Chapter 6
Inventory Policies for Non-instantaneous Deteriorating Items with Random Start Time of Deterioration

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Abstract An inventory model for non-instantaneous deteriorating items with random start time of deterioration is investigated in this paper. For many products, the start time of deterioration cannot be predicted due to physical nature of the product. In this paper, products in the inventory system are considered to be deteriorated at a constant rate after a certain random time of inventory received by the retailer. Demand for the product is considered to be price sensitive. Two scenarios viz. with preservation technology investments and without preservation technology investments are compared to obtain retailer's optimal policies which include optimal cycle time, preservation cost, and selling price. The objective is to maximize total profit of retailers with respect to cycle time, selling price, and preservation technology investments. The results indicate that use of preservation technology helps retailers to generate more profit.

Keywords Non-instantaneous deterioration · Random start time of deterioration · Preservation technology · Price sensitive demand · Inventory policies

MSC 90B05

6.1 Introduction

Product demand has been always one of the major concerns for inventory managers. Demand for the product gets affected by various parameters such as stock, time, selling price, quality, different promotional offers, etc. It is very essential to select the precise demand pattern to make optimal inventory decisions. There are certain products for which the demand pattern is very sensitive to the product price. In such demand pattern, notable change can be observed in the product demand as the selling

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price of the product changes. Increase in selling price is useful in generating revenue but it leads to a decrease in the demand of the product. On the other hand, reduction in product price may attract customers to buy the product but it may be harmful for overall profit of the firm. Deterioration is defined, in general, as spoilage, damage, decay, perishing, fungus, or evaporation of inventory goods. Deterioration of the product may start instant after the production process or it may start later at a certain fixed time or random time after the production. Deterioration affects quality and/or quality of the product that causes loss of goodwill and reduction in the profit of the firm. Inventory managers may use preservation technology to reduce the deterioration rate of product.

Sarkar (2012) investigated an inventory model with delay in payments and time-varying deterioration rate. Dye (2013) studied the effect of preservation technology on a non-instantaneous deteriorating inventory model. Dye and Hsieh (2013) considered instantaneous deterioration with time-dependent demand for inventory model to obtain optimum policies. Hsieh and Dye (2013) gave a production-inventory model incorporating the effect of preservation technology investment where they considered the time fluctuating demand. Shah et al. (2013) gave optimal inventory policies for single-supplier single-buyer deteriorating items with price-sensitive stock-dependent demand and order-linked trade credit. Shah et al. (2021) studied an inventory model for instantaneously deteriorating items with use of preservation technology investments. In development of the model, they considered promotional efforts to promote sales for retailer and quantity discounts from supplier to encourage the retailer for a large order. Singh and Sharma (2013) gave a global optimizing policy for decaying items with ramp type demand considering preservation technology investments. Mishra (2014) studied deteriorating inventory model with controllable deterioration rate for time-dependent demand and time-varying holding cost. Shah and Shah (2014) studied inventory model for deteriorating items with price-sensitive stock-dependent demand under inflation. Tayal et al. (2014) and Zhang et al. (2014) studied inventory model with preservation technology investment with different demand types for instantaneous deteriorating items. Liu et al. (2015) gave joint dynamic pricing and investment strategy for perishable foods with price-quality-dependent demand. Sarkar et al. (2015) investigated inventory model with trade credit policy and variable deterioration for products with maximum lifetime. Singh and Rathore (2015) gave optimum payment policy with preservation technology investment and shortages under trade credit. Tsao (2016) studied joint location inventory and preservation decisions for non-instantaneous deterioration items under delay in payments. Bardhan et al. (2019) considered stock-dependent demand for non-instantaneous deteriorating items.

Most of the researches have been carried out considering instantaneous or non-instantaneous with deterministic start time of deterioration. However, this assumption seems unrealistic. It is not possible to predict exact start time of deterioration. Rahim et al. (2000) considered deterioration starting at a random point to study the inventory model, however they did not consider the idea of preservation technology. Pal et al. (2018) considered deterioration to start at random point with preservation technology where they considered constant demand. Tai et al. (2019) investigated joint inspection and inventory control for deteriorating items with random maximum life time.

In this paper, an inventory model for non-instantaneous deteriorating items with random start time of deterioration is considered. Market demand pattern is considered to be price sensitive. Further, the products are considered to be non-instantaneous deteriorating items with constant rate of deterioration. Retailer may invest in preservation technology to reduce the deterioration rate. With consideration of all these parameters authors aim to maximize total profit and examine optimal decisions for retailer. A numerical example is provided to validate the mathematical model. Moreover, a sensitivity analysis has been carried out to analyze the effect of changes in various inventory parameters on decision variables as well as the total profit, where one inventory parameter is varied by −20, −10, 10, and 20%. Comparison of both cases 'with preservation' and 'without preservation' have been analyzed to decide which of them is more beneficial for the retailer.

6.2 Assumptions and Notations

Authors use the following assumptions and notations in development of mathematical models.

(1) Replenishment rate is infinite and there is no lead time.
(2) The demand rate is $R(p) = \alpha - \beta p\ ;\ \alpha, \beta > 0$ where, α is scale demand and β is price sensitivity factor.
(3) Products are considered to be non-instantaneous deteriorating with constant rate of deterioration.
(4) Inventory model is for a single cycle $[0, T]$, which includes two phases: (i) $[0, x]$ where there is no deterioration and (ii) $[x, T]$ where products deteriorate at a constant rate. The point in time x at which deterioration starts is a random variable with positive range.
(5) There is no replacement or repair for deteriorated items in the inventory system.
(6) The proportion of reduced deterioration rate $m(\xi)$ is a continuous, concave, increasing function of the retailer's capital investments ξ with $m(0) = 0$ and $\lim\limits_{\xi \to \infty} m(\xi) = 1$. Further, we assume $m'(\xi) > 0$ to ensure that it is worth to invest money in preservation technology and $m''(\xi) < 0$ to ensure diminishing return from capital investments on preservation.

Following notations have been used in the development of the mathematical model.

Decision variables:

p Selling price is \$/unit.
T Cycle length of inventory.
ξ Preservation technology investment in \$/unit.

Other inventory parameters:

A	Ordering cost in \$/order.
c	Cost price in \$/unit.
h	Holding cost in \$ per unit per unit time.
$I_1(t)$	Inventory level during time interval $[0, x]$.
$I_2(t)$	Inventory level during time interval $[x, T]$.
$R(p)$	Price-sensitive demand rate.
x	Point in time when deterioration begins, a random variable over (a, b) with $pdf\ f(\cdot)$ and $cdf\ F(\cdot)$.
θ	Deterioration rate $(0 < \theta < 1)$.
$m(\xi)$	Proportion of reduced deterioration rate $(0 \leq m(\xi) \leq 1)$.

Objective function:

$\Pr(T, p, \xi)$	Average total profit of retailer with preservation technology investments.

6.3 Mathematical Model

Graphical representation of the Inventory model is shown in Fig. 6.1.

Figure 6.1 shows structure of the inventory model. Products in the system are considered as non-instantaneous deteriorating in nature. As per our assumption deterioration starts at random time x, hence there is no deterioration in the time interval $[0, x]$ and inventory level decreases due to the demand only, Whereas during $[x, T]$ inventory level is depleted due to combined effect of demand and deterioration.

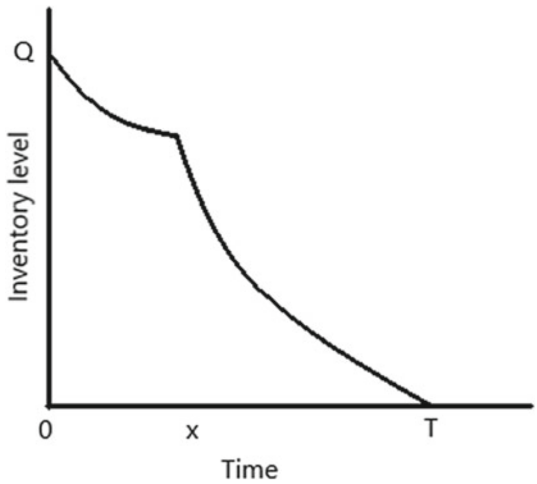

Fig. 6.1 Graphical representation of the inventory model

Corresponding inventory levels at any point of time t in respective intervals are governed by the differential equations,

$$\frac{\mathrm{d}}{dt} I_1(t) = -R(p), \quad 0 \le t \le x \tag{6.1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} I_2(t) = -R(p) - (1 - m(\xi)) \cdot \theta \cdot I_2(t), \quad x \le t \le T \tag{6.2}$$

Using the boundary condition $I_2(T) = 0$ and continuity of the demand function for solving Eqs. (6.1) and (6.2) inventory levels $I_1(t)$ and $I_2(t)$ for corresponding time interval can be obtained as below:

For $0 \le t \le x$,

$$I_1(t) = (\alpha - \beta p)\left(x - t + \frac{e^{(1 - m(\xi))\theta(t-x)} - 1}{(1 - m(\xi))\theta}\right) \tag{6.3}$$

And for $x \le t \le T$,

$$I_2(t) = (\alpha - \beta p)\left(\frac{e^{(1 - m(\xi))\theta(T-t)} - 1}{(1 - m(\xi))\theta}\right) \tag{6.4}$$

The total inventory during the interval [0, T] is as given below:

$$I(t) = \int_0^x I_1(t)\mathrm{d}t + \int_x^T I_2(t)\mathrm{d}t \tag{6.5}$$

The ordering quantity is given as,

$$Q = I_1(0) = \int_a^b (\alpha - \beta p)\left(x + \frac{\mathrm{e}^{(1-m(\xi))\theta(-x)} - 1}{(1 - m(\xi))\theta}\right) f(x)\mathrm{d}x \tag{6.6}$$

The holding cost is:

$$\mathrm{HC} = h \int_a^b \left[\int_0^x I_1(t)\mathrm{d}t + \int_x^T I_2(t)\mathrm{d}t\right] f(x)\mathrm{d}x \tag{6.7}$$

The total preservation cost is:

$$\mathrm{PTC} = \xi \int_a^b \left[\int_0^x I_1(t)\mathrm{d}t + \int_x^T I_2(t)\mathrm{d}t\right] f(x)\mathrm{d}x \tag{6.88}$$

Total sales revenue is:

$$\mathrm{TR} = p\int_0^T R(p)t\,\mathrm{d}t \tag{6.9}$$

The total profit of complete inventory cycle $[0, T]$ is given as

$$\Pr(T, p, \xi) = \frac{1}{T}(\mathrm{TR} - A - cQ - \mathrm{HC} - \mathrm{PTC}) \tag{6.10}$$

6.4 Numerical Example

Authors now illustrate the inventory model with numerical examples. The objective is to maximize total profit of the retailer which can be obtained by differentiating Eq. (6.10) with respect to decision variables T, p,ξ and setting them zero in order to get solution. This is shown in the following procedure.

Step 1: Allocate values to all inventory parameters other than decision variables.

Step 2: Work out $\frac{\partial \Pr}{\partial \xi} = 0$, $\frac{\partial \Pr}{\partial p} = 0$ and $\frac{\partial \Pr}{\partial T} = 0$ to get optimum values of decision variables, T p and ξ respectively.

Step 3: Substitute values of decision variables obtained above in Eq. (6.10) to get optimum value of total profit of the retailer.

Consider the following example to validate the mathematical formulation.

Example Let $A = \$\ 5000$ per order, $a = 5$ days, $b = 10$ days, $c = \$\,30$ per unit, $h = \$\ 5$/unit/day, $\theta = 0.2$, $\alpha = 300$, $\beta = 2$. Demand $R(p) = \alpha - \beta p$ units/day. Authors have considered the reduced deterioration rate $m(\xi) = 1 - \mathrm{e}^{(-k\cdot\xi)}$; where $k = 0.06$ is the simulation coefficient representing the change in the reduced deterioration rate per unit change in capital (Dye 2013). Moreover, authors assume the probability density function of x, $f(x) = \begin{cases} \frac{2x}{b^2-a^2} & ;\ a \le x \le b \\ 0 & ;\ otherwise \end{cases}$ where $a = 5 \le x \le 10 = b$, with mean $\mu = 7.7777$ and standard deviation $\sigma = 1.4163$. The form of pdf is selected in such a way that probability of product will start deteriorating, increases with time.

By following the procedure mentioned above to get the optimal values of all the decision variables, optimal values of decision variables and the total profit are obtained as mentioned in Table 6.1 for both scenarios with preservation and without preservation.

Concavity of the profit function can be seen from the following graphs in Fig. 6.2.

Figure 6.2 shows concavity of the profit function with preservation technology investments. Figure 6.2a represents concavity of the profit function with respect to selling price and cycle time, Fig. 6.2b shows that profit function is concave with

Table 6.1 Optimal values for the inventory model

Decision variables	Preservation cost (ξ)	Retail price (p)	Cycle time (T) (days)	Total profit
With preservation technology investment	\$4.33	\$85.73	28.89	\$119, 187
Without preservation technology investment	NA	\$88.48	20.46	\$77, 169

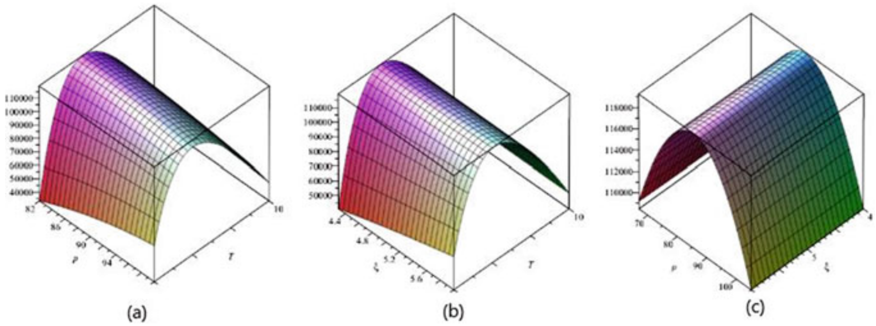

Fig. 6.2 Concavity of profit function with respect to decision variables (with preservation)

respect to cycle time and preservation cost and Fig. 6.2c depicts concavity of profit function with respect to selling price and preservation cost. Thus, Fig. 6.2 assures the concavity of profit function with respect to all the decision variables.

Next, authors proceed to determine the sensitivity of total profit of retailer, preservation technology cost, cycle time, and selling price with respect to change in other inventory parameters by −20, −10, 10, and 20% as shown in Table 6.2.

Table 6.2 characterizes sensitivity analysis of decision variables and total profit with respect to change in other inventory parameters. Table 6.2 shows that total profit is very responsive to the parameters a, b, h, α, β, θ and k. Parameters A and c have negligible outcome on profit. Increase in a, b, α and k results in increase in total profit. On the other side, the total profit decreases with increase in the parameters h, β and θ. Similarly, the sensitivity of preservation cost can be seen in the Table 6.2. Preservation cost increases with increase in the parameters a and β, while increase in the parameters b, c, h, α, θ and k reduces preservation technology investments. There is ignorable effect of change in A on preservation cost. Moreover, cycle time is very sensitive to the parameters a, b, k, α, β and θ. Other parameter's effect is negligible to the cycle time. Cycle time increases with increase in a, b and α while it decreases with increase in h, k, β and θ. Selling price is very responsive to all the parameters except A. It can be observed that with respect to increase in c and α,

Table 6.2 Impact of change in various inventory parameters on decision variables

Inventory parameters	Decision variables	Percentage change in various inventory parameters				
		−20%	−10%	0	10%	20%
Ordering cost/order (A)	T	28.8989	28.8993	28.8998	28.9002	28.9007
	P	85.7328	85.7333	85.7338	85.7343	85.7348
	ξ	4.3315	4.3303	4.3291	4.328	4.3268
	Profit	119,222	119,205	119,187.2	119,170	119,153
Lower limit of deterioration interval (a)	T	28.4	28.65	28.8998	29.16	29.42
	P	85.94	85.84	85.7338	85.62	85.51
	ξ	4.283	4.309	4.3291	4.343	4.351
	Profit	116,375	117,754	119,187.2	120,666	122,181
Upper limit of deterioration interval (b)	T	27.82	28.36	28.8998	29.43	29.96
	P	86.46	86.09	85.7338	85.4	85.09
	ξ	4.72	4.53	4.3291	4.12	3.9
	Profit	112,115	115,669	119,187.2	122,655	126,061
Cost price/unit (c)	T	28.9033	28.9015	28.8998	28.8981	28.8964
	P	85.714	85.724	85.7338	85.744	85.754
	ξ	4.355	4.342	4.3291	4.316	4.304
	Profit	119,275	119,231	119,187.2	119,143	119,099
Holding cost/unit (h)	T	30.2	29.54	28.8998	28.28	27.67
	P	86	85.86	85.7338	85.6	85.47
	ξ	5.45	4.89	4.3291	3.77	3.2
	Profit	123,528	121,329	119,187.2	117,101	115,069
Preservation efficiency scale (k)	T	29.71	29.21	28.8998	28.63	28.36
	P	88.13	86.68	85.7338	85.03	84.47
	ξ	10.82	6.75	4.3291	2.7	1.52
	Profit	113,572	116,932	119,187.2	120,673	121,587
Constant demand rate co-efficient (α)	T	28.38	28.57	28.8998	29.26	29.61
	P	70.2	77.89	85.7338	93.65	101.6
	ξ	7.34	5.47	4.3291	3.53	2.93
	Profit	70,199	93,026	119,187.2	148,768	181,832
Selling price dependent demand rate co-efficient (β)	T	29.79	29.3	28.8998	28.6	28.41
	P	105.59	94.54	85.7338	78.59	72.73
	ξ	2.68	3.45	4.3291	5.35	6.59
	Profit	159,716	137,025	119,187.2	104,809	92,968
Natural deterioration rate (θ)	T	34.14	31.23	28.8998	26.99	25.4
	P	86.72	86.2	85.7338	85.32	84.95
	ξ	4.75	4.54	4.3291	4.13	3.94
	Profit	136,578	126,966	119,187.2	112,744	107,306

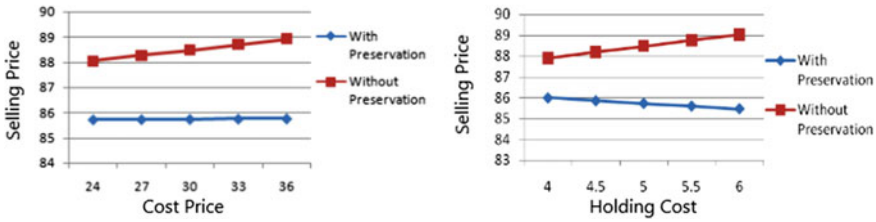

Fig. 6.3 Sensitivity of selling price with respect to cost price and holding cost (with preservation and without preservation)

selling price also increases. On the other side, increase in a, b, h, β, θ and k result in decrease in the selling price. It can be observed from the graph that Total profit and cycle time do not respond significantly to the change in A. Profit and cycle time both increases with increase a, b and α. On the other hand, increase in c, h, β and θ results in decrease in both the profit and cycle time.

The retailer should wisely decide the investment amount for preservation technology to reduce the deterioration rate so as the total cost does not increase and the total profit can be maximized. Sensitivity analysis of selling price, cycle time, and total profit with preservation technology investments and without preservation technology investments is shown in following Figs. 6.3, 6.4, 6.5, 6.6, 6.7, 6.8 and 6.9.

Figure 6.3 shows that selling price is equally sensitive with respect to cost price in both situations. Increase in cost price gives rise to increase in selling price, which is slightly less in with preservation compared to without preservation case. Similarly, increase in holding cost results the hike in selling price. Selling price without preservation case remains higher than the preservation technology.

Figure 6.4 shows that selling price increases with increase in α and decreases with an increase in β. This is clearly reflected in the graph above. It can be noted that selling price in with preservation case is slightly less than the selling price in without preservation case.

Figure 6.5 depicts sensitivity of cycle time with respect to cost price and holding cost. First graph represents effect of change in cost price on cycle time and second

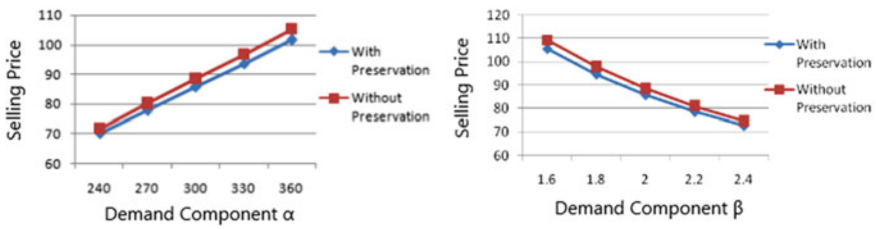

Fig. 6.4 Sensitivity of selling price with respect to demand components (with preservation and without preservation)

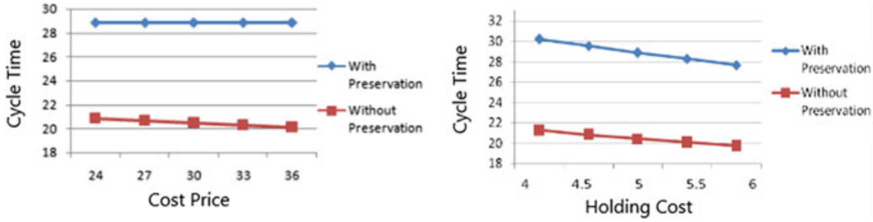

Fig. 6.5 Sensitivity of cycle time with respect to cost price and holding cost (with preservation and without preservation)

graph shows effect of change in holding cost on cycle time. In both scenarios 'with preservation' and 'without preservation' the cycle time remains almost the same with increase in cost price, while cycle time decreases with increase in holding cost in both cases. Cycle time remains higher in preservation case compared to no-preservation.

Figure 6.6 characterizes the change in cycle time with respect to change in demand components. Cycle time increases with increase in α. Cycle time remains higher in preservation case compared to without preservation because the preservation technology let the product last for a longer time. On the other side, increase in β results into decrease in the cycle time.

Figure 6.7 represents the effect of change in total profit with respect to cost price and holding cost. First graph represents effect in total profit due to increase in cost price and second graph shows effect of increase of holding cost on total profit.

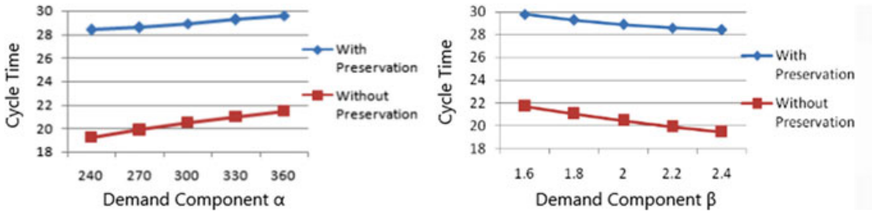

Fig. 6.6 Sensitivity of cycle time with respect to demand components (with preservation and without preservation)

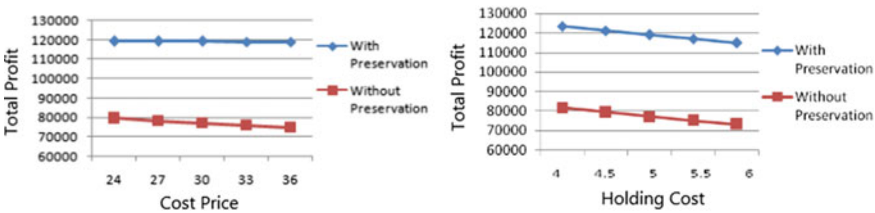

Fig. 6.7 Sensitivity of total profit with respect to cost price and holding cost (with preservation and without preservation)

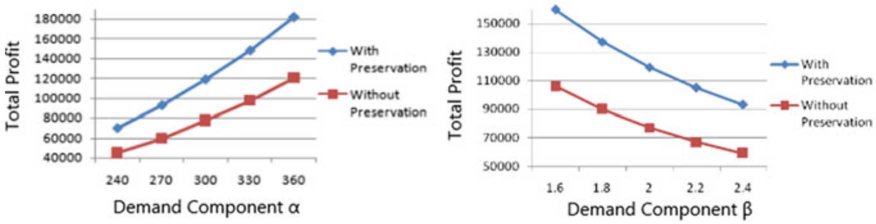

Fig. 6.8 Sensitivity of Total Profit with respect to demand components (with preservation and without preservation)

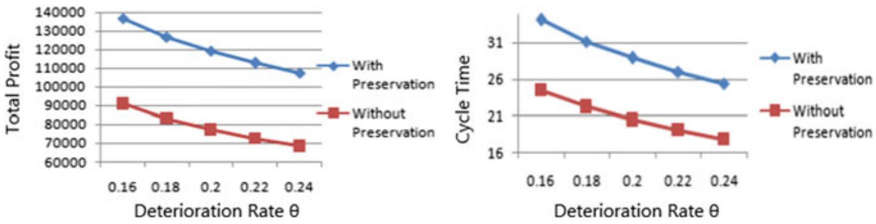

Fig. 6.9 Sensitivity of Total Profit and Cycle Time with respect to deterioration (with preservation and without preservation)

Increase in the costs associated with inventory results in decrease in total profit which is clearly reflected in both graphs. The total profit in preservation is higher than profit with no preservation.

Figure 6.8 shows how the total profit changes with respect to change in demand components. Total profit increases with the increase in α and decreases with increase in β in both the scenario 'with preservation' and 'without preservation'. Total profit is higher in preservation technology case compared to no-preservation case.

Figure 6.9 depicts how the change in the deterioration rate affects the total profit and the cycle time. First graph shows change in total profit with respect to increase in deterioration rate. Here it can be noticed that with increase in the rate of deterioration total profit decreases. However in 'with preservation' due to preservation technology the decrease in total profit is lower compared to 'without preservation' case. Second graph denotes the change in cycle time with respect to increase in deterioration. With increase in the deterioration rate, the cycle time decreases in both scenarios. Due to preservation technology the decrease in 'with preservation' is less than that in 'without preservation'.

6.5 Conclusion

Authors have studied non-instantaneous deteriorating products with random start time of deterioration with preservation technology investments. Demand of the product is considered to be price sensitive. Study of the model includes comparison of ‘with preservation investments’ and ‘without preservation investments’ through the graphs and detailed analysis has been carried out. The model is validated through numeric example. Sensitivity analysis has been carried out to check the effect of different parameters on decision variables. It is observed from the study that investments in preservation technology give better profit than the non-preservation technology case. However, the retailer needs to take care of investments in preservation technology because it helps in reducing the deterioration rate but the higher amount of investments can increase the capital cost and decrease the total profit.

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