Chapter 17
Developing the Competencies of Mathematics Teacher-Researchers

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Abstract Instead of seeing teachers solely as instructors in the classrooms, there is a growing trend to position teachers as agents of change, who collaborate with different stakeholders to innovate and improve their teaching practices. These changing demands of educational systems have placed increased emphasis on developing teacher-researchers who are able to adopt an inquiry stance in their mathematics teaching. In this chapter, we first give an overview of the crucial role of teacher-researchers by drawing on relevant literature and looking back at the key shifts in teacher development. Next, we describe some of the key competencies of a teacher-researcher. Following this, we describe how mathematics teachers develop these competencies in Singapore before we look forward to how mathematics educators can continue to address some of the challenges in developing the competencies of mathematics teacher-researchers.

Keywords Learning from teaching · Mathematics teacher noticing · Teaching as inquiry · Teacher education · Teacher professional development

17.1 Looking Back: Why Do We Need Mathematics Teacher-Researchers?

The vision for the mathematics curriculum in a changing world challenges teachers to go beyond teaching to the tests and instead, think more deeply about the kind of skills students need to master to thrive in this age of unprecedented changes. To this end, teachers have to continuously update their knowledge to include more research-based or evidence-based teaching strategies. Drawing from analyses of PISA data, the OECD (2016) suggests mathematics teachers can think more deeply about what

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they teach, whom they teach, and how they teach to raise the quality of their instruction. There are two key levers for raising the quality of mathematics instruction: research about how students best learn mathematics and collaboration with other teachers to improve teaching (p. 25). These levers position teachers as active agents of change, rather than passive recipients of research, a shift towards a teaching as inquiry paradigm. As highlighted by Cochran-Smith and Lytle (1999), this inquiry stance sees teachers "learning to how to teach and improve one's teaching by collecting and analyzing the 'data' of daily life in schools" (p. 17). In some ways, this resonates with Berthoff's (1987) views of teacher as a researcher who generates practice-grounded theories through dialogue with other teachers and interrogation of existing teaching practices. Since the 1990s, this shift to seeing teachers as teacher-researchers has continued to feature prominently in efforts to improve the quality of teaching (Cochran-Smith & Lytle, 1999; Stigler & Hiebert, 1999; Timperley, Wilson, Barrar, & Fung, 2007).

Against this international backdrop, the Singapore educational landscape has also evolved rapidly in the last five decades. From the survival-driven phase (1959–1978); through the efficiency-driven phase (1979–1996); to the ability-based, aspiration-driven phase (1997–2011); student-centric, values-driven phase (2012–2018); and now "Learn for Life: Remaking Pathways" (2019 onwards), our education system has always focused on improving the quality of educational experiences for all our pupils. (For readers interested in how the education phases influenced science and mathematics education, see Chaps. 6 and 7, respectively.) Through the years, Singapore has moved from providing a comprehensive and strong basic education for every child to developing each child to the best of his/her potential through a focus on innovation, creativity, and research (Ministry of Education-Singapore, 2013). Consequently, the role of teachers in Singapore has shifted from being providers of quality content towards being facilitators of quality learning who orchestrate high-quality interactions between students and teachers in the classrooms. With the aim of supporting schools to engage students in learning, there were efforts to reduce curriculum content to create *white space* for teachers to customise and create instructional materials for their profile of students (Ministry of Education-Singapore, 2013). These efforts were accompanied by a push for teachers to adopt a wider range of pedagogical and assessment approaches. In addition, *time-tabled time* was introduced to provide time and space for teachers to discuss, plan, and reflect on their lessons. All these initiatives were also bolstered by the formation of professional learning teams in many schools (Chua, 2009). These professional learning teams are tasked to inquire into current teaching practices of different subjects in their schools and explore the theory-practice nexus in teaching and learning.

These changes necessitate the development of research competencies amongst teachers. In the case of mathematics teachers, they have to move from adopting research-based teaching strategies such as the Singapore Model Method and the Concrete-Pictorial-Abstract instructional heuristic (see Chap. 9) to examining the effectiveness of these approaches for their specific student profiles and exploring other strategies for teaching mathematics through practitioner inquiry. As part of the

implementation of white space and time-tabled time, many mathematics teachers have begun to explore the use of action research and other job-embedded professional activities such as lesson study (Lim, Lee, Saito, & Syed Haron, 2011), which involve de-privatisation of classrooms—a feature of professional learning associated with high performing education systems (Vieluf, Kaplan, Klieme, & Bayer, 2012). De-privatising classrooms and engaging in reflective inquiry about teaching practices provide opportunities for teachers to learn how to teach better from their own experiences, other teachers' experiences, and research findings (Mason, 2002). Such inquiry stance is essential for teachers to understand implications from research in order to apply them to develop new pedagogical approaches (Timperley et al., 2007).

17.2 How Do We Develop Competencies of Teacher-Researchers?

Having looked back at the fundamental shift from mathematics teachers to teacher-researchers, we now turn to describe how teachers' research competencies are developed in Singapore. In this section, we will first elaborate on the three critical competencies of teacher-researchers before we describe how these competencies are developed through three avenues. First, we describe how the Ministry of Education (MOE) provided top-down support for bottom-up initiatives during the Teach Less Learn More (TLLM) movement from 2005 to 2011. Next, we highlight initiatives by the National Institute of Education (NIE) to equip every NIE undergraduate student-teachers with educational research skills and how NIE's postgraduate programmes provide a platform for in-service teachers to further hone their research competencies. Last but not least, we highlight how some of the teachers' research competencies are developed through their participation in research projects.

17.2.1 Competencies of Teacher-Researchers

Langrall (2006) stated that teachers have often been referred to as consumers of research rather than producers of research, and she added that for most teachers "the process as well as the product of their inquiry is tacit" (p. 1). While teachers appreciate the value of research, they know that their primary role is to teach and not to do research. What do teachers who are also researchers have to do? Cochran-Smith (2006) highlighted that "Teachers who are researchers continuously pose problems, identify discrepancies between theory and practice, and challenge common routines. They continuously ask questions about teaching and learning and they do not

flinch from self-critical reflection…" (p. xv). More specifically, Langrall (pp. 1–2) stressed on the following competencies:

- Reading and reflecting on research and other literature in the field
- Interpreting findings from the research literature to influence their instructional practice;
- Participating in study groups with their colleagues
- Generating research questions for themselves and others to investigate
- Participating in research studies and professional development projects led by other researchers
- Designing and implementing their own studies and sharing their findings with others

By developing these competencies, teachers can begin to hone their research skills and work at the theory-practice nexus, where they learn to translate what they find through their own studies into changes in their own practices. Recognising that these competencies are critical for improving teaching, the Ministry of Education (Singapore) seeded the development of these research skills in their teachers through the Teach Less Learn More (TLLM) movement.

17.2.2 Teach Less Learn More (TLLM): Top-Down Support for Bottom-Up Initiatives

The TLLM movement was launched in 2005, as the education system moved into the ability-driven phase, to improve the quality of classroom interactions by making learning more engaging, enjoyable, and meaningful for students (Ministry of Education-Singapore, 2013). This movement came about as a result of the introduction of an ability-driven education, which suggested a need to harness the diverse talents and abilities of teachers in schools towards the goal of delivering the best learning environment for all students (Crawford, 2002). As such, the Ministry of Education (Singapore) supported teachers to innovate and improve their teaching practices through the Research Activist (RA) scheme, as part of the TLLM *Ignite!* initiatives from 2006 to 2011. Teachers, identified to be research activists, were attached to the MOE for 2 full days per week over a period of 40 weeks, ensuring that they had time and space to think more deeply about teaching and learning issues, and work on their proposed school-based curriculum innovations (SCIs), which were targeted at addressing their students' learning needs. During the 2 days, the RAs were trained by academics in curriculum design and research methodologies. The training covered a variety of curriculum theories and design frameworks, as well as both quantitative and qualitative research methods. Besides a seed funding and additional training workshops on specific pedagogy, these RAs also had access to relevant curriculum partners and consultants who are experts in the content, curriculum, or pedagogy. To facilitate professional conversations, these RAs

were placed in a network, comprising other RAs doing similar SCIs, under the facilitation of a MOE curriculum officer. In addition, these RAs were given platforms such as local conferences to present and share their SCIs.

Although this movement may seem like a massive undertaking, Singapore's fidelity to the movement's intent is quite a strong one because all stakeholders involved contributed actively to the TLLM *Ignite!* initiatives, knowing that they serve the greater good for Singapore students and the nation. Consequently, all these initiatives provided a much-needed top-down support for the RAs' self-initiated projects and prepared the ground for developing the competencies of mathematics teacher-researchers. In total, there were 327 TLLM *Ignite!* projects, of which about 25% were mathematics-focused. Even in schools where the projects were focused on other disciplines, the research expertise gathered by the RAs would be helpful for initiating mathematics-focused SCIs subsequently. As the RAs embarked on their SCIs, they had opportunities to apply their learning to design, develop, and implement their SCIs. Doing so provides a time and space for mathematics teachers to engage with the six steps in the research process (Creswell & Guetterman, 2019):

1. Identifying a research problem;
2. Reviewing the literature;
3. Specifying a purpose for research;
4. Collecting data;
5. Analyzing and interpreting the data; and
6. Reporting and evaluating research. (p. 7)

These steps are aligned with the skillset identified by Langrall (2006), and teachers have opportunities to work through these skills through their SCI. By focusing the SCI on a teaching or learning issue specific to their school, the school's RA works with a team of teachers to study the selected issue by reading relevant research articles; develop an evidence-based intervention; collect, analyse, and interpret data from the intervention; and report their findings to ascertain what they have learned from the implementation of the SCI. These activities mirror what Cochran-Smith and Lytle (1999) have highlighted about learning how to teach by collecting and analysing data. Although the TLLM *Ignite!* initiatives had ended in 2012, these initiatives seeded the development of research competencies in many schools and heightened the level of professionalism of many teachers.

17.2.3 Developing In-service Teachers' Research Competencies

The heightened level of professionalism amongst many teachers have help raised the level of professional discourse and have led to more teachers pursuing postgraduate degrees at the National Institute of Education (NIE), where they could deepen their mastery of both their research competencies and content knowledge

(Ministry of Education-Singapore, 2013). As mentioned earlier, most pre-service courses for teachers did not include a research component. More than a decade ago, Foong (2007) stated that there is an emerging trend in Singapore in teacher professional development to pursue master's programmes for the opportunity to learn about and do research. Through the Professional Development Continuum Model (PDCM) and subsequently the enhanced Professional Development Continuum Model, the Ministry of Education in Singapore has encouraged in-service teachers at all levels to take masters courses to upgrade their qualifications and to develop their research competencies through courses run at the National Institute of Education (NIE). Other than doctoral courses such as Doctor of Philosophy (PhD) or Doctor in Education (EdD), in-service teachers can also enrol for the following masters programmes: Master of Education (MEd-Mathematics), Master of Science (MSc-Mathematics for Educators), or the Master of Arts (MA). Research is a strong component of each of these programmes. The MEd and MSc programmes are each based on the completion of 30 academic units (AU) worth of courses (1 AU = 13 hours of coursework).

17.2.3.1 Master of Education (Mathematics)

This specialisation in the Master of Education programme provides coursework that develops knowledge of mathematics as a subject and its pedagogy. It develops reflective practitioners of Mathematics education, prepares teachers for career development in such capacities as the MOE's master teacher or senior specialist tracks, and provides induction into mathematics education research. These MEd courses can be completed through coursework only or through a combination of coursework and dissertation. Students enrolled in this programme can complete either six courses with a dissertation (dissertation option) or complete seven courses (Coursework only option). A compulsory course MED 900 Educational Inquiry offers teachers opportunities to learn about educational research methodology, which lays the foundation for the dissertation and Integrative Project. Those selecting the coursework-only option will take a special course titled MED 902 Integrative Project as one of the seven courses. The other courses provide opportunities for teacher candidates to explore research and issues specific to the learning and teaching of mathematics. It is worthwhile to note that all of the other specialisation elective courses lean heavily towards reading and interpreting research findings.

17.2.3.2 Master of Science (Mathematics for Educators)

Unlike the MEd which focuses on mathematics education courses, the MSc (Mathematics for Educators) focuses on mathematical content. The programme is designed to cater to the professional needs of mathematics educators and emphasises the acquisition of wide and in-depth content knowledge in mathematics as well as its linkages to mathematics teaching. This provides an avenue for teachers to

deepen their research capabilities in mathematics. Candidates in the course will have the opportunity to study courses in different areas of mathematics, conducted by active working mathematicians, and work on a research project in mathematics. The underlying assumption is that teachers who command a strong mastery of mathematics will enable them to teach better and to promote higher-order thinking amongst students in the learning of mathematics. All candidates for this course have to complete one compulsory 2 AU course on mathematical research methods (MSM 900) and seven specialisation elective courses to be chosen from Level 1 and Level 2 courses, with no more than three from Level 1 courses. Candidates acquire skills in reading and interpreting research in the content area of mathematics. Although this programme is not explicitly tied to improving the quality of teaching, a good understanding of mathematics is crucial for handling various tasks related to mathematics education, such as the design of contemporary and rigorous curriculum, assessment of mathematics learning, and development of teaching resources.

17.2.3.3 Master of Arts (Mathematics Education) and Master of Science (Mathematics)

Both of these programmes require candidates to complete a supervised thesis of about 40,000 to 50,000 words in an approved area in mathematics education and mathematics, respectively. Being a research-intensive programme, graduate students will have the opportunity to publish journal articles, book chapters, or other academic papers. These two programmes thus provide a platform for teachers to do research and disseminate their findings beyond the classrooms.

Each of these courses becomes part of the larger ecosystem in which in-service teachers have many opportunities to develop their research competencies in both mathematics and mathematics education. The teacher-as-researcher ecosystem, which was seeded by the TLLM initiative, has come a long way since 2005. As of December 2017, there are 5165 teachers with a Master's degree and 140 teachers with a PhD (out of 33,163 teachers) across all schools (Ministry of Education-Singapore, 2018, p. 12). That is, about 16% of the teachers have post-graduate qualifications. These figures suggest that schools have access to research expertise and are well positioned to take advantage of the research capability to embark on their own investigations of teaching practices.

17.2.4 Developing Pre-service Teachers' Research Competencies

The theory-practice nexus should not only be seen from the perspective of researchers at universities developing theories and teachers in schools as the implementers and users of research ideas. To empower teachers, it is important that teachers

develop their own research skills for them to undertake research projects either individually, in small collegial groups, or together with experts from universities. Schools in Singapore were encouraged to implement personal or team action projects (see Foong, 2007). However, although an action research project may be directly relevant to a teacher's practice, it has a very limited scope.

For the paradigm of teaching as inquiry to take root, it is also important that this inquiry stance can be developed in our pre-service teachers. To this end, a core course, Educational Research, in the NIE's Enhanced BA/BSc (Education) programme was launched in 2015. The course was designed to equip student teachers with an understanding of the purposes, processes, and outcomes of academic and educational research, with a strong focus on methods of designing, collecting, analysing, and interpreting data. This introductory research course is offered to all student teachers and provides an opportunity for student teachers to be guided by NIE faculty members as they explore a topic of mutual interest and experience the educational research process.

For the pre-service teachers taking the 16-month Post-graduate Diploma in Education programme, they have a 4-week observation attachment in schools to explore the connections between educational theory taught in the NIE and the teaching practices in schools. By engaging in observations of teachers in schools, and discussions with the lecturers during the attachment, there are opportunities to explore the different perspectives of teaching. More importantly, the student-teachers get to see how the lecturers at the NIE and teachers in school model pedagogical reasoning (Shulman, 1987) as they reflect upon their instructional decisions.

Together with the TLLM *Ignite!* initiatives, the pre-service teacher and in-service teacher programmes at the NIE provide the necessary platforms to develop the competencies of our teacher-researchers. Although the notion of teaching as inquiry may not be explicitly introduced to our teachers at the NIE, the structure of the programmes inculcates an inquiry mindset in our student-teachers to examine more deeply how our students learn mathematics and how teachers can approach teaching by exploring the connections between theory and practice.

17.2.5 Participation in Research Projects

Aligned with the MOE's vision of raising the quality of teachers (Heng, 2012; Ministry of Education-Singapore, 2013), the Office of Educational Research (OER) at the NIE had established two research centres—Centre for Research in Pedagogy and Practice (CRPP) in 2003 and Centre for Research in Child Development in 2017—to spearhead research projects in education. In addition to improving quality of instructional practices, these projects also provide opportunities for teachers to be research collaborators with educational researchers.

From 2008 to 2016, there were 161 OER research projects with school involvement. Amongst these projects, there were at least 53 noteworthy mathematics education projects in various fields such as mathematical problem-solving (Leong

et al., 2016), metacognition (Lee, Yeo, & Hong, 2014), teaching practices (Kaur, 2010), productive failure (Kapur, Lee, & Lee, 2018), and teacher noticing (Choy & Dindyal, 2017) involving about 500 mathematics teachers. Besides these, mathematics teachers are also involved in projects involving lesson study, which is a professional development platform for teachers to research their own practices (Jiang, Choy, & Lee, 2019). Teachers' participation in these projects affords opportunities to learn from their practices and will continue to be an important way to develop our teacher-researchers.

17.3 Looking Forward: What's Next for Developing Mathematics Teacher-Researchers?

Since the beginning of the TLLM movement in 2005, teachers are encouraged to adopt a more inquiry stance in their teaching. As highlighted, more teachers are involved in developing and implementing SCIs, taking up post-graduate studies, participating in regular professional development activities such as lesson study, and collaborating with researchers in other research projects (Ministry of Education-Singapore, 2013). Although this is an encouraging trend, participation in these research-focused professional development does not guarantee that teachers would learn from their practices and hone their research competencies. In this section, we will examine some of the issues and challenges when developing mathematics teacher-researchers, before we discuss some of the ways to address these issues.

17.3.1 Issues and Challenges

As argued by Lampert (2010), what matters is not the kind of professional development activities but what teachers focus on and how they engage with the activities within the contexts of learning communities. Drawing on Mason's (2002) idea that professional learning takes place in three worlds of experiences—world of personal experiences, one's colleagues' experiences, and the world of theories and observations (p. 93)—we will highlight the two main challenges with regard to developing teacher-researchers to improve teaching and learning.

First, teachers may not always recognise the possibilities to act differently from what they are currently doing. The ability to recognise possibilities is crucial for changing teaching practices. As Mason stated, the ability to recognise possibilities to act differently lies at the intersection of the three worlds of experience, which underscore the importance of reflection during collaborative professional development activities. However, it may be case that teachers may miss critical points during collaborative reflection. For example, Choy (2016b) highlighted how teachers may miss the subtle nuances of the mathematics concepts during lesson study

discussions, and other researchers have emphasised the key roles played by *knowledgeable others* during lesson study to enhance the quality of discussions (Jiang et al., 2019; Takahashi & McDougal, 2016). Although one may argue that the resistance to change practices may have risen solely from teachers' lack of mathematical knowledge for teaching, there are other factors such as persistent beliefs about mathematics, teaching, and learning which may hinder teachers' ability to change their practices (Choy, 2015, 2016a, 2016b). The challenge remains: How do we, as mathematics educators, support our teachers to develop the professional vision (Goodwin, 1994) to discern critical instructional details about mathematics, students' learning of mathematics, and their own teaching practices?

Second, as highlighted, teacher-researchers often need external expertise or resource support as they embark on teacher-initiated action research projects or other activities such as lesson study. The key issue is that teachers may not always have access to the relevant expertise. In addition, it may not be feasible or sustainable to have one external expert with each professional learning team in schools. How can we develop a more sustainable model for professional learning as teachers continue to be taken on the role of teacher-researchers? What kind of resources can we provide or co-construct to support the work that teacher-researchers do? And how can we enhance the existing ecosystem to encourage synergy and collaboration between teachers, researchers, and other professional learning facilitators, such as the Academy of Singapore Teachers? These are critical questions that need to be answered as we move forward in our journey.

17.3.2 The Way Forward

Notwithstanding the challenges, Singapore mathematics teachers can continue this journey of *learning to teach* through a teacher-researcher stance by building on the existing ecosystem of professional development and learning. To address the challenges, what is needed is not more hours of professional learning. Rather, the key is to develop a *sustainable* professional learning model, in which teacher-researchers learn from their teaching through the three worlds of experiences. How this model may look like is the focus of a current development project at the NIE (AFD 06/17 CBH). In addition, to deepen professional learning of teacher-researchers, mathematics educators need to sharpen the professional vision of our teachers, that is, to sharpen what teachers see and how they make sense of these observations to make instructional decisions—or what researchers termed as teacher noticing (Sherin, Jacobs, & Philipp, 2011). However, enhancing teacher noticing alone may not be sufficient. Instead, it is necessary to enhance teachers' ability to notice productively (Choy, Thomas, & Yoon, 2017), where teachers' noticing results in teachers making pedagogically productive instructional decisions. Doing this requires teachers to hone their pedagogical reasoning (Shulman, 1987), which may be a critical pathway to improve the quality of mathematics instruction. But how can teachers' noticing expertise be developed? How do we sharpen teachers' pedagogical reasoning? What

Shulman (1987) implied in his model of pedagogical reasoning and action is that teachers can learn from their own teaching or the idea of *docendo discimus*—by teaching, we learn. If we were to examine the processes of pedagogical reasoning and action, it became apparent that the model revolves around a teacher's day-to-day teaching activities. This has important implications for us as mathematics educators. Exploring how we, as mathematics educators, can support teacher-researchers to teach better and learn better from their own practices will certainly chart the directions of future research in mathematics teacher education.

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