# Chapter 11
# Conclusion and Future Work

In this chapter, some concluding remarks and related future work are given.

## 11.1 Conclusion

This book is mainly concerned with the complex networked systems under imperfect communication constraints. The research problems proposed in Sect. 1.2 have been respectively discussed in this book.

(a) Chapter 2 is mainly devoted to studying Problem 1.2. Under arbitrary finite communication delays, the consensus problem in directed static networks is studied. It has been proved that consensus can be realized whatever the finite communication delays are.
(b) Two types of imperfect communication, i.e., quantization and communication delays, are studied in Chaps. 3 and 4. In Chap. 3, Problem 1.2 is investigated in detail and a unified framework for continuous-time multi-agent consensus problem with quantization and communication delays is developed. By using the nonsmooth technique, practical consensus of the multi-agent systems is obtained. It is interesting to observe that the quantization parameter decides the size of the practical consensus set and communication delays only affect the center of the practical consensus set. Chapter 4 is devoted to studying Problem 1.2. Without considering input quantization, consensus problems with communication quantization and communication delays simultaneously are considered. Discrete-time protocol and continuous-time protocol are, respectively, discussed in Sects. 4.1 and 4.2. We have proved that under the connected network topology, the multi-agent network can achieve consensus.
(c) Chapter 5 is concerned with Problem 1.2. Compared with the traditional periodic sampled-data control or time-driven control method, the event-driven control method is more flexible and robust in some real multi-agent systems.



J. Lu et al., *Collective Behavior in Complex Networked Systems under Imperfect Communication*, https://doi.org/10.1007/978-981-16-1506-1_11

In Chap. 5, discrete-time and continuous-time multi-agent consensus problems under event-triggered control are studied, respectively. Discrete-time consensus protocol with communication delays is discussed in Sect. 5.1. In Sect. 5.2, continuous-time event-triggered and self-triggered consensus protocols are proposed, respectively. It is also shown that the Zeno behavior can be excluded under our proposed event-based protocol with communication delays.

(d) Chapters 6 and 7 are concerned with Problem 1.2 and Problem 1.2, respectively. Continuous-time bipartite consensus and fixed-time/finite-time bipartite consensus problems in networks of agents with antagonistic interactions and communication delays are investigated. Effective consensus protocols are designed to realize expected collective behaviors. By the Lyapunov stability method and homogeneity of function analysis, the relation between the order of $\phi(x)$ and the speed of convergence is obtained.

(e) Chapters 8 and 9 are devoted to studying Problem 1.2. In Chap. 8, the exponential synchronization behavior of a general complex dynamical network is investigated. Some sufficient conditions are proposed to guarantee the globally exponential synchronization of the network. Moreover, one quantity is distilled from the coupling matrix to characterize the synchronizability of corresponding dynamical networks. Chapter 9 mainly studies the pinning cluster synchronization of coupled neural networks by a novel event-triggered mechanism. An effective distributed event-triggered scheme is proposed to realize expected cluster synchronization and meanwhile exclude the Zeno behavior. Under event-triggered mechanism, some controllers will be pinned to certain selected nodes in coupled neural networks to realize expected cluster synchronization.

(f) Chapter 10 is devoted to Problem 1.2. First, consensus analysis of nonlinear multi-agent network with arbitrary communication topology is given, which uses the global information of the network. For large scale multi-agent networks, to reduce the size of the networks and meanwhile conserve the consensus property, a new network reduction approach is proposed which only uses the local information of the network in the reduction process. Furthermore, based on the network reduction method discussed in this chapter, a novel consensus recovery approach is provided to improve the reliability of the network system and preserve the consensus property under node failure.

## 11.2 Future Work

Some related topics for future research are listed as follows:

(1) In this book, we only study the first-order multi-agent consensus problem under imperfect communications. In the future, we will extend the results of this book to the high-order multi-agent consensus problems. In particular, we will design new consensus protocols for higher-order multi-agent systems and investigate how quantization and communication delays affect the final

consensus results. Moreover, we will design event-triggered consensus protocol for higher-order multi-agent networks and derive distributed event-triggered conditions for reaching consensus.

(2) In Chaps. 3 and 4, the effect of two kinds of uniform quantization is analyzed in detail. In future work, we will focus on the effect of other different types of quantization, such as logarithm quantization, in the distributed complex networked systems. Moreover, in many real-world complex networks, individuals in the network are often able to build new links or suppress old ones among themselves as time goes on. Hence, collective dynamical behavior problems with quantization and communication delays under time-varying communication topology will be considered in future research.

(3) In Chaps. 6 and 7, bipartite consensus of multi-agent systems with antagonistic interactions and communication delays is studied. In the future, we will extend the results of this book to the bipartite synchronization problems. Furthermore, we will focus on more imperfect communication on the cooperative-competitive complex networks, such as quantization, data dropout, noise, etc.

(4) Chapters 8 and 9 mainly concentrate on the synchronization problem of complex dynamical networks with communication delays. We will further consider the collective behaviors of stochastic complex networks with communication delays in our future work. Furthermore, the application of network synchronization problem with communication delays also will be studied, such as distributed Kalman filtering for sensor networks, distributed fault diagnosis, etc.

(5) In Chap. 10, network reduction and recovery methods were proposed for multi-agent consensus with node failure. In the future, we will also consider the reverse problem of the network reduction process, i.e., what would happen if some new nodes were added to the multi-agent network? The application of network reduction and its converse process in some practical problems, such as optimal allocation of sensors network, distributed fault diagnosis and consensus maintenance, and so on, shall all be explored in the future work.