrix}$	$\begin{pmatrix} 0.892 \\ 0.0108 \\ 0.4519 \end{pmatrix}$	$\begin{pmatrix} 0.1 \\ 0.9 \\ 0.4243 \end{pmatrix}$
X_{13}	$\begin{pmatrix} 0 \\ 0.9996 \\ 0.0283 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0.801 \\ 0.5987 \end{pmatrix}$	$\begin{pmatrix} 0.6 \\ 0.35 \\ 0.7194 \end{pmatrix}$	$\begin{pmatrix} 0.006 \\ 0.992 \\ 0.1261 \end{pmatrix}$	$\begin{pmatrix} 0.9915 \\ 0.0085 \\ 0.1298 \end{pmatrix}$	$\begin{pmatrix} 0.1669 \\ 0.8212 \\ 0.5457 \end{pmatrix}$	$\begin{pmatrix} 0.0354 \\ 0.9291 \\ 0.3681 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.7448 \\ 0.0709 \\ 0.6635 \end{pmatrix}$	$\begin{pmatrix} 0.9244 \\ 0.0108 \\ 0.3813 \end{pmatrix}$	$\begin{pmatrix} 0.9 \\ 0.1 \\ 0.4243 \end{pmatrix}$
X_{14}	$\begin{pmatrix} 0.0002 \\ 0.9701 \\ 0.2427 \end{pmatrix}$	$\begin{pmatrix} 0.0199 \\ 0.5223 \\ 0.8525 \end{pmatrix}$	$\begin{pmatrix} 0.2 \\ 0.75 \\ 0.6305 \end{pmatrix}$	$\begin{pmatrix} 0.008 \\ 0.9801 \\ 0.1983 \end{pmatrix}$	$\begin{pmatrix} 0.9915 \\ 0.0004 \\ 0.1301 \end{pmatrix}$	$\begin{pmatrix} 0.1192 \\ 0.8569 \\ 0.5015 \end{pmatrix}$	$\begin{pmatrix} 0.0354 \\ 0.8937 \\ 0.4473 \end{pmatrix}$	$\begin{pmatrix} 0.0001 \\ 0.9999 \\ 0.0141 \end{pmatrix}$	$\begin{pmatrix} 0.9575 \\ 0.0284 \\ 0.2870 \end{pmatrix}$	$\begin{pmatrix} 0.9935 \\ 0.0032 \\ 0.1138 \end{pmatrix}$	$\begin{pmatrix} 0.6 \\ 0.35 \\ 0.7194 \end{pmatrix}$

Step 1. Form the Pythagorean fuzzy general decision matrix and determine the weights of the criteria.

The general decision matrix is given in Table 2.

Step 2. Apply the ratio system technique using (16).

$$\tilde{U}_i^A = \text{PFYWA}\left(\tilde{X}_{ij} | j = 1, 2, \ldots, m; w\right).$$

Step 3. Apply the reference point technique.

(i) Form the weighted general decision matrix.
(ii) Define the best rating (1, 0, 0) and the worst rating (0, 1, 0).
(iii) Find the degree of similarity between the ratings of each alternative for the evaluation criteria and the best rating.

$$\mu_i = S_i^+\left(\tilde{\mathcal{X}}_{ij}, \tilde{R}_j^+\right) = 1 - d_i^+\left(\tilde{\mathcal{X}}_{ij}, \tilde{R}_j^+\right),$$

where $d_i^+\left(\tilde{\mathcal{X}}_{ij}, \tilde{R}_j^+\right)$ is the normalized Euclidean distance (36).

(iv) Find the degree of similarity between the ratings of each alternative for the evaluation criteria and the worst rating.

$$\upsilon_i = S_i^-\left(\tilde{\mathcal{X}}_{ij}, \tilde{R}_j^-\right) = 1 - d_i^-\left(\tilde{\mathcal{X}}_{ij}, \tilde{R}_j^-\right),$$

where $d_i^-\left(\tilde{\mathcal{X}}_{ij}, \tilde{R}_j^-\right)$ is the normalized Euclidean distance (37).

(v) Set the fuzzy distance from the reference point.

$$\tilde{U}_i^R = (\mu_i, \upsilon_i).$$

Step 4. Apply the full multiplicative form approach using (17).

$$\tilde{U}_i^M = PFYWG\left(\tilde{X}_{ij} | j = 1, 2, \ldots, m; w\right).$$

Step 5. Calculate the overall fuzzy utility scores using (4).

$$\tilde{U}_i^T = PFWA_Y\left(\tilde{U}_i^A, \tilde{U}_i^R, \tilde{U}_i^M | \omega_A, \omega_R, \omega_M\right),$$

where $\omega_A = \omega_R = \omega_M$, and$\omega_A + \omega_R + \omega_M = 1$.

Step 6. Rank the alternatives by the overall fuzzy utility scores using the score function (18) and the accuracy function (19).

The results of the three techniques, the overall fuzzy scores, and the ranking are summarized in Table 3.

The best ESTs obtained by the proposed PF-MULTIMOORA are NaNiCl, Ni–Cd, and ZnBr. Meanwhile, the worst ESTs are Pb-acid, VRB, and SMES. The best ESTs obtained by the IF-MULTIMOORA [68] is Molten salt, NaNiCl, and ZnBr. The worst three technologies coincide in the two methods.

The problem is resolved using the normalized Hausdorff and the normalized Hamming distances in the reference point approach to compare the ranking results. The rankings obtained by the PF-MULTIMOORA using these distance measures are given in Table 4. The results reveal that the ranking remains unchanged using different normalized distance measures.

Table 3 Results using the normalized Euclidean distance in the reference point approach

	AU	RPU (Euclidean)	MU	Total	Score	Rank
Hydrogen	(0.7189, 0.5219)	(0.2089, 0.7024)	(0.4667, 0.7039)	(0.4879, 0.6802)	−0.2247	9
PHS	(0.6773, 0.5314)	(0.2252, 0.6218)	(0.4121, 0.7553)	(0.4698, 0.6878)	−0.2523	10
CAES	(0.7291, 0.5866)	(0.2179, 0.6303)	(0.4551, 0.7882)	(0.5048, 0.7265)	−0.2729	11
Flywheel	(0.7880, 0.3809)	(0.1984, 0.7124)	(0.6129, 0.6607)	(0.5582, 0.6250)	−0.0790	4
SMES	(0.6351, 0.5884)	(0.1680, 0.7605)	(0.3724, 0.8070)	(0.4164, 0.7570)	−0.3997	14
Super cap	(0.6738, 0.3919)	(0.1977, 0.6978)	(0.5459, 0.6377)	(0.4907, 0.6113)	−0.1329	6
Pb-acid	(0.7573, 0.5816)	(0.2015, 0.7245)	(0.4856, 0.8034)	(0.5064, 0.7408)	−0.2922	12
Ni–Cd	(0.8339, 0.3646)	(0.2221, 0.6908)	(0.6852, 0.6696)	(0.6034, 0.6116)	−0.0099	2
Li-ion	(0.6695, 0.4192)	(0.1975, 0.7241)	(0.5530, 0.6742)	(0.4934, 0.6396)	−0.1656	8
NaS	(0.7039, 0.4217)	(0.1911, 0.7483)	(0.5832, 0.6721)	(0.5146, 0.6470)	−0.1538	7
NaNiCl	(0.8710, 0.3662)	(0.2282, 0.6812)	(0.6970, 0.6740)	(0.6261, 0.6148)	0.01399	1
VRB	(0.6418, 0.6096)	(0.1633, 0.7636)	(0.4352, 0.7970)	(0.4399, 0.7472)	−0.3649	13
ZnBr	(0.8135, 0.3711)	(0.2119, 0.6946)	(0.6621, 0.6628)	(0.5866, 0.6152)	−0.0345	3
Molten salt	(0.7887, 0.4279)	(0.2202, 0.6948)	(0.5887, 0.6592)	(0.5575, 0.6323)	−0.0890	5

Table 4 Results using normalized Hamming and normalized Hausdorff distances

	RPU (Hamming)	Total	Score	Rank	RPU (Hausdorff)	Total	Score	Rank
Hydrogen	(0.2796, 0.8169)	(0.4643, 0.6421)	−0.1967	9	(0.2237, 0.6674)	(0.4963, 0.6305)	−0.1772	9
PHS	(0.3215, 0.7788)	(0.4378, 0.6355)	−0.2123	10	(0.2468, 0.6031)	(0.4456, 0.6293)	−0.1975	10
CAES	(0.3319, 0.8069)	(0.4669, 0.6677)	−0.2278	11	(0.2302, 0.6039)	(0.4710, 0.6589)	−0.2124	11
Flywheel	(0.2754, 0.8353)	(0.5326, 0.5841)	−0.0575	4	(0.2120, 0.6730)	(0.5371, 0.5710)	−0.0375	4
SMES	(0.2430, 0.8779)	(0.3914, 0.7179)	−0.3622	14	(0.1854, 0.7336)	(0.3972, 0.7090)	−0.3448	14
Super cap	(0.2538, 0.8062)	(0.4720, 0.5752)	−0.1081	6	(0.2306, 0.6697)	(0.4830, 0.5659)	−0.0869	6
Pb-acid	(0.2779, 0.8395)	(0.4810,0.7025)	−0.2621	12	(0.2093, 0.6797)	(0.4836, 0.6875)	−0.2389	12
Ni–Cd	(0.2930, 0.8024)	(0.5798, 0.5744)	−0.0062	2	(0.2331, 0.6433)	(0.5835, 0.5586)	0.0284	2
Li-ion	(0.2593, 0.8272)	(0.4728, 0.6052)	−0.1427	8	(0.2171, 0.6916)	(0.4794, 0.5944)	−0.1236	8
NaS	(0.2581, 0.8491)	(0.4922, 0.6134)	−0.1340	7	(0.2032, 0.7096)	(0.4963, 0.6005)	−0.1143	7
NaNiCl	(0.3122, 0.8062)	(0.5981, 0.5732)	0.0292	1	(0.2306, 0.6254)	(0.5989, 0.6005)	0.0511	1
VRB	(0.2440, 0.8375)	(0.4130, 0.6927)	−0.3029	13	(0.2106, 0.6496)	(0.4288, 0.6847)	−0.2849	13
ZnBr	(0.2860, 0.8138)	(0.5619, 0.5756)	−0.0155	3	(0.2257, 0.6470)	(0.5665, 0.5597)	0.0077	3
Molten salt	(0.2967, 0.8116)	(0.5320, 0.5934)	−0.0691	5	(0.2271, 0.6513)	(0.5343, 0.5789)	−0.0496	5

The effect of the different strategies: technical, economic, and environmental strategies, on the ranking of the ESTs is also studied. The three distance measures are also used to rank the alternatives. The ranking using the environmental strategy is the same as the balanced strategy utilizing the three distance measures. Regarding the technical and economic strategies, the top-ranked three technologies and the worst-ranked three technologies are the same for the three distance measures. Slight differences are observed in the results from the balanced strategy in the moderately performing technologies. From four to six alternatives exchange rankings, within one or two places forward and backwards according to the used distance measure.

The ranking of the ESTs for the different strategies using the three distance measures is quite consistent. In the economic and environmental strategies, the ranking of the three distance measures is the same. In the technical strategy, only the 8th and 9th alternatives exchanged ranks. PHS is ranked the 8th and NaS is ranked the 9th using the normalized Euclidean and normalized Hamming distance. Meanwhile, PHS is ranked the 9th and NaS is ranked the 8th using the normalized Hausdorff distance. The results are summarized in Tables 5, 6, and 7.

The ranking of the proposed PF-MULTIMOORA and the IF-MULTIMOORA [68] for the different strategies are given in Table 8.

The results of the three approaches in Table 3 are defuzzified as given in Table 9, and then the alternatives are ranked using the dominance theory. From Table 10, the generally dominating rule is applied to rank the alternatives. The ranking is the

Table 5 Results using technical strategy

	Technical					
	Normalized Hamming distance		Normalized Euclidean distance		Normalized Hamming distance	
	Score	Rank	Score	Rank	Score	Rank
Hydrogen	−0.2662	10	−0.2371	10	−0.2164	10
PHS	−0.2649	9	−0.2258	8	−0.2126	8
CAES	−0.3246	11	−0.2777	11	−0.2694	11
Flywheel	−0.2107	5	−0.1889	5	−0.1716	5
SMES	−0.4378	14	−0.4080	14	−0.3932	14
Super cap	−0.2179	6	−0.1919	6	−0.1731	6
Pb-acid	−0.3315	12	−0.3033	12	−0.2815	12
Ni–Cd	−0.1250	2	−0.1074	2	−0.0869	2
Li-ion	−0.2424	7	−0.2207	7	−0.2028	7
NaS	−0.2469	8	−0.2308	9	−0.2133	9
NaNiCl	−0.1148	1	−0.0982	1	−0.0774	1
VRB	−0.4014	13	−0.3470	13	−0.3239	13
ZnBr	−0.1535	3	−0.1337	3	−0.1137	3
Molten salt	−0.1968	4	−0.1743	4	−0.1548	4

Table 6 Results using economic strategy

	Economic					
	Normalized Hamming distance		Normalized Euclidean distance		Normalized Hausdorff distance	
	Score	Rank	Score	Rank	Score	Rank
Hydrogen	−0.1146	9	−0.0887	9	−0.0704	9
PHS	−0.1470	11	−0.1118	11	−0.0943	11
CAES	−0.1357	10	−0.0998	10	−0.0824	10
Flywheel	−0.0154	5	0.0052	5	0.0268	5
SMES	−0.2878	14	−0.2455	14	−0.2251	14
Super cap	−0.0609	6	−0.0360	6	−0.0124	6
Pb-acid	−0.1610	12	−0.1321	12	−0.1066	12
Ni–Cd	0.0526	2	0.0677	2	0.0924	2
Li-ion	−0.1048	8	−0.0799	8	−0.0574	8
NaS	−0.0733	7	−0.0520	7	−0.0300	7
NaNiCl	0.0937	1	0.1060	1	0.1269	1
VRB	−0.2519	13	−0.2013	13	−0.1733	13
ZnBr	0.0260	3	0.0439	3	0.0691	3
Molten salt	0.0120	4	0.0288	4	0.0468	4

Table 7 Results using environmental strategy

	Environmental					
	Normalized Hamming distance		Normalized Euclidean distance		Normalized Hausdorff distance	
	Score	Rank	Score	Rank	Score	Rank
Hydrogen	−0.3009	9	−0.2729	9	−0.2538	9
PHS	−0.3537	10	−0.3081	10	−0.2970	10
CAES	−0.3669	11	−0.3160	11	−0.3031	11
Flywheel	−0.0066	4	0.0107	4	0.0283	4
SMES	−0.4814	14	−0.4483	14	−0.4335	14
Super cap	−0.1203	6	−0.0988	6	−0.0801	6
Pb-acid	−0.3942	12	−0.3654	12	−0.3438	12
Ni–Cd	0.0465	2	0.0594	2	0.0788	2
Li-ion	−0.1481	8	−0.1280	8	−0.1096	8
NaS	−0.1388	7	−0.1204	7	−0.1024	7
NaNiCl	0.0667	1	0.0788	1	0.0980	1
VRB	−0.4470	13	−0.3862	13	−0.3650	13
ZnBr	0.0281	3	0.0434	3	0.0636	3
Molten salt	−0.0836	5	−0.0656	5	−0.0477	5

Table 8 Ranking results for different strategies using PF-MULTIMOORA and IF-MULTIMOORA

Technology	Balanced		Technical		Economic		Environmental	
	PF	IF	PF	IF	PF	IF	PF	IF
Hydrogen	9	6	10	6	9	5	9	9
PHS	10	9	8	5	11	9	10	11
CAES	11	8	11	4	10	7	11	10
Flywheel	4	5	5	10	5	6	4	5
SMES	14	14	14	14	14	14	14	14
Super cap	6	11	6	11	6	11	6	8
Pb-acid	12	12	12	9	12	10	12	12
Ni–Cd	2	4	2	2	2	4	2	4
Li-ion	8	10	7	8	8	12	8	7
NaS	7	7	9	12	7	8	7	6
NaNiCl	1	2	1	7	1	2	1	1
VRB	13	13	13	13	13	13	13	13
ZnBr	3	3	3	3	3	3	3	2
Molten salt	5	1	4	1	4	1	5	3

Table 9 The scores of the three approaches using the Euclidean distance

Technology	AU score	RPU score	MU score
Hydrogen	0.2444	−0.4497	−0.2777
PHS	0.1763	−0.3360	−0.4007
CAES	0.1875	−0.3498	−0.4141
Flywheel	0.4759	−0.4682	−0.0609
SMES	0.0571	−0.5501	−0.5126
Super cap	0.3004	−0.4478	−0.1087
Pb-acid	0.2352	−0.4843	−0.4096
Ni–Cd	0.5625	−0.4284	−0.0211
Li-ion	0.2725	−0.4853	−0.1487
NaS	0.3176	−0.5234	−0.1116
NaNiCl	0.6245	−0.4120	0.0315
VRB	0.0403	−0.5564	−0.4458
ZnBr	0.5241	−0.4376	0
Molten salt	0.4389	−0.4343	−0.0880

same till the ninth place. A slight change is observed in the least ranked ESTs. PHS and Pb-acid exchange the 10th and 12th place, and SMES and VRB exchange the 13th and 14th place.

Table 10 The solution using the dominance theory

Rank	AU ranking	RPU ranking	MU ranking	Total rank
1	NaNiCl	PHS	NaNiCl	NaNiCl
2	Ni–Cd	CAES	Ni–Cd	Ni–Cd
3	ZnBr	NaNiCl	ZnBr	ZnBr
4	Flywheel	Ni–Cd	Flywheel	Flywheel
5	Molten salt	Molten salt	Molten salt	Molten salt
6	NaS	ZnBr	Super cap	Super cap
7	Super cap	Super cap	NaS	NaS
8	Li-ion	Hydrogen	Li-ion	Li-ion
9	Hydrogen	Flywheel	Hydrogen	Hydrogen
10	Pb-acid	Pb-acid	PHS	Pb-acid
11	CAES	Li-ion	Pb-acid	PHS
12	PHS	NaS	CAES	CAES
13	SMES	SMES	VRB	SMES
14	VRB	VRB	SMES	VRB

6 Conclusion

The MULTIMOORA method is one of the most practical MCDM methods that has been used to solve complicated decision-making problems. In this chapter, a new version of MULTIMOORA is developed to increase its efficiency and accuracy for solving large scale MCDM applications in the Pythagorean fuzzy environment. The proposed PF-MULTIMOORA exploits newly proposed aggregation operators that guarantee fair treatment among the evaluation criteria. Therefore, the proposed method avoids any biased treatment and false ranking that might occur in certain situations. When applying the reference point technique the distance is defined on a fuzzy basis rather than a crisp basis. Hence, instead of utilizing one reference point, i.e., the best rating, two reference points are utilized: the best and worst ratings. To avoid the complications of the dominance theory, the aggregation approach is applied. So, the aggregation approach is carried out using the fuzzy results of the three techniques. Thus, defuzzification is employed only in the final step for ranking in which the accuracy function can be also utilized with the score function to make the comparison more discriminatory.

Energy storage technologies were evaluated using the developed PF-MULTIMOORA. Sodium-nickel chloride, nickel–cadmium, and zinc–bromine were the top-ranked energy storage technologies. Meanwhile, lead-acid, vanadium redox, and superconducting magnetic energy storage were the worst-ranked technologies.

The dominance theory was applied to rank the alternatives instead of aggregating the three approaches. The result revealed that the alternatives till the ninth place are unchanged. A slight change is observed in the least ranked ESTs. It was clear that ranking by aggregating the three approaches is more direct and simpler than the dominance theory.

Besides the balanced weighting strategy, the effect of technical, economic, and environmental strategies on the ranking of the ESTs was studied. Only slight differences were observed in the results from the balanced strategy in the moderately performing technologies, while the three top technologies and three worst technologies remain unchanged.

In addition to the normalized Euclidean distance, another two distance measures were examined in the reference point technique, the normalized Hausdorff and the normalized Hamming distances. The ranking of the ESTs for the different weighting strategies using the two distance measures was quite similar to the ranking using the normalized Euclidean distance.

The contribution of the study can be summarized as follows. First, in the reference point technique, two reference points are used instead of one. Hence, the distance can be expressed using PFSs. Second, the study exploits aggregation operators in the ratio system approach and the full multiplicative form approach that prevent erroneous decisions.

The proposed PF-MULTIMOORA is restricted to using the theoretical reference point to guarantee that the resulting fuzzy distance is a PFS. The Empirical reference point cannot be utilized, which is a limitation in the proposed method. Future research will focus on expressing fuzzy distances using both theoretical and empirical reference points. Also, reference point techniques namely, PF-TOPSIS and PF-VIKOR will be implemented using fuzzy distances to study its performance compared with using crisp distances.

References

1. Asante D, He Z, Adjei NO, Asante B (2020) Exploring the barriers to renewable energy adoption utilising MULTIMOORA—EDAS method. Energy Policy 142:111479. https://doi.org/10.1016/j.enpol.2020.111479
2. Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20(1):87–96
3. Atanassov KT (1999) Intuitionistic Fuzzy sets: theory and applications. Springer Pyhsica-Verlag Heidelberg, New York, NY
4. Brauers WKM, Zavadskas EK (2006) The MOORA method and its application to privatization in a transition economy. Control Cybern 35(2):445–496
5. Brauers WKM, Zavadskas EK (2010) Project management by MULTIMOORA as an instrument for transition economies. Technol Econ Dev Econ Baltic J Sustain 16(1):5–24
6. Brauers WKM, Zavadskas EK (2012) Robustness of MULTIMOORA: a method for multi-objective optimization. Informatica 23(1):1–25
7. Chen S, Wang J, Wang T (2019) Cloud-based ERP system selection based on extended probabilistic linguistic MULTIMOORA method and Choquet integral operator. Comp Appl Math 38. https://doi.org/10.1007/s40314-019-0839-z.
8. Chen Y, Ran Y, Wang Z, Li X, Yang X, Zhang G (2020) An extended MULTIMOORA method based on OWGA operator and Choquet integral for risk prioritization identification of failure modes. Eng Appl Artif Intell 91:103605. https://doi.org/10.1016/j.engappai.2020.103605

9. Çolak M, Kaya I (2020) Multi-criteria evaluation of energy storage technologies based on hesitant fuzzy information: a case study for Turkey. J Energy Storage 28:101211. https://doi.org/10.1016/j.est.2020.101211
10. Crampes C, Trochet J-M (2019) Economics of stationary electricity storage with various charge and discharge durations. J Energy Storage J Energy Storage 24:100746. https://doi.org/10.1016/j.est.2019.04.020
11. Dahooie JH, Zavadskas EK, Firoozfar HR, Vanaki AS, Mohammadi N, Brauers WKM (2019) An improved fuzzy MULTIMOORA approach for multi-criteria decision making based on objective weighting method (CCSD) and its application to technological forecasting method selection. Eng Appl Artif Intell 79:114–128
12. Dai W, Zhong Q, Qi C (2020) Multi-stage multi-attribute decision-making method based on the prospect theory and triangular fuzzy MULTIMOORA. Soft Comput 24:9429–9440
13. Dehghani-Sanij AR, Tharumalingama E, Dusseault MB, Fraser R (2019) Study of energy storage systems and environmental challenges of batteries. Renew Sustain Energy Rev 104:192–208
14. Dizdar EN, Ünver M (2019) The assessment of occupational safety and health in Turkey by applying a decision-making method; MULTIMOORA. Hum Ecol Risk Assess. https://doi.org/10.1080/10807039.2019.1600399
15. Edington ANC (2019) The role of long duration energy storage in decarbonizing power systems. A thesis Submitted to the Institute for Data, Systems, and Society in partial fulfillment of the requirements for the degree of Master of Science in Technology and Policy at the Massachusetts Institute of Technology.
16. Ejegwa PA (2020) Distance and similarity measures for Pythagorean fuzzy sets. Granular Comput 5:225–238. https://doi.org/10.1007/s41066-018-00149-z
17. Fedajev A, Stanujkic D, Karabašević D, Brauers WKM, Zavadskas EK (2020) Assessment of progress towards "Europe 2020" strategy targets by using the MULTIMOORA method and the Shannon Entropy Index. J Clean Prod 244:118895. https://doi.org/10.1016/j.jclepro.2019.118895
18. Garg H (2016) A novel accuracy function under interval-valued Pythagorean fuzzy environment for solving multi-criteria decision making problem J. Intel Fuzzy Syst. 31(1):529–540
19. Garg H (2016) A new generalized Pythagorean fuzzy information aggregation using Einstein operations and its application to decision making. Int J Intel Syst. 31(9):886–920
20. Garg H (2018) Generalized Pythagorean fuzzy geometric interactive aggregation operators using Einstein operations and their application to decision making. J Experi Theor Artif Intel 30(6):763–794
21. Garg H (2018) New exponential operational laws and their aggregation operators for interval-valued Pythagorean fuzzy multicriteria decision-making. Int J Intel Syst. 33(3):653–683
22. Garg H (2019) New logarithmic operational laws and their aggregation operators for Pythagorean fuzzy set and their applications. Int J Intell Syst 34(1):82–106
23. Garg H (2019) Novel neutrality operation–based Pythagorean fuzzy geometric aggregation operators for multiple attribute group decision analysis. J Intell Syst 34(10):2459–2489
24. Garg H (2020) Neutrality operations-based Pythagorean fuzzy aggregation operators and its applications to multiple attribute group decision-making process. J Amb Intell Human Comput 11:3021–3041
25. Gündoğdu FK (2020) A spherical fuzzy extension of MULTIMOORA method. J Intell Fuzzy Syst 38(1):963–978
26. Gündoğdu FK, Kahraman C (2019) Spherical fuzzy sets and spherical fuzzy TOPSIS method. J Intell Fuzzy Syst 36(1):337–352
27. Guney MS, Tepe Y (2017) Classification and assessment of energy storage systems. Renew Sustain Energy Rev 75:1187–1197
28. Hafezalkotob A, Hafezalkotob A, Liao H, Herrera F (2019) An overview of MULTIMOORA for multi-criteria decision-making: theory, developments, applications, and challenges. Inform Fusion 51:145–177

29. Hussian Z, Yang M-S (2019) Distance and similarity measures of Pythagorean fuzzy sets based on the Hausdorff metric with application to fuzzy TOPSIS. Int J Intell Syst. 34(10):2633–2654
30. Khan N, Dilshad S, Khalid R, Kalair AR, Abas N (2019) Review of energy storage and transportation of energy. Energy Storage. https://doi.org/10.1002/est2.49
31. Krishan O, Suhag S (2018) An updated review of energy storage systems: classification and applications in distributed generation power systems incorporating renewable energy resources. Energy Res 43(12):6171–6210
32. Lee J, Srimuka P, Fleischmann S, Su X, Hatton TA, Presser V (2019) Redox-electrolytes for non-flow electrochemical energy storage: a critical review and best practice. Prog Mater Sci 101:46–89
33. Li X-H, Huang L, Li Q, Chen Liu H-C (2020) Passenger satisfaction evaluation of public transportation using Pythagorean fuzzy MULTIMOORA method under large group environment. Sustainability 12:4996. https://doi.org/10.3390/su12124996
34. Liang D, Darko AP, Xu Z, Wang M (2019) Aggregation of dual hesitant fuzzy heterogeneous related information with extended Bonferroni mean and its application to MULTIMOORA. Comput Ind Eng 135:156–176
35. Liang D, Darko AP, Zeng J (2019) Interval-valued Pythagorean fuzzy power average-based MULTIMOORA method for multi-criteria decision-making. J Exp Theor Artif Intell. https://doi.org/10.1080/0952813X.2019.1694589
36. Liang W, Zhao G, Hong C (2019) Selecting the optimal mining method with extended multi-objective optimization by ratio analysis plus the full multiplicative form (MULTIMOORA) approach. Neural Comput Appl 31:5871–5886
37. Liao H, Qin R, Gao C, Wu X, Hafezalkotob A, Herrera F (2019) Score-HeDLiSF: a score function of hesitant fuzzy linguistic term set based on hesitant degrees and linguistic scale functions: An application to unbalanced hesitant fuzzy linguistic MULTIMOORA. Inform Fusion 48:39–54
38. Lin M, Huang C, Xu Z (2020) MULTIMOORA based MCDM model for site selection of car sharing station under picture fuzzy environment. Sustain Cities Soc 53:101873. https://doi.org/10.1016/j.scs.2019.101873
39. Liu P, Li Y (2019) An extended MULTIMOORA method for probabilistic linguistic multicriteria group decision-making based on prospect theory. Comput Ind Eng 136:528–545
40. Luo L, Zhang C, Liao H (2019) Distance-based intuitionistic multiplicative MULTIMOORA method integrating a novel weight-determining method for multiple criteria group decision making. Comput Ind Eng 131:82–98
41. Ma Z, Xu Z (2016) Symmetric Pythagorean fuzzy weighted geometric/averaging operators and their application in multi-criteria decision-making problems. Int J Intel Syst. 31(12):1198–1219
42. Mahmoud M, Ramadan M, Olabi A, Pullen K, Naher S (2020) A review of mechanical energy storage systems combined with wind and solar applications. Energy Convers Manag 210:112670. https://doi.org/10.1016/j.enconman.2020.112670
43. Medina P, Bizuayehu AW, Catalão JPS, Rodrigues EMG, Contreras J (2014) Electrical energy storage systems: technologies' state-of-the-art, techno-economic benefits and applications analysis. In: 47th Hawaii international conference on system sciences, Waikoloa, HI, pp 2295–2304. https://doi.org/10.1109/HICSS.2014.290.
44. Mohler D, Sowder D (2017) Energy storage and the need for flexibility on the grid. renewable energy integration: practical management of variability, uncertainty, and flexibility in power grids, 2nd edn, Chapter 23, pp 309–316
45. Omrani H, Alizadeh A, Amini M (2020) A new approach based on BWM and MULTIMOORA methods for calculating semi-human development index: an application for provinces of Iran. Socio-Econ Plan Sci 70:100689. https://doi.org/10.1016/j.seps.2019.02.004
46. Peng X, Yang Y (2015) Some results for Pythagorean fuzzy sets. Int J Intell Syst 30(11):1133–1160
47. Peng X, Yang Y (2016) Pythagorean fuzzy Choquet integral based MABAC method for multi attribute group decision making. Int J Int Syst. 31(10):989–1020

48. Pérez-Domínguez L, Rodríguez-Picón LA, Alvarado-Iniesta A, David Luviano Cruz DL, Xu Z (2018) MOORA under Pythagorean fuzzy set for multiple criteria decision making. Complexity 2602376. https://doi.org/10.1155/2018/2602376
49. Rahimi S, Hafezalkotob A, Monavari SM, Hafezalkotob A, Rahimi R (2020) Sustainable landfill site selection for municipal solid waste based on a hybrid decision-making approach: fuzzy group BWM-MULTIMOORA-GIS. J Clean Prod 248:119186. https://doi.org/10.1016/j.jclepro.2019.119186
50. Sarbu I, Sebarchievici C (2018) A comprehensive review of thermal energy storage. Sustainability 10:191. https://doi.org/10.3390/su10010191
51. Shahzadi G, Akram M, Al-Kenani AN (2020) Decision-making approach under Pythagorean fuzzy Yager weighted operators. Mathematics 8:70. https://doi.org/10.3390/math8010070
52. Siksnelyte I, Zavadskas EK, Bausys R, Streimikiene D (2019) Implementation of EU energy policy priorities in the Baltic Sea Region countries: sustainability assessment based on neutrosophic MULTIMOORA method. Energy Policy 125:90–102
53. Smarandache F (1998) A unifying field in logics: neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability. American Research Press, Rehoboth
54. Souzangarzadeh H, Jahan A, Rezvani MJ et al (2020) Multi-objective optimization of cylindrical segmented tubes as energy absorbers under oblique crushes: D-optimal design and integration of MULTIMOORA with combinative weighting. Struct Multidisc Optim. https://doi.org/10.1007/s00158-020-02486-7
55. Tavana M, Shaabani A, Mohammadabadi SM, Varzganid N (2020) An integrated fuzzy AHP-fuzzy MULTIMOORA model for supply chain risk-benefit assessment and supplier selection. Int J Syst Sci: Oper Logist. https://doi.org/10.1080/23302674.2020.1737754
56. Torra V (2010) Hesitant fuzzy sets. Int J Intell Syst 25:529–539
57. Torra V, Narukawa Y (2009) On hesitant fuzzy sets and decision. In: IEEE international conference on fuzzy systems, Jeju Island, Korea, 20–24 August 2009
58. Wang L, Garg H, Li N (2020) Pythagorean fuzzy interactive Hamacher power aggregation operators for assessment of express service quality with entropy weight. Soft Comput. https://doi.org/10.1007/s00500-020-05193-z
59. Wei G, Lu M (2018) Pythagorean fuzzy Maclaurin symmetric mean operators in multiple attribute decision making. Int J Intel Syst 33(5):1043–1070
60. Xian S, Liu Z, Gou X, Wan W (2020) Interval 2-tuple Pythagorean fuzzy linguistic MULTIMOORA method with CIA and their application to MCGDM. Int J Intell Syst 35(4):650–681
61. Yager RR (2013) Pythagorean fuzzy subsets. In: Proceedings of joint IFSA world congress and NAFIPS annual meeting, Edmonton, Canada, 24–28 June 2013, pp 57–61
62. Yager RR (2014) Pythagorean membership grades in multicriteria decision making. IEEE Trans Fuzzy Syst 22:958–965
63. Yager RR, Abbasov AM (2013) Pythagorean membership grades, complex numbers, and decision making. Int J Intell Syst 28(5):436–452
64. Yörükoğlu M, Aydın S (2020) Wind turbine selection by using MULTIMOORA method. Energy. https://doi.org/10.1007/s12667-020-00387-8
65. Zadeh LH (1965) Fuzzy sets. Inf. Control 8(3):338–353
66. Zadeh LH (1975) The concept of a linguistic variable and its applications to approximate reasoning. Inf Sci 8:199–249
67. Zafirakis DP (2010) Overview of energy storage technologies for renewable energy systems. In: Kaldellis JK (ed) Stand-alone and hybrid wind energy systems technology, energy storage and applications. Woodhead Publishing Series in Energy, pp 29–80
68. Zeng W, Li D, Yin Q (2018) Distance and similarity measures of Pythagorean fuzzy sets and their applications to multiple criteria group decision making. Int J Intell Syst 33(11):2236–2254
69. Zhang C, Chen C, Streimikiene D, Balezentis T (2019) Intuitionistic fuzzy MULTIMOORA approach for multi-criteria assessment of the energy storage technologies. Appl Soft Comput J 79:410–423

70. Zhang X (2016) A novel approach based on similarity measure for Pythagorean fuzzy multiple criteria group decision making. Int J Intel Syst. 31(6):593–611
71. Zhang X, Xu Z (2014) Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets. Int J Intell Syst 29(12):1061–1078

Iman Mohamad Sharaf received the B.Sc. degree in Electronics and Communication Engineering from Cairo University. She also received the M.Sc. and the Ph.D. degrees in Engineering Mathematics from Cairo University, Egypt. Currently, she is an Associate Professor of Engineering Mathematics at the Department of Basic Sciences, the Higher Technological Institute, Tenth of Ramadan City, Egypt. She was the Head of the department for the academic year 2011/2012. She authored and co-authored several research papers published in refereed International Journals and international book series. Her Research interests include: Mathematical programming, multi-criteria decision making, fuzzy systems, computational intelligence, and soft computing.