Chapter 2
An Inventory Model for Stock and Time-Dependent Demand with Cash Discount Policy Under Learning Effect and Partial Backlogging

Nidhi Handa, S. R. Singh, and Chandni Katariya

Abstract This study considers an inventory model with stock and time-dependent demand. Stock level always plays a very vital role and affects the demand rate. Vendors usually offer different schemes to attract more customers. In this paper, the scheme of cash discount is working as promotional tool for increasing demand rate. Shortages are allowed with partial backlogging and backlogging rate present in the model is assumed as a waiting time-dependent function. To make the study more realistic learning effect is applied on holding cost. Three cases for the allowed trade credit period are described in the present paper. To illustrate the model numerical example for different cases have been discussed by using Mathematica 11.3. sensitivity analysis with respect to distinct parameters is carried out for the feasibility and the applicability of the model.

Keywords Inventory model · Learning effect · Deterioration · Stock and time-dependent demand · Cash discount · Trade credit · Partial backlogging · Shortages

2.1 Introduction

The role of demand is very vital while developing an inventory model, Available stock and time are the factors that always influence the demand. Khurana and Chaudhary (2016) proposed an inventory model using stock and price-dependent demand for deteriorating items under shortage backordering. Giri et al. (2017) introduced a vendor–buyer supply chain inventory model using time-dependent demand under preservation technology. Khurana and Chaudhary (2018) developed a deteriorating inventory model for stock and time-dependent with partial backlogging. Bardhan

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et al. (2019) introduced a non-instantaneous deteriorating inventory model for stock-dependent demand under preservation technology. Handa et al. (2020) worked on an EOQ model with stock-dependent demand for trade credit policy under shortages.

Deterioration is another important factor whose part in the construction of an inventory model is very useful. Deterioration can be defined as the reduction, or spoilage in the original value of the product. Skouri et al. (2009) developed some inventory policies under Weibull deterioration rate for ramp type demand. Chowdhury et al. (2014) formulated an inventory model for price and stock-dependent demand. Mahapatra et al. (2017) introduced a model using deteriorating items based on reliability-dependent demand under partial backlogging. Rastogi et al. (2018) developed an inventory policy for non-instantaneous deteriorating items using price-sensitive demand with partial backordering.

In the construction of an inventory model, the basic assumption is that when the stock out situation occurs then the shortages that take place are either completely lost or completely backlogged which is not realistic. At the arrival of the stock some customers are interested to come back, which is known as partial backlogging. Roy and Chaudhuri (2011) studied an inventory model using price-dependent demand, Weibull deterioration, and partial backlogging. Kumar and Singh (2014) presented a two-warehouse inventory model in which demand depends upon stock level under partial backordering. Geetha and Udayakumar (2016) formulated inventory policies for non-instantaneous deteriorating products under multivariate demand rate and partial backorder. Khanna et al. (2017) proposed an inventory model using selling price-dependent demand for imperfect items under shortage backordering and trade credit. Kumar et al. (2020) studied the effect of preservation and learning on partial backordering inventory model for deteriorating items with the environment of the Covid-19 pandemic.

In today's competitive market the trade credit period offered by the seller has become a very useful incentive policy for attracting new customers. Singh et al. (2016) proposed an EOQ model allowing stock-dependent demand under trade credit policy. Shaikh (2017) introduced a deteriorating inventory model based on advertisement and price-dependent demand using partial backlogging and mixed type of trade credit. Tripathi et al. (2018) studied an inventory model for time-varying holding cost with stock dependent demand having different. Shaikh et al. (2019) introduce a Weibull distributed deteriorating inventory model allowing multivariate demand rate and trade credit period.

Learning is a realistic phenomenon that occurs naturally. Generally, it is seen that when workers accomplish the same procedure repeatedly then they learn how to performs more efficiently such phenomenon is called learning effect. Singh et al. (2013) presented an inventory model for imperfect products under the effect of inflation and learning. Singh and Rathore (2016) formulated a reverse logistic inventory model with preservation and inflation under learning effect. Goyal et al. (2017) proposed an EOQ model using advertisement-based demand under learning effect and partial backorder. Singh et al. (2020) introduced a reverse logistic inventory model for variable production under learning effect.

This paper represents an inventory model considering stock and time-dependent demand, cash discount, and partial backlogging. To make the study more realistic learning effect is applied on holding cost. Different cases for the allowed trade credit period are also described in the model. To improve the efficiency of the model numerical example for different cases and sensitivity analysis for distinct value of parameters have been discussed.

2.2 Assumptions

1. Demand used in the model is a function of stock and time i.e. $(\delta + \beta t + \gamma E_1(t))$.
2. Items used in the model are of decaying nature.
3. No replacement policy is allowed for deteriorating products in whole cycle period.
4. Shortages are considered with partial backlogging.
5. Deteriorating rate is constant.
6. Backlogging rate present in the model is assumed as a waiting time-dependent function.
7. This model incorporates the effect of learning on holding cost.
8. Trade credit period is allowed in the model.

2.3 Notations

Notations used in the model.

$E(t)$	level of inventory at any time t
δ, β, γ	coefficients of demand
Q_1	initial stock level
Q_2	backorder quantity during stock out
k	rate of deterioration
$\phi(\eta)$	rate of backlogging
η	waiting time up to next arrival lot
T	cycle time
u_1	time at which level of inventory becomes zero
$h_f + \frac{h_g}{n^\lambda}$	per unit holding cost under learning effect where $\lambda > 0$
s_r	shortage cost per unit
d	per unit deterioration cost
l_r	per unit lost sale cost
c	purchasing cost per unit
A	per order ordering cost
p	selling price per unit
$U.T.P_x.$	unit time profit

M	allowed trade credit period
I_c	rate of interest charged
I_e	rate of interest earned
y	rate of cash discount.

2.4 Mathematical Modelling

Figure 2.1 represents the behavior of inventory system with respect to time. Q_1 denotes the initial inventory level at $t = 0$. Level of inventory depletes in the interval $[0, u_1]$ for the reason of deterioration and demand. At $t = u_1$, inventory level turns into zero, and after that shortages occur with partial backlogging. The depletion of the inventory is shown in Fig. 2.1

Differential equations of the inventory system can be represented as follows

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} + kE_1 = -(\delta + \beta t + \gamma E_1(t)) \quad 0 \le t \le u_1 \tag{2.1}$$

$$\frac{\mathrm{d}E_2}{\mathrm{d}t} = -(\delta + \beta t) \quad u_1 \le t \le T \tag{2.2}$$

Boundary equations are given as follows:

$$E_1(u_1) = E_2(u_1) = 0$$

Solution of Eqs. (2.1) and (2.2) are given by

$$E_1(t) = \left[\delta(u_1 - t) + \frac{\beta}{2}\left(u_1^2 - t^2\right) + (k+\gamma)\left\{\frac{\delta}{2}\left(u_1^2 - t^2\right) + \frac{\beta}{2}\left(u_1^3 - t^3\right)\right\}\right]\mathrm{e}^{-(k+\gamma)t} \quad 0 \le t \le u_1 \tag{2.3}$$

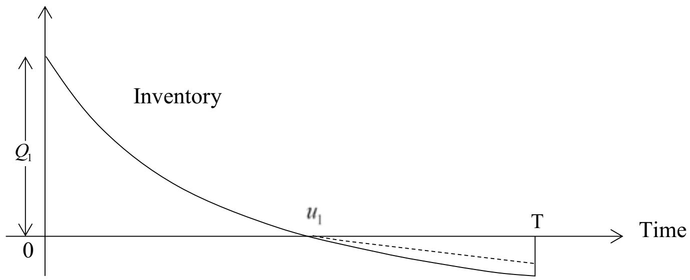


Fig. 2.1 Inventory time graph of system

$$E_2(t) = \left[\delta(u_1 - t) + \frac{\beta}{2}\left(u_1^2 - t^2\right)\right] \quad u_1 \le t \le T \tag{2.4}$$

2.5 Associated Costs

Ordering Cost:

Ordering cost per order for the system is taken as follows:

$$O.C_x. = A \tag{2.5}$$

Purchasing Cost:

Q_1 denotes the initial inventory level at $t = 0$ and Q_2 for the duration $[u_1, T]$.

$$E_1(0) = Q_1 = \left\{\delta u_1 + \beta\frac{u_1^2}{2} + (k+\gamma)\left(\delta\frac{u_1^2}{2} + \beta\frac{u_1^2}{3}\right)\right\} \tag{2.6}$$

$$Q_2 = \int_{u_1}^{T} (\delta + \beta t)\phi(\eta)\mathrm{d}t \tag{2.7}$$

$$= \left\{\frac{\delta}{2}\left(T^2 - u_1^2\right) + \frac{\beta}{3}\left(T^3 - u_1^3\right)\right\} \tag{2.8}$$

$$P.C_x. = \{Q_1 + Q_2\}c \tag{2.9}$$

Hence, the purchasing cost of the system is given by

$$P.C_x = \left\{\delta u_1 + \beta\frac{u_1^2}{2} + (k+\gamma)\left(\delta\frac{u_1^2}{2} + \beta\frac{u_1^2}{3}\right) + \frac{\delta}{2}\left(T^2 - u_1^2\right) + \frac{\beta}{3}\left(T^3 - u_1^3\right)\right\}c \tag{2.10}$$

Sales Revenue:

Sales revenue can be taken as follows:

$$S.R_x. = (Q_1 + Q_2)p \tag{2.11}$$

$$S.R_x. = \left\{\delta u_1 + \beta\frac{u_1^2}{2} + (k+\gamma)\left(\delta\frac{u_1^2}{2} + \beta\frac{u_1^2}{3}\right) + \frac{\delta}{2}\left(T^2 - u_1^2\right) + \frac{\beta}{3}\left(T^3 - u_1^3\right)\right\}p \tag{2.12}$$

Holding Cost:

Holding cost is considered in the duration when the system holds the inventory. Holding cost is taken as follows:

$$H.C_x. = \left(h_f + \frac{h_g}{n^\lambda}\right)\int\limits_0^{u_1} E_1(t)\mathrm{d}t \tag{2.13}$$

$$H.C_x. = \left(h_f + \frac{h_g}{n^\lambda}\right)\left\{\delta\frac{u_1^2}{2} + \beta\frac{u_1^3}{3} + (k+\gamma)\left(\delta\frac{u_1^3}{6} + \beta\frac{u_1^4}{8}\right)\right\} \tag{2.14}$$

Shortage Cost:

In the inventory system shortages occur during the stock out condition when goods are not available to fulfil the customers demand. Shortage cost of the system is taken as follows:

$$S.C_x. = s_r\int\limits_{u_1}^{T}(\delta + \beta t)\mathrm{d}t \tag{2.15}$$

$$S.C_x. = \left\{\delta(T - u_1) + \frac{\beta}{2}\left(T^2 - u_1^2\right)\right\}s_r \tag{2.16}$$

Lost Sale Cost:

In the inventory system lost sale cost is considered during the stock out condition when some customers fulfil their demand from other places. Lost sale cost is taken as follows:

$$L.S.C_x. = l_r\int\limits_{u_1}^{T}(\delta + \beta t)(1 - \phi(\eta))\mathrm{d}t \tag{2.17}$$

$$L.S.C_x. = l_r\left\{\delta\frac{T^2}{2} + \beta\frac{T^3}{6} - \delta u_1 T - \beta T\frac{u_1^2}{2} + \delta\frac{u_1^2}{2} + \beta\frac{u_1^3}{3}\right\} \tag{2.18}$$

Deterioration Cost:

Deterioration cost is considered for those products that are deteriorated or decayed in the system. The deterioration cost is taken as follows:

$$D.C_x. = d\left\{E_1(0) - \int\limits_0^{u_1}(\delta + \beta t)\mathrm{d}t\right\} \tag{2.19}$$

$$D.C_x. = d(k+\gamma)\left(\delta\frac{u_1^2}{2} + \beta\frac{u_1^3}{3}\right) \tag{2.20}$$

2.6 Permissible Delay

Trade credit period is the useful incentive policy for attracting more customers. In this time period vendor allows a certain time limit to retailer to pay all his dues. If the retailer pays all his dues before the credit limit then there will be no interest charged otherwise interest will be charged on unpaid amount. Retailer can also earn interest on sales revenue.

Two cases for allowed trade credit period are given as follows:

Case 1: When $M \geq u_1$ (Fig. 2.2).

For this case vendor has enough amount to settle all his payments since the credit limit period is more than the period of sold out all the stock. In this case, interest charged would be zero and interest earned in the duration [0, M] is given as follows.

$$I.V_1. = pI_e\int_0^{u_1}(\delta+\beta t+\gamma E(t))\mathrm{d}t + (M-u_1)\int_0^{u_1}(\delta+\beta t+\gamma E(t))\mathrm{d}t \tag{2.21}$$

$$\begin{aligned} pI_e\Bigg\{&\frac{\delta u_1^2}{2}+\frac{\beta u_1^3}{3}-\gamma\Bigg(\frac{\delta(k+\gamma)+\beta}{8}u_1^4+\frac{\delta u_1^3}{6}+\frac{\beta(k+\gamma)}{10}u_1^5 \\ &-\frac{\delta(k+\gamma)}{12}u_1^4-\frac{(\delta(k+\gamma)^2+\beta(k+\gamma))}{15}u_1^5 \\ &-\frac{\beta(k+\gamma)^2}{9}u_1^6\Bigg)\Bigg\}+\{(M-u_1) \\ &\Bigg(\delta u_1+\frac{\beta u_1^2}{2}-\gamma\Bigg(\frac{\delta u_1^2}{2}+\frac{\delta(k+\gamma)+\beta}{3}u_1^3-\delta(k+\gamma)\frac{u_1^3}{6} \\ &-\frac{(\delta(k+\gamma)^2+\beta(k+\gamma))}{8}u_1^4-\frac{\beta(k+\gamma)^2}{10}u_1^5\Bigg)\Bigg\} \end{aligned} \tag{2.22}$$

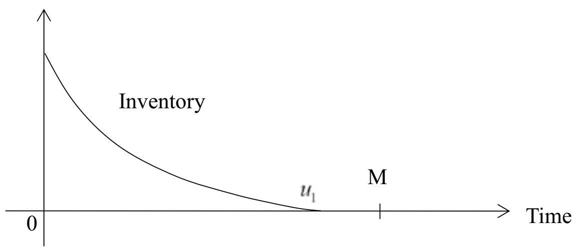


Fig. 2.2 Inventory time graph when ($M \geq u_1$)

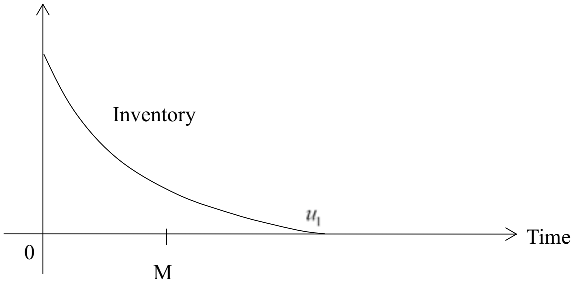


Fig. 2.3 Inventory time graph when $M < u_1$

And interest charged is given as follows:

$$I.C_1 = 0$$

Case 2: When $M < u_1$ (Fig. 2.3)

For this case, vendor has to settle all his payments before to sold out all the stock. For interest earned and interest charged two following cases take the place:

Case 2.1: When M $< u_1$ and

$$pD\,[0, M] + I.V_{2.1}\,[0, M] \geq cE\,(0): \tag{2.23}$$

For this case, vendor has enough amount to settle all his payments. Interest charged would be zero for this case, but interest would be earned in the duration $[0, M]$.

$$I.C_{2.1} = 0 \tag{2.24}$$

$$I.V_{2.1} = pI_e \int\limits_0^M (\delta + \beta t + \gamma E(t))t\mathrm{d}t \tag{2.25}$$

$$\begin{aligned} = pI_e \Bigg\{ & \frac{\delta M^2}{2} + \frac{\beta M^3}{3} - \gamma \Bigg(\frac{\delta(k+\gamma)+\beta}{8} M^4 \\ & + \frac{\delta M^3}{6} + \frac{\beta(k+\gamma)}{10} M^5 - \frac{\delta(k+\gamma)}{12} M^4 \\ & - \frac{(\delta(k+\gamma)^2 + \beta(k+\gamma))}{15} M^5 - \frac{\beta(k+\gamma)^2}{9} M^6 \Bigg) \Bigg\} \end{aligned} \tag{2.26}$$

Case 2.2: When $M < u_1$ and

$$pD\,[0, M] + I.V_{2.2}\,[0, M] < cE\,(0): \tag{2.27}$$

For this case, vendor has not enough amount to settle all his payments so interest would be charged on unpaid amount. In the duration $[0, M]$ earned interest is given by as follows:

$$
\begin{aligned}
I.V_{2.2} &= pI_e \int_0^M (\delta + \beta t + \gamma E(t))t\,\mathrm{d}t \\
&= pI_e \left\{ \frac{\delta M^2}{2} + \frac{\beta M^3}{3} - \gamma \left(\frac{\delta(k+\gamma)+\beta}{8} M^4 \right.\right. \\
&\quad + \frac{\delta M^3}{6} + \frac{\beta(k+\gamma)}{10} M^5 - \frac{\delta(k+\gamma)}{12} M^4 \\
&\quad \left.\left. - \frac{(\delta(k+\gamma)^2 + \beta(k+\gamma))}{15} M^5 - \frac{\beta(k+\gamma)^2}{9} M^6 \right) \right\}
\end{aligned} \tag{2.28}
$$

Interest charged on unpaid amount is given by as follows:

$$I.C_{2.2} = B.I_c \tag{2.29}$$

$$B = cE_1(0) - \{pD\,[0, M] + I.V_{2.2}\,[0, M]\} \tag{2.30}$$

$$
\begin{aligned}
&= \left\{ \left[c\left(\delta u_1 + \frac{\beta u_1^2}{2} + (k+\gamma)\left(\frac{\delta u_1^2}{2} + \frac{\beta u_1^3}{3} \right) \right] - p\left[\delta M + \frac{\beta M^2}{2} \right.\right.\right. \\
&\quad - \gamma \left(\frac{\delta M^2}{2} + \frac{\delta(k+\gamma)+\beta}{3} M^3 + \frac{\beta(k+c)}{4} M^4 - \delta(k+\gamma)\frac{M^3}{6} \right. \\
&\quad \left.\left. - \frac{(\delta(k+\gamma)^2 + \beta(k+\gamma))}{8} M^4 \right) - \frac{\beta(k+\gamma)^2}{10} M^5 \right) \Bigg] \\
&\quad - pI_e \left[\frac{\delta M^2}{2} + \frac{\beta M^3}{3} - \gamma \left(\frac{\delta(k+\gamma)+\beta}{8} M^4 \right.\right. \\
&\quad + \frac{\delta M^3}{6} + \frac{\beta(k+\gamma)}{10} M^5 - \frac{\delta(k+\gamma)}{12} M^4 \\
&\quad \left.\left.\left. - \frac{(\delta(k+\gamma)^2 + \beta(k+\gamma))}{15} M^5 - \frac{\beta(k+\gamma)^2}{9} M^6 \right) \right] \right\}
\end{aligned}
$$

Case 3: When cash discount facility is given:

For this case, retailer provides cash discount at a rate of $y\%$ to settle all his dues at the arrival of the stock. Interest earn would be

$$I.V_3 = pI_e \int_0^T (\delta + \beta t + \gamma E(t))\mathrm{d}t \tag{2.31}$$

$$pI_e\left\{\delta T+\frac{\beta T^2}{2}-c\left(\frac{\delta u_1^2}{2}+\frac{\delta(k+\gamma)+\beta}{3}u_1^3+\frac{\beta(k+\gamma)}{4}u_1^4\right.\right.$$
$$\left.\left.-\delta(k+\gamma)\frac{u_1^3}{6}-\frac{(\delta(k+\gamma)^2+\beta(k+\gamma))}{8}u_1^4\right)\right\}$$

Purchasing cost for this case would be

$$P.C_x=\left\{\delta u_1+\beta\frac{u_1^2}{2}+(k+\gamma)\left(\delta\frac{u_1^2}{2}+\beta\frac{u_1^2}{3}\right)\right.$$
$$\left.+\frac{\delta}{2}\left(T^2-u_1^2\right)+\frac{\beta}{3}\left(T^3-u_1^3\right)\right\}c\left(1-\frac{y}{100}\right) \tag{2.32}$$

2.7 Unit Time Profit

Unit time profit for the system is given by as follows:

$$U.T.P_x=\frac{1}{T}\{S.R_x.-P.C_x.-H.C_x.-D.C_x.$$
$$-L.S.C_x.-S.C_x.-O.C_x.-I.C.+I.V\} \tag{2.33}$$

2.8 Numerical Example

Case 1: When $M\geq u_1$

$A=300$ per/order, $c=22$ Rs/unit, $d=21$, $k=0.001$, $T=28$ days, $M=22$ days, $\delta=300$ units, $\beta=0.1$, $\gamma=0.01$, $l_r=7$ Rs/unit, $s_r=5$ Rs/unit, $h_f=0.22$ Rs/unit, $h_g=0.15$ Rs/unit, $p=30$ Rs/unit, $I_e=0.02$, $n=2$, $\lambda=0.1$.

After solving Eq. (2.33) with the help of corresponding parameters optimal value of $u_1=19.6461$ days and $U.T.P_x=2077.92$ Rs. and optimal ordered quantity $Q=8702.17$ units.

The behavior of the system for $U.T.P_x.$ is given by Figs. 2.4 and 2.5 with the help of Mathematica 11.3.

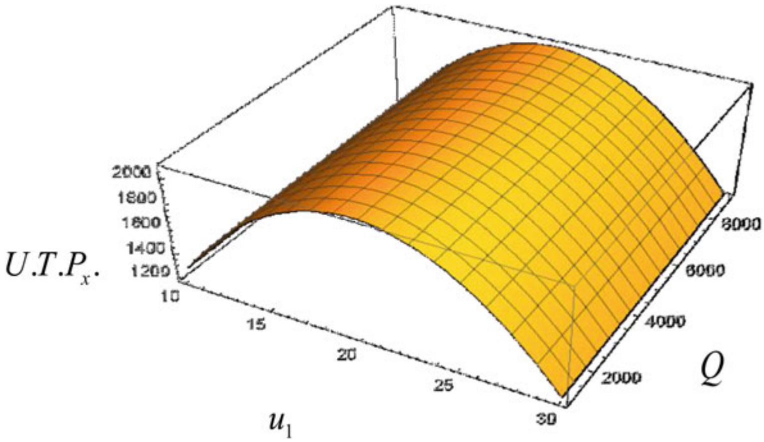
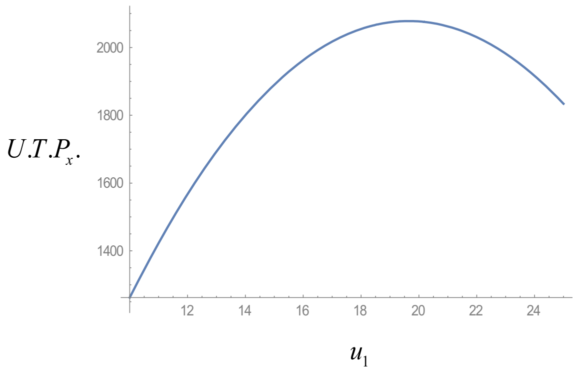


Fig. 2.4 Behavior of $U.T.P_x$. with respect to u_1 and Q

Fig. 2.5 Behavior of $U.T.P_x$. with rest to u_1



Case 2.1: When $M < u_1$ and

$$pD\,[0, M] + IV_{2.1}\,[0, M] \geq cE\,(0):$$

$A = 300$ per/order, $c = 22$ Rs/unit, $d = 21$, $k = 0.001$, $T = 28$ days, $M = 17$ days, $\delta = 300$ units, $\beta = 0.1$, $\gamma = 0.01$, $l_r = 7$ Rs/unit, $s_r = 5$ Rs/unit, $h_f = 0.22$ Rs/unit, $h_g = 0.15$ Rs/unit, $p = 30$ Rs/unit, $I_e = 0.02$, $n = 2$, $\lambda = 0.1$.

After solving Eq. (2.33) with the help of corresponding parameters optimal value of $u_1 = 18.5963$ days and $U.T.P_x = 1533.71$ Rs. and optimal ordered quantity $Q = 8535.0$ units.

The behavior of the system for $U.T.P_x$. is given by Figs. 2.6 and 2.7 with the help of Mathematica 11.3.

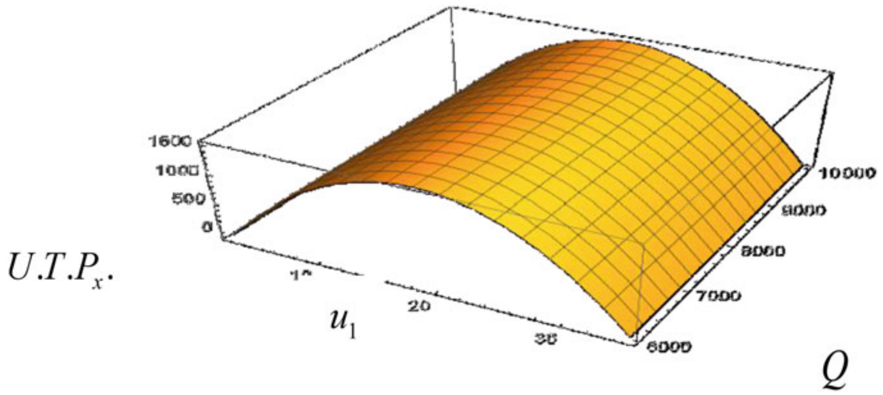


Fig. 2.6 Behavior of $U.T.P_x.$ with respect to u_1 and Q

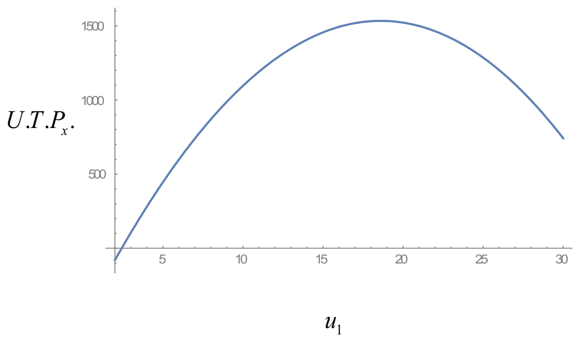


Fig. 2.7 Behavior of $U.T.P_x.$ with respect to u_1

Case 2.2: When M $< u_1$ and

$$pD\,[0, M] + IV_{2.2}\,[0, M] < cE\,(0):$$

$A = 300$ per/order, $c = 22$ Rs/unit, $d = 21$, $k = 0.001$, $T = 28$ days, $M = 17$ days, $\delta = 300$ units, $\beta = 0.1$, $\gamma = 0.01$, $l_r = 7$ Rs/unit, $s_r = 5$ Rs/unit, $h_f = 0.22$ Rs/unit, $h_g = 0.15$ Rs/unit, $p = 30$ Rs/unit, $I_e = 0.02$, $n = 2$, $\lambda = 0.1$, $I_c = 0.016$.

After solving Eq. (2.33) with the help of corresponding parameters optimal value of $u_1 = 18.5961$ days and $U.T.P_x = 1627.54$ Rs. and optimal ordered quantity $Q =$ 8534.97 units.

The behavior of the system for $U.T.P_x.$ is given by Figs. 2.8 and 2.9 with the help of Mathematica 11.3.

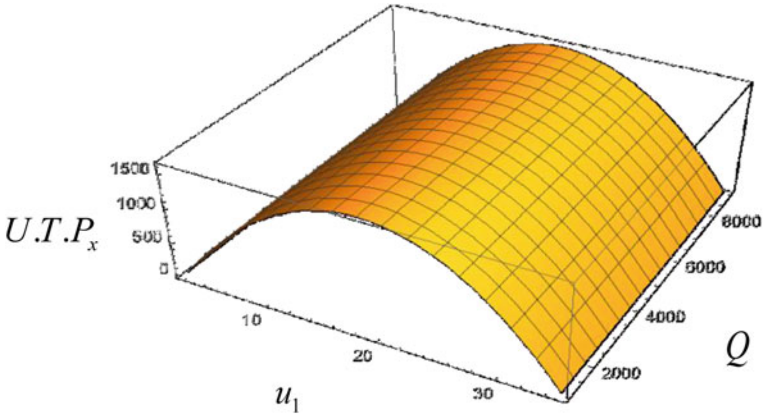


Fig. 2.8 Behavior of $U.T.P_x$. with respect to u_1 and Q

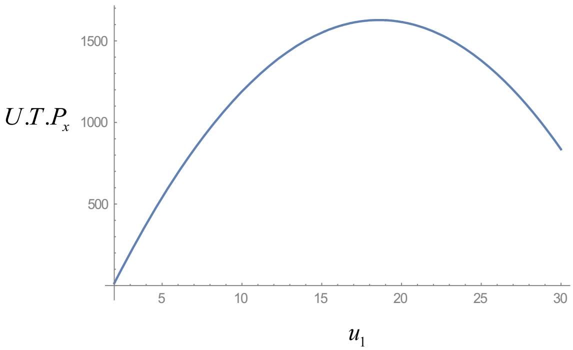


Fig. 2.9 Behavior of $U.T.P_x$. with respect to u_1

Case 3: When cash discount facility is given:

$A = 300$ per/order, $c = 22$ Rs/unit, $d = 21$, $k = 0.001$, $T = 28$ days, $\delta = 300$ units, $\beta = 0.1$, $\gamma = 0.01$, $l_r = 7$ Rs/unit, $s_r = 5$ Rs/unit, $h_f = 0.22$ Rs/unit, $h_g = 0.15$ Rs/unit, $p = 30$ Rs/unit, $I_e = 0.02$, $n = 2$, $\lambda = 0.1$, $y = 0.02$.

After solving this model with the help of corresponding parameters optimal value of $u_1 = 18.4906$ days and $U.T.P_x = 826.307$ Rs. and optimal ordered quantity $Q = 8517.72$ units.

The behavior of the system for $U.T.P_x$. is given by Figs. 2.10 and 2.11 with the help of Mathematica 11.3.

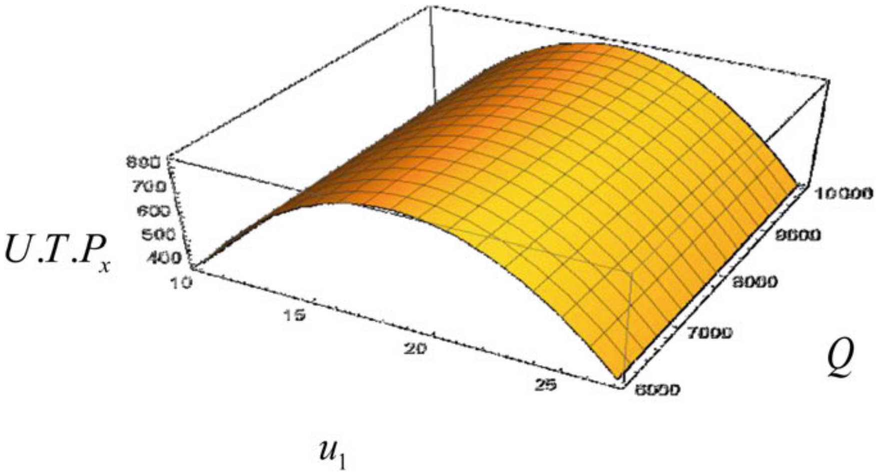
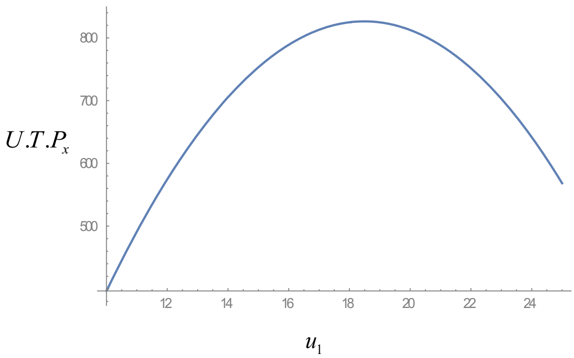


Fig. 2.10 Behavior of $U.T.P_x$. with respect to u_1 and Q

Fig. 2.11 Behavior of $U.T.P_x$. with respect to u_1



2.9 Sensitivity Analysis

Sensitivity analysis for distinct parameters are specified as follows:

Case 1: When M $\geq u_1$ (Tables 2.1, 2.2, 2.3, 2.4, 2.5 and 2.6).

Case 2: When $M < u_1$ (Tables 2.7, 2.8, 2.9, 2.10, 2.11 and 2.12).

2.10 Observations

- Tables 2.1 and 2.7 represent the effect of a on u_1 and on $U.T.P_r$, it is observed that after an increment in a, value of u_1 in both the tables remain unaffected while

Table 2.1 Variation in optimal solution for demand parameter (δ)

% change in (δ) (%)	(δ)	u_1	$U.T.P_r.$
−20	240	19.6461	2080.06
−15	255	19.6461	2079.52
−10	270	19.6461	2078.99
−5	285	19.6461	2078.45
0	300	19.6461	2077.92
5	315	19.6461	2077.38
10	330	19.6461	2076.85
15	345	19.6461	2076.31
20	360	19.6461	2075.77

Table 2.2 Variation in optimal solution for shortage parameter (s_r)

% change in (s_r) (%)	(s_r)	u_1	$U.T.P_r.$
−20	4	19.021	2171.51
−15	4.25	19.1771	2147.48
−10	4.5	19.3332	2123.87
−5	4.75	19.4896	2100.68
0	5	19.6461	2077.92
5	5.25	19.8027	2055.57
10	5.5	19.9595	2033.65
15	5.75	20.1165	2012.16
20	6	20.2737	1991.08

Table 2.3 Variation in optimal solution for lost sale cost parameter (l_r)

% change in (l_r) (%)	(l_r)	u_1	$U.T.P_r.$
−20	5.6	19.3762	2097.36
−15	5.95	19.4453	2092.38
−10	6.3	19.5132	2087.48
−5	6.65	19.5802	2082.66
0	7	19.6461	2077.92
5	7.35	19.711	2073.25
10	7.7	19.7749	2068.65
15	8.05	19.8379	2064.12
20	8.4	19.8999	2059.66

some decrement in $U.T.P_r$ in Table 2.1 and some increment in $U.T.P_r$ in Table 2.7 are detected.

Table 2.4 Variation in optimal solution for deterioration cost parameter (d)

% change in (d) (%)	(d)	u_1	$U.T.P_r.$
−20	16.8	20.2326	2176.73
−15	17.85	20.0825	2151.48
−10	18.9	19.9347	2126.6
−5	19.95	19.7893	2102.08
0	21	19.6461	2077.92
5	22.05	19.505	2054.1
10	23.1	19.378	2032.63
15	24.15	19.2293	2007.49
20	25.2	19.0946	1984.67

Table 2.5 Variation in optimal solution for deterioration parameter (k)

% change in (k) (%)	(k)	u_1	$U.T.P_r.$
−20	0.00096	18.3506	1869.34
−15	0.00102	18.2662	1855.59
−10	0.00108	18.1827	1841.98
−5	0.00114	18.1002	1828.50
0	0.0012	19.6461	2077.92
5	0.00126	19.636	2076.31
10	0.00132	19.6259	2074.71
15	0.00138	19.6158	2073.1
20	0.00144	19.6058	2071.5

Table 2.6 Variation in optimal solution for interest earned parameter (I_e)

% change in (I_e) (%)	(I_e)	u_1	$U.T.P_r.$
−20	0.016	19.4902	1729.38
−15	0.017	19.531	1863.73
−10	0.018	19.5705	1935.1
−5	0.019	19.6089	2006.5
0	0.02	19.6461	2077.92
5	0.021	19.6822	2149.36
10	0.022	19.7172	2220.83
15	0.023	19.7513	2292.32
20	0.024	19.7844	2363.83

Table 2.7 Variation in optimal solution for demand parameter (δ)

% change in (δ) (%)	(δ)	u_1	$U.T.P_r.$
−20	240	18.5961	1300.82
−15	255	18.5961	1382.54
−10	270	18.5961	1464.21
−5	285	18.5961	1545.87
0	300	18.5961	1627.54
5	315	18.5961	1709.2
10	330	18.5961	1790.87
15	345	18.5961	1872.53
20	360	18.5961	1954.2

Table 2.8 Variation in optimal solution for lost sale cost parameter (l_r)

% change in (l_r) (%)	(l_r)	u_1	$U.T.P_r.$
−20	5.6	18.1531	1652.52
−15	5.95	18.2678	1646.06
−10	6.3	18.3798	1639.74
−5	6.65	18.4892	1633.57
0	7	18.5961	1627.54
5	7.35	18.7006	1621.64
10	7.7	18.8027	1615.87
15	8.05	18.9111	1609.75
20	8.4	19.0003	1604.7

Table 2.9 Variation in optimal solution for deterioration cost parameter (d)

% change in (d) (%)	(d)	u_1	$U.T.P_r.$
−20	16.8	19.4008	1717.21
−15	17.85	19.1934	1694.07
−10	18.9	18.9902	1671.42
−5	19.95	18.7911	1649.25
0	21	18.5961	1627.54
5	22.05	18.4049	1606.27
10	23.1	18.2175	1585.44
15	24.15	18.0337	1565.03
20	25.2	17.8535	1545.03

Table 2.10 Variation in optimal solution for shortage cost parameter (s_r)

% change in (s_r) (%)	(s_r)	u_1	$U.T.P_r.$
−20	4	17.6956	1733.93
−15	4.25	17.921	1706.42
−10	4.5	18.1462	1679.52
−5	4.75	18.3712	1653.23
0	5	18.5961	1627.54
5	5.25	18.8208	1602.46
10	5.5	19.0452	1577.11
15	5.75	19.2696	1554.11
20	6	19.4937	1530.84

Table 2.11 Variation in optimal solution for deterioration parameter (k)

% change in (k) (%)	(k)	u_1	$U.T.P_r.$
−20	0.00096	19.1732	1692.05
−15	0.00102	18.9201	1675.32
−10	0.00108	18.5521	1674.23
−5	0.00114	18.4421	1652.25
0	0.0012	18.5961	1627.54
5	0.00126	18.4121	1620.02
10	0.00132	18.2121	1518.20
15	0.00138	18.0121	1515.12
20	0.00144	17.4224	1512.12

Table 2.12 Variation in optimal solution for interest earned parameter (I_e)

% change in (I_e) (%)	(I_e)	u_1	$U.T.P_r$
−20	0.016	18.5961	1449.16
−15	0.017	18.5961	1493.75
−10	0.018	18.5961	1538.35
−5	0.019	18.5961	1582.94
0	0.02	18.5961	1627.54
5	0.021	18.5961	1672.13
10	0.022	18.5961	1716.73
15	0.023	18.5961	1761.32
20	0.024	18.5961	1805.92

- Tables 2.2 and 2.10 represent the effect of s_r on u_1 and on $U.T.P_r$, it is observed that after an increment in s_r, some increment in u_1 and some decrement in $U.T.P_r$ in both the tables are detected.
- Tables 2.3 and 2.8 represent the effect of l_r on u_1 and on $U.T.P_r$, it is observed that after an increment in l_r, some increment in u_1 and some decrement in $U.T.P_r$ in both the tables are detected.
- Tables 2.4 and 2.9 represent the effect of d on u_1 and on $U.T.P_r$, it is observed that after an increment in d, some decrement in u_1 and $U.T.P_r$ in both the tables are detected.
- Tables 2.5 and 2.11 represent the effect of k on u_1 and on $U.T.P_r$, it is observed that after an increment in k, some increment in u_1 and $U.T.P_r$ in Table 2.5 while some decrement in u_1 and $U.T.P_r$ in Table 2.11 are detected.
- Tables 2.6 and 2.12 represent the effect of I_e on u_1 and on $U.T.P_r$, it is observed that after an increment in I_e, value of u_1 remains unaffected in Table 2.12 while some increment in u_1 in Table 2.6 and $U.T.P_r$ in both the tables are detected.

2.11 Conclusions

Present paper is concerned with inventory policies for variable demand under some real-life situations like cash discount and learning effect. Shortages are also allowed with partial backlogging and backlogging rate present in the model is assumed as a waiting time-dependent function. All these facts together make this study very unique and straight forward. To improve the efficiency of the model numerical examples for different cases and sensitivity analysis for distinct value of parameters have been discussed with the help of Mathematica 11.3. This Model further can be modified for different demands, deterioration, and more cases of backlogging rate. Also, can be extended for different realistic approaches such as inflationary environment and preservation technology.

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