

# Modified Particle Swarm Optimization (MPSO)-Based Short-Term Hydro–Thermal–Wind Generation Scheduling Considering Uncertainty of Wind Energy



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**Abstract** Integration of renewable in hydro–thermal scheduling considering economic and environmental factors forms a multi-objective nonlinear optimization problem involving many equality and inequality constraints. Main objective of this problem is to minimize emission as well as generation cost on short-term basis maintaining all system constraints. In this research, a framework for hydro–thermal–wind generation scheduling (HTWGS) has been proposed using a modified particle swarm optimization (MPSO) algorithm. Results showed that this algorithm provides better result while various complex constraints were considered in the HTWGS problem.

**Keywords** Hydro–thermal–wind scheduling · Modified particle swarm optimization

## Nomenclature

$i, j, k$	Index of thermal, hydro, wind power unit, respectively
$C_T, F_T, W_T$	Total cost, fuel cost, and wind cost, respectively
$N_t, N_h, N_w$	Total number of thermal, hydro, wind units, respectively
$\tau, T$	Time sub-interval and scheduling period, respectively
Up	Index of upstream reservoir
$Q_{hj,\tau}, I_{hj,\tau}$	Discharge and inflow rate of $j^{\text{th}}$ hydro unit $\tau$ , respectively
$P_{ti,\tau}, P_{hj,\tau}, P_{wk,\tau}$	Thermal, hydro and wind of $i^{\text{th}}, j^{\text{th}}$ and $k^{\text{th}}$ at $\tau$ , respectively
$V_{hj,\tau}, S_{hj,\tau}$	Reservoir volume and spillage of $j^{\text{th}}$ hydro unit $\tau$ , respectively
$P_{d,\tau}, P_{L,\tau}$	Total demand and transmission loss at $\tau$
$OE_{wk,\tau}, UE_{wk,\tau}$	Over and under estimation cost of $k^{\text{th}}$ wind at $\tau$ , respectively
$\alpha_i, \beta_i, \gamma_i, \delta_i, \varepsilon_i$	Emission coefficient of $i^{\text{th}}$ thermal unit

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$a_i, b_i, c_i, d_i$	Fuel cost coefficient of $i$ th thermal unit
$e_i, h_i$	Coefficient of the valve point effect of $i$ th thermal unit
$C_{(1-6)j}$	Hydro power output coefficient of $j$ th hydro unit
$P_{ti}^{\min}, P_{ti}^{\max}$	Minimum and maximum power limit of $i$ th thermal unit
$P_{hj}^{\min}, P_{hj}^{\max}$	Minimum and maximum power limit of $j$ th hydro unit
$Q_{hj}^{\min}, Q_{hj}^{\max}$	Minimum and maximum discharge limit of $j$ th hydro reservoir
$V_{hj}^{\min}, V_{hj}^{\max}$	Minimum and maximum volume limit of $j$ th hydro reservoir
$V_{hj}^{\min}, V_{hj}^{\max}$	Minimum and maximum volume limit of $j$ th hydro reservoir
$V_{hj}^{\text{begin}}, V_{hj}^{\text{end}}$	Initial and final storage volume of $j$ th hydro reservoir.

## 1 Introduction

In recent times, global warming has become a matter of great concern due to increase in power demand involving more pollution. To address this problem, an optimum operation of a thermal-renewable energy mixture is a promising option.

Inclusion of solar and wind energy into the energy sector has been proved as more cost-effective, necessitating its enclosure in the scheduling progression.

Genetic algorithm (GA) gives satisfactory results in various areas such as optimal solution of scheduling problem [1–3], hydro generator governor tuning [4], and economic dispatch [5]. However, in the literature, different classes of empirical algorithms such as genetic algorithm (GA) approach based on differential evolution (DV) [6, 7], particle swarm optimization (PSO) [8], modified dynamic neighborhood learning-based particle swarm optimization (PSO) [9], simulated annealing (SA) [10], evolutionary programming (EP) [11], modified differential evaluation (MDE) [12], and some other population-based optimization techniques have proved their effectiveness particularly in solving short-term hydro–thermal scheduling (STHTS) problems. Recently, random optimization methodologies and many other empirical algorithms based on natural phenomenon like adaptive chaotic artificial bee colony (ACABC) algorithm, artificial immune system (AIS) have given better result in solving STHTS problem. Recently, many other empirical algorithms inspired by natural phenomenon and random optimization methodology [13, 14] have been applied successfully in ST-HTWS problems.

Derived from the relationship among uncertainty budget of renewable energy, number of intermittent power supplies and upper bound of constraints-violating probability of spinning reserve capacity, the uncertain-budget decision is guided, and the blindness of decision can be reduced. The computational steps of MPSO, the contradiction between optimization depth and velocity generally existed in swarm intelligence evolutionary algorithms. The proposed technique in the test systems and its simulation results are discussed and summarized in the conclusions in this paper.

## 2 Mathematical Formulation of Generation Scheduling

This article demonstrates the scheduling formulation of a hydro–thermal–wind generation scheduling problem considering various economics and environmental factors. Due to its impulsive nature, renewable resources make the generation scheduling problem more challenging.

### 2.1 Formulation of Multi-objective Function

Cost involved in a hydro system is independent of its output, and hence, in the proposed HTWS scheduling, overall generation cost involves coal cost involved in thermal plant along with miscellaneous cost involved in solar and wind power.

The optimization involved in this problem is minimization of the generation cost of thermal, wind, and solar power plants along with maintaining minimum emission by considering different constraints involved in the proposed scheduling.

To achieve this, a nonlinear multi-objective function can be mathematically formulated as follows:

$$\text{Min} \dots C_T(F_T, E_T, W_T) \quad (1)$$

$$\text{Min} \dots C_T = \sum_{\tau=1}^T \left( \sum_{i=1}^{N_t} (P_{ii,\tau} + E_{i\tau}) + \sum_{k=1}^{N_w} (P_{wk,\tau} C_{wk} + OEC_{wk,\tau} + UEC_{wk,\tau}) \right) \quad (2)$$

The hydro units power output is expressed as a function of reservoir volume and head given by

$$P_{hj,\tau} = (c_{1j} V_{hj,\tau}^2 + c_{2j} Q_{hj,\tau}^2 + c_{3j} V_{hj,\tau} Q_{hj,\tau} + c_{4j} V_{hj,\tau} + c_{5j} Q_{hj,\tau} + c_{6j}) \quad (3)$$

Thus, the multi-objective function (1) can be modified as

$$\text{Minimize } C_T(F_T + h_i \times E_T + W_T) \quad (4)$$

Fuel cost of the thermal power plant can be expressed mathematically as a quadratic function of the real power output including valve point effects [2]. This can be mathematically formulated as follows:

$$F_T = \sum_{\tau=1}^T \left( \sum_{i=1}^{N_t} [a_i P_{ii,\tau}^2 + b_i P_{ii,\tau} + c_i + |e_i \sin(f_i (P_{ii}^{\min} - P_{ii,\tau}))|] \right) \quad (5)$$

Emission from thermal power plant depends on its output by the penalty factor  $h_i$ . Overall emission of pollutant  $E_T$  can be expressed mathematically as

$$E_T = \sum_{\tau=1}^T \left( \sum_{i=1}^{N_i} [\alpha_i P_{ii,\tau}^2 + \beta_i P_{ii,\tau} + \gamma_i + \varepsilon_i \exp(\delta_i P_{ii,\tau})] \right) \text{ lb/h} \quad (6)$$

Wind velocity is the deterministic factor for wind power generation. Total operating cost for a wind extraction unit consists of three components: (a) direct cost, (b) underestimation cost, and (c) overestimation cost [46]. The concerned cost function can be formulated mathematically as

$$W_T = \sum_{\tau=1}^T \left( \sum_{k=1}^{N_w} C_{wk} P_{wk,\tau} + \text{OEC}_{wk,\tau} + \text{UEC}_{wk,\tau} \right) \quad (7)$$

## 2.2 Constraints

Constraints related to the proposed HTWS problem mainly are generator capacity (operating limits), storage volume of the reservoir, discharge limit, power balance, and water balance constraints.

Dynamic water balance equation of the reservoir can be written as

$$V_{hj,\tau} = V_{hj,\tau-1} + I_{hj,\tau} - Q_{hj,\tau} - S_{hj,\tau} + \sum_m^{R_{uj}} Q_{hm}(\tau-t_{mj}) + S_{hm}(\tau-t_{mj}) \quad (8)$$

Initial and final reservoir storage volume is expressed as

$$V_{hj,0} = V_{hj,begin} \quad (9)$$

$$V_{hj,T} = V_{hj,end} \quad (10)$$

Thermal power unit generation limit is given as

$$P_{ii}^{\min} \leq P_{ii} \leq P_{ii}^{\max} \quad (i = 1, 2, 3 \dots N_i) \quad (11)$$

Hydro power unit generation limit is given as

$$P_{hj}^{\min} \leq P_{hj} \leq P_{hj}^{\max} \quad (j = 1, 2, 3 \dots N_j) \quad (12)$$

Wind power unit generation limit is given as

$$0 \leq P_{wk} \leq P_{wk}^{\text{rated}} \quad (k = 1, 2, \dots N_w) \quad (13)$$

Reservoir storage volume limit is given below

$$V_{hj,\tau}^{\min} \leq V_{hj,\tau} \leq V_{hj,\tau}^{\max} \quad (14)$$

Reservoir storage discharge limit is given below

$$Q_{hj,\tau}^{\min} \leq Q_{hj,\tau} \leq Q_{hj,\tau}^{\max} \quad (15)$$

Power system power balance constraint is given as

$$\sum_{i=1}^{N_t} P_{ti,\tau} + \sum_{j=1}^{N_h} P_{hj,\tau} + \sum_{k=1}^{N_w} P_{wk,\tau} = P_{D,\tau} + P_{L,\tau} \quad (16)$$

### 3 Modified Particle Swarm Optimization (MPSO) Algorithm

Conventional PSO maintains a random search considering random values in velocity equation for each particle. In such a case, calculation of velocity for each particle assigns different random values.

Whereas in modified particle swarm optimization (MPSO) algorithm, a unique random value is fixed to enhance individual searching (pbest) for the population in one iteration. Similarly, each particle is assigned with different random values during global search (gbest) of velocity equation. MPSO shows improved result for individual searching, thereby providing more optimal solutions.

According to MPSO, velocity update equation can be written as

$$v_k^{(r+1)} = C_f \left[ wt v_i^{(r)} + c_1 rand^{(r)} (pbest_k - x_k^{(r)}) + c_2 rand_k^{(r)} (gbest_k - x_k^{(r)}) \right] \quad (17)$$

The computational steps of the MPSO methods are as follows:

Step 1: The algorithm starts with initialization of the particles. The initial velocity is generated for all the particles.

Step 2: Compute penalty factor for all thermal power plants.

Step 3: Calculate the hydro power plant's output, and apply the respective power inequality constraints.

Step 4: Compute the fuel cost and emission of thermal power plants.

Step 5: Compute the wind power generation cost.

Step 6: Calculate the fitness of the particles, considering all costs and equality constraints. Set the present value of each particle as its best position, *pbest*.

Step 7: Check for the lowest value of particle best position. Set the value as *gbest*.

Step 8: Calculate the updated velocity of each individual by Eq. (17)

Step 9: Update each individual position.

Step 10: Calculate the new fitness value for each particle. Replace the old *pbest* value with new one, if the present value shows improvement over the previous value.

Step 11: Replace the *gbest* with the lowest value from the new *pbest*, if the present value shows improvement over the previous value.

Step 12: Repeat step 8–11 until the equality constraints fall within a specified tolerance limits or maximum number of iterations reached.

Particle giving latest *gbest* value provides optimum schedule of generation.

## 4 Simulation and Test Results

In the present study, objective function is treated along with the penalty factor. In this analysis, maximum penalty factor approach is used as it offers an acceptable solution for the problem of emission and fuel cost.

Main problem involved in applying any heuristic algorithm is the parameter setting. Range selection for such parameters is considered by considering the concerned values in the literature, and then, a fine-tuning is carried out by a trial-and-error method.

Proposed test system for the present research involves four hydro plants, three thermal plants, and two wind plants. The schematic diagram is shown in Fig. 1.

The hourly water discharge from hydro plant is shown in Fig. 2. Hourly water discharge and reservoir storage volume are tabulated in Table 1. The storage volume limitation was addressed by adjusting the water discharge from each reservoir.

Optimal demand allocation for hydro–thermal–wind system and corresponding economic and emission values from the simulation are tabulated in the following tables. Optimal hydro–thermal–wind generation scheduling for the test system is depicted in Table 2. This analysis considers various economic and emission factors obtained from MPSO method.

The simulations were carried out in MATLAB 2019a platform for 50 iterations, and results were analyzed based on the best, average, and worst case with standard deviation. It is imperative to note that MPSO provides competent and effective solution from quality and consistency point of view.

Optimal load allocation among thermal, wind, and hydro system on daily basis is shown in Fig. 3. The comparison regarding total fuel cost is shown in Table 3. It is evident that MPSO provides a better the optimal generation schedule is shown in Fig. 4.

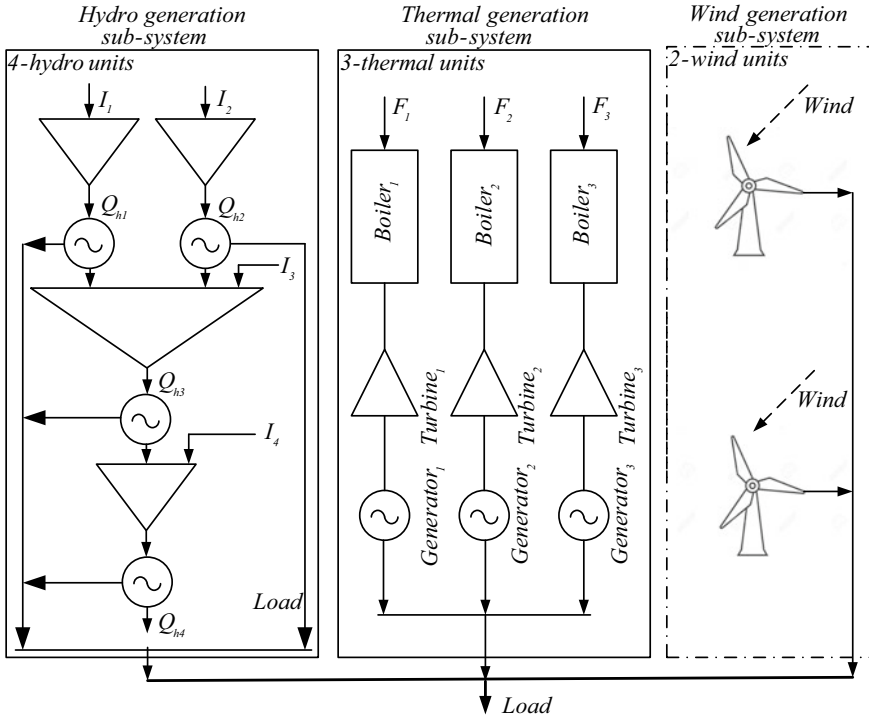


Fig. 1 Schematic diagram of the hydro-thermal-wind (HTW) test system

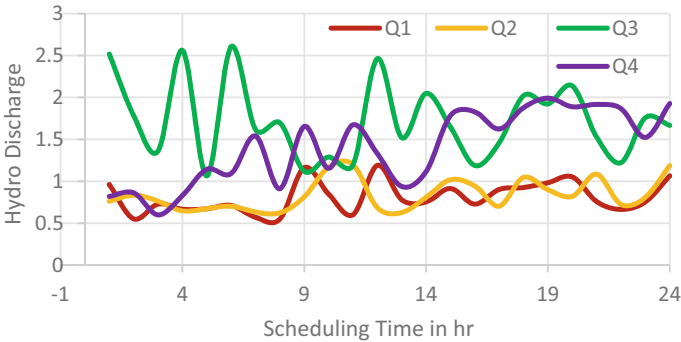


Fig. 2 Hydro plant water discharge curve of test system

### 5 Conclusion

Present study investigates the effectiveness of certain empirical algorithm belonging to different empirical groups for a solution of optimal generation of an HTWGS

**Table 1** Hourly water discharge and reservoir storage volume obtained using the MPSO algorithm of test system

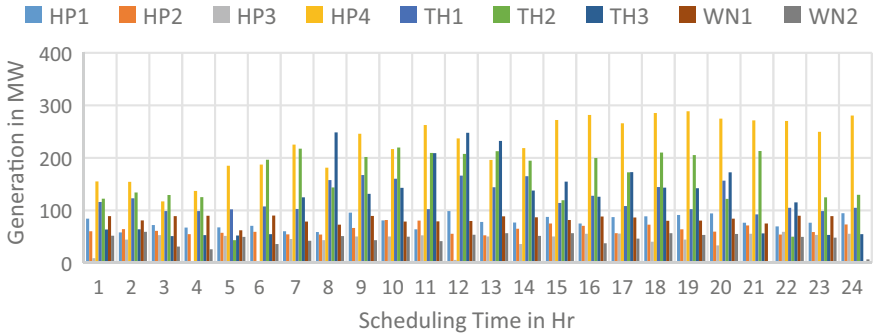
Hour	Water Discharge ( $\times 10^5$ m <sup>3</sup> /h)				Reservoir storage volume ( $\times 10^6$ m <sup>3</sup> )			
	$Q_1$	$Q_2$	$Q_3$	$Q_4$	$V_1$	$V_2$	$V_3$	$V_4$
1	0.9617	0.7633	2.5171	0.8202	1.0038	0.8037	1.5293	1.1460
2	0.5524	0.8347	1.7838	0.8641	1.0386	0.8002	1.4329	1.0836
3	0.7294	0.7643	1.3603	0.6003	1.0456	0.8138	1.4331	1.0395
4	0.6663	0.6470	2.5616	0.8300	1.0490	0.8391	1.3285	0.9565
5	0.6735	0.6756	1.0645	1.1442	1.0417	0.8515	1.4084	1.0938
6	0.7127	0.7036	2.6060	1.0924	1.0404	0.8512	1.3309	1.1630
7	0.5718	0.6349	1.6176	1.5409	1.0632	0.8477	1.3312	1.1449
8	0.5503	0.6239	1.6986	0.9114	1.0982	0.8553	1.3201	1.3099
9	1.1621	0.8146	1.1191	1.6544	1.0820	0.8538	1.3458	1.2509
10	0.8480	1.1697	1.2908	1.1552	1.1072	0.8269	1.3452	1.3960
11	0.6015	1.1994	1.1935	1.6738	1.1670	0.7969	1.4145	1.3904
12	1.1905	0.6861	2.4622	1.3302	1.1480	0.8083	1.3545	1.4272
13	0.7805	0.6280	1.5206	0.9385	1.1799	0.8255	1.4195	1.4453
14	0.7549	0.8099	2.0484	1.1162	1.2244	0.8345	1.4837	1.4628
15	0.9149	1.0200	1.6477	1.7839	1.2430	0.8225	1.4956	1.4037
16	0.7282	0.9450	1.1889	1.8279	1.2701	0.8080	1.5350	1.4672
17	0.9037	0.7044	1.4592	1.6237	1.2698	0.8076	1.5815	1.4569
18	0.9265	1.0479	2.0216	1.8794	1.2571	0.7628	1.5742	1.4738
19	0.9830	0.8998	1.9232	1.9921	1.2288	0.7428	1.5767	1.4393
20	1.0534	0.8214	2.1381	1.8888	1.1835	0.7407	1.5360	1.3693
21	0.7585	1.0850	1.5261	1.9170	1.1776	0.7222	1.6065	1.3390
22	0.6654	0.7240	1.2229	1.8673	1.1911	0.7398	1.6996	1.1337
23	0.7561	0.8097	1.7591	1.5235	1.2055	0.7388	1.6916	1.3789
24	1.0655	1.1878	1.6668	1.9274	1.2000	0.7000	1.7000	1.4000

system considering various environmental and economic factors. Modified particle swarm optimization (MPSO) method is proposed in this purpose. In the present analysis, maximum penalty factor approach was used as it converts the multi-objective economic and emission function into a single objective one. Simulations also verified that MPSO demonstrated a better performance than the other selected algorithm in terms of solution quality as well as consistency. The proposed method is very effective as it takes less time due to less computational steps involved in the analysis. Besides, the method is easy to implement which makes the algorithm suitable for addressing large-scale hydro–thermal–wind optimal scheduling problem.



**Table 2** Optimal generation schedule of the hydro–thermal–wind (HTW) system obtained using the MPSO algorithm for test system

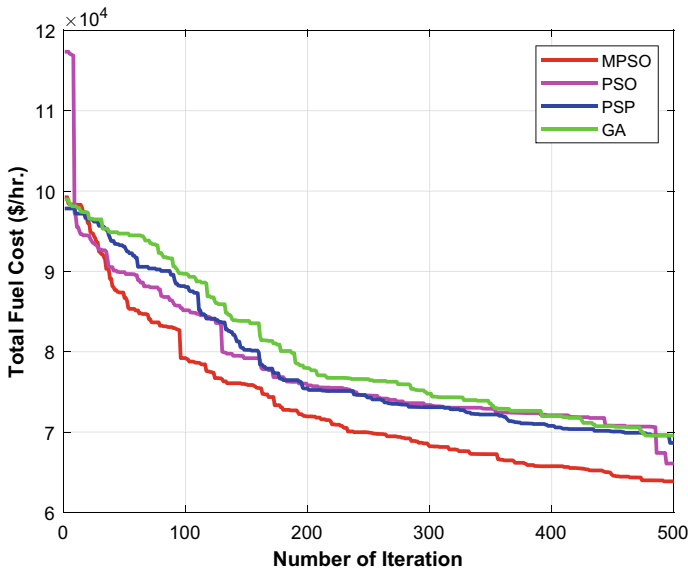
Hr.	Hydro Gen. (MW)				Thermal Gen. (MW)			Wind Gen (MW)	
	HP <sub>1</sub>	HP <sub>2</sub>	HP <sub>3</sub>	HP <sub>4</sub>	TP <sub>1</sub>	TP <sub>2</sub>	TP <sub>3</sub>	WP <sub>1</sub>	WP <sub>2</sub>
1	84.311	60.018	8.9457	154.974	115.951	122.030	63.519	88.592	51.655
2	57.805	64.023	44.393	153.923	122.783	133.843	63.809	80.651	58.766
3	71.660	60.692	52.555	116.998	98.6265	128.984	50.935	88.748	30.799
4	67.146	54.541	0	136.567	98.6379	124.735	52.786	89.523	26.060
5	67.519	57.190	51.267	185.015	101.860	43.3162	52.042	62.125	49.663
6	70.355	59.023	0	187.139	107.191	196.366	54.469	89.761	35.691
7	59.899	54.187	45.476	225.293	102.717	217.345	124.70	78.330	42.039
8	58.628	53.839	42.981	181.227	157.493	143.593	248.55	72.739	50.946
9	95.421	66.088	50.103	245.835	167.217	201.579	131.43	89.024	43.296
10	80.923	81.495	50.351	216.458	159.862	219.653	142.90	78.652	49.699
11	63.842	80.568	52.327	262.537	102.230	209.194	209.06	79.162	41.068
12	98.491	55.386	5.3777	237.051	165.893	207.301	247.42	79.808	53.263
13	77.808	52.457	50.31	196.169	143.989	212.412	232.16	88.277	56.400
14	76.514	64.680	35.712	218.178	164.828	194.381	137.81	86.664	51.221
15	87.428	74.975	50.145	271.726	114.252	118.971	154.33	81.547	56.618
16	74.852	70.434	55.259	281.431	127.552	199.547	126.04	88.001	36.871
17	87.055	56.541	55.651	265.569	108.361	172.001	172.64	86.176	46.001
18	88.304	72.282	40.066	285.466	144.315	209.921	143.16	80.032	56.446
19	91.127	63.828	44.210	288.401	102.133	205.063	141.78	80.490	52.951
20	93.823	59.409	33.163	274.599	156.528	121.667	172.29	83.895	54.620
21	76.225	70.978	55.453	271.000	92.1141	212.701	56.250	74.990	0.2848
22	69.334	53.529	58.751	269.861	104.717	49.8435	114.99	89.471	49.491
23	76.402	58.617	52.832	249.585	98.5397	124.364	52.987	88.921	47.749
24	94.237	73.188	55.256	280.329	104.616	129.222	54.510	2.3243	6.3150



**Fig. 3** Optimal power generation schedules from the MPSO algorithm over 24 h. time span of test system

**Table 3** Statistical analysis of the heuristic algorithms in terms of total fuel cost

Method	Fuel cost (\$/h)			
	Best	Average	Worst	Std. Dev.
MPSO	66083.66	66086.74	66089.37	1.6586
PSO	68646.80	68649.49	68652.15	1.6634
GA	71016.97	71021.02	71025.93	2.8973



**Fig. 4** Convergence characteristics of MPSO, PSO, PSP and GA algorithms in terms of total fuel cost of test system

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