Chapter 8
Supply Chain Coordination for Deteriorating Product with Price and Stock-Dependent Demand Rate Under the Supplier's Quantity Discount

Chetan A. Jhaveri and Anuja A. Gupta

Abstract In this research paper, optimal ordering and pricing strategy for deteriorating products is developed when demand of a product depends on selling price and stock availability. Without supply chain coordination, the buyer makes policy to maximize its own profit which may not be beneficial to the vendor. Vendor can offer quantity discount as an incentive to encourage buyer to participate in the coordinated strategy. To coordinate the vendor–buyer decisions, two coordination policies are presented in this paper. First, coordinated supply chain strategy is developed to show that integrated supply chain can get higher channel profits. Later, coordinated supply chain with quantity discount strategy is derived and the total profits under the two policies are compared. The numerical example demonstrates that the vendor–buyer coordination along with quantity discount results in an extra total profit and hence it is significant to consider the coordinated vendor–buyer supply chain strategy with quantity discount. Sensitivity analysis is carried out to understand the effect of various key parameters on the optimal solution.

Keywords Price-dependent demand · Stock-dependent demand · Deterioration · Supply chain coordination · Quantity discount · All-units quantity discounts

8.1 Introduction

Supply chain management can be explained as the systematic coordination of all the business processes like procurement of raw material, selection of vendor, product design, inventory management, manufacturing, and end-customer delivery. Supply chain management has been defined by Lambert et al. (1998) as the coordination of key business processes starting from raw material procurement till end-customer delivery of the product or service in such a way that it adds value to the customers

C. A. Jhaveri (✉) · A. A. Gupta
Institute of Management, Nirma University, Ahmedabad, India
e-mail: chetan@nirmauni.ac.in

A. A. Gupta
e-mail: anuja.gupta@nirmauni.ac.in

N. H. Shah et al. (eds.), *Decision Making in Inventory Management*,
Inventory Optimization, https://doi.org/10.1007/978-981-16-1729-4_8

as well as all the other stakeholders of the organization. A supportive relationship between the buyer and supplier would include mutual trust, sharing information, resource, and profit. This strong relationship is essential so as to have a successful supply chain network (Yang 2004). As a result, a mutually beneficial environment is created between the parties by increasing their joint profits that help the buyer in providing a faster response to the customer demand.

Supply chain coordination is an integral part of an organization, which is used to coordinate and focus on all the relevant resources on the supply chain thus optimizing the use of the available resources and capabilities involved in the overall supply chain. According to Yang (2004), there is rise in the attention given for coordinating the supply chain in organizations due to reasons like depletion in the resources, increase in competition, globalization trend, increasing costs, faster response times, and decreasing product life cycles. Increasing the speed at which materials move in the supply chain would help to reduce the stock level, which would further lead to cost savings for the company.

Nowadays retail stores display a wide array and variety of products of various color, brand, price, and flavor. This is because the companies have observed that a broader collection of products help them to attract more customers into purchasing them. Thus, demand for product is influenced by display stock and does not remain constant. Practically not all the products in the market can have a constant demand, hence it arises the need for development of inventory control models to tackle variable demands. In the past, studies have been done on inventory and pricing strategies for price-dependent demand and supplier's quantity discount schemes. Also, it is observed that product's price as well as its stock-display level affects its demand. It is believed that a large pile of stock display of a particular product in the supermarket will influence customers to purchase it as compared to a product that has a small pile on display.

Retail price of a product has a direct relationship with the demand rate while an inverse relationship with quantity discount price. In order to motivate buyers while making purchasing decisions, often lower costs per unit of goods or materials are offered when purchased in larger quantities. Thus quantity discount is offered by the vendors to persuade buyers into purchasing larger quantities. In the last few years, ecommerce has revolutionized the entire retail industry with the use of quantity discount schemes.

The main goal of this research study is to illustrate the importance of a coordinated supply chain while managing the inventory for deteriorating products having both price as well as stock-dependent demand rate considering the quantity discount scheme of supplier. This has been done by developing a mathematical model for a supply chain system, which is further explained using a numerical illustration to investigate the managerial implication. The second section of this paper contains relevant literature review. Third section includes the mathematical modeling towards the research objective. This section also explains various assumptions and parameters used for modelling. Solution algorithm and a numerical example have been presented in sections four and five respectively. Finally concluding remarks and suggestions for the analyzed model have been provided in the last section.

8.2 Literature Review

In this section, various relevant literatures have been discussed and classified based on the type of inventory models.

8.2.1 Inventory Models Considering Variable Demand

Most of the products in the market have a variable demand that is affected by many factors like price, availability, discounts, quality, and stock at display. Thus, there is a need to formulate models based on such factors to manage the inventory so that situations such as over-stocking and under-stocking don't arise. Sarker et al. (1997) have developed a model to achieve the optimal lot-size and order-level for a certain type of goods having varied demand due to decline in quality level. In this model authors have considered two cases wherein they have considered demand to be constant as well as dependent on the stock level. Various other researches have been done where demand depends on the stock, time, or price. Such literatures have been discussed further in this section.

8.2.2 Inventory Models Assuming Price Dependent Demand

For the price-sensitive demand, Li et al. (1996) developed a lot-for-lot joint pricing policy and discussed the benefits obtained as a result of coordination between the buyer and supplier. For the items having linear price function for demand, Wee (1997) came up with an optimal replenishment policy with an objective to maximize the net profit. For the products having constant demand rate, Wee (1998) came up with lot-for-lot discount pricing policy. But neither of these papers considered integrating quantity discount policy with the price-sensitive demand. Qin et al. (2007) developed inventory models with price sensitive demand rate in a coordinated supply chain system. Alfares and Ghaithan (2016) extended the research by Alfares (2015) by considering the price-dependent demand to the existing model.

8.2.3 Inventory Models with Stock-Dependent Demand

Large pile of stock is kept in the display in the supermarket to attract more customers into buying that product mainly because of the variety, visibility, and popularity. Also, it is observed that a low stock display would give out the perception of the product being of low quality or less sold. Thus it can be said that the demand rate for certain types of goods is influenced by the level of stock kept in display in the supermarkets. Stock-dependent consumption rate inventory model was developed by Gupta and Vrat (1986). Their model was anchored on the initial order quantity demand rate instead of the immediate inventory level requirements. Teng and Chang (2005) derived an economic production quantity (EPQ) model to show the dependence of demand rate for specific types of items on selling price per unit and on-display stock with an objective to maximize the profit as well. Goyal and Chang (2009) derived a model to identify the optimal ordering quantity for the buyer as the demand rate depends on the display stock level. Mandal and Phaujdar (1989), Datta and Pal (1990), Urban (2005), Hou and Lin (2006), Chang et al. (2010), Datta and Paul (2001), Sajadieh (2010) have developed and analysed various inventory models considering stock-dependent demand.

8.2.4 Deteriorating Products

As most of the physical products are deteriorating over time, in the recent years, the maintenance of inventories for deteriorating items have received much attention from several researchers. When the utility or usefulness of an item decreases through ways of evaporation, spoilage, or decay; it is known as deterioration of an item. Deterioration may happen during usual period of storage for several products like electronic components, chemicals, drugs, foods, films, etc. Hence, the loss occur due to deterioration of item cannot be ignored. Thus, deterioration of physical goods in the inventory system is a very realistic feature and several researchers realized the necessity to take this fact into consideration while developing inventory models. Giri et al. (1996) developed an inventory model by considering demand for deteriorating items to be stock dependent with a constant rate of deterioration. An objective of this study was to maximize the total profit and find out the appropriate number of orders in the finite planning horizon. Yang and Wee (2000) presented policies for deteriorating items having constant demand rate. Lee and Dye (2012) formulated a deteriorating inventory model having stock-dependent demand. The objective of this model was to know the strategies for optimal replenishments along with maximizing the total profit per unit time. A lot of models for deteriorating items and stock-dependent demand rate in the literature have aimed towards minimizing the inventory costs, but Pando et al. (2018) has considered the rate of deterioration per unit time to be constant part of inventory level with an objective to maximize the total profit per unit time. To study more on deteriorating items literatures can be reviewed from the research done by Raafat (1991), Wee (1999), Yang and Wee (2005) and Sarkar et al. (2013).

8.2.5 Supply Chain Coordination

With the increased market competition in the present global markets, organizations are compelled to closely work in collaboration with their suppliers and immediate customers. It is also observed that through better coordination of the supply chain, stocks across the supply chain can be more efficiently managed. In the lack of coordination in the supply chain, each player will act independently to maximize their profit. This may not be the beneficial to the other players of the chain and hence it may result in poor performance of the entire supply chain. The supply chain coordination between the vendor and buyer was first studied by Clark and Scarf (1960); wherein it was assumed that buyer is the sole decision maker of the entire ordering process and hence the solution obtained from such models were not economical for the vendor. Enumerable studies have been done on supply chain coordination. In most cases the resulting profits are distributed equally among supplier and retailer, thus benefiting both the entities. There should be a proper flow of information among the parties in order to have successful supply chain coordination. If one of the parties has better information than others, that might turn out to be his strategic advantage, and might use that information to gain cooperation from other parties. In such cases, the less informed parties try to offer incentives so as to provoke the other party to disclose his private information. The information shared by the parties affects the managers while decision-making. Thus in order to avoid these situations, there should be a mutual flow of information among the parties to maintain the supply chain coordination. Researchers like Goyal and Gupta (1989), Vishwanathan (1998) have come up with inventory models that are applicable to such kind of problems that involve supply chain coordination between vendor and buyer.

8.2.6 Inventory Models with Quantity Discount

Researchers recognized that quantity discounts on selling price can provide economic advantages like lower unit purchase cost and lower procurement costs for both vendor and buyer. Some researchers investigated the integrated buyer-vendor inventory problems considering quantity discounts. A fixed order quantity decision model considering the discounting scheme was developed by Lal and Staelin (1984) to benefit the buyers. Vendor oriented optimal quantity discount policy to maximize vendor's profit with no additional cost to the buyer, was studied by many researchers; Monahan (1984) was amongst those early researchers. Monahan's model was generalized and taken further by Lee and Rosenblatt (1986); who developed a fixed order quantity decision model with a discounting scheme that would benefit the buyers. To find out replenishment interval and discount price for any desirable negotiation factor an algorithm was developed by Chakravarty and Martin (1988). Joglekar (1988) has commented on the work done by Monahan (1984) and then explained the model using a numerical illustration. This algorithm was a scheme to build up a mutual cost sharing scheme between buyers and sellers. A simple approach has been proposed by Goyal and Gupta (1990) to identify the optimal order quantity when discounts are offered by the vendor on larger purchases by the buyer. To determine an optimal pricing and replenishment strategy, Weng and Wong (1993) developed a general discount model considering all-unit quantity. For their model Weng and Wong considered demand to be price sensitive. Vendor's quantity discount was considered by Weng (1995) in another study from the point of view of cutting down vendor's operating cost along with increasing buyer's demand. Burwell et al. (1997) developed an inventory model for price-dependent demand considering all-unit quantity discount with an objective to determine the selling price and the optimal lot size. This model by Burwell et al. (1997) was modified by Chang (2013) with an objective to maximize the profit and to determine the accurate optimized values for the lot size and the selling price. Various other inventory models have been developed by Li and Huang (1995), Corbett and Groot (2000), Qi et al. (2004), Li and Liu (2006), Transchel and Minner (2008), Datta and Paul (2001), Zhan et al. (2014), Yin et al. (2015), Alfares and Ghaithan (2016) considering the quantity discount offered by the vendor to the buyer. A manager can use order size-based quantity discounts to achieve channel coordination. Very few inventory models have been developed in the recent literature considering quantity discount scheme. Thus, in this paper, the authors have considered quantity discount as one of the parameters that affects supply chain coordination for deteriorating products while determining the demand rate.

In the literature, several research studies on inventory models were found to be developed for quantity discount and stock-dependent demand while considering supply chain coordination between the vendor and the buyer. There were also models on deterioration, variable demand and price dependent demand, but not a single model has considered all these factors simultaneously. Thus in this research paper, the authors have developed an inventory model for deteriorating products with stock and price dependent demand rate considering quantity discount scheme offered by suppliers to the buyer with the presence of supply chain coordination between the two parties. Table 8.1 summarizes the literatures reviewed for this paper on the basis of various features.

8.3 Mathematical Modelling and Analysis

Following assumptions are used to derive the mathematical models in this paper:

(a) The rate of replenishment and lead time are considered to be instantaneous and constant respectively.
(b) The rate of demand decreases linearly with retail price of the product.
(c) All-unit quantity discount is offered by the vendor to the buyer.
(d) The buyer and the vendor share their complete information with each other.
(e) Shortage is not permitted.
(f) A sole unit having a steady deterioration rate is considered.

Table 8.1 Summary of literature review based on various features

Authors	Supply chain coordination	Price-dependent demand	Stock-level dependent demand	Quantity discounts	Deterioration
Alfares (2015)			✓	✓	
Alfares and Ghaithan (2016)		✓		✓	
Chakravarty and Martin (1988)				✓	
Chang et al. (2010)			✓		✓
Chang (2013)		✓		✓	
Clark and Scarf (1960)	✓				
Corbett and De Groote (2000)	✓			✓	
Datta and Pal (1990)			✓		
Datta and Paul (2001)		✓	✓		
Dye and Yang (2016)		✓			✓
Giri et al. (1996)			✓		✓
Gupta and Vrat (1986)			✓		
Goyal (1977)	✓				
Goyal and Gupta (1989)	✓				
Goyal and Chang (2009)			✓		
Hou and Lin (2006)		✓	✓		✓
Joglekar (1988)				✓	
Lal and Staelin (1984)		✓		✓	
Lambert et al. (1998)	✓				

(continued)

Table 8.1 (continued)

Authors	Supply chain coordination	Price-dependent demand	Stock-level dependent demand	Quantity discounts	Deterioration
Lee and Rosenblatt (1986)				✓	
Lee and Dye (2012)			✓		✓
Li and Huang (1995)	✓			✓	
Li et al. (1996)	✓	✓			
Li and Liu (2006)	✓			✓	
Mandal and Phaujdar (1989)			✓		✓
Monahan (1984)				✓	
Pando et al. (2018)			✓		✓
Qi et al. (2004)	✓			✓	
Qin et al. (2007)	✓	✓		✓	
Raafat (1991)					✓
Sajadieh et al. (2010)	✓		✓		
Sarkar et al. (2013)		✓			✓
Sarker et al. (1997)			✓		✓
Teng and Chang (2005)		✓	✓		✓
Transchel and Mirner (2008)		✓		✓	
Urban (2005)			✓		
Viswanathan (1998)	✓				
Wee (1997)		✓			✓
Wee (1998)		✓		✓	✓
Wee (1999)		✓		✓	✓

(continued)

Table 8.1 (continued)

Authors	Supply chain coordination	Price-dependent demand	Stock-level dependent demand	Quantity discounts	Deterioration
Weng and Wong (1993)		✓		✓	
Weng (1995)		✓		✓	
Yang and Wee (2000)	✓				✓
Yang (2004)		✓		✓	✓
Yang and Wee (2005)	✓				✓
Yin et al. (2015)	✓			✓	
Zhang et al. (2014)	✓			✓	

Source own

(g) Deterioration of the units will be considered only after they enter the inventory.
(h) The deteriorated units cannot be repaired or replaced.
(i) Carrying cost will be applied only to the good units.
(j) Supply chain system with single buyer and single vendor is considered.

In this paper, three different cases have been discussed. The vendor–buyer collaboration and quantity discount have not been considered in the first case, while in the second case vendor–buyer integration without quantity discount has been considered. Finally in the third case, buyer-vendor integration as well as quantity discount have been considered simultaneously.

Following parameters related to the vendor are considered for the research:

$I_{vi}(t)$	Level of stock for case i, $i = 1, 2, 3$
C_v	Setup cost, \$ per cycle
C_{vb}	Fixed cost to process each buyer's order
P_v	Unit cost for the vendor
F_v	Cost of carrying inventory in percentage per year and per dollar
TC_{vi}	Total cost per year for case i, $i = 1, 2, 3$
TP_{vi}	Total profit per year for case i, $i = 1, 2, 3$
S_v	Extra profit sharing for case 3 as compared to case 1 ($S_v = \mathrm{TP}_{v3} - \mathrm{TP}_{v1}$)

Other parameters related to the buyer are as follows:

$I_{bi}(t)$	Level of Inventory for case i, $i = 1, 2, 3$
C_b	Buyer's ordering cost, \$ per order
P_{bi}	Purchase price per unit for the buyer for case i, $i = 1, 2$
F_b	Cost of carrying inventory in percentage per year and per dollar
TC_{bi}	Total cost per year for case i, $i = 1, 2, 3$
TP_{bi}	Total profit per year for case i, $i = 1, 2, 3$
S_b	Extra profit sharing for case 3 as compared to case 1 ($S_b = \mathrm{TP}_{b3} - \mathrm{TP}_{b1}$)

Following are the variable parameters:

T_{bi}	Replenishment period for the buyer for case i, $i = 1, 2, 3$
n_i	Number of replenishments from the vendor to the buyer per cycle for case i
T_{vi}	Replenishment period for the vendor for case i, $i = 1, 2, 3$
P_m	Retail price for end customer
d	Price-dependent demand rate per year
P_{b3}	Purchase unit price for the buyer for case 3

Other parameters related to buyer and the vendor are as follows:

a	Scale parameter for demand rate
b	Price–dependent parameter for demand rate
β	Stock-dependent selling rate parameter
θ	Constant rate of deterioration of on-hand-inventory
TC_i	Total cost per annum (TC_{vi} and TC_{bi}) for case i
TP_i	Total profit per annum (TP_{vi} and TP_{bi}) for case i
γ	Vendor and buyer's extra profit sharing negotiation factor for extra profit sharing

The inventory level decreases due to the demand and constant deterioration of available stock. Differential equation for inventory system of buyer can be presented as

$$\frac{\mathrm{d}I_{bi}(t)}{\mathrm{d}t} + \theta I_{bi}(t) = -(\alpha + \beta I_{bi}(t)),\ 0 \le t \le T_{bi} \tag{8.1}$$

The boundary condition will take place when $I_{bi}(T_{bi}) = 0$.
The buyer's inventory level using Spiegel (1960) is

$$I_{bi}(t) = \frac{\alpha}{\theta + \beta}\left(e^{(\theta+\beta)(T_{bi-t})} - 1\right) \tag{8.2}$$

Case 1: Supply chain system with the absence of both channel coordination and quantity discount.

The total cost for the system is,

$$\mathrm{TC}_{b1} = \text{Buyer's order cost} + \text{Inventory carrying cost} + \text{Buyer's purchasing cost}$$

$$\mathrm{TC}_{b1} = \left[C_b + P_{b1}F_{b1}\int_0^{T_{b1}} I_{b1}(t)\mathrm{d}t + P_{b1}I_{b1}\ (0)\right]/T_{b1}$$

$$\mathrm{TC}_{b1} = \frac{\left[C_b + P_{b1}F_{b1}\left(\frac{\alpha}{(\theta+\beta)^2}\right)\left(e^{(\theta+\beta).T_{b1}} - (\theta+\beta)T_{b1} - 1\right) + P_{b1}\left(\frac{\alpha}{\theta+\beta}\right)\left(e^{(\theta+\beta).T_{b1}} - 1\right)\right]}{T_{b1}} \tag{8.3}$$

The three terms in Eq. (8.3) represents cost of ordering, holding cost, and the cost of purchasing, respectively. Using Taylor series approximation, $e^{(\theta+\beta)Tb1}$ in Eq. (8.3) is replaced by $1 + (\theta + \beta)T_{b1} + ½\,((\theta + \beta)T_{b1})^2 + {}^1/_{3!}\,((\theta + \beta)T_{b1})^3$, for $(\theta + \beta)T_{b1} << 1$. In Taylor series the fourth term's percentage error is

$$\frac{\frac{(\theta+\beta)^3T_{b1}^3}{3!}}{1 + (\theta+\beta)T_{b1} + \frac{(\theta+\beta)^2T_{b1}^2}{2!} + \frac{(\theta+\beta)^3T_{b1}^3}{3!}}$$

For the small value of $(\theta + \beta)T_{b1}$, the percentage error is very small. It will be even smaller for term higher than four. Hence the term four and onwards are neglected from equation.

The approximated total cost of buyer is,

$$\mathrm{TC}_{b1} \cong \left[\frac{C_b}{T_{b1}} + P_{b1} \times F_{b1}\ x\ \frac{\alpha}{2} \times T_{b1} + P_{b1} \times \alpha\left(1 + \frac{(\theta+\beta)}{2}\ T_{b1}\right)\right] \tag{8.4}$$

According to the model's assumption; the demand rate has a linearly decreasing function of the retail price while an increasing function of stock-dependent selling rate.

$$d = \alpha + \beta I_{b1}(t) \tag{8.5}$$

where, $\alpha = a - bP_m$.

Buyer's total profit can be calculated by deducting his total cost from his total sales revenue

$$\mathrm{TP}_{b1} = (\text{Sales revenue per time unit}) - \mathrm{TC}_{b1}$$

Now,

$$\mathrm{SR} = \frac{P_m}{T_{b1}} \int_0^{T_{b1}} (\alpha + \beta I_{b1}(t)\mathrm{d}t)$$

$$\mathrm{SR} = \frac{P_m}{T_{b1}} \left[\alpha T_{b1} + \frac{\beta\alpha}{(\theta+\beta)^2} \left(e^{(\theta+\beta)(T_{b1})} - (\theta+\beta)T_{b1} - 1 \right) \right]$$

Using Taylor's series approximation;

$$\mathrm{SR} = P_m \times \alpha \left(1 + \frac{\beta T_{b1}}{2} \right)$$

$$\mathrm{TP}_{b1} = P_m \times \alpha \left(1 + \frac{\beta T_{b1}}{2} \right) - \mathrm{TC}_{b1} \tag{8.6}$$

We get the following results by taking first derivatives of TP_{b1} with respect to T_{b1} and P_m, and equating these equations to zero.

$$\frac{\partial \mathrm{TP}_{b1}}{\partial T_{b1}} = 0 \tag{8.7}$$

$$\frac{\partial \mathrm{TP}_{b1}}{\partial P_m} = 0 \tag{8.8}$$

The optimal values of T_{b1} and P_m which are denoted by T_{b1}^* and P_m^*, will be derived numerically as the solutions obtained in Eqs. (8.7) and (8.8) are not in a closed form.

By using Eqs. (8.4) and (8.5) buyer's optimal total cost is derived for $(\theta + \beta)T_{b1}$ << 1 as follows:

$$\mathrm{TC}_{b1}^*\left(T_{b1}^*, P_m^*\right) \cong \left[\frac{C_b}{T_{b1}^*} + P_{b1} \times F_b \times \frac{\left(a - bP_m^*\right)}{2} \times T_{b1}^* + P_{b1} \times \left(a - bP_m^*\right) \times \left(1 + \frac{(\theta+\beta)}{2} T_{b1}^*\right) \right] \tag{8.9}$$

The replenishment period for the vendor can be calculated as

$$T_{v1} = n_1 T_{b1}^*, \tag{8.10}$$

where n_1 represents the positive integer.

The inventory level for the vendor is

$$I_{v1}(t) = \frac{\alpha}{\theta+\beta} \left[e^{(\theta+\beta)\left(n_1 T_{b1}^* - t\right)} - 1 \right], \tag{8.11}$$

where

$$0 \le t \le n_1 T_{b1}^* .$$

As shown in Eq. (8.11) there is an exponential decrease in the inventory level of the vendor. Using Eqs. (8.11) and (8.2), vendor's annual total cost can be derived as follows:

$$\mathrm{TC}_{v1} = \frac{1}{n_1 T_{b1}^*}\left[C_v + n_1 C_{vb} + P_v F_v\left(\int_0^{n_1 T_{b1}^*} I_{v1}(t)\mathrm{d}t - n_1 \int_0^{T_{b1}^*} I_{b1}(t)\mathrm{d}t\right) + P_v I_{v1}(0)\right]$$

$$\mathrm{TC}_{v1} \cong \frac{C_v + n_1 C_{vb}}{n_1 T_{b1}^*} + \frac{P_v F_v \alpha (n_1 - 1) T_{b1}^*}{2} + P_v \alpha\left[1 + \frac{(\theta + \beta)}{2} n_1 T_{b1}^*\right] \tag{8.12}$$

In Eq. (8.12), the first two terms are costs related to the ordering, the next term is saw-tooth shape inventory holding cost while the last term represents costs related to purchasing.

$TP_{v1} =$ (Sales revenue per time unit) $-$ TC_{v1} Annual total profit for the vendor is

$$\mathrm{TP}_{v1} = \frac{P_{b1} I_{b1}^*(0)}{T_{b1}^*} - \mathrm{TC}_{v1} \approx P_{b1}\alpha\left(1 + \frac{(\theta + \beta)}{2}\, T_{b1}^*\right) - \mathrm{TC}_{v1} \tag{8.13}$$

Here, $P_{b1}\alpha\left(1 + \frac{(\theta+\beta)}{2}\, T_{b1}^*\right)$ is the approximated sales revenue for the vendor. Total profit of vendor presented in Eq. (8.13) is a function of a one variable n_1. For the vendor's total profit, the optimal policy can be formulated as

$$\text{Maximize } \mathrm{TP}_{v1}(n_1) \;\text{ for }\; n = 1, 2, 3, \ldots. \tag{8.14}$$

As n_1 is a discrete integer, the following condition must be satisfied for the optimal value of n_1, which is denoted by n_1^*:

$$\mathrm{TP}_{v1}\left(n_1^* - 1\right) \le \mathrm{TP}_{v1}\left(n_1^*\right) \ge \mathrm{TP}_{v1}\left(n_1^* + 1\right) \tag{8.2.15}$$

Vendor–buyer system's total profit can be derived using the following equation, when quantity discount and buyer-vendor coordination is not considered

$$\mathrm{TP}_1 = \mathrm{TP}_{b1}\left(T_{\mathrm{b}1}^* P_m^*\right) + \mathrm{TP}_{v1}\left(n_1^*\right) \tag{8.16}$$

In case 1, each player makes strategic decisions independently, without considering vendor-buyer coordination. The total annual profit without coordination presented in Eq. (8.16) is a function of multiple decision variables T_{b1}, P_m and n_1. Buyer first optimizes the decision variables T_{b1} and P_m; whereas vendor optimizes the decision variable n_1.

Case 2: Supply chain system considers channel coordination without vendor's quantity discount

The aim of vendor-buyer coordination is to maximize total channel profit by sharing profit, cost, demand, and stock-related information. This coordination also supports in responding to the customer demand quickly.

Based on Eqs. (8.4) and (8.12), following are the total costs for buyer and vendor, respectively

$$\mathrm{TC}_{b2} = \left[\frac{C_b}{T_{b2}} + P_{b2} \times F_{b2} \times \frac{\alpha}{2} \times T_{b2} + P_{b2} \times \alpha\left(1 + \frac{(\theta+\beta)}{2} T_{b2}\right)\right] \tag{8.17}$$

$$\mathrm{TC}_{v2} = \frac{C_v + n_2 C_{vb}}{n_2 T_{b2}} + \frac{P_v F_v \alpha (n_2 - 1) T_{b2}}{2} + P_v \alpha\left[1 + \frac{(\theta+\beta)}{2} n_2 T_{b2}\right] \tag{8.18}$$

The sum of Eqs. (8.17) and (8.18) represents the coordinated total cost. Based on Eqs. (8.6) and (8.13), following are the profits for buyer and vendor respectively

$$\mathrm{TP}_{b2} = (\text{Sales revenue per time unit}) - \mathrm{TC}_{b2}$$

where,

$$\mathrm{SR} = \frac{P_m}{T_{b2}}\left[\alpha.T_{b2} + \frac{\beta\alpha}{(\theta+\beta)^2}\left(e^{(\theta+\beta)(T_{b2})} - (\theta+\beta)T_{b2} - 1\right)\right]$$

Using Taylor's series approximation, SR can be expressed as,

$$\mathrm{SR} = P_m \times \alpha\left(1 + \frac{\beta T_{b2}}{2}\right)$$

Thus,

$$\mathrm{TP}_{b2} = P_m \times \alpha\left(1 + \frac{\beta T_{b2}}{2}\right) - \mathrm{TC}_{b2} \tag{8.19}$$

$$\mathrm{TP}_{v2} = \frac{P_{b2} I_{b2}(0)}{T_{b2}} - \mathrm{TC}_{v2} \approx P_{b2}\alpha\left(1 + \frac{(\theta+\beta)}{2} T_{b2}\right) - \mathrm{TC}_{v2} \tag{8.20}$$

The total coordinated profit is $\mathrm{TP}_2 = \mathrm{TP}_{b2} + \mathrm{TP}_{v2}$.

Now the objective is to maximize the total coordinated profit,

$$\text{i.e., Max } \mathrm{T}P_2(T_{b2}, P_m, n_2) = \mathrm{TP}_{b2}\left(T_{b2}, P_m\right) + \mathrm{TP}_{v2}\left(n_2\right) \tag{8.21}$$

In case 2 vendor-buyer coordination is considered. Joint optimization has been done for the three decision variables T_{b2}, P_m and n_2 rather than optimizing independently as done in case 1.

Case 3: Supply chain system when vendor–buyer coordination and quantity discount are considered simultaneously.

In quantity discount scheme, the discount price, P_{b3} is smaller than the unit price, P_{b1} offered in case 1 and 2. Following equation represents the lot size per shipment Q for the buyer:

$$Q = I_{b3}(t = 0) = \frac{\alpha}{\theta + \beta}\left(e^{(\theta+\beta)(T_{b3})} - 1\right) \tag{8.22}$$

The delivery quantity from vendor to the buyer per annum can be derived as follows:

$$\frac{I_{b3}(0)}{T_{b3}} = \frac{\alpha}{T_{b3}(\theta + \beta)}\left(e^{(\theta+\beta)(T_{b3})} - 1\right) \approx d\left(1 + \frac{(\theta + \beta)}{2}\,T_{b3}\right) \tag{8.23}$$

Likewise, following are the annual total cost for buyer and vendor respectively:

$$\mathrm{TC}_{b3} \cong \left[\frac{C_b}{T_{b3}} + P_{b3} \times F_{b3}\, x\, \frac{\alpha}{2} \times T_{b3} + P_{b3} \times \alpha\left(1 + \frac{(\theta + \beta)}{2}\,T_{b3}\right)\right] \tag{8.24}$$

$$\begin{aligned}\mathrm{TC}_{v3} = {} & \frac{C_v + n_3 C_{vb}}{n_3 T_{b3}} + \frac{P_v F_v \alpha (n_3 - 1) T_{b3}}{2} + P_v\alpha\left[1 + \frac{(\theta + \beta)}{2} n_3 T_{b3}\right] \\ & + (P_{b1} - P_{b3})\left(1 + \frac{(\theta + \beta)}{2}\,T_{b3}\right)\end{aligned} \tag{8.25}$$

When the vendor offers a quantity discount, there is an additional cost which is shown as the last term in Eq. (8.25). Following is the total profit of buyer and vendor respectively:

$$\mathrm{TP}_{b3} = \text{Sales revenue per time unit(SR)} - \mathrm{TC}_{b3} \tag{8.26}$$

$$\mathrm{TP}_{v3} = \frac{P_{b3} I_{b3}(0)}{T_{b3}} - TC_{v3} = P_{b3}.\alpha\left(1 + \frac{(\theta + \beta)}{2}\,T_{b3}\right) - \mathrm{TC}_{v3} \tag{8.27}$$

The difference between TP_{b3} and TP_{b1} is the buyer's extra profit, denoted by S_b is shown below:

$$S_b = \mathrm{TP}_{b3} - \mathrm{TP}_{b1} \tag{8.28}$$

The vendor's extra profit is the difference between TP_{v3} and TP_{v1}, defined by S_v.

$$S_v = \mathrm{TP}_{v3} - \mathrm{TP}_{v1} \tag{8.29}$$

The uncoordinated total profit in case 1 (TP_1) is less than the coordinated total profit in case 2 (TP_2), also the coordinated total profit in case 3 (TP_3) is greater than that of TP_2; hence it can be said that TP_3 is greater than TP_1. This relationship between the total profit of case 3 and case 1 for both the vendor and the buyer, denoted as S_b and S_v is defined as:

$$S_v = \gamma S_b, \gamma \ \geq 0, \tag{8.30}$$

where, γ = negotiation factor.

When the negotiation factor $\gamma = 0$, all the extra profit is given to the buyer. When $\gamma = 1$, all the extra profit is distributed equally between the buyer and vendor. While if $\gamma > 1$, all extra profit is given to the vendor. Following is the optimization problem for case 3:

$$\text{Maximize } \mathrm{TP}_3(T_{b3}, P_m, n_3) = \mathrm{TP}_{b3}(T_{b3}, P_m) + \mathrm{TP}_{v3}\ (n_3) \tag{8.31}$$

Here, TP_3 is the function of the three variables n_3, T_{b3} and P_m .

8.4 Solution Procedure

For case 1, value of n_1 is to be determined such that TP_1 presented as Eq. (8.16) can be maximized. Here T_{b1} and P_m are optimized by buyer first and then variable n_1 is optimized by the vendor such that Eqs. (8.14) and (8.15) are satisfied.

For case 2, value of n_2 is to be determined such that TP_2 (8.21) can be maximized. Following procedure can be used to derive n_2 i.e. the number of delivery per order, as it is a discrete variable:

(a) Given a range of n_2 values, first with respect to P_m and T_{b2} obtain the partial derivative of TP_2 and equate them to zero; for a given range of n_2 values. For each n_2, $P_m(n_2)$ and $T_{b2}(n_2)$ are the optimal value of Pm and T_{b2} respectively.

(b) Derive $n_2{}^*$, the optimal value of n_2, such that

$$\begin{aligned}\mathrm{TP}_2\big(T_{b2}\big(n_2^* - 1\big),\ n_2^* - 1,\ P_m\big(n_2^* - 1\big)\big) &\leq TP_2\big(T_{b2}\big(n_2^*\big),\ n_2^*,\ P_m\big(n_2^*\big)\big)\\ &\geq TP_2\big(T_{b2}\big(n_2^* + 1\big),\ n_2^* + 1,\ P_m\big(n_2^* + 1\big)\big)\end{aligned}$$

For case 3, Eq. (8.31) has to be maximized to determine the value of decision variable n_3. In order to maximize the total profit TP_3; find partial derivatives of TP_3 with respect to T_{b3} and P_m need to be set equal to zero as shown below:

$$\frac{\partial \mathrm{TP}_3}{\partial T_{b3}} = 0 \tag{8.32}$$

$$\frac{\partial \mathrm{TP}_3}{\partial P_m} = 0 \tag{8.33}$$

In case 3, quantity discount is offered to the buyer, thus solution procedure in case 3 is different than that in case 2. While applying the procedure, the solution obtained from Eqs. (8.31) to (8.33) must be rounded up. The values of P_m, T_{b3} and $\mathrm{TP}_3(P_m,\ T_{b3})$ should be rounded to the nearest two decimals, while the order quantity, Q should be rounded to the nearest integer.

For case 3, Eq. (8.31) has to be maximized to determine the value of n_3. Following procedure will be used to derive the value of n_3 in case 3. Given a range of n_3 values, first find the partial derivative of TP_3 with respect to T_{b3} and P_m. Equate these equations to zero and solve to get the value of T_{b3} and P_m.

Step 1: For a given range of n_3 values, optimal values of $T_{\mathrm{b}3}$ and P_m can be obtained using the following procedure:

a. Put $\mathrm{TP}_{3\max} = 0$ and $j = J$

b. Solve for P_m and T_{b3} after replacing all the given values (a, b, β, θ,) and $P_{b3} = c_j$ in Eqs. (8.32) and (8.33). Obtain order quantity Q from Eq. (8.22).
The obtained solution will be feasible if Q lies in the correct purchase cost range i.e. $q_{j-1} \le Q < q_j$. To calculate $\mathrm{TP}_3(P_m,\ T_{b3})$ put the optimal values of T_{b3} and P_m in Eq. (8.31). Set $\mathrm{TP}_{3\max} = \mathrm{TP}_3(P_m,\ T_{b3})$ if $\mathrm{TP}_3(P_m,\ T_{b3}) > \mathrm{TP}_{3\max}$. Next, go to step (e).
The obtained solution is not feasible if order quantity Q does not fall in the right purchasing cost range. In that case, follow step (c).

c. Since value of Q is obtained in step (b) does not fall in the range $q_{j-1} \le Q < q_j$, it is not a feasible quantity. To take advantage of price discount the order quantity must be at price break i.e. $Q = q_{j-1}$. Substitute this value of Q in the equation of P_m (see appendix).
Solve for T_{b3} by substituting $Q = q_{j-1},\ P_{b3} = c_j$ and other given values (a, b, β, θ) along with P_m into Eq. (8.32). To calculate $\mathrm{TP}_{3j}(P_m,\ T_{b3})$ put the values of $Q = q_{j-1}$ and the corresponding values of T_{b3} and P_m obtained above into Eq. (8.31). Set $\mathrm{TP}_{3\max} = \mathrm{TP}_{3j}(P_m,\ T_{b3})$ if $\mathrm{TP}_{3j} > \mathrm{TP}_{3\max}$. Go to step (d).

d. Set $j = j - 1$ if $j \ge 2$ and go to step (b).
Follow step (e) if $j = 1$.

e. The obtained solution is the feasible solution associated with $\mathrm{TP}_{3\max}$. By specifying the optimal values of T_{b3}, P_m, $\mathrm{TP}_{3j}(P_m,\ T_{b3})$, the obtained solution can be defined for a given value of n_3. This ends the process.

Step 2:

n_3^* is the optimal value of n_3 which can be derived by satisfying following condition:

$$\begin{aligned}&\mathrm{TP}_3\big(T_{b3}\big(n_3^* - 1\big),\ n_3^* - 1,\ P_m\big(n_3^* - 1\big)\big) \le \mathrm{TP}_3\big(T_{b3}\big(n_3^*\big),\ n_3^*,\ P_m\big(n_3^*\big)\big)\\ &\quad \ge \mathrm{TP}_3\big(T_{b3}\big(n_3^* + 1\big),\ n_3^* + 1,\ P_m\big(n_3^* + 1\big)\big)\end{aligned} \tag{8.34}$$

8.5 Numerical Example

The solution procedure discussed in the previous section can be explained through the following numerical example. Data which are considered to illustrate the derived model and the proposed algorithm are as follows:

Scale parameter, $a = 2000$.
Price-dependent parameter, $b = 33$.
Stock-dependent selling rate parameter, $\beta = 0.03$.
Carrying cost for vendor, in percentage per annum per dollar, $F_v = 0.2$
Setup cost for vendor, $C_v = \$6000$.
Fixed cost for vendor to process each order placed by buyer, $C_{vb} = \$100$.
Unit cost for vendor, $P_v = \$20$.
Carrying cost for buyer, in percentage per annum per dollar, $F_b = 0.2$
Buyer's ordering cost, $C_b = \$100$.
Purchased unit price for buyer without price discount, $P_{b1} = P_{b2} = \$33$.
Deterioration rate, $\theta = 0.05$.
Negotiation factor, $\gamma = 0$ or 1.

As per the model assumption, all-unit discount scheme is being offered by the vendor to the buyer wherein the buyer gets discount based on the quantity purchased by him.

Following is the price range, based on which per unit cost for the buyer can be determined:

No. of units		Cost per unit	
q_1	0–299	c_1	33
q_2	300–599	c_2	31.5
q_3	More than 599	c_3	30

The computational results are presented in Table 8.2. The annual demand, buyer's unit purchase price and replenishment period, number of replenishments from vendor, the optimum retail price of product, and associated total annual profit for buyer and vendor for all the three cases are presented in Table 8.2.

The number of replenishments for case 1, i.e,. supply chain without integration is $n = 9$; the associated retail price and buyer's replenishment period are \$47.30 and 0.2413 years are also shown in Table 8.2. The corresponding annual demand for the product is 441 units. The total annual profit for buyer and vendor are \$5450 and \$213 respectively. The total annual profit for the supply chain without integration is \$ 5663.

For case 2, when supply chain coordination is considered, the optimal values of the decision parameters retail price and buyer's replenishment period are \$43.34 and 0.6533 years. The number of replenishment from vendor to buyer 'n' is 3 and the annual demand of the product is 575 units. The total annual profit for buyer and vendor are \$4262 and \$2300 respectively. The optimal value of coordinated channel's total annual profit is \$6562. The total annual profit for the coordinated channel is \$899 is higher than the total profit of supply chain without coordination. Due to channel coordination vendor profit is increased from \$213 to \$2300 whereas buyers profit is declined from \$5450 to \$4262. Since coordination in the supply chain is beneficial to vendor only, buyer would not like to participate in the coordinated strategy and resist to share the information.

To encourage the buyer to participate in the channel coordination, vendor may offer quantity discount and can share profit benefit with the buyer; which is earned due to coordination strategy. When supply chain coordination and quantity discount are considered simultaneously, the channel's annual total profit is increased to \$6629 with the optimal unit discounted purchase price of \$31.50. The percentage of extra total profit (PETP_3) is 17.06% which is higher than 15.87%, the percentage of extra total profit (PETP_2) when coordination is considered without discount policy.

From Table 8.2, it can be observed that the vendor can earn greater profits by the adoption of an appropriate discount strategy. The increase in the channel annual total profit from case 1 to case 3 is \$ 966 (\$6629–\$5663). Due to supply chain coordination, in case 2 and case 3; vendor's extra profit S_v is increased by \$2087 and \$1826, whereas buyer's extra profit Sb is negative as profit is decreased by \$1188 and \$860 respectively. If all extra profit earned in case 3 is offered to the buyer (i.e. negotiation factor $\gamma = 0$), then buyer and vendor's annual total profit will be \$5556 and \$1073, which is higher than case 1, where coordination and quantity discount are not considered in supply chain system. Adoption of coordination along with quantity discount policy is beneficial to both vendor and buyer.

The numerical results obtained through the above solution procedure shows that TP_n is strictly concave in T_b and P_m (Fig. 8.1). Hence, the local maximum value of objective function obtained here from proposed solution procedure is indeed the global maximum solution.

8.5.1 Sensitivity Analysis

The relative impact of various parameters on the optimal solution obtained in case 3 is studied through sensitivity analysis. The sensitivity analysis is performed by changing value of each given parameters by −20%, −10%, +10%, and +20%, taking one parameter at a time and keeping the value of other parameters unchanged. The results of the sensitivity analysis are given in Tables 8.3, 8.4, 8.5, 8.6 and 8.7. The results of the sensitivity analysis show the impact of changes in the key parameters on the decision variables P_m, n_3, d, P_{b3}, T_{b3}, TP_1, TP_2 and TP_3.

From the results shown in Table 8.3, it is observed that PETP_3 changes significantly in the range 9% to 49%, when the price-sensitive parameter b changes. The change in b and PETP_3 is positively correlated. This indicates that when b increases, it is more significant to consider coordination strategy with price discount (Fig. 8.2).

It can be observed from Tables 8.4 and 8.5, when C_v, C_b and C_{vb} increases, total annual profit decrease but PETP3 increases. Hence, it is very important to take into

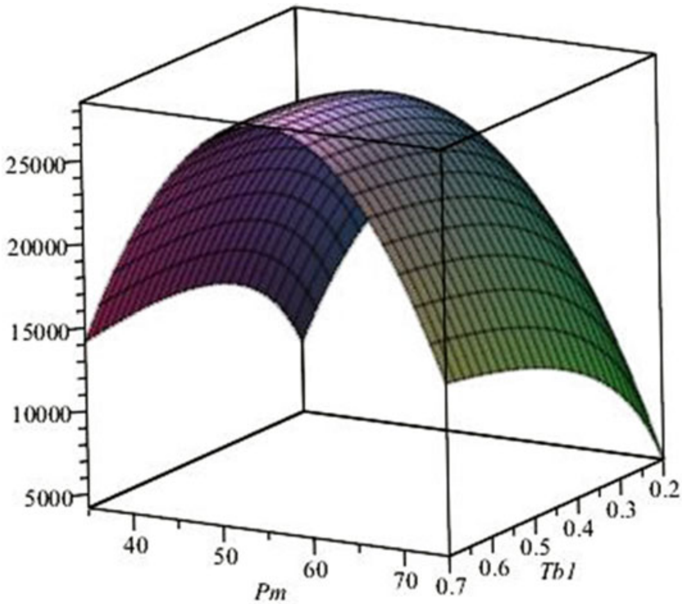

Fig. 8.1 Concavity of total profit function. *Source* own

Table 8.2 The optimal solution at various cases when $\theta = 0.05$

Case i	$i = 1$	$i = 2$	$i = 3$
P_m	47.30	43.34	43.35
d	441	575	578
P_{bi}	33	33	31.50
n_i	9	3	2
T_{bi}	0.2413	0.6533	0.9599
TP_{bi}	5450	4262	4590
TP_{vi}	213	2300	2039
TP_i	5663	6562	6629
$PETP_i$	–	0.1587	0.1706

$PETP_i$: Percentage of extra total profit for case i compared to case 1;
$PETP_i = (TP_i - TP_1)/TP_1$
Source own

account both the integration and the quantity discount when the costs related to order processing for the player of supply chain increase.

From Table 8.6, we can see that total annual profit decreases significantly, and PETP3 increases when rate of deterioration θ increases. This shows that supply chain coordination with quantity discount strategy is advisable when rate of deterioration increases over the time period.

Table 8.3 Sensitivity analysis for price-dependent parameter b

b	P_m	d	P_{b3}	n_3	T_{b3}	TP_1	TP_2 ($PETP_2$)		TP_3 ($PETP_3$)	
26.4	50.70	671	30	2	0.9103	12,177	13,099	(7.57%)	13,276	(9.03%)
29.7	46.58	625	30	2	0.9378	8487	9400	(10.78%)	9564	(12.69%)
33	43.35	578	31.5	2	0.9599	5663	6562	(15.87%)	6629	(17.06%)
36.3	40.72	529	31.5	2	0.9995	3479	4352	(25.09%)	4409	(26.73%)
39.6	38.54	479	31.5	3	0.7195	1786	2617	(46.53%)	2668	(49.38%)

Source own

The results shown in Table 8.7 indicate that with increase in the stock-dependent selling rate parameter β, PETP3 increases significantly. Hence, it is preferable to adopt coordination with discount policy when stock-dependent selling rate parameter β increases.

As price-sensitive parameter b, rate of deterioration θ and C_V increases, demand decreases significantly whereas if stock-dependent selling rate parameter β, C_{vb} and C_b increases, demand also increases. The effect of stock-dependent selling rate parameter β is more significant on T_{b3} and n. Retail price of product is more sensitive to price-sensitive parameter b, C_{vb} and C_b as compared to other parameters (Figs. 8.3 and 8.4).

8.6 Conclusions

This study presents coordinated supply chain system with variable demand rate and a variable unit purchase cost. In this study, more realistic model parameters like stock-dependent selling rate and deterioration are considered in deriving the model. A model has been derived, and an efficient solution procedure has been discussed to determine the optimal unit retail selling price and replenishment cycle. The impact of price-sensitive parameter b, deterioration, stock-dependent selling rate on total annual profit, demand of product, and retail selling price are reported. The results indicate that supply chain coordination with quantity discount increases the extra total profit gain of about 17.06%.

Supply chain coordination helps in optimizing the overall system rather than its individual players and not only increases total annual profits but also reduce variability in demand and inventory level, resulting in more efficient supply chain. The result of sensitivity analysis shows that the effects of price-sensitive parameter, stock-dependent selling rate and deterioration on the total annual profit are very significant, and hence cannot be ignored while deriving the supply chain model.

Table 8.4 Sensitivity analysis for C_V

C_v	P_m	d	P_{b3}	n_3	T_{v3}	T_{b3}	TP_1	TP_2 ($PETP_2$)		TP_3 ($PETP_3$)	
4800	43.13	591	30	1	1.6584	1.6584	6236	7215	(15.70%)	7309	(17.2065%)
5400	43.20	583	31.5	2	1.8190	0.9095	5940	6877	(15.77%)	6950	(17.0034%)
6000	43.35	578	31.5	2	1.9198	0.9599	5663	6562	(15.87%)	6629	(17.0581%)
6600	43.47	574	30	2	2.0400	1.0200	5394	6264	(16.13%)	6410	(18.8357%)
7200	43.61	570	30	2	2.1350	1.0675	5146	5978	(16.17%)	6122	(18.9662%)

$T_{vi} = T_{bi} \times n_i$

Source own

Table 8.5 Sensitivity analysis for C_{vb} and C_b

C_{vb}, C_b	P_m	d	P_{b3}	n_3	T_{b3}	TP_1	TP_2 ($PETP_2$)		TP_3 ($PETP_3$)	
80, 80	43.29	577	31.5	3	0.6516	5769	6624	(14.82%)	6680	(15.79%)
90, 90	43.34	578	31.5	2	0.9566	5712	6593	(15.42%)	6650	(16.42%)
100, 100	43.35	578	31.5	2	0.9599	5663	6562	(15.87%)	6629	(17.06%)
110, 110	41.80	629	30	2	0.9319	5613	6532	(16.37%)	6619	(17.92%)
120, 120	41.93	625	30	2	0.9383	5564	6505	(16.91%)	6610	(18.80%)

Source own

In this study, the problem of simultaneously determining a pricing and ordering strategy for deteriorating product is addressed. The models can be applied for efficient supplier management in system like super market and stationery stores to determine optimal ordering and pricing policy. Retailer can use this model to optimize this retail unit price and inventory control variables.

The above model can be extended by considering different form of demand rate like nonlinear function of inventory level or retail price. Also, consideration of shortages, permissible delay in payment in the model can help to extend the model further. Additionally, this model can be extended further for deteriorating product with a two-parameter Weibull distribution.

Table 8.6 Sensitivity analysis for θ

θ	P_m	d	P_{b3}	n_3	T_{v3}	T_{b3}	TP_1	TP_2 ($PETP_2$)		TP_3 ($PETP_3$)	
0.01	43.11	586	30	2	2.0742	1.0371	6144	7026	(14.36%)	7172	(16.73%)
0.04	42.20	579	30	2	1.9114	0.9557	5780	6675	(15.48%)	6788	(17.44%)
0.045	41.93	578	30	2	1.8808	0.9404	5722	6618	(15.66%)	6710	(17.27%)
0.05	43.35	578	31.5	2	1.9198	0.9599	5663	6562	(15.87%)	6629	(17.06%)
0.055	43.50	580	30	1	1.8535	1.8535	5606	6507	(16.07%)	6575	(17.29%)
0.06	43.53	579	30	1	1.8406	1.8406	5549	6452	(16.27%)	6523	(17.55%)
0.10	43.71	572	30	1	1.7466	1.7466	5110	6030	(18.00%)	6121	(19.78%)

Source own

Table 8.7 Sensitivity analysis for stock-dependent selling rate parameter β

β	P_m	d	P_{b3}	n_3	T_{b3}	TP_1	TP_2 ($PETP_2$)		TP_3 ($PETP_3$)	
0.024	43.2902	576	31.5	3	0.6595	5701	6581	(15.44%)	6637	(16.42%)
0.027	43.3039	576	31.5	3	0.6588	5682	6572	(15.66%)	6628	(16.65%)
0.03	43.35	578	31.5	2	0.9599	5663	6562	(15.87%)	6629	(17.06%)
0.033	43.49	582	30	1	1.8780	5644	6553	(16.11%)	6665	(18.09%)
0.036	43.50	584	30	1	1.8893	5625	6552	(16.48%)	6702	(19.15%)
0.05	43.5491	590	30	1	1.9451	5533	6566	(18.67%)	6880	(24.34%)

Source own

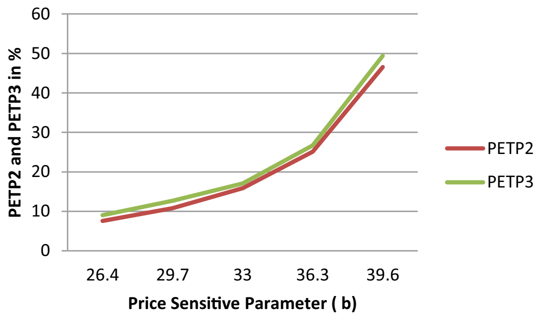

Fig. 8.2 $PETP_2$ and $PETP_3$ versus price-sensitive parameter. *Source* own

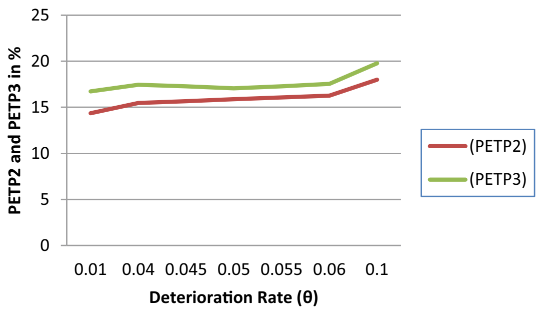

Fig. 8.3 $PETP_2$ and $PETP_3$ versus deterioration rate. *Source* own

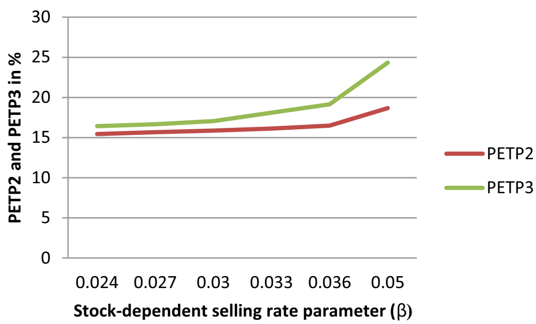

Fig. 8.4 $PETP_2$ and $PETP_3$ versus Stock-dependent selling rate. *Source* own

Appendix

From Eq. (8.22), P_m can be expressed as a function at Q and T_{b3} as follows:

$$Q = I_{mb} = (a - b.P_m)T_{b3}[1 + (\theta + \beta)T_{b3}] \quad \text{(Using Taylor series approximation)}$$

Using above equation, P_m can be expressed as:

$$P_m = \frac{1}{b}\left[a - \frac{Q}{T_{b3}\left(1 + \frac{(\theta+\beta)T_{b3}}{2}\right)}\right]$$

References

Alfares HK (2015) Maximum-profit inventory model with stock-dependent demand, time-dependent holding cost, and all-units quantity discounts. Math Model Anal 20(6):715–736

Alfares H, Ghaithan A (2016) Inventory and pricing model with price-dependent demand, time-varying holding cost, and quantity discounts. Comput Ind Eng 94:170–177. https://doi.org/10.1016/j.cie.2016.02.009

Burwell TH, Dave DS, Fitzpatrick KE, Roy MR (1997) Economic lot size model for price-dependent demand under quantity and freight discounts. Int J Prod Econ 48(2):141–155

Chakravarty A, Martin G (1988) An optimal joint buyer-seller discount pricing model. Comput Oper Res 15(3):271–281. https://doi.org/10.1016/0305-0548(88)90040-8

Chang CT, Teng JT, Goyal SK (2010) Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. Int J Prod Econ 123(1):62–68

Chang HC (2013) A note on an economic lot size model for price-dependent demand under quantity and freight discounts. Int J Prod Econ 144(1):175–179. https://doi.org/10.1016/j.ijpe.2013.02.001

Clark AJ, Scarf H (1960) Optimal policies for a multi-echelon inventory problem. Manage Sci 6(4):475–490

Corbett CJ, De Groote X (2000) A supplier's optimal quantity discount policy under asymmetric information. Manage Sci 46(3):444–450

Datta TK, Pal AK (1990) A note on an inventory model with inventory-level-dependent demand rate. J Oper Res Soc 41(10):971–975

Datta TK, Paul K (2001) An inventory system with stock-dependent, price-sensitive demand rate. Prod Plan Control 12(1):13–20

Dye CY, Yang CT (2016) Optimal dynamic pricing and preservation technology investment for deteriorating products with reference price effects. Omega 62:52–67

Giri B, Pal S, Goswami A, Chaudhuri K (1996) An inventory model for deteriorating items with stock-dependent demand rate. Eur J Oper Res 95(3):604–610. https://doi.org/10.1016/0377-2217(95)00309-6

Goyal SK (1977) An integrated inventory model for a single supplier-single customer problem. Int J Prod Res 15(1):107–111. https://doi.org/10.1080/00207547708943107

Goyal SK, Gupta YP (1989) Integrated inventory models: the buyer-vendor coordination. Eur J Oper Res 41(3):261–269. https://doi.org/10.1016/0377-2217(89)90247-6

Goyal SK, Gupta OK (1990) A simple approach to the discount purchase problem. J Oper Res Soc 41(12):1169–1170

Goyal SK, Chang CT (2009) Optimal ordering and transfer policy for an inventory with stock dependent demand. Eur J Oper Res 196(1):177–185

Gupta R, Vrat P (1986) Inventory model for stock-dependent consumption rate. Opsearch 23(1):19–24

Hou KL, Lin LC (2006) An EOQ model for deteriorating items with price-and stock-dependent selling rates under inflation and time value of money. Int J Syst Sci 37(15):1131–1139

Joglekar PN (1988) Note—comments on "A quantity discount pricing model to increase vendor profits." Manage Sci 34(11):1391–1398

Lal R, Staelin R (1984) An approach for developing an optimal discount pricing policy. Manage Sci 30(12):1524–1539. https://doi.org/10.1287/mnsc.30.12.1524

Lambert DM, Cooper MC, Pagh JD (1998) Supply chain management: implementation issues and research opportunities. Int J Logist Manage 9(2):1–20

Lee HL, Rosenblatt MJ (1986) A generalized quantity discount pricing model to increase supplier's profits. Manage Sci 32(9):1177–1185. https://doi.org/10.1287/mnsc.32.9.1177

Lee Y, Dye C (2012) An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate. Comput Ind Eng 63(2):474–482. https://doi.org/10.1016/j.cie.2012.04.006

Li SX, Huang Z (1995) Managing buyer-seller system cooperation with quantity discount considerations. Comput Oper Res 22(9):947–958

Li SX, Huang Z, Ashley A (1996) Inventory, channel coordination and bargaining in a manufacturer-retailer system. Ann Oper Res 68(1):47–60

Li J, Liu L (2006) Supply chain coordination with quantity discount policy. Int J Prod Econ 101(1):89–98

Mandal BA, Phaujdar S (1989) An inventory model for deteriorating items and stock-dependent consumption rate. J Oper Res Soc 40(5):483–488

Monahan J (1984) A quantity discount pricing model to increase vendor profits. Manage Sci 30(6):720–726. https://doi.org/10.1287/mnsc.30.6.720

Pando V, San-José L, García-Laguna J, Sicilia J (2018) Optimal lot-size policy for deteriorating items with stock-dependent demand considering profit maximization. Comput Industr Eng 117:81–93. https://doi.org/10.1016/j.cie.2018.01.008

Qi X, Bard J, Yu G (2004) Supply chain coordination with demand disruptions. Omega 32(4):301–312. https://doi.org/10.1016/j.omega.2003.12.002

Qin Y, Tang H, Guo C (2007) Channel coordination and volume discounts with price-sensitive demand. Int J Prod Econ 105(1):43–53

Raafat F (1991) Survey of literature on continuously deteriorating inventory models. J Oper Res Soc 42(1):27–37

Sajadieh MS, Thorstenson A, Jokar MRA (2010) An integrated vendor–buyer model with stock-dependent demand. Transp Res Part E: Logist Transp Rev 46(6):963–974

Sarkar B, Saren S, Wee HM (2013) An inventory model with variable demand, component cost and selling price for deteriorating items. Econ Model 30:306–310. https://doi.org/10.1016/j.econmod.2012.09.002

Sarker BR, Mukherjee S, Balan CV (1997) An order-level lot size inventory model with inventory-dependent demand and deterioration. Int J Prod Econ 48:227–236

Spiegel M (1960) Applied differential equations. Prentice, Englewood Cliffs

Teng J, Chang C (2005) Economic production quantity models for deteriorating items with price-and stock-dependent demand. Comput Oper Res 32(2):297–308. https://doi.org/10.1016/s0305-0548(03)00237-5

Transchel S, Mirner S (2008) Coordinated lot-sizing and dynamic pricing under a supplier all-units quantity discount. Bus Res 1(1):125–141. https://doi.org/10.1007/bf03342706

Urban TL (2005) Inventory models with inventory-level-dependent demand: a comprehensive review and unifying theory. Eur J Oper Res 162(3):792–804

Viswanathan S (1998) Optimal strategy for the integrated vendor-buyer inventory model. Eur J Oper Res 105(1):38–42

Wee HM (1997) A replenishment policy for items with a price-dependent demand and a varying rate of deterioration. Prod Plan Control 8(5):494–499. https://doi.org/10.1080/095372897235073

Wee HM (1998) Optimal buyer-seller discount pricing and ordering policy for deteriorating items. Eng Econ 43(2):151–168

Wee H (1999) Deteriorating inventory model with quantity discount, pricing and partial backordering. Int J Prod Econ 59(1–3):511–518. https://doi.org/10.1016/s0925-5273(98)00113-3

Weng KZ, Wong RT (1993) General models for the supplier's all-unit quantity discount policy. Nav Res Logist 40(7):971–991. https://doi.org/10.1002/1520-6750(199312)40:7%3c971::aid-nav3220400708%3e3.0.co;2-t

Weng KZ (1995) Modeling quantity discounts under general price-sensitive demand functions: optimal policies and relationships. Eur J Oper Res 86(2):300–314. https://doi.org/10.1016/0377-2217(94)00104-k

Yang PC, Wee HM (2000) Economic ordering policy of deteriorated item for vendor and buyer: an integrated approach. Prod Plan Control 11(5):474–480

Yang P (2004) Pricing strategy for deteriorating items using quantity discount when demand is price sensitive. Eur J Oper Res 157(2):389–397. https://doi.org/10.1016/s0377-2217(03)00241-8

Yang PC, Wee HM (2005) A win-win strategy for an integrated vendor-buyer deteriorating inventory system. Math Model Anal, 541–546

Yin S, Nishi T, Grossmann IE (2015) Optimal quantity discount coordination for supply chain optimization with one manufacturer and multiple suppliers under demand uncertainty. Int J Adv Manuf Technol 76(5–8):1173–1184

Zhang Q, Luo J, Duan Y (2014) Buyer–vendor coordination for fixed lifetime product with quantity discount under finite production rate. Int J Syst Sci 47(4):821–834. https://doi.org/10.1080/00207721.2014.906684