

Drop Test Impact Analysis—Experimental and Numerical Evaluations



S. I. B. Syed Abdullah

Abstract The susceptibility of composites to impact damage is one of the main drawbacks for the material to be used as a structural component. This is especially true for Low Velocity Impact (LVI) loads, in which the laminate may appear pristine on surface, but considerable damage is present internally. This is often known as the Barely Visible Impact Damage (BVID), in which this type of damage is a hidden menace which is often detrimental to the overall integrity of the structure. Therefore, it is important to fully understand the mechanics of failure to improve the damage tolerance and impact performance of the laminate. In addition, a comprehensive understand of the mechanics of failure would enable a physically based constitutive equations to be derived, thus reducing the number of experimental tests required, leading to a reduction in the total design cost. In this chapter, a review will be made on the LVI testing on composite materials, particularly on the experimental and numerical evaluations of failure and damage under LVI loadings.

Keywords Low Velocity Impact · Finite Element Methods · Damage mechanics · Fracture mechanics · Delamination

1 Introduction

The use of Fibre Reinforced Polymers (FRPs) is rapidly increasing due to its high strength-to-weight ratio. Furthermore, its superior corrosion resistance as well as improved fatigue performance often makes it highly desirable in many industrial applications (Cantwell and Morton 1991). However, the susceptibility of FRPs to impact damage is one of the major downfall for material selection by structural designers. Low Velocity Impact (LVI), usually from large mass impactors with velocities of up to 70 m/s (Cantwell and Morton 1991; Davies and Olsson 2004), could present a significant threat to the structure. This may come in the form of in-plane fibre damage, delamination, or debonding between components, and is usually called Barely Visible Impact Damage (BVID). BVID normally cannot be detected by the

S. I. B. Syed Abdullah (✉)

Department of Aeronautics, Imperial College London, London, UK

naked eye and requires the use of Non-Destructive Testing (NDT) such as Ultrasonic C-Scan to capture the extent of damage (normally delamination).

Information obtained from experimental results are often translated into mathematical models which can then be implemented as numerical models for use in design tools such as the Finite Element Method (FEM). In recent years, the advancement in numerical modelling allows for a more accurate damage prediction, thus allowing engineers to not ‘over-design’ a structure to maintain its integrity under various loading conditions.

In the following sub-sections, a review will be made on the impact response of composite materials under LVI loading, with an emphasis on the experimental and numerical correlation.

2 Low Velocity Impact on Composite Structures

The response of composite laminates from transverse impact loading is known to vary with the speed of impact (Abrate 1998; Davies and Olsson 2004). In Low Velocity Impact (LVI) conditions, boundary effects usually dominate since the impact duration is longer between the laminate and the impactor. Conversely, stress wave effects usually dominate in High Velocity Impact (HVI), since the impact times are usually shorter than LVI. Furthermore, damage patterns observed in the two cases are often different. In LVI, global damage modes may be observed, since large deflections often occur, which depend highly on the shear properties (both in-plane and interlaminar) of the material. Compared to LVI, the type of damage found on HVI laminates are often highly localised at the region of impact (Fig. 1).

Barely Visible Impact Damage (BVID), which occurs under LVI conditions, is often the hidden menace, causing a significant degree of damage in the composite laminate. BVID almost always results in delamination, severely compromising the integrity of the structure. (Davies et al. 1994) proposed a simple fracture mechanics-based model to predict the critical load in which delamination onset will occur in low velocity impact loading. The critical load, also called the Delamination Threshold Load (DTL), P_c , is given by:

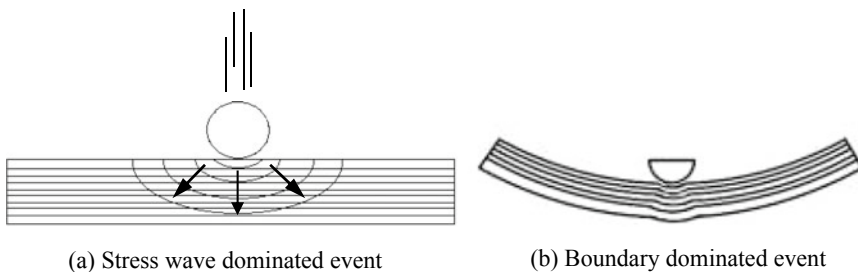


Fig. 1 Response types for impacted composite structures (Cantwell and Morton 1991)

$$P_c^2 = \frac{8\pi^2 E_f h^3}{9(1 - \nu^2)} G_{IIc} \tag{1}$$

where P_c is the critical load to initiate delamination, G_{IIc} is the Mode II critical strain energy, ν is the Poisson’s ratio, E_f is the laminate flexural modulus, and h is the laminate thickness. The high dependence on the shear properties for LVI loading is apparent, resulting in the inclusion of Mode II critical strain energy in Eq. (1). This is not surprising since LVI is a flexural dominated event, hence interlaminar shear properties are central to the impact performance of laminated composites. However, the analytical relation proposed by Davies is only limited to classical composite types, such as Carbon Fibre/Epoxy (CF/Epoxy) and Glass Fibre/Epoxy (GF/Epoxy). In addition, Eq. (1) is also restricted to UD fabric architecture. For advanced fabric architecture, such as woven or Non-Crimp Fabrics (NCFs), Eq. (1) will yield inaccurate results.

Figure 2 (Syed Abdullah 2019) presents the LVI response for three different composite laminates, namely IM7/8552 (CF/Epoxy), S2-Glass/MTM57 (GF/Epoxy), and Vectran/MTM57 (Thermotropic Liquid Crystal Polymer, TLCP/Epoxy) under 40 J of impact energy. Note that S2-Glass/Epoxy and Vectran/Epoxy are based on the NCF architecture, whilst IM7/8552 is based on

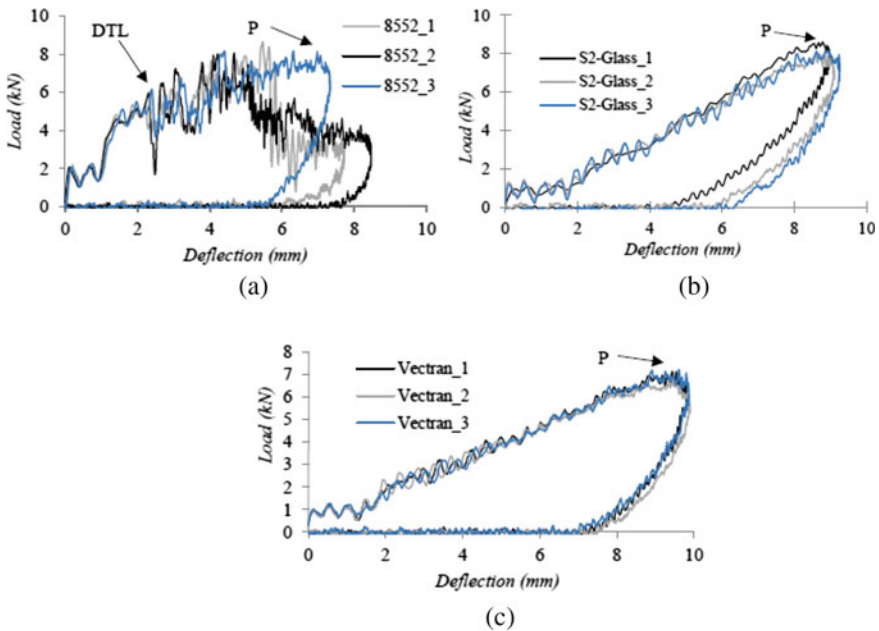


Fig. 2 LVI response for three different composite laminates under 40 J of impact energy **a** IM7/8552, **b** S2-class/MTM57, **c** vectran/MTM57 (Syed Abdullah 2019; Syed Abdullah et al. 2021)

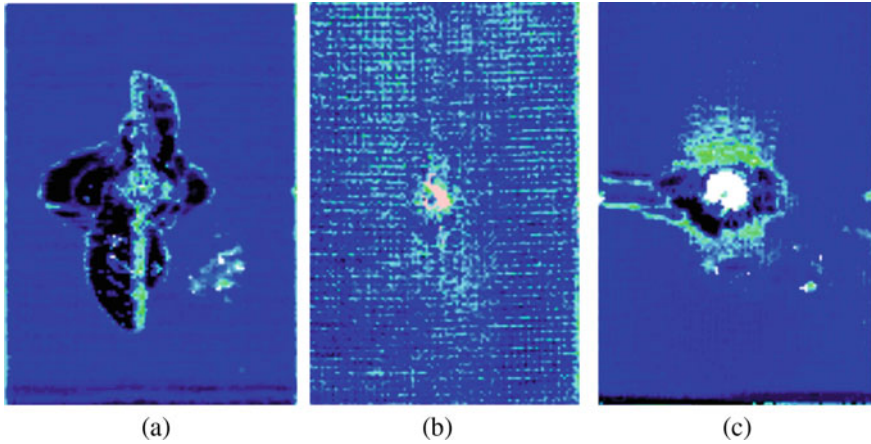


Fig. 3 Ultrasonic C-scan image of damaged composite under 40 J LVI loading. **a** IM7/8552, **b** S2-glass/MTM57, **c** vectran/MTM57 (Syed Abdullah 2019; Syed Abdullah et al. 2021)

the UD architecture. A sharp load drop can be clearly seen in the early stage of impact in the load–deflection response of IM7/8552, Fig. 2a, whilst no observable drop can be found for S2-Glass/MTM57 and Vectran/MTM57, Fig. 2b, c, respectively. This is consistent with the findings obtained from the ultrasonic C-scan images taken from the damaged laminates, Fig. 3. From Fig. 3, it can be clearly seen that considerable delamination was present on the IM7/8552 laminate, Fig. 3a, whilst minimal delamination was observed on the S2-Glass/MTM57 and Vectran/MTM57 laminates—Fig. 3a, b, respectively.

Table 1 presents the calculated DTL using Eq. (1). Additionally, the experimentally measured DTL based on the load–deflection response presented in Fig. 2 is also included in Table 1. While the prediction of the DTL using Eq. (1) was considerably accurate for IM7/8552, this is not the case for Vectran/MTM57 and S2-Glass/MTM57 laminates. This is due to several reasons. First, the model depends on the flexural stiffness of the composite. Whilst the calculated flexural stiffness may be reliable for S2-Glass/MTM57, this may not be the case for Vectran/MTM57. The low compressive properties, both stiffness and strength, suggest that the flexural stiffness is driven by tensile strain. Secondly, the Poisson’s ratio for both laminates was significantly lower than IM7/8552, due to the fabric architecture of S2-Glass/MTM57 and Vectran/MTM57, compared to IM7/8552.

Table 1 Delamination threshold load: measured and calculated (Syed Abdullah 2019)

Material	Measured DTL (kN)	Calculated DTL (kN)
IM7/8552	5.78	6.11
S2-glass/MTM57	N/A	N/A
Vectran/MTM57	N/A	N/A

Other analytical models, such as those based on the spring-mass systems (Davies and Olsson 2004), or indentation models derived from classical Hertzian contact models, are often used to capture the inelastic unloading commonly observed in tougher composites, such as those based on polymer fibres or thermoplastic matrices (Fig. 4).

An example of inelastic unloading is shown in Fig. 2a, b, where it can be observed that the load–deflection curve does not unload back directly to the origin, indicating that the laminates have absorbed a finite amount of energy transferred from the impactor during the impact event. For S2-Glass/Epoxy, damage is present in the form of matrix cracks and fibre kinking (compression failure), whilst for Vectran/MTM57, permanent indentation on the laminate front face is observed, with minimal yarn splitting and delamination on the front face of the laminate.

The analytical models described previously are often used to predict the LVI response of composite laminates. The main advantage of these models is the insight provided by closed form expressions, which directly shows the influence of different parameters. For instance, the influence of G_{IIc} in Eq. (1) is evident due to the flexural dominated response under LVI events. However, the analytical models are almost always limited to a simple geometry, and no or limited ability to model damage

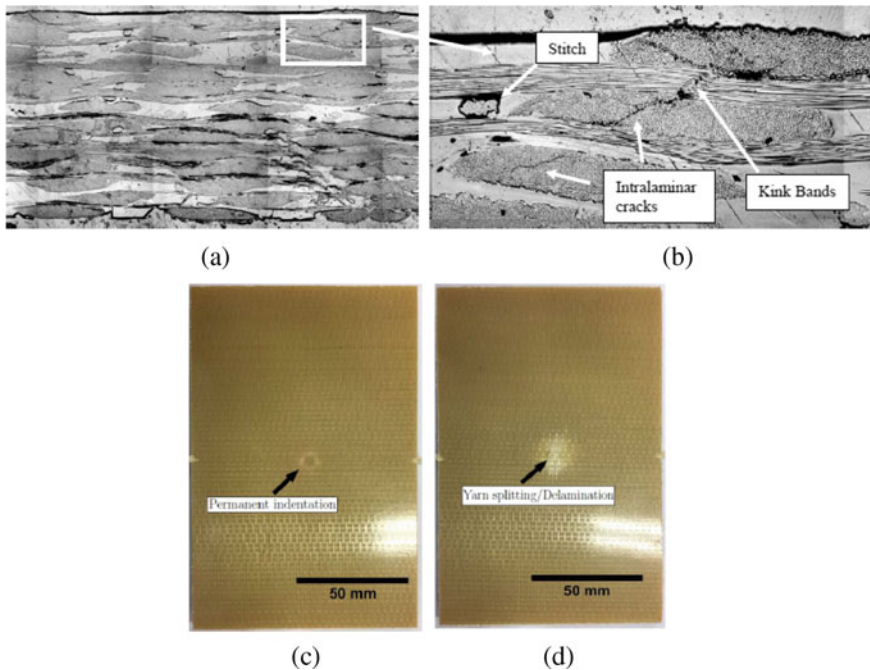


Fig. 4 Damage observed on impacted S2-Glass/MTM57 and Vectran/MTM57 laminates, **a** cross-sectional image around the area of impact, **b** close-up image of the white square in **a**, **c** front face of Vectran/MTM57 damaged laminate **c** back face of Vectran/MTM57 (Syed Abdullah 2019; Syed Abdullah et al. 2021)

growth. Therefore, these models are more suited to predict the impact response up to damage initiation, rather than for simulation of the actual growth. Ultimately, computational mechanics is often selected to model the full impact event, enabling a more accurate damage prediction capability.

3 Numerical Modelling of Composite Structures Under Low Velocity Impact

The use of numerical models for the prediction of mechanical properties or impact response can considerably reduce the time and cost related to composite structural design. Numerical techniques such as the Finite Element Method (FEM), is often utilised to predict damage due to impact loading in a composite material. In recent years, the use of energy-based models has seen an appreciable increase due to their accuracy and reliability in damage prediction. Mathematical models such as the Classical Laminate Theory (CLT) are used in conjunction with energy-based theories such as fracture mechanics to describe laminate failure. For example, the energy required for fibre failure is taken as the energy obtained from standard fracture mechanics tests such as the Compact Tension (CT) or Compact Compression (CC). This energy is commonly known as the strain energy release rate, G_c , and is assumed to be the area enclosed under the stress versus strain curve of the relevant modes. A typical stress–strain–damage relationship for a linear-elastic based composite laminates (such as CF/Epoxy and GF/Epoxy) is shown in Fig. 5.

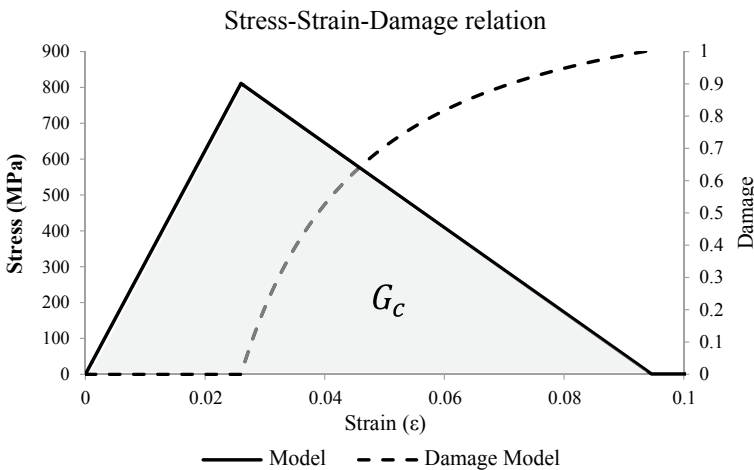


Fig. 5 Typical stress–strain–damage relationship for a linear-elastic composite laminate (Syed Abdullah et al. 2021)

The concept of degradation is essentially part of a more general Continuum Damage Mechanics (CDM) approach, which was first introduced by (Kachanov 1999) and later by (Rabotnov 1969) when attempting to describe the creep behaviour in metals. CDM is an attractive approach since it provides a method in which accurate determination of the material condition can be made—from a pristine condition (no damage) until final failure (full damage). The earlier approach of modelling laminated composites using stress (or strain) based criteria was found to be theoretically inaccurate (Tsai and Wu 1971; Hashin 1980; Chang and Chang 1987), in which the stress is immediately reduced to zero upon reaching its threshold strength. This is a gross over-simplification which neglects the post-failure behaviour of a laminated composite.

The earliest implementation of energy-based damage mechanics approach is proposed by (Ladeveze and LeDantec 1992). In-plane testing of various laminate orientation was simulated using the CDM approach and later compared with the experimental results. Damage onset was taken as the failure strength of relevant modes, and then linearly degraded until zero. Excellent correlation between the experimental and simulation results was obtained. Later, (Matzenmiller et al. 1995) utilise the CDM approach by considering the post-failure behaviour as a function of the Weibull distribution of strength. (Williams and Vaziri 1995) implemented the approach suggested by (Matzenmiller et al. 1995) into LS-Dyna as a plane stress material model. Following this, Iannucci et al. (2001) (Iannucci and Ankersen 2006; Iannucci and Willows 2006, 2007; Iannucci et al. 2009) employed the CDM approach to model thin laminated composites (UD and woven) under LVI and HVI loadings. All numerical prediction including the force–time/displacement histories were in close agreement with the experimental results.

The linear degradation model, originally proposed by (Bažant and Oh 1983), has been widely used due to its ability to reproduce excellent force–time/deflection histories (Davies and Zhang 1995; Davies and Olsson 2004). However, this approach is rather simplified and does not involve any physical reasoning. For this reason, Maimi et al. (2007a, b) proposed a linear-exponential softening law, Fig. 6a, which

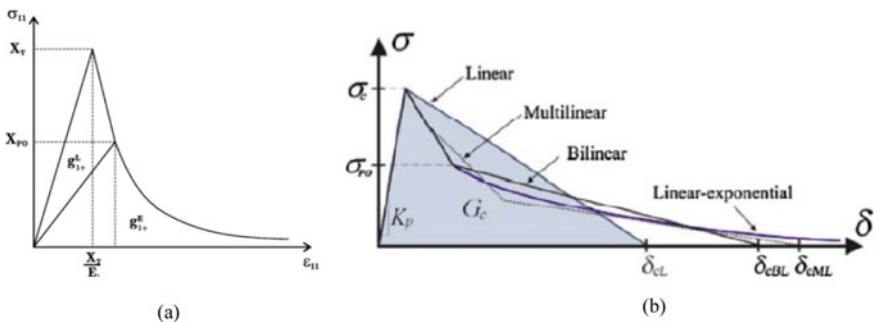


Fig. 6 Alternative softening laws for fibre tension and compression failures, **a** exponential softening law, **b** linear/bi-linear/multi-linear softening laws (Dávila et al. 2009)

was later super-imposed by (Dávila et al. 2009) to a bi-linear softening law involving two linear curves to represent damage propagation—illustrated in Fig. 6b. These softening laws were based on the processes typically involved in fibre failure (i.e. matrix failure, fibre-bridging, fibre pull-out etc.). For instance, at damage onset, stress will be linearly degraded until reaching the fibre pull-out strength, X_{po} , followed by an exponential softening until zero stress.

These alternative softening laws not only provide a sound physical justification for damage degradation but were also reported to yield more accurate results. This was shown by Davila and co-workers (Dávila et al. 2009), when comparing the linear and bi-linear softening laws in a Compact Tension (CT) simulation. It was found that the bi-linear softening law was able to closely predict the shape of the Resistance curve (R-curve) with the experimental results if compared to the linear softening law.

Apart from describing an accurate determination of the material condition, the CDM approach also allows for the alleviation of mesh dependency in numerical analysis. This is because CDM is based on a representative unit volume and therefore the energy related to this unit volume must be made constant regardless of the element dimension. Thus, a length parameter is introduced, typically known as the element characteristic length, l_c , to ensure constant energy dissipation throughout the analysis. This approach is commonly known as the smeared formulation, whereby the fracture energy is smeared over the full volume of the element.

However, the calculation of l_c is not always straightforward and may require a separate algorithm in the material model. Some of the methods proposed by previous researchers include the use of the element shape functions, or purely from a geometrical approach. This was discussed by (Ehrich 2013), which investigated both strategies for the l_c calculation. For the former (using geometrical approach), the nodal coordinates from the element nodal connectivity were accessed and then stored as a history variable, which was then called back to calculate the coordinate global to local coordinate transformation based on the ply angle, θ , given by:

$$x' = \cos(\theta)x + \sin(\theta)y \quad (2)$$

$$y' = -\sin(\theta)x + \cos(\theta)y \quad (3)$$

where (x', y') are the material coordinates, and (x, y) , are the global coordinates. Upon transformation, the characteristic element length, l_c can be calculated using the desired approach (geometry or element shape functions). For the geometrical approach, l_c will be calculated from the element sub-intervals giving the characteristic length for each strip, Fig. 7a, which is then used to calculate the l_c for the entire element.

Conversely, when utilising the element shape function approach, l_c , is calculated using the isoparametric nodal coordinates which is used in the partial differential equation originally proposed by (Oliver 1989) given by:

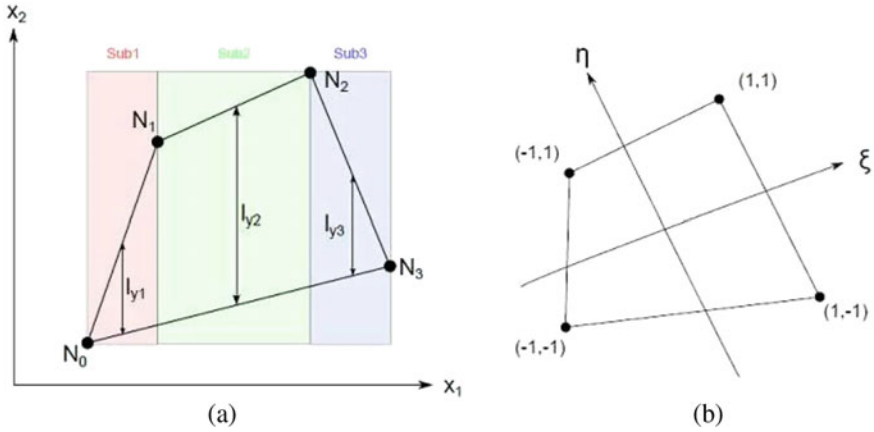


Fig. 7 Calculation of l_c from different approaches, **a** geometrical, **b** element shape function (isoparametric coordinates) (Ehrich 2013)

$$g_f = G_c \frac{\partial v(x', y')}{\partial x'} = \frac{G_c}{l_c} \tag{4}$$

where g_f is the specific fracture energy, and G_c is the critical fracture energy of the material.

Donadon et al. (2008), (Donadon and Iannucci 2006) utilised the smeared formulation to accommodate irregular/non-structured mesh, by utilising the element shape functions approach to calculate l_c for use in the smeared formulation. (Ehrich 2013) has also implemented these modifications into plane stress elements in Abaqus explicit. (Raimondo et al. 2012) proposed an alternative approach in alleviating mesh dependency. The method is based on the maximum crack density, which is suggested to be a material characteristic in a composite laminate. Hence, a crack density parameter was introduced into their degradation model to account for the total energy dissipated in one element.

4 Modelling Damage

In the context of CDM, damage was initiated when the current stress reaches the maximum strength of the material ($d = 0$), and then degraded to zero stress ($d = 1$). Each failure modes (tension, compression, and shear) are typically assigned with a damage variable, hence allowing the current state of the model to be examined. Failure normally initiate as matrix cracks and then followed by delamination, similar to the experimental observation. While the interaction between matrix failure and delamination are sometimes treated separately, a study performed by (Bouvet et al. 2012), attempts to capture this interaction by using discrete intra-ply (matrix)

zero-thickness cohesive elements (or cohesive surfaces). These elements are based on non-linear springs and follow the well-known bi-linear traction–separation law, similar to the one proposed by (Bažant and Oh 1983). The authors reported a remarkable correlation between the experimental results, in spite of a couple of drawbacks. First, the approach is computationally costly due to the number of cohesive algorithms utilised in the model. Secondly, and most importantly, the authors reported a significant mesh sensitivity in the model. Perhaps it is worth noting that the model is restricted to UD materials due to the inherent nature of the approach. In a follow-up study, (Bouvet et al. 2013) utilised the previous model to simulate the Compression After Impact (CAI) response of CF/Epoxy laminates. Similar to the previous study, both the impact and the CAI response were in excellent agreement with the experimental observations. Recently, Lopes and co-workers (Lopes et al. 2016) utilised the approach proposed by (Bouvet et al. 2012) in modelling intra-ply failure. For the in-plane response, the authors employed the constitutive relations proposed by Maimi et al. (2007a, b). Although the approach is computationally costly, accurate prediction of damage types, in particular matrix cracking and delamination were achieved.

5 Conclusion

The susceptibility of composite materials to impact induced loads remains one of its greatest drawbacks. The ability of composite materials to absorb impact energies are often inferior to that of metallic materials such as steel and aluminium. One of the main reasons for this weakness is due to the large energy absorbing potential by way of plasticity of metallic materials, when compared to polymer composites. For instance, (Karthikeyan et al. 2013) investigated the ballistic performance of cured IM7/8552 (CF/Epoxy) with 304 stainless steel plates. It was concluded that the uncured 304 stainless steel plates showed significant inelastic deformation before penetration, resulting in superior ballistic performance. In contrast, the brittle nature of IM7/8552 restricts its energy absorbing potential to the elastic regime, limiting in a poor impact performance compared with the metallic system. Thus, it is essential that the performance of composite materials under impact induced loads is improved.

Since the last few decades, experimental campaigns have provided a clear insight into the mechanics of failure of composite structures under impact loads. This information enables constitutive relations to be derived, which is then used to predict failure of the composites under impact loads. In addition, the advancement of analytical and numerical techniques in recent years have greatly helped in reducing the costs associated with structural design using composite structures. The use of polymer fibre-based composites such as Vectran and Dyneema have seen a considerable increase owing to its superior performance under impact loads. However, many aspects of these composites remain unclear and not fully understood. Thus, the need to fully understand the mechanics of failure is essential for a number of reasons. First, with a comprehensive understanding, engineers could exploit the full potential of the

composite, without the need to ‘over-design’ to ensure that the composite structure meets all safety requirements. This would result in a more cost-effective design, with minimal material consumption and fewer man-hours. Secondly, an accurate prediction of failure can be made, leading to a more reliable design since the mechanics of the composite are fully understood. Thirdly, physically based constitutive equations can be derived, which can be implemented as a material model for Finite Element Method (FEM) modelling. The material model can then be used to reduce the number of experimental tests required, leading to a reduction in the total design cost.

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