New Frontiers in Regional Science: Asian Perspectives 43

Kenichiro Ikeshita
Daisuke Ikazaki *Editors*

Globalization, Population, and Regional Growth in the Knowledge-Based Economy

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Chapter 1
The Issues of Regional Development

Kenichiro Ikeshita

Abstract This chapter provides an overview of the topics covered in this book. The content of the book is divided into three parts: (1) analysis of globalization and institutions, (2) macroeconomic analysis of family, and (3) dynamic analysis of education and inequality. In this chapter, the content and contributions of each chapter are briefly described. All of these topics are important when we consider the development of regional economies, and this book provides a recent analysis of these topics.

Keywords Regional development · Globalization · Institution · Macroeconomics of family · Knowledge-based economy · Education · Inequality

1.1 Globalization and Institutions

The rapid process of globalization is one of the most important characteristics of the post-WWII economy. Globalization has facilitated trade in goods and services, investment, and the movement of ideas across borders. As a result of globalization, many countries have achieved rapid economic growth. The Asian region, in particular, has been an area that has benefited most from free trade and achieved economic development.

On the other hand, Asian economies currently face many challenges. The most serious problem is the stagnation of globalization. In the short term, the pandemic caused by the novel coronavirus has disrupted global value chains, greatly reducing trade and restricting the movement of people around the world. Japan is facing a major recession, and other emerging economies in Asia are also expected to experience significant declines in their economic growth rates.

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Moreover, since the global financial crisis of 2008–2009, many people have opposed globalization because it has widened the income gap between rich and poor people. Especially in developed countries, many argue that globalization has deprived low-skilled workers of jobs and widened the income gap. In addition, the growing dysfunction of the World Trade Organization (WTO) in recent years has been another factor that has hindered the progress of globalization.

It is important to examine the various impacts of globalization on the economy. Whether globalization brings about economic growth has been an important question in economics. However, while there exist various models and empirical results in the literature, no clear conclusion has yet been reached. Chapter 2 of this book delves into this issue. In this chapter, the author provides an important framework for analyzing the effects of globalization on economic growth. He specifically analyzes the relationship between foreign direct investment (FDI) and economic growth, taking into account the heterogeneity of firms. The author shows that liberalization of FDI is more conducive to economic growth than trade liberalization. This consequence is important not only in terms of trade policy but also in terms of growth policy and may advance future research.

In an increasingly globalized world, it is important to set up an appropriate research and development (R&D) system. This is because ideas produced as a result of R&D are nonrival, and therefore, the profitability of R&D increases as the market size increases. The most important policy issue for R&D systems is the design of an effective intellectual property rights (IPR) protection system. Designing an appropriate IPR system is important for promoting economic growth. However, since IPR protection systems legally protect returns on research efforts, industries and firms heavily dependent on R&D may engage in political activities, such as lobbying and political contributions, to achieve stronger IPR policies. In 2015, the healthcare industry, which includes pharmaceutical and medical companies, spent the largest amount ever spent on lobbying in the United States. Does corporate political activity promote or hinder economic growth? Chapter 3 attempts to answer this question using an endogenous growth model. In this chapter, the author shows that when governments are less transparent and corrupt, firms make political contributions to the government in order to increase the profitability of R&D. At the same time, the government may also strengthen IPR protection in order to obtain political contributions. This result applies the political economy of IPR protection policy to a growth model and presents an important perspective on growth theory.

While globalization is a global trend, there is a great diversity of institutions among nations. The question of which institutions promote economic growth is a major issue in development economics. Chapter 4 considers this question from both theoretical and empirical perspectives. In Chap. 4, the author focuses on the "resource curse." The resource curse is a phenomenon in which countries with a richer supply of natural resources suffer from worsening poverty and slower economic development. This phenomenon has been observed in resource-rich African countries. The author examines the impact of resource endowment on economic development by incorporating an institutional approach to the "Big push model." The results show that the resource curse occurs in resource-rich countries when the quality of institutions is low. Furthermore, using provincial data on the Chinese economy, the author reveals that China also faces the resource curse.

1.2 Macroeconomics of Family

In recent years, economists have focused on the macroeconomics of family and household decision-making. The emergence of this field of economic research is not surprising because microfoundations have become an essential part of macroeconomic research.

The most important economic issue related to the macroeconomics of family is the decline in the fertility rate. Recently, not only developed countries but also many Asian countries are facing a decline in fertility rates, and this problem is expected to lead to major structural changes in their economies.

While many economists have tried to explain the causes of this decline in fertility rates, the problem of childless households has received little attention. Chapter 5 addresses this issue using an overlapping generations model. Specifically, by introducing preference heterogeneity into the model, the author analyzes a situation in which some households become childless. The author shows that a rise in the childlessness rate decreases the total fertility rate in the economy and tends to have negative effects on the utility of all households in the steady state. Later in the chapter, the author shows that child allowances benefit not only households with children but also households without children. These consequences are important because they reveal the comprehensive social benefits of child allowances.

Labor supply is also an important economic activity for households. In particular, governments in developed countries begin to consider labor policies that take into account work-life balance. The purpose of work-life balance policies is to increase people's life satisfaction by improving the balance between work and private life. However, in order to determine whether work-life balance policies are comprehensively desirable, it is necessary to examine them using economic analysis. Chapter 6 presents the theoretical foundations of work-life balance and the results of the empirical analysis. In particular, the author successfully explains the mechanism by which working hours decrease with economic development based on a standard neoclassical growth model. Chapter 6 also examines the relationship between work-life balance indicators and life satisfaction using data from the Better Life Index published by the Organization for Economic Co-operation and Development (OECD). The results do not support the hypothesis that the work-life balance indicator affects the life satisfaction of either men or women. In addition, analysis of the regional characteristics of the 34 countries comprising the OECD found no indication of regional differences affecting the life satisfaction of either men or women. The author obtained these empirical results by analyzing the relationship between work-life balance and life satisfaction through a rigorous econometric procedure. This study is one of the pioneering studies in this field.

1.3 Knowledge-Based Economy, Education, and Inequality

With the spread of the novel coronavirus, the digitalization of society is expected to proceed more rapidly. As a result, a knowledge-based society will be formed in the near future. A knowledge-based society is one in which economic activities are based not on the production and transaction of goods and services or investment in physical capital, but on the generation and use of data and the invention and sharing of ideas. The most important aspect for the realization of a knowledge-based society is the development of workers that possess not only advanced knowledge but also communication skills and creativity. In the field of economics, there has been a tremendous amount of research on the relationship between investment in education and economic development. This book presents several important perspectives on educational investment and economic development. In particular, Chaps. 7–9 all deal with the issue of education and economic development using overlapping generations models.

One important issue in the economics of education is effective educational policies. Chapter 7 addresses this question. In this chapter, the authors use an overlapping generations model to analyze the economic impact of a policy on how many children should be in the classroom (class-size policy). The analysis describes various life patterns and shows that the economy may have two steady-state equilibria with different income distributions. In addition, if the steady state corresponds to a smaller income difference, a small class-size policy has the effect of reducing income inequality. This study reveals the conditions under which a small class-size policy is effective in economic development. This makes a significant contribution to the research on education and economic development.

Chapter 8 applies the overlapping generations game to the issue of educational investment. It analyzes the relationship between household educational choices and poverty. With respect to the decision to invest in education, parental incentives to invest in education are important. Especially in developing countries, parents may be unable to trust that their children will be able to support themselves in the future even if they send their children to school. Thus, parents may have an incentive to reduce educational investments in their children. The author analyzes the problem of intergenerational conflict among family members by applying a game theory framework to the overlapping generations model. The results show that investment in education is not likely to be an equilibrium in this game. This consequence is important because it reveals that it is extremely difficult for households to escape the poverty trap.

Finally, Chap. 9 discusses the relationship between education and technological change. Specifically, the author focuses on skill-biased technological change and educational costs. Chapter 9 incorporates these factors into an overlapping generations model to analyze the impact of technological progress on income inequality. This research finds that the economy converges monotonically to a steady state if the educational costs do not increase sharply with technological progress. In this case, the model is able to explain the "skill-premium puzzle" under the condition that human capital makes a significant contribution to technological progress. In contrast, if educational costs increase sharply with technological progress, economic growth, mobility, and inequality will exhibit cyclical behaviors. In addition, if human capital has a significant impact on technological progress, a temporary increase in exogenous technological progress can reduce aggregate growth not only in the transition process but also in the steady state. Chapter 9 successfully integrates skill-biased technological change and intergenerational income mobility. This makes a great contribution to the study of the dynamics of intergenerational mobility.

Part I
Globalization and Institution

Chapter 2
Growth and International Knowledge Spillovers with Firm Heterogeneity

Hideaki Uchida

Abstract Using theoretical analysis, this study explores the effects of trade and foreign direct investment (FDI) liberalization on productivity growth in a model with heterogeneous firms. Baldwin and Robert-Nicoud (2008, BRN hereafter) show that freer trade has an ambiguous impact on growth rate in a heterogeneous firms model. Their main findings are in contrast to the notions shared by most literature on the endogenous growth of homogeneous firms, which says that trade liberalization promotes economic growth. Although BRN mainly focus on the diffusion of international knowledge by trade, we can see that some countries benefit not only from international trade but also from FDI by foreign firms in the process of development. That is, FDI liberalization can influence knowledge spillovers among local companies and promote the development of the host country. In contrast to BRN, this chapter considers whether trade and FDI liberalization could push up economic growth in the steady state or not. This chapter finds that FDI liberalization tends to have more positive growth promoting effects than trade liberalization does. On the other hand, trade liberalization is prone to have an ambiguous impact on growth rates.

Keywords Endogenous growth · Firm heterogeneity · Knowledge spillovers · Trade liberalization · FDI liberalization

2.1 Introduction

International trade and foreign direct investment (FDI) have grown at high rates in the last few decades. However, recent empirical evidence reveals that multinational sales remain concentrated in a few top firms. Table 2.1 below presents the percentage of total manufacturing exports accounted for by the top exporters, ranked in terms of

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Table 2.1 Top exporters' shares in total exports, manufacturing sector

Country	Rank order in exports		
	Top 1%	Top 5%	Top 10%
Japan	62	85	92
Germany	59	81	90
France	44 (68)	73 (88)	84 (94)
United Kingdom	42	69	80
Italy	32	59	72
Hungary	77	91	96
Belgium	48	73	84
Norway	53	81	91

Source: Wakasugi et al. (2008) for Japan, Mayer and Ottaviano (2007) for other countries

Table 2.2 Export and FDI premium

Country	Employment premium	Value added premium	Wage premium	Capital intensity premium	Skill intensity premium
Exporter's premium					
Japan	3.02	5.22	1.25	1.29	1.53
Germany	2.99		1.02		
France	2.24	2.68	1.09	1.49	
United Kingdom	1.01	1.29	1.15		
Italy	2.42	2.14	1.07	1.01	1.25
Hungary	5.31	13.53	1.44	0.79	
Belgium	9.16	14.80	1.26	1.04	
Norway	6.11	7.95	1.08	1.01	
FDI-maker's premium					
Japan	4.79	8.79	1.26	1.53	1.52
Germany	13.19				
France	18.45	22.68	1.13	1.52	
Belgium	16.45	24.65	1.53	1.03	
Norway	8.28	11.00	1.34	0.87	

Source: Wakasugi et al. (2008) for Japan, Mayer, and Ottaviano (2007) for other countries

their individual exports in each country. In all countries, the top 10% of exporters are found to be responsible for the vast majority of the total sales of exports. For example, in Japan, firms in the top 10% account for 92% of all exports.

Melitz (2003) introduces the heterogeneous productivities of firms, and theoretically demonstrates that the choices of a firm between non-exports and exports are related to its productivity. Helpman et al. (2004) extend the analysis of Melitz (2003) by including FDI. Mayer and Ottaviano (2007) and Wakasugi et al. (2008) apply the results of Melitz (2003) and Helpman et al. (2004) to European and Japanese firms and examine the characteristics of a few internationalized firms.

Table 2.2 below shows the export premium, which is measured in terms of the ratio of the average value of exporters to that of non-exporters for employment, value added, wages paid, capital intensity, and skill intensity. The FDI premium is similar to the export premium. It can be seen that these ratios are greater than 1 for almost all cases. This means that exporting firms employ more workers, increase value added, pay higher wages, and are more capital and skill intensive when compared to non-exporting firms. Furthermore, FDI premium ratios are greater than export premium ratios, that is, this means that firms engaging in FDI employ more workers, add more value added, pay higher wages, and are more capital and skill intensive when compared to exporting firms.

Therefore, a few highly productive firms are responsible for the vast majority of international economic transactions. BRN (2008) examine the growth effects of freer trade embedding the heterogeneity of firms in a product-innovation endogenous economic growth model as developed by Romer (1990) and Grossman and Helpman (1991). Gustafsson and Segerstrom (2010) present the improved version of the BRN (2008) model by eliminating the strong scale effect, which means that a larger economy should grow faster. Ourens (2016) proposes a revised version of the BRN (2008) model with a modified welfare analysis. This study improves the BRN (2008) model by embedding FDI. Borensztein et al. (1998) show that FDI is an important vehicle for the trasfer of technology. We then examine the effects of trade and FDI liberalization on economic growth in three analytically tractable cases out of the five that were considered in BRN (2008). In a heterogeneous firm model, trade and FDI openness have effects on the intensity of competition in each market and the distribution of operating firms which affects the innovation rate. The main purpose of this study is to analyze the effect of trade and FDI liberalization policy on economic growth. As a result, this study shows that international economic liberalization can promote economic growth even if international knowledge spillovers do not occur. Coe and Helpman (1995) argue that a country's productivity depends on its own R&D efforts as well as the R&D of trade partners. However, Keller (2002) shows that technology diffusion is not global but local to a substantial extent.

The rest of this chapter is organized as follows. Section 2.2 describes the setup of the model. Section 2.3 presents the effects of trade and FDI liberalization on economic growth. Section 2.4 presents concluding remarks.

2.2 Setup of the Model

There are two symmetric economies with a single primary factor, labor, a Dixit-Stiglitz[1] consumption goods sector with monopolistic competition, and an innovation sector in which firms create knowledge through R&D.

[1]See Dixit and Stiglitz (1977) for details.

2.2.1 Consumer Behavior

The representative household maximizes utility over an infinite horizon. Intertemporal preferences are given by

$$\int_0^{\infty} \mathrm{e}^{-\rho t} \log D(t) \mathrm{d}t \tag{2.1}$$

where $D(t)$ represents an index of consumption at time t, and ρ is the subjective discount rate. The natural logarithm of the consumption index measures instantaneous utility at a moment in time. Households can purchase at time t all varieties that have been developed by the R&D sector prior to t. We take the product space to be continuous and express the measure of products invented before time t by $m(t)$. It is supposed that D has a constant and equal elasticity of substitution between every pair of goods. Specifically, D is defined as

$$D = \left[\int_0^m x(j)^{\alpha} \mathrm{d}j \right]^{1/\alpha} \tag{2.2}$$

where $x(j)$ denotes consumption of variety j. With these preferences, it is shown that the elasticity of substitution between any two products is $\sigma = \frac{1}{1-\alpha} > 1$, and the demand function of each differentiated good is

$$x(j) = \frac{Ep(j)^{-\sigma}}{P^{1-\sigma}} \tag{2.3}$$

where E is an amount of expenditure spent by a household, $p(j)$ is the price of that variety, and P is an aggregate price index given by

$$P = \left[\int_0^m p(j)^{1-\sigma} \mathrm{d}j \right]^{1/(1-\sigma)} \tag{2.4}$$

The representative household maximizes Eq. (2.1) subject to an intertemporal budget constraint. Using $D = E/P$, we can rewrite equation

$$\int_0^{\infty} \mathrm{e}^{-\rho t} [\log E(t) - \log P(t)] \mathrm{d}t \tag{2.5}$$

The maximization of Eq. (2.5), subject to an intertemporal budget constraint, yields the Euler equation

$$\frac{\dot{E}}{E} = r - \rho \tag{2.6}$$

where r is the interest rate at which households can borrow or lend freely.

2.2.2 Producer Behavior

There is a continuum of firms, each of them produces a different variety j. Production requires only labor, which is inelastically supplied at its aggregate level L. Labor is perfectly mobile within a country and is paid at the common wage rate w per unit of labor. Firm technology is represented by a cost function that exhibits constant marginal cost. Each firm has a different productivity level indicated by a which represents the marginal cost incurred in producing one unit of a differentiated good. Profits of a firm j with the unit labor requirement $a(j)$ from domestic sales are given by

$$\pi^D(j) = p(j)x(j) - wa(j)x(j) \tag{2.7}$$

where w is the common wage rate, hereafter normalized to one. The per-unit trade costs are modeled in the iceberg formulation such that $\tau > 1$ units of a good must be shipped for one unit to reach its destination. Similarly, profits of a firm j with the unit labor requirement $a(j)$ from exports are given by

$$\pi^X(j) = p(j)x(j) - \tau a(j)x(j) \tag{2.8}$$

Profits of a firm j from FDI are given by Eq. (2.7), since FDI firms do not bear iceberg trade costs.

From the demand function as Eq. (2.3), regardless of its productivity, each firm chooses the same profit maximization markup as equal to $1/\alpha$. This yields the following profit maximizing prices by domestic, exporting and FDI firms

$$p^D(j) = p^F(j) = \frac{a(j)}{\alpha}, \quad p^X(j) = \frac{\tau a(j)}{\alpha} \tag{2.9}$$

Using the number of available varieties and a distribution of productivity levels, the aggregate price index defined in Eq. (2.4) is given by

$$P = \frac{n^{1/(1-\sigma)}}{\alpha}\left[\int_0^\infty a^{1-\sigma}\mu_{\mathrm{D}}(a)\mathrm{d}a + p_{\mathrm{X}}\phi\int_0^\infty a^{1-\sigma}\mu_{\mathrm{X}}(a)\mathrm{d}a + p_{\mathrm{F}}\int_0^\infty a^{1-\sigma}\mu_{\mathrm{F}}(a)\mathrm{d}a\right]^{1/(1-\sigma)} \tag{2.10}$$

where $\mu_{\mathrm{D}}(a)$ is a distribution of productivity levels of domestic firms, $\mu_{\mathrm{X}}(a)$ is that of exporting firms, and $\mu_{\mathrm{F}}(a)$ is that of FDI firms, and $p_{\mathrm{X}}(p_{\mathrm{F}})$ represents the ex ante probability that one of these successful entry firms will export (FDI). Thus, $p_{\mathrm{X}}n(p_{\mathrm{F}}n)$ is the number of foreign firms exporting (FDI) to the home country. Furthermore, using the pricing rule (Eq. 2.9), this price index can be written as $P = n^{1/(1-\alpha)}p(\widetilde{a})$, where $\widetilde{a}$ is a weighted average of the productivity levels of operating firms and is defined by

$$\widetilde{a} = \left[\int_0^{\infty} a^{1-\sigma}\mu_{\mathrm{D}}(a)\mathrm{d}a + p_{\mathrm{X}}\phi\int_0^{\infty} a^{1-\sigma}\mu_{\mathrm{X}}(a)\mathrm{d}a + p_{\mathrm{F}}\int_0^{\infty} a^{1-\sigma}\mu_{\mathrm{F}}(a)\mathrm{d}a\right]^{1/(1-\sigma)} \tag{2.11}$$

This implies that the profit earned from selling in a domestic market and exporting can be rewritten as

$$\pi_a^D = \pi_a^F = \frac{(1-\alpha)E}{n}\left[\frac{a}{\widetilde{a}}\right]^{1-\sigma} \tag{2.12}$$

$$\pi_a^X = \frac{(1-\alpha)E}{n}\left[\frac{\tau a}{\widetilde{a}}\right]^{1-\sigma} \tag{2.13}$$

Using the profit of each firm, aggregate profit Π is derived as $\Pi = (1-\alpha)E$. Therefore, the profit of a firm selling in a domestic market with the average productivity is $\pi_{\widetilde{a}}^D = (1-\alpha)E/n$, and aggregate profit can be expressed as $\Pi = n\pi_{\widetilde{a}}^D$.

Let $v_a(s)$ denote the value of a firm operating at time s whose productivity level is a. We assume that the stock market value of the firm at time s equals the present discounted value of its infinite stream of profits subsequent to s. That is,

$$v_a(s) = \int_s^{\infty} \mathrm{e}^{-[R(t)-R(s)]}\pi_a(t)\mathrm{d}t \tag{2.14}$$

where $R(t)$ represents the cumulative discount factor applicable to profits earned at time t. Differentiation of Eq. (2.14) with respect to s yields the following no-arbitrage condition

$$\pi_a + \dot{v}_a = rv_a \tag{2.15}$$

Solving for v_a yields

$$v_a = \frac{\pi_a}{r - \dot{v}_a / v_a} \tag{2.16}$$

2.2.3 Firm Entry

There is a large pool of potential entrants into the industry. Firms must first make an initial investment in order to enter. To develop a new variety, a firm needs to create f_I units of knowledge in the R&D sector. The unit labor requirement associated with creating knowledge is $1/K_n$, that is, it takes $1/K_n$ units labor to create one unit of knowledge, where K_n represents a collection of ideas and methods that will be useful to later generations of innovators. Individual firms treat K_n as a parameter, but it can change over time due to knowledge spillovers. Thus, the labor requirement of developing a new variety is f_I/K_n. It also represents the sunk cost incurred in developing a new variety because the wage rate is normalized to one. Knowledge creation is also involved in adapting a variety to market-specific standards, regulations, and norms. To sell a new variety in a domestic market, a firm needs to create f_{D} units of knowledge. To export a new variety in a foreign market, a firm needs to create f_{X} ($>f_{\mathrm{D}}$) units of knowledge. Thus, the labor requirement and the cost of selling a new variety in the domestic market is f_{D}/K_n, and for exporting is f_{X}/K_n. Once a firm has developed a new variety, it learns the unit labor requirement a, associated with its production. The unit labor requirement a is drawn from a probability density function $g(a)$. $g(a)$ is defined on $[0, \overline{a}]$, and has a corresponding cumulative distribution function $G(a)$. Once drawn, the unit labor requirement of a firm associated with producing a particular variety does not change over time. Upon entry with a low productivity draw, a firm may decide to immediately exit and not produce.

Given that the firms with threshold values a_{D}, a_{X}, and a_{F} are indifferent between entering and not entering the domestic and foreign markets, respectively, the costs of entering must be equal to the value of present discounted profits,

$$v_{a_{\mathrm{D}}} = \frac{f_{\mathrm{D}}}{K_n}, \quad v_{a_{\mathrm{X}}} = \frac{f_{\mathrm{X}}}{K_n}, \quad v_{a_{\mathrm{F}}} = \frac{f_{\mathrm{F}}}{K_n} \tag{2.17}$$

Substituting the profit flows Eqs. (2.12) and (2.13) into these three equations yields the market entry conditions,

$$\begin{aligned}
\frac{(1-\alpha)E/n}{r+\dot{K}_n/K_n}\left[\frac{a_{\mathrm{D}}}{\widetilde{a}}\right]^{1-\sigma} &= \frac{f_{\mathrm{D}}}{K_n},\\
\frac{(1-\alpha)E/n}{r+\dot{K}_n/K_n}\left[\frac{\tau a_{\mathrm{X}}}{\widetilde{a}}\right]^{1-\sigma} &= \frac{f_{\mathrm{X}}}{K_n},\\
\frac{(1-\alpha)E/n}{r+\dot{K}_n/K_n}\left[\frac{a_{\mathrm{F}}}{\widetilde{a}}\right]^{1-\sigma} &= \frac{f_{\mathrm{F}}}{K_n}
\end{aligned} \tag{2.18}$$

Equation (2.18) corresponds to the ZCP conditions mentioned in Melitz (2003). Hereafter, a_{D}, a_{X}, and a_{F} satisfying Eq. (2.18) are referred to as the cutoff levels. These conditions, taken together, imply that

$$a_{\mathrm{X}} = a_{\mathrm{D}}\left(\frac{T}{\phi}\right)^{1/(1-\sigma)}, \quad a_{\mathrm{F}} = a_{\mathrm{D}}T\prime^{1/(1-\sigma)} \tag{2.19}$$

where $\phi \equiv \tau^{1-\sigma}$ and $T \equiv f_{\mathrm{X}}/f_{\mathrm{D}}$ are measures of the freeness of trade, and $T' \equiv f_{\mathrm{F}}/f_{\mathrm{D}}$ is that of FDI.

Any entering firm drawing a productivity level $a > a_{\mathrm{D}}$ will immediately exit and never produce. The productivity distribution of operating firms must then be determined by the initial productivity draw, conditional on successful entry. Hence, $\mu_{\mathrm{z}}(a)$ ($\mathrm{z} = \mathrm{D}, \mathrm{X}, \mathrm{F}$) is the conditional distribution of $g(a)$ on $[0,\ \ a_{\mathrm{z}}](\mathrm{z} = \mathrm{D}, \mathrm{X}, \mathrm{F})$,

$$\mu_{\mathrm{D}}(a) = \begin{cases} \dfrac{g(a)}{G(a_{\mathrm{D}})} & \text{if } \ a \le a_{\mathrm{D}} \\ 0 & \text{if } \ a > a_{\mathrm{D}} \end{cases} \tag{2.20}$$

$$\mu_{\mathrm{X}}(a) = \begin{cases} \dfrac{g(a)}{G(a_{\mathrm{X}})} & \text{if } \ a_{\mathrm{F}} < a \le a_{\mathrm{X}} \\ 0 & \text{otherwise} \end{cases} \tag{2.21}$$

$$\mu_{\mathrm{F}}(a) = \begin{cases} \dfrac{g(a)}{G(a_{\mathrm{F}})} & \text{if } \ a \le a_{\mathrm{F}} \\ 0 & \text{if } \ a > a_{\mathrm{F}} \end{cases} \tag{2.22}$$

Noting $p_{\mathrm{X}} = (G(a_{\mathrm{X}}) - G(a_{\mathrm{F}}))/G(a_{\mathrm{D}})$ and $p_{\mathrm{F}} = G(a_{\mathrm{F}})/G(a_{\mathrm{D}})$, these conditional distributions derive the average productivity level $\widetilde{a}$ as

$$\widetilde{a} = \left(\frac{1}{G(a_{\mathrm{D}})}\right)^{\frac{1}{1-\sigma}}\left[\int_0^{a_{\mathrm{D}}} a^{1-\sigma}g(a)\mathrm{d}a + \phi\int_0^{a_{\mathrm{X}}} a^{1-\sigma}g(a)\mathrm{d}a + \int_0^{a_{\mathrm{F}}} a^{1-\sigma}g(a)\mathrm{d}a\right]^{\frac{1}{1-\sigma}} \tag{2.23}$$

Equation (2.23) shows how this endogenous range affects the average productivity level.

We assume that there is free entry by firms into the innovation market. Since all operating firms, other than the cutoff firm, earn positive profits, the average profit level of operating firms must be positive. The expectation of future positive profits is the only reason why firms consider sinking the R&D investment cost that is required for entry. Free entry ensures that ex ante expected discounted profits must equal ex ante expected fixed costs of developing a profitable variety,[2]

$$\frac{(1-\alpha)E/n}{r+\dot{K}_n/K_n}=\frac{\overline{f}}{K_n} \tag{2.24}$$

where

$$\overline{f}=\left(\frac{1}{G(a_{\mathrm{D}})}f_I+f_{\mathrm{D}}+\frac{G(a_{\mathrm{X}})-G(a_{\mathrm{F}})}{G(a_{\mathrm{D}})}f_{\mathrm{X}}+\frac{G(a_{\mathrm{F}})}{G(a_{\mathrm{D}})}f_{\mathrm{F}}\right) \tag{2.25}$$

The first term on the right-hand side of Eq. (2.25) represents the expected cost of innovation, where $1/G(a_{\mathrm{D}})$ can be seen as the number of attempts that are necessary before a profitable variety is discovered. The second term f_{D} is the fixed cost of local market adaptation paid by all producing firms. The third and fourth terms are the expected fixed costs associated with adapting a variety to the foreign market. $(G(a_{\mathrm{X}}) - Ga_{\mathrm{F}})/G(a_{\mathrm{D}})$ represents the likelihood of having developed a variety that is profitable enough to export, given that domestic market entry has taken place. Similarly, $G(a_{\mathrm{F}})/G(a_{\mathrm{D}})$ is that for FDI. Hence, $\overline{f}$ is the ex ante expected fixed cost of developing a profitable variety at time t, measured in units of knowledge created.

The right-hand side of Eq. (2.24) also represents the expected labor requirement of creating a variety, since the wage rate is normalized to one. The flow of new varieties is determined by the labor devoted to R&D divided by the expected labor requirement for the successful development of a new variety,

$$\dot{n}=\frac{L_I}{\overline{f}/K_n} \tag{2.26}$$

where L_I is all the labor devoted to R&D in the entire economy. Hence, the economy-wide amount of labor used in the R&D sector is expressed as a function of the growth rate of varieties,

$$L_I=\frac{n\overline{f}}{K_n}g \tag{2.27}$$

where $g \equiv \dot{n}/n$ is the growth rate of varieties.

[2] See Appendix A.1 for details.

2.3 Steady-State Equilibrium of the Model

In this section, the steady-state equilibrium implications of trade liberalization are studied. We assume hereafter $G(a)$ follows a Pareto distribution to get explicit solutions for cutoff productivity levels satisfying Eq. (2.18).

$$G(a) = \left(\frac{a}{\overline{a}}\right)^k, \qquad 0 \le a \le \overline{a} \tag{2.28}$$

where k and $\overline{a}$ are the shape and scale parameters of distribution, respectively. By choice of units, $\overline{a}$ is normalized to unity without loss of generality.

With Eqs. (2.19) and (2.28), we solve for the average productivities of domestic selling, exporting and FDI firms as functions of the cutoff level a_{D}.

Thus, Eq. (2.23) yields[3]

$$\widetilde{a} = \left(\frac{\beta}{\beta - 1}\right)^{1/(1-\sigma)} \left[1 + T^{1-\beta}\phi^{\beta} + (1-\phi)T\prime^{1-\beta}\right]^{1/(1-\sigma)} a_{\mathrm{D}} \tag{2.29}$$

where $\beta \equiv k(\sigma - 1) > 1$.

Using Eqs. (2.18), (2.24), and (2.29), the steady-state cutoff unit labor requirement for selling in domestic market is obtained, as

$$a_{\mathrm{D}} = \left(\frac{(\beta-1)f_I}{\left(1 + T^{1-\beta}\phi^{\beta} + (1 - \beta\phi + (\beta-1)(T/T'))T\prime^{1-\beta}\right)f_{\mathrm{D}}}\right)^{1/k} \tag{2.30}$$

Then, a_{X} and a_{F} are determined by using Eq. (2.19) to substitute for a_{D} in Eq. (2.30). The steady-state cutoff unit labor requirements for exporting and FDI are

$$a_{\mathrm{X}} = \left(\frac{T^{1-\beta}\phi^{\beta}(\beta-1)f_I}{\left(1 + T^{1-\beta}\phi^{\beta} + (1 - \beta\phi + (\beta-1)(T/T'))T\prime^{1-\beta}\right)f_{\mathrm{X}}}\right)^{1/k} \tag{2.31}$$

$$a_{\mathrm{F}} = \left(\frac{T\prime^{1-\beta}(\beta-1)f_I}{\left(1 + T^{1-\beta}\phi^{\beta} + (1 - \beta\phi + (\beta-1)(T/T'))T\prime^{1-\beta}\right)f_{\mathrm{F}}}\right)^{1/k} \tag{2.32}$$

The increased exposure to trade forces the least productive firms to exit and generates entry of new firms into the export market.

The labor market is perfectly competitive. L units of labor services are applied to R&D and the production of differentiated goods. Thus, labor market equilibrium requires

[3]See Appendix A.2 for details.

$$L_I + L_M = L \tag{2.33}$$

where L_M is the economy-wide amount of labor used to produce varieties. L_M also represents aggregate payments to production workers. It must match the difference between aggregate revenue and profit, $L_M = E - (1 - \alpha)E = \alpha E$. Using this result and Eq. (2.27), the labor market clearing condition can be written as

$$\frac{n\bar{f}}{K_n} g + \alpha E = L \tag{2.34}$$

Using Eqs. (2.24) and (2.34), it follows

$$g = \frac{K_n L}{n\bar{f}} - \alpha \frac{1}{(1-\alpha)} \left(r + \frac{\dot{K}_n}{K_n} \right) \tag{2.35}$$

where

$$\bar{f} = \left(\frac{\beta \left(1 + T^{1-\beta} \phi^{\beta} + (1-\phi) T \prime^{1-\beta} \right) f_{\mathrm{D}}}{\beta - 1} \right) \tag{2.36}$$

In the following, we specify the formulation of public knowledge. In all cases, note n=n* because we assume two countries are symmetrical. In addition, $\dot{K}_n / K_n$ equals the growth rate of varieties g in the steady state. Depending on the specification of public knowledge, the growth rate of variety can be derived using Eq. (2.35).

2.3.1 Grossman-Helpman Model

In Grossman and Helpman (1991), researchers in each country learn not only from the R&D projects undertaken in their own countries but also from those that are conducted abroad. With international diffusion of technical information, the knowledge stock in each country is given by

$$K_n = n + \lambda n^* = (1 + \lambda) n \tag{2.37}$$

where $0 \leq \lambda \leq 1$ is the proportion of knowledge available in the other country. Substituting Eq. (2.37) into Eq. (2.35), the growth rate of varieties in this case is

$$g = (1-\alpha)\frac{(\beta-1)(1+\lambda)}{\beta\left(1+T^{1-\beta}\phi^{\beta}+(1-\phi)T\prime^{1-\beta}\right)f_{\mathrm{D}}}L-\alpha\rho \tag{2.38}$$

Lower trade and FDI fixed costs unambiguously have anti-growth effects. The effect of reducing iceberg trade costs depends on the relative levels of fixed and trade costs. With some calculation, we can confirm that the effect of reducing the iceberg cost on the economic growth rate is

$$\frac{\partial g}{\partial \phi} = \begin{cases} +, & \text{if } \phi < \frac{T}{T'}\left(\frac{1}{\beta}\right)^{1/(\beta-1)}, \\ 0, & \text{if } \phi = \frac{T}{T'}\left(\frac{1}{\beta}\right)^{1/(\beta-1)}, \\ -, & \text{if } \phi > \frac{T}{T'}\left(\frac{1}{\beta}\right)^{1/(\beta-1)}. \end{cases} \tag{2.39}$$

Noting $\phi = \tau^{1-\sigma}$, it means that trade liberalization with respect to the reduction of the iceberg trade cost promotes economic growth when the iceberg cost is high. We summarize the result of the analysis with a Grossman-Helpman type knowledge formulation as follows.

Proposition 2.1. *When public knowledge is related with the number of varieties, reducing trade and FDI fixed costs necessarily lowers the economic growth rate in the steady state. Reducing the iceberg cost raises the economic growth rate if and only if the iceberg cost is relatively high.*

2.3.2 Efficiency-Linked Knowledge Spillover Model

Considering that the R&D sector learns the existing available knowledge, the public knowledge spillovers can be influenced by not only the number of varieties but also the productivities of operating firms. Therefore, the public knowledge in this case is

$$\begin{aligned} K_n = n\tilde{a}^{1-\sigma} = n\left(\frac{\beta}{\beta-1}\right)[1+T^{1-\beta}\phi^{\beta}+(1-\phi)T'^{1-\beta}] \\ \times \left(\frac{(\beta-1)\,f_I}{(1+T^{1-\beta}\phi^{\beta}+(1-\beta\phi+(\beta-1)(T/T'))T\prime^{1-\beta})\,f_{\mathrm{D}}}\right)^{-1/\beta} \end{aligned} \tag{2.40}$$

Substituting Eq. (2.40) into Eq. (2.35), the growth rate of varieties in this case is

$$g = (1-\alpha)\left(\frac{\left(1 + T^{1-\beta}\phi^{\beta} + (1-\beta\phi + (\beta-1)(T/T'))T\prime^{1-\beta}\right)f_{\mathrm{D}}}{(\beta-1)f_I}\right)^{1/\beta}\frac{L}{f_{\mathrm{D}}} - \alpha\rho. \tag{2.41}$$

Reducing FDI fixed costs unambiguously raises the economic growth rate in the steady state. On the other hand, the effect of trade liberalization is ambiguously as follows,

$$\frac{\partial g}{\partial \phi} = \begin{cases} +, & \text{if } \phi > \dfrac{T}{T'}, \\ 0, & \text{if } \phi = \dfrac{T}{T'}, \\ -, & \text{if } \phi < \dfrac{T}{T'}. \end{cases} \tag{2.42}$$

$$\frac{\partial g}{\partial T} = \begin{cases} +, & \text{if } T > \phi T', \\ 0, & \text{if } T = \phi T', \\ -, & \text{if } T < \phi T'. \end{cases} \tag{2.43}$$

Lower iceberg trade costs raise the economic growth rate if and only if the iceberg cost is relatively low. Lower trade fixed costs raise the economic growth rate if and only if that cost is relatively low. Note that Eq. (2.40) means there is no international knowledge spillovers.

2.3.3 Lab-Equipment Model

In Rivela-Batiz and Romer (1991a, b), it is supposed that knowledge is produced using the final good. It means that public knowledge is positively related to the quantity of final goods. Thus, it eventually implies that public knowledge is inversely related to price index of CES (constant elasticity of substitution) composite.

$$K_n = P^{1-\sigma} = \frac{n\tilde{a}^{1-\sigma}}{\alpha^{1-\sigma}} \tag{2.44}$$

Substituting Eq. (2.44) into Eq. (2.35), the growth rate of varieties in this case is

$$g = (1-\alpha)\left(\frac{\left(1 + T^{1-\beta}\phi^{\beta} + (1-\beta\phi + (\beta-1)(T/T'))T\prime^{1-\beta}\right)f_D}{(\beta-1)f_I}\right)^{1/\beta}\frac{L}{\alpha^{1-\sigma}f_D} - \alpha\rho. \tag{2.45}$$

The comparative effects of trade and FDI liberalization are essentially similar with that in the efficiency-linked knowledge spillovers model; we therefore omit the analysis with this model.

We have the following proposition on the effect of trade and FDI liberalization with the efficiency-linked knowledge spillovers model and the lab-equipment model.

Proposition 2.2. *When public knowledge is inversely related with the number and the productivities of operating firms or price index of CES composite, FDI liberalization unambiguously promotes economic growth. On the other hand, reducing the iceberg cost lowers the economic growth rate if and only if the iceberg cost is low.*

From the result, we can conclude the following implications. FDI liberalization definitely makes firms in the most productive range more active, and thus, the innovator can gain the benefits of a more productive knowledge spillover. On the contrary, trade liberalization can have ambiguous effects. Trade liberalization has led to domestic firms exiting from the market. They also have the most productive FDI firms to transit into the exporting range.

2.3.4 The Growth Effects of Trade and FDI Liberalization

Effects of changes of iceberg or fixed costs on the economic growth rate in Eq. (2.35) are divided into two channels which are through $\bar{f}$ or K_n/n. In any specifications, lower T or T' unambiguously increases the ex ante expected fixed cost of developing a profitable variety $\bar{f}$ in Eq. (2.36). Thus, it always has an anti-growth effect because exporting and FDI require much larger amounts of knowledge to adapt varieties into foreign market. Reducing iceberg trade cost ($\mathrm{d}\phi > 0$) increases $\bar{f}$ if ϕ is relatively large.

Different effects of changes of iceberg or fixed costs on the economic growth rate stem from the K_n/n channel. As in the Grossman-Helpman model, under Eq. (2.37), the changes of ϕ, T and T' have no impact on the term K_n/n. The overall impact comes from the $\bar{f}$ channel. In this case, lower T or T' has unambiguously anti-growth effect.

As in the efficiency-linked model, under Eq. (2.40), the K_n/n channel can have the pro-growth effects. Trade or FDI liberalization makes productive firms more active on average and this promotes R&D because the public knowledge is associated with average productivity. If the pro-growth effect is sufficiently strong to overwhelm the anti-growth effect, the overall effect on economic growth is positive.

As in the lab-equipment model, under Eq. (2.44), the effects on economic growth through two channels are similar to those in the efficiency-linked model.

2.4 Conclusion

This study developed an endogenous growth model with firm heterogeneity, in which firms can choose to sell in their domestic market or to export or engage in FDI. We explore the growth effects of trade and FDI liberalization. Our main finding is that FDI liberalization improves industry productivity and promotes economic growth when knowledge spillover is linked with efficiency of firms or physical final goods. On the other hand, freer trade has ambiguous growth effects. If trade liberalization disturbs firms engaging in FDI, it may hinder economic growth.

Lower T or T' unambiguously increases the ex ante expected fixed cost of developing a profitable variety; thus it has always an anti-growth effect. Reducing iceberg trade cost ($d\phi > 0$) increases $\bar{f}$ if ϕ is relatively large. Effects of changes through the K_n/n channel depend on the specifications of K_n. In the Grossman-Helpman model, the changes of trade and FDI liberalization have no impact on the term K_n/n. The overall impact comes from the $\bar{f}$ channel. On the other hand, in the efficiency-linked or lab-equipment models, the K_n/n channel can have pro-growth effects and if this effect is sufficiently strong to overwhelm the anti-growth effect, the overall effect on economic growth is positive.

This study assumes that two economies are symmetric, which means that these countries are at the same stages of development. However, trade and FDI liberalization work asymmetrically between developed and developing countries. We leave this for future research.

Appendix

A.1 Derivation of the Free Entry Condition in Eq. (2.24)

Free entry ensures that ex ante expected discounted profits must equal *ex ante* expected fixed costs of developing a profitable variety.

$$
\begin{aligned}
&G(a_{\mathrm{D}})\left\{\int_0^{a_{\mathrm{D}}}\frac{(1-\alpha)E/n}{r+\dot{K}_n/K_n}\left[\frac{a}{\tilde{\tilde{a}}}\right]^{1-\sigma}\frac{g(a)}{G(a_{\mathrm{D}})}\mathrm{d}a-\frac{f_{\mathrm{D}}}{K_n}\right\}+(G(a_{\mathrm{X}})-G(a_{\mathrm{X}}))\\
&\quad\times\left\{\int_{a_{\mathrm{X}}}^{a_{\mathrm{F}}}\frac{(1-\alpha)E/n}{r+\dot{K}_n/K_n}\left[\frac{\tau a}{\tilde{\tilde{a}}}\right]^{1-\sigma}\frac{g(a)}{G(a_{\mathrm{X}})-G(a_{\mathrm{F}})}\mathrm{d}a-\frac{f_{\mathrm{X}}}{K_n}\right\}+G(a_{\mathrm{F}})\\
&\quad\times\left\{\int_0^{a_{\mathrm{F}}}\frac{(1-\alpha)E/n}{r+\dot{K}_n/K_n}\left[\frac{a}{\tilde{\tilde{a}}}\right]^{1-\sigma}\frac{g(a)}{G(a_{\mathrm{F}})}\mathrm{d}a-\frac{f_{\mathrm{F}}}{K_n}\right\}\\
&\quad=\frac{f_I}{K_n}\\
&\quad\Leftrightarrow\frac{(1-\alpha)E/n}{r+\dot{K}_n/K_n}\left[\frac{1}{\tilde{\tilde{a}}}\right]^{1-\sigma}\left[\int_0^{\infty}a^{1-\sigma}\mu_{\mathrm{D}}(a)\mathrm{d}a+p_{\mathrm{X}}\phi\int_0^{\infty}a^{1-\sigma}\mu_{\mathrm{X}}(a)\mathrm{d}a+p_{\mathrm{F}}\int_0^{\infty}a^{1-\sigma}\mu_{\mathrm{F}}(a)\mathrm{d}a\right]\\
&\quad=\frac{1}{K_n}\left(\frac{1}{G(a_{\mathrm{D}})}f_I+f_{\mathrm{D}}+\frac{G(a_{\mathrm{X}})-G(a_{\mathrm{F}})}{G(a_{\mathrm{D}})}f_{\mathrm{X}}+\frac{G(a_{\mathrm{F}})}{G(a_{\mathrm{D}})}f_{\mathrm{F}}\right)\Leftrightarrow\frac{(1-\alpha)E/n}{r+\dot{K}_n/K_n}=\frac{\bar{f}}{K_n}
\end{aligned}
$$

A.2 Derivation of the Average Productivities in Eq. (2.29)

$$
\begin{aligned}
\tilde{a}&=\left(\frac{1}{G(a_{\mathrm{D}})}\right)^{1/(1-\sigma)}\left[\int_0^{a_{\mathrm{D}}}a^{1-\sigma}g(a)\mathrm{d}a+\phi\int_{a_{\mathrm{F}}}^{a_{\mathrm{X}}}a^{1-\sigma}g(a)\mathrm{d}a+\int_0^{a_{\mathrm{F}}}a^{1-\sigma}g(a)\mathrm{d}a\right]^{1/(1-\sigma)}\\
&=\left(\frac{1}{a_{\mathrm{D}}^k}\right)^{1/(1-\sigma)}\left[\int_0^{a_{\mathrm{D}}}a^{1-\sigma}ka^{k-1}\mathrm{d}a+\phi\int_{a_{\mathrm{F}}}^{a_{\mathrm{X}}}a^{1-\sigma}ka^{k-1}\mathrm{d}a+\int_0^{a_{\mathrm{F}}}a^{1-\sigma}ka^{k-1}\mathrm{d}a\right]^{1/(1-\sigma)}\\
&=\left(\frac{k}{a_{\mathrm{D}}^k}\right)^{1/(1-\sigma)}\left[\int_0^{a_{\mathrm{D}}}a^{k-\sigma}\mathrm{d}a+\phi\int_{a_{\mathrm{F}}}^{a_{\mathrm{X}}}a^{k-\sigma}\mathrm{d}a+\int_0^{a_{\mathrm{F}}}a^{k-\sigma}\mathrm{d}a\right]^{1/(1-\sigma)}\\
&=\left(\frac{k}{a_{\mathrm{D}}^k}\right)^{1/(1-\sigma)}\left[\left[\frac{1}{k-\sigma+1}a^{k-\sigma+1}\right]_0^{a_{\mathrm{D}}}+\phi\left[\frac{1}{k-\sigma+1}a^{k-\sigma+1}\right]_{a_{\mathrm{F}}}^{a_{\mathrm{X}}}+\left[\frac{1}{k-\sigma+1}a^{k-\sigma+1}\right]_0^{a_{\mathrm{F}}}\right]^{1/(1-\sigma)}\\
&=\left(\frac{k}{a_{\mathrm{D}}^k}\right)^{1/(1-\sigma)}\left[\frac{1}{k-\sigma+1}a_{\mathrm{D}}^{k-\sigma+1}+\phi\frac{1}{k-\sigma+1}a_{\mathrm{F}}^{k-\sigma+1}+(1-\phi)\frac{1}{k-\sigma+1}a_{\mathrm{F}}^{k-\sigma+1}\right]^{1/(1-\sigma)}\\
&=\left(\frac{k}{k-\sigma+1}\right)^{1/(1-\sigma)}\left[a_{\mathrm{D}}^{1-\sigma}+\phi a_{\mathrm{D}}^{1-\sigma}\left(\frac{T}{\phi}\right)^{(k-\sigma+1)/(1-\sigma)}+(1-\phi)a_{\mathrm{D}}^{1-\sigma}T\prime^{(k-\sigma+1)/(1-\sigma)}\right]^{\frac{1}{1-\sigma}}\\
&=\left(\frac{\beta}{\beta-1}\right)^{1/(1-\sigma)}\left[1+T^{1-\beta}\phi^{\beta}+(1-\phi)T\prime^{1-\beta}\right]^{1/(1-\sigma)}a_{\mathrm{D}}
\end{aligned}
$$

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Chapter 3
Political Economy of Patent Policy and Economic Growth

Kenichiro Ikeshita

Abstract This study examines how firms' campaign contributions to political parties affect patent policy, innovation, and welfare in an economy. We find that when the governing party is corrupt and places great importance on contributions, it strengthens patent protection and increases innovation. In addition, we find that a higher fraction of campaign contributions in firm profits encourages innovation and the governing party chooses stronger patent protection when it is corrupt. This implies that if campaign contributions are important for policy-making, the governing party has an incentive to choose a strong patent protection level.

Keywords Endogenous growth · Innovation · Intellectual property rights · Campaign contributions · Political economy

3.1 Introduction

Intellectual property rights (hereafter, IPR) protection is an important policy issue for governments. In the last two decades, many developed countries such as the United States and Japan have strengthened their IPR regimes. Additionally, since the Trade-Related Aspects of Intellectual Property Rights agreements have been approved as part of the final Uruguay Round trade talks, many less developed countries have also strengthened their IPR protection. In developed countries, a strong IPR policy enhances research and development (R&D) and increases productivity and the long-term growth rate. In less developed countries, a strong IPR policy attracts foreign investment and technology, which promotes economic growth.

The above discussion leads to the following hypothesis: firms or industries dependent on their research divisions have incentives for carrying out political activities to achieve stronger patent policies. Political activities by firms usually

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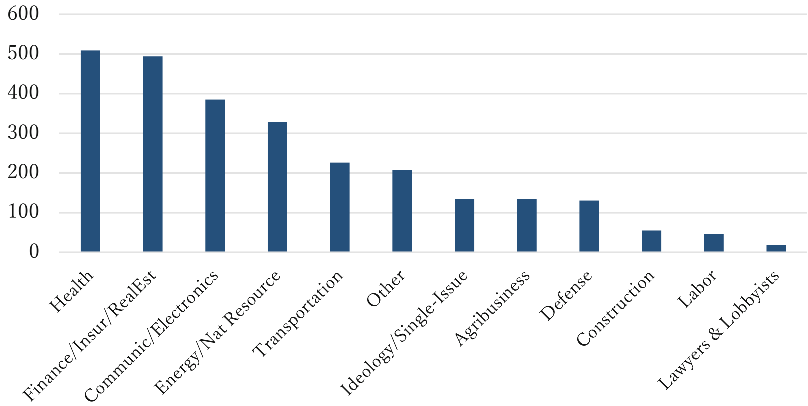


Fig. 3.1 Lobbying expenditure by industry in 2015. (Source: Center for Responsive Politics 2015)

take the form of campaign contributions and government lobbying. Figure 3.1 presents lobbying expenditures by industry in 2015 as evidence to support this hypothesis. According to the Center for Responsive Politics (2015), the total amount spent on lobbying in the United States in 2015 was 32.2 billion dollars. As Fig. 3.1 suggests, at 509 million dollars, the health industry, including pharmaceutical and medical companies, spent the largest amount on lobbying among all industries. The next was communications/electronics, spending 385 million dollars for lobbying. As these industries are R&D intensive, they have strong incentives to use political tools to influence the government's policy-making process.

Many researchers discuss the effect of patent policies on development and economic growth. Among theoretical studies, Judd (1985) uses an exogenous growth model to examine how patent length affects the market equilibrium path. After the development of the endogenous growth theory by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), numerous studies have investigated the relationship between patent policies and economic growth. Iwaisako and Futagami (2003) use a variety-expansion growth model based on Romer (1990) to show that the optimal patent length to maximize social welfare is finite. Lai and Qiu (2003) and Grossman and Lai (2004) develop the North-South trade model to analyze the international effects of patent protection, and show that governments of developed countries choose stronger patent protection than those of developing countries. Their analyses, however, assume that governments maximize household utility as a measure of social welfare and ignore incentives for firms to engage in political activities. In contrast, Eicher and Garcìa-Penalõsa (2008) show how private incentives to protect patent rights affect economic growth. They show that multiple steady states can emerge in their model; one steady state is high-growth equilibrium characterized by stronger patent protection, and the other is no-growth equilibrium in the absence of patent protection. However, they do not consider the process of policy-making explicitly. This implies that they cannot examine the interaction between patent holders and governments.

In this chapter, we clarify how patent protection distorted by campaign contributions affects innovation and economic growth. To achieve this goal, we provide a theoretical analysis based on the variety-expansion model developed by Grossman and Helpman (1991, Chap. 3). Our model is closely related to that of Eicher and Garcìa-Penalõsa (2008), in that, the firms make efforts to protect IPR. However, we assume that the governing party decides its IPR protection after taking into account household utility and the amount of campaign contributions. This assumption is different from that in other literature. In general, campaign contributions are regarded as a rent-seeking activity.[1] Many researchers have investigated the relationship between rent-seeking activities and economic development. One important finding in the literature is that rent-seeking activities tend to lower the rate of economic growth, because they distort the allocation of resources and weaken capital accumulation and research activity. However, campaign contributions aimed at strengthening IPR protection may enhance incentives for research activity and promote economic growth.

Our main results are as follows. First, we find if the governing party is corrupt and places great importance on campaign contributions, it strengthens patent protection and increases innovation. Second, we show that a higher fraction of campaign contributions in firm profits encourages innovation and the governing party chooses stronger patent protection when it is corrupt. This implies that if campaign contributions are important for the policy-making process, the governing party tends to have an incentive to choose a very strong patent protection level.

This chapter is organized as follows. Section 3.2 describes the baseline model and identifies the equilibrium path. Section 3.3 examines how campaign contributions by patent holders affect patent policy and innovation. Section 3.4 provides the concluding remarks.

3.2 A Baseline Model of Innovation and Intellectual Property Rights

3.2.1 Household

We consider a closed economy with identical households. This economy has a fixed number of households. We normalize the total number of households to unity. We assume that each household is endowed L units of labor and supplies their labor inelastically. Households consume two kinds of goods, homogenous goods and

[1]For example, based on an empirical analysis, Mauro (1995) finds that rent-seeking activities have a negative effect on economic growth. Murphy et al. (1991) argue that rent-seeking activities reward talent more than entrepreneurship does. This implies that rent-seeking activities discourage economic development.

continuum of differentiated goods. The representative household maximizes lifetime utility over an infinite horizon. The lifetime utility U^h is given by:

$$U^h = \int_0^\infty [\beta \log D(\tau) + (1-\beta)\log Y(\tau)]\mathrm{e}^{-\rho\tau}\mathrm{d}\tau, \tag{3.1}$$

where ρ is a discount rate. $D(t)$ is the consumption index of differentiated goods at time t and $Y(t)$ is consumption of homogenous goods. $\beta \in (0, 1)$ is a parameter that determines the shares of goods in expenditure. Many kinds of differentiated goods exist in this economy and $n(t)$ denotes a measure of differentiated products invented before time t. Specific differentiated good is indexed by $j \in [0, n(t)]$. $D(t)$ is represented by a Dixit and Stiglitz-type function:

$$D(t) = \left[\int_0^{n(t)} x(j,t)^\alpha \mathrm{d}j\right]^{1/\alpha}, \tag{3.2}$$

where $x(j, t)$ is consumption of jth variety of differentiated product at time t and $\alpha \in (0, 1)$ is a parameter which determines the price elasticity of demand.

Under these assumptions, the household's optimization problem can be broken down into two stages. At the first stage, the household chooses the optimal time path of expenditure in order to maximize Eq. (3.1) subject to the following lifetime budget constraint:

$$\int_0^\infty \mathrm{e}^{-\int_0^\tau r(s)\mathrm{d}s} E(\tau)\mathrm{d}\tau = \int_0^\infty \mathrm{e}^{-\int_0^\tau r(s)\mathrm{d}s}[w(\tau) - T(\tau)]\mathrm{d}\tau + W(0), \tag{3.3}$$

where $E(t)$ is instantaneous expenditure, $r(t)$ is the interest rate, $w(t)$ is the wage, $T(t)$ is lump-sum tax, and $W(0)$ is initial asset holding. At the second stage, the household determines how to allocate a given expenditure across differentiated goods and homogenous goods. By solving the second optimization problem, the demands for differentiated goods and homogenous goods are given by:

$$x(j,t) = \frac{\beta E(t)}{\int_0^{n(t)} p(j',t)^{-\frac{\alpha}{1-\alpha}}\mathrm{d}j'} p(j,t)^{-\frac{1}{1-\alpha}}, \tag{3.4}$$

$$Y(t) = (1-\beta)E(\tau). \tag{3.5}$$

Next, we return to the first stage of the household's optimization. To derive the time path of expenditure, we define the ideal price index of $D(t)$ as

$$P_D(t) = \left[\int_0^{n(t)} p(j,t)^{-\frac{\alpha}{1-\alpha}} \mathrm{d}j \right]^{-\frac{1-\alpha}{\alpha}}. \quad (3.6)$$

Using this price index, we rewrite the flow of utility as $\log \beta^{\beta}(1-\beta)^{1-\beta} + \log E(t) - \beta \log P_{\mathrm{D}}(t)$. Therefore, the solution of the above dynamic optimization problem is given by the following Euler equation:

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho. \quad (3.7)$$

3.2.2 *Intellectual Property Rights Policy*

The literature provides different ways to formulate intellectual property rights in an economic model. For example, various features of intellectual property and patent legislations (namely, *patent duration, patent breadth, exogenous rate of imitation,* and *cost of imitation*) are considered and analyzed. In this study, we assume that the government chooses a measure of its enforcement policy, given by $\omega \in [0, 1]$ where ω represents the probability that an invented good is protected from competition by a secured patent. Once a patent is enforced by the government, the patent holder can enjoy exclusive right to produce and sell the good. For simplicity, we assume that the life of a patent that the government decides to enforce is infinite. Therefore, there are differentiated products protected by patents with a fraction of ω and not protected with a fraction of $1 - \omega$.[2]

To enforce ω, the government must employ labor. The lump-sum tax collected from households is used to employ these labor forces. Especially, we assume $\gamma\omega^{\theta}$ units of labor are employed by the government to enforce ω. It represents the social cost of patent enforcement. Moreover, we assume $\theta > 1$, which implies that the marginal social cost of patent enforcement is increasing.

3.2.3 *Producers*

Labor is the only factor of production. We assume that one unit of labor is required to produce both differentiated and homogenous goods. The homogenous good is always produced in competitive markets. As the homogenous good is the numeraire,

[2]In the case of finite patent length, the relationship between the rate of innovation and patent policy is derived as an implicit function. This consequence makes the analysis more complicated. As for the difficulty caused by the assumption of finite patent length, see Iwaisako and Futagami (2003).

its price is equal to the unit cost of producing it. This implies that $w(t) = 1$, where $w(t)$ is the wage rate.

Firms in the differentiated goods sector produce their goods based on the design created by R&D activity. As for the products protected by patents, the price of such products is given as $p_m = 1/\alpha$, based on Eq. (3.4), because the patent holders have exclusive rights to produce and sell those goods. $x_m(t)$ denotes a demand for differentiated goods protected by patents and $\pi(t)$ denotes a patent profit. Thus, the profit is given as

$$\pi(t) = \left(\frac{1-\alpha}{\alpha}\right) x_m(t). \tag{3.8}$$

The technology of producing a product is immediately imitated if a patent is not enforced for the product. Thus, the price of differentiated goods without patent is $p_c = 1$ because these goods are produced in a competitive market. $x_c(t)$ denotes demand for products that are not patent-protected. Obviously, products without patent protection do not generate profits for their producers.

3.2.4 Research and Development Sector

In this study, new designs of differentiated goods are invented by R&D. $v(t)$ denotes the value of a new design of differentiated goods, which is equivalent to a sum of the discounted value of the expected profits from time t. Therefore, we have

$$v(t) = \int_t^{\infty} \pi(t) \mathrm{e}^{-\int_t^s r(s') \mathrm{d}s'} \mathrm{d}s. \tag{3.9}$$

By differentiating $v(t)$ with respect to t, we derive the no-arbitrage condition as

$$\dot{v}(t) + \pi(t) = r(t) v(t). \tag{3.10}$$

Next, we consider the technology of product development. If a firm engaging in R&D activity employs $L_n(t)$ units of labor, the firm can produce $\dot{n}(t)$ units of new designs of differentiated goods by the following knowledge creation function:

$$\dot{n}(t) = \frac{n(t)}{a} L_n(t), \tag{3.11}$$

where a represents a parameter that determines the productivity of product development. We assume that firms can conduct R&D activity freely, and they finance the costs of product development by issuing equities. Once a firm succeeds in developing a new design of a differentiated product, this design creates a value of $v(t)$ when it is patent-protected. As ω is the probability that an invented good is protected by a patent, the expected value of the new design is given by $\omega v(t)$. However, as $a/n(t)$ units of labor are required to invent a new blueprint, in equilibrium, the value of the patent must not exceed that cost, along with innovation with positive and finite use of labor in product development.

$$\omega v(t) \leq \frac{w(t)a}{n(t)} = \frac{a}{n(t)}, \quad \text{with equality whenever } \dot{n}(t) > 0. \tag{3.12}$$

3.2.5 National Income

The final equilibrium condition equates savings with investment. The total income of this economy consists of wages from labor supply and dividend from equities. As mentioned before, the measure of products protected by patents is $\omega n(t)$. The aggregate income is given as $w(t)L + \omega n(t)\pi(t)$. In this study, we assume that the government collects tax from households in order to enforce patent policy. Therefore, disposable income is $w(t)L + \omega n(t)\pi(t) - w(t)\gamma\omega^{\theta}$. Savings is the difference between disposable income and aggregate expenditure $E(t)$. The savings finance the research investment through the financial market. Thus, we derive the equilibrium condition as follows:

$$w(t)L + \omega n(t)\pi(t) = E(t) + \frac{w(t)a\dot{n}(t)}{n(t)} + w(t)\gamma\omega^{\theta}. \tag{3.13}$$

3.2.6 Equilibrium Path

In this section, we derive the equilibrium path of the economy. Dividing both sides of Eq. (3.10) by $v(t)$ gives

$$\frac{\pi(t)}{v(t)} + \frac{\dot{v}(t)}{v(t)} = r(t). \tag{3.14}$$

Next, we consider the demand for differentiated goods. Using $p_c = 1$ and $p_m = 1/\alpha$, the demand function for differentiated goods protected by patents is

$$x_m(t) = \frac{\alpha\beta E(t)}{[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}]n(t)}. \tag{3.15}$$

Similarly, the demand function for differentiated goods not protected by patents is

$$x_c(t) = \frac{\alpha^{-\frac{\alpha}{1-\alpha}}\beta E(t)}{[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}]n(t)}. \tag{3.16}$$

By substituting Eq. (3.15) into Eq. (3.8), the profit $\pi(t)$ is expressed as:

$$\pi(t) = \frac{(1-\alpha)\beta E(t)}{[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}]n(t)}. \tag{3.17}$$

Using this expression, we can rewrite the no-arbitrage condition into Eq. (3.14) as:

$$\frac{(1-\alpha)\beta E(t)}{[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}]n(t)v(t)} + \frac{\dot{v}(t)}{v(t)} = r(t). \tag{3.18}$$

Here we define $z(t) \equiv E(t)/(n(t)v(t))$. By using Eqs. (3.7) and (3.18), the change in $z(t)$ is expressed by the following differential equation:

$$\frac{\dot{z}(t)}{z(t)} = \frac{(1-\alpha)\beta z(t)}{[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}]} - \frac{\dot{n}(t)}{n(t)} - \rho. \tag{3.19}$$

Simultaneously, dividing Eq. (3.13) by $n(t)v(t)$ and using $w(t) = 1$ and Eq. (3.12) yield

$$\frac{\dot{n}(t)}{n(t)} = \frac{L - \gamma\omega^{\theta}}{a} + \frac{(1-\alpha)\beta z(t)}{[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}]} - \frac{z(t)}{\omega}. \tag{3.20}$$

Substituting Eq. (3.20) into Eq. (3.19), we have

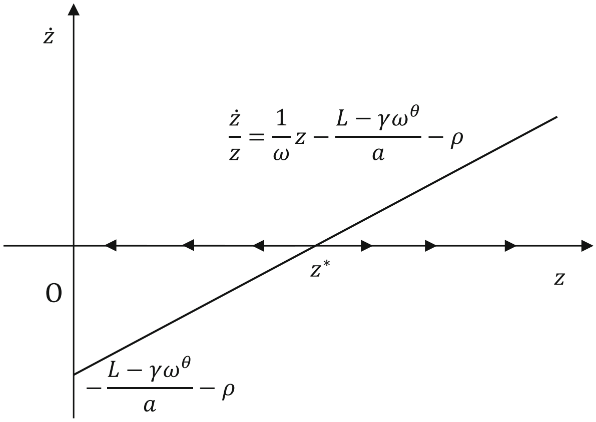
$$\frac{\dot{z}(t)}{z(t)} = \frac{z(t)}{\omega} - \frac{L - \gamma\omega^{\theta}}{a} - \rho. \tag{3.21}$$

The steady state of the economy is determined so that $\dot{z} = 0$. We let z^* denote the value of $z(t)$ in the steady state. Therefore, we can derive z^* as:

$$z^* = \frac{\omega\left(L - \gamma\omega^{\theta} + a\rho\right)}{a}. \tag{3.22}$$

The phase diagram of this model is shown in Fig. 3.2. We can easily see that the unique steady state z^* is unstable because $1/\omega$ is positive. Since we can interpret $z(t)$ as the inverse of total asset measured by utility, $z(t)$ is a variable that can jump. Therefore, $z(t)$ jumps to the steady-state value z^* in this economy. This implies that

Fig. 3.2 Phase diagram of z



our model has no transitional dynamics and the equilibrium path jumps to the steady state instantaneously.[3]

Next, we derive the growth rate of $n(t)$ in the steady state, g. Substituting Eq. (3.12) into the expression of z^* given by Eq. (3.22), we derive $E(t)$ in the steady state. Since we can show that the expenditure $E(t)$ is constant and depends on ω, the steady-state value of expenditure is expressed as:

$$E(\omega) = L - \gamma\omega^{\theta} + a\rho. \tag{3.23}$$

The expenditure $E(\omega)$ is a decreasing function of ω. If the government wants to strengthen patent protection, it has to collect more tax. This tax collection reduces the household's disposable income and expenditure.

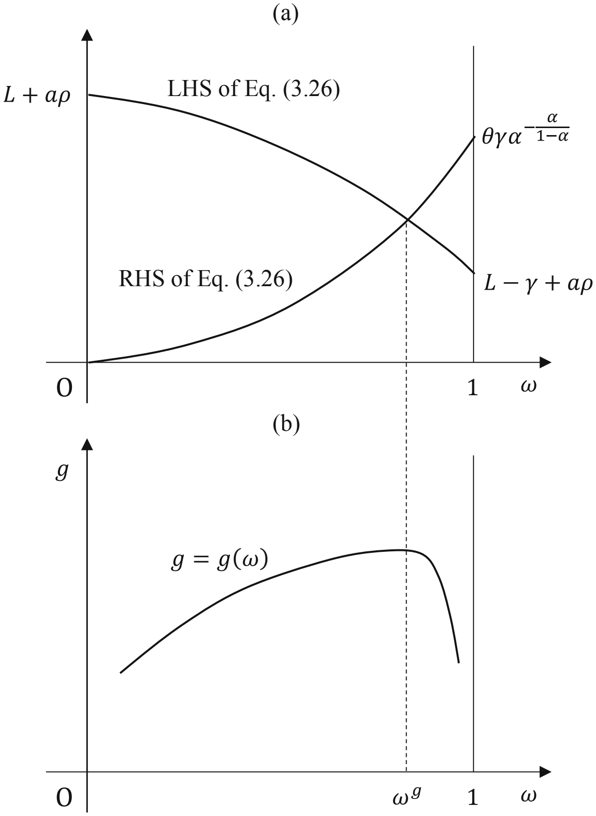
Substituting Eq. (3.22) into Eq. (3.20) yields the growth rate of $n(t)$ in the steady state as follows:

$$g = \frac{(1-\alpha)\omega\beta\left(L - \gamma\omega^{\theta} + a\rho\right)}{\left[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right]a} - \rho. \tag{3.24}$$

We call g the rate of innovation in the steady state. We now analyze how the strength of IPR protection affects innovation and welfare. The rate of innovation g is a function of the effort of patent enforcement ω, giving us $g = \mathrm{g}(\omega)$. Differentiating g with respect to ω yields:

[3]In other words, the equilibrium path that is not in the steady state cannot satisfy rational expectations. For details, see Grossman and Helpman (1991).

Fig. 3.3 Determination of growth-maximizing patent protection



$$\frac{\mathrm{d}g(\omega)}{\mathrm{d}\omega} = \frac{\beta(1-\alpha)}{a}\frac{\alpha^{-\frac{\alpha}{1-\alpha}}\left(L - \gamma\omega^{\theta} + a\rho\right) - \theta\gamma\omega^{\theta}\left[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right]}{\left[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right]^2}. \tag{3.25}$$

Let ω^g denote the patent protection that maximizes the rate of innovation. Based on Eq. (3.25), ω^g must satisfy the relationship:

$$\alpha^{-\frac{\alpha}{1-\alpha}}\left(L - \gamma\omega^{\theta} + a\rho\right) = \theta\gamma\omega^{\theta}\left[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right]. \tag{3.26}$$

Figure 3.3 shows the relationship between the rate of innovation and patent protection. From Fig. 3.3a, we find that there is a unique interior solution of ω^g when the cost of enforcing the patent policy (γ and ω) is large enough. Moreover, we easily find that the growth-maximizing level of patent protection is larger when the economy has a larger population (L), higher productivity of research activity (a), and lower cost of patent protection (γ).[4]

3.2.7 Benevolent Governing Party

We now analyze how patent policies are determined. Here, we consider the case where the governing party is benevolent and maximizes the household's utility U^h. In this case, the party's objective function W is given by

$$W = U^h. \tag{3.27}$$

For simplicity, we assume that the party set ω at time 0 and does not change the patent policy after that.

After a fair amount of calculation, we derive U^h as follows:

$$U^h(\omega) = \frac{\beta}{\rho}\left(\frac{1-\alpha}{\alpha}\right)\left\{\frac{g(\omega)}{\rho} + \log\left[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right]\right\} + \frac{1}{\rho}E(\omega) + \Lambda_0. \tag{3.28}$$

Let ω^b denote the level of patent protection that maximizes the welfare represented by Eq. (3.28). We find the condition that ω^b must satisfy by solving $\mathrm{d}W(\omega)/\mathrm{d}\omega = 0$ as:

$$\begin{aligned}
\alpha^{-\frac{\alpha}{1-\alpha}}\left(L - \gamma\omega^\theta + a\rho\right) &= \theta\gamma\omega^\theta\left[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right] \\
&+ \frac{\rho a}{\beta(1-\alpha)}\left(\alpha^{-\frac{\alpha}{1-\alpha}} - 1\right)\left[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right] \\
&+ \frac{\rho^2 a\alpha}{\beta^2(1-\alpha)^2}\,\frac{\theta\gamma\omega^{\theta-1}\left[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right]^2}{L - \gamma\omega^\theta + a\rho}.
\end{aligned} \tag{3.29}$$

The second term of Eq. (3.29) shows that deadweight loss increases with patent protection. The third term implies that expenditure is a decreasing function of ω because more resources are needed to strengthen patent protection. The second and third terms of Eq. (3.29) are positive. Therefore, we can state the following proposition.

Proposition 3.1 *Welfare-maximizing patent protection is weaker than growth-maximizing patent protection.*

[4]On the flip side, the rate of innovation becomes zero when the level of patent protection is too weak. From Eq. (3.24), we can derive an infimum in which the rate of innovation is positive. Here, we focus on the case where the rate of innovation is positive because we are interested in an equilibrium path where R&D is conducted.

Fig. 3.4 Determination of welfare-maximizing patent protection

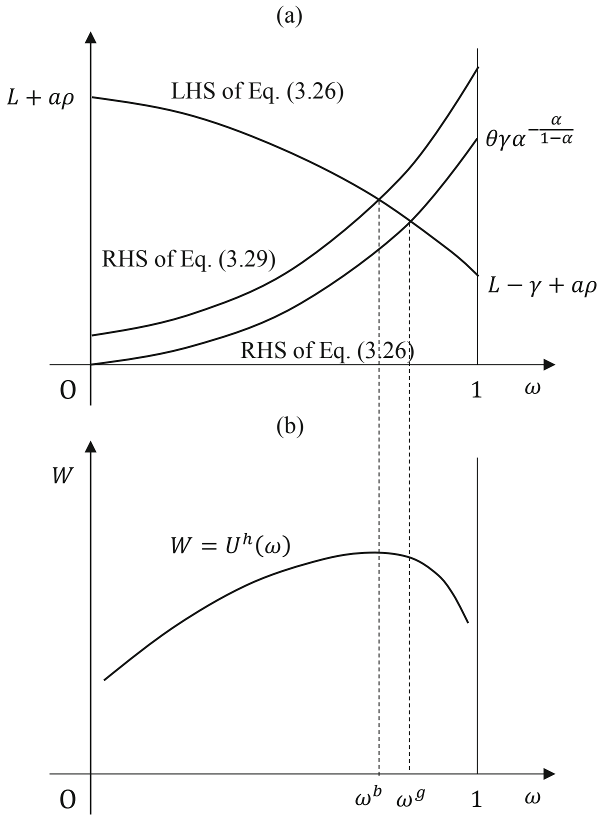


Figure 3.4 shows the relationship between IPR protection and welfare. When the scale of economy is larger (larger L), the cost of enforcing patent protection is smaller (smaller γ) and the consumer prefers differentiated goods (larger β), and the welfare-maximizing level of patent protection is higher.

3.3 Campaign Contributions

3.3.1 Introducing Campaign Contributions

In this section, we consider the possibility of campaign contributions. We assume that firms that produce differentiated goods protected by patents engage in lobbying activities to strengthen their monopolistic power. In particular, firms offer a fraction of their profits to the governing party as campaign contributions. When the governing party is corrupt and willing to receive the contributions, such political activities may distort the IPR policy.

In this study, each firm that produces differentiated goods protected by patent gains a constant fraction ϕ of profit. Therefore, the firm offers a fraction $(1-\phi)$ of profit to the governing party. In this case, for the market value of a new design of differentiated goods, Eq. (3.9) is modified as

$$v(t) = \int_t^\infty \phi\pi(t) \mathrm{e}^{-\int_t^s r(s')\mathrm{d}s'} \mathrm{d}s. \tag{3.30}$$

From Eq. (3.30), compared to when there is no campaign contribution, we find that the value of new design is smaller. By differentiating $v(t)$ with respect to t, we derive the no-arbitrage condition as

$$\dot{v}(t) + \phi\pi(t) = r(t)v(t). \tag{3.31}$$

In addition, the equilibrium condition of national income, Eq. (3.13), must be changed. In this section, we have assumed that a constant fraction of profits is donated to the governing party. Therefore, households receive $\omega\phi n(t)\pi(t)$ as a dividend and Eq. (3.13) is modified as follows:

$$w(t)L + \omega\phi n(t)\pi(t) = E(t) + \frac{w(t)a\dot{n}(t)}{n(t)} + w(t)\gamma\omega^\theta. \tag{3.32}$$

Next, we analyze the equilibrium path of the economy. As we can derive the equilibrium path as the same as in Sect. 3.2, we omit the derivations of the equilibrium path. As in Sect. 3.2, this economy has no transitional dynamics and jumps to its steady state. We can show that the expenditure $E(\omega)$ is same as Eq. (3.23). Further, the rate of innovation in the steady state is given by

$$g = \frac{\phi(1-\alpha)\omega\beta\left(L - \gamma\omega^\theta + a\rho\right)}{[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}]a} - \rho. \tag{3.33}$$

Since the rate of innovation g is a function of the effort of patent enforcement ω and ϕ, we can write $g = g(\omega, \phi)$. Differentiating g with respect to ω yields

$$\frac{\partial g(\omega,\phi)}{\partial\omega} = \frac{\phi\beta(1-\alpha)}{a} \frac{\alpha^{-\frac{\alpha}{1-\alpha}}\left(L - \gamma\omega^\theta + a\rho\right) - \theta\gamma\omega^\theta\left[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right]}{[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}]^2}. \tag{3.34}$$

Similarly, differentiating g with respect to ϕ yields

$$\frac{\partial g(\omega,\phi)}{\partial\phi} = \frac{(1-\alpha)\omega\beta(L - \gamma\omega^\theta + a\rho)}{[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}]a}. \tag{3.35}$$

Equation (3.34) implies that growth-maximizing patent protection is the same as in the case without campaign contributions. In other words, ϕ does not affect growth-maximizing patent protection. Then again, Eq. (3.35) is positive. When the fraction of campaign contributions in the profits is smaller, the incentive to engage in research activity becomes higher. This effect stimulates innovation in the economy.

3.3.2 Government with Campaign Contributions

Next, we examine how the governing party chooses its patent policy when campaign contribution exists. The party decides the patent protection considering not only households' utility but also the amount of contributions. In this section, utility function of the party is given as the average of households' utility and sum of discounted value of contributions.

$$W = \zeta U^h + (1-\zeta)U^g, \tag{3.36}$$

where ζ is a parameter that determines the degree of corruption of the governing party. A small ζ implies that the governing party values the amount of campaign contributions. We define U^g as:

$$U^g = \int_0^\infty \mathrm{e}^{-\int_0^\tau r(s)\mathrm{d}s}(1-\phi)\omega n(\tau)\pi(\tau)\mathrm{d}\tau. \tag{3.37}$$

Using Eq. (3.33), the amount of campaign contributions is expressed as:

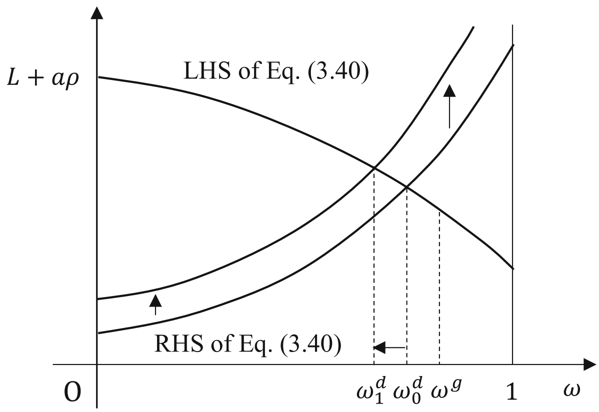
$$\begin{aligned}(1-\phi)\omega n(t)v(t) &= \frac{(1-\phi)\omega(1-\alpha)\beta E(\omega)}{[\omega+(1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}]} \\ &= \frac{a(1-\phi)(g+\rho)}{\phi}.\end{aligned} \tag{3.38}$$

Substituting Eq. (3.38) into Eq. (3.37) and using $r(t) = \rho$ on the equilibrium path reveals that U^g is simplified as:

$$U^g = \frac{a(1-\phi)(g+\rho)}{\rho\phi}. \tag{3.39}$$

Equations (3.28)–(3.39) imply that the objective function of the governing party, given as Eq. (3.36), is a function of ω. Let ω^d denote the level of patent protection that maximizes the welfare represented by Eq. (3.36). We find the condition that ω^d satisfies by solving $\mathrm{d}W(\omega)/\mathrm{d}\omega = 0$ as follows:

Fig. 3.5 An increase in ζ



$$\alpha^{-\frac{\alpha}{1-\alpha}}\left(L-\gamma\omega^{\theta}+a\rho\right)=\theta\gamma\omega^{\theta}\left[\omega+(1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right] + \rho a\Gamma(\zeta,\phi)\left(\alpha^{-\frac{\alpha}{1-\alpha}}-1\right)\left[\omega+(1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right] + \frac{\rho a\alpha\Gamma(\zeta,\phi)}{\beta(1-\alpha)}\frac{\theta\gamma\omega^{\theta-1}\left[\omega+(1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right]^{2}}{L-\gamma\omega^{\theta}+a\rho}, \tag{3.40}$$

where $\Gamma(\zeta,\phi)$ is defined as

$$\Gamma(\zeta,\phi)=\frac{\zeta}{\beta(1-\alpha)\zeta\phi+\rho a(1-\zeta)(1-\phi)}. \tag{3.41}$$

Using Eqs. (3.40) and (3.41), we can examine how the campaign contributions affect patent policy. First, we focus on the effect of ζ, which is a parameter that determines the preferences of the governing party. A larger ζ implies that the party places more emphasis on the household's utility. Differentiating Eq. (3.41) with respect to ζ yields

$$\frac{\partial\Gamma(\zeta,\phi)}{\partial\zeta}=\frac{\rho a(1-\phi)}{[\beta(1-\alpha)\zeta\phi+\rho a(1-\zeta)(1-\phi)]^{2}}. \tag{3.42}$$

Equation (3.42) is positive. This shows that when ζ is large, the second and third terms of Eq. (3.40) become large, and ω^{d} becomes smaller (See Fig. 3.5). It implies that if the governing party is not corrupt and places emphasis on households' utility, the party chooses weaker patent protection. In contrast, the corrupt party chooses higher patent protection. We can show that $\Gamma(0,\phi)=0$ for any ϕ. This implies that the second and third terms of Eq. (3.40) are zero and that Eq. (3.40) corresponds to Eq. (3.26). Intuitively, when the party is corrupt, it is likely to have too much incentive to strengthen patent protection in order to receive more contributions from the firms. Therefore, we obtain the following proposition.

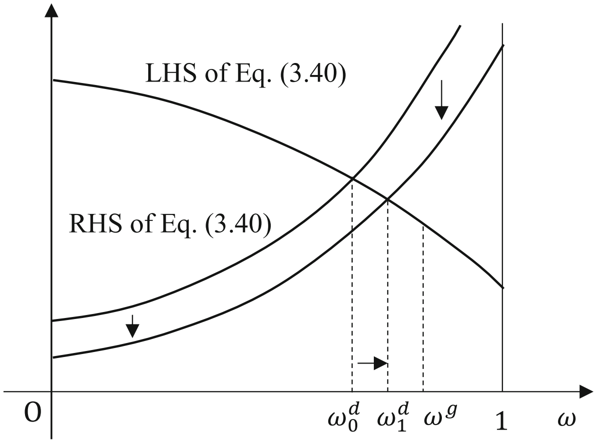


Fig. 3.6 An increase in ϕ ($\zeta > \zeta^*$)

Proposition 3.2 *If the governing party is corrupt and gives a lot of attention to campaign contributions, it strengthens patent protection and increases innovation.*

Next, we examine the effect of ϕ. The parameter ϕ represents the share of campaign contributions to the profit. A larger ϕ implies that the firms donate a large proportion of their profits to the governing party. Differentiating Eq. (3.41) with respect to ϕ yields

$$\frac{\partial \Gamma(\zeta, \phi)}{\partial \phi} = -\frac{\zeta[\beta(1-\alpha)\zeta - \rho a(1-\zeta)]}{[\beta(1-\alpha)\zeta\phi + \rho a(1-\zeta)(1-\phi)]^2}. \tag{3.43}$$

Therefore, the sign of Eq. (3.43) depends on the relation between ζ and ζ^* defined as

$$\zeta^* = \frac{\rho a}{\beta(1-\alpha) + \rho a}. \tag{3.44}$$

The above shows that $\partial \Gamma(\zeta, \phi)/\partial \phi < 0$ when $\zeta > \zeta^*$.

In this case, an increase in ϕ reduces the second and third terms of Eq. (3.40), and the governing party chooses higher patent protection (See Fig. 3.6). However, $\partial \Gamma(\zeta, \phi)/\partial \phi > 0$ when $\zeta < \zeta^*$. In this case, an increase in ϕ enhances the second and third terms of Eq. (3.40), and the party chooses weaker patent protection (See Fig. 3.7).

Intuitively, we can find two effects when ϕ becomes higher. First, when ϕ is higher, an increase in ω raises the rate of innovation $g(\omega)$ more sharply. The higher rate of innovation increases the household's utility and campaign contributions. In other words, an increase in φ raises the marginal benefit of strengthening patent protection. Second, an increase in ϕ directly reduces the size of campaign contributions. This effect decreases the marginal benefit of strengthening patent protection. When $\zeta > \zeta^*$, the first effect dominates and the marginal benefit of strengthening

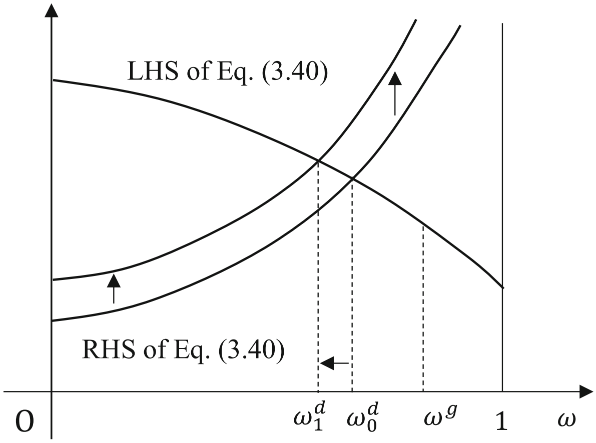


Fig. 3.7 An increase in ϕ ($\zeta < \zeta^*$)

patent protection increases because the benevolent governing party seriously considers household utility. This implies that when $\zeta > \zeta^*$, a decrease in ϕ, which corresponds to a higher proportion of campaign contributions, only discourages innovation and the party chooses weaker patent protection. In contrast, when $\zeta < \zeta$, the second effect dominates, and the marginal benefit of strengthening patent protection decreases because the party gives a lot of attention to campaign contributions. As a consequence, when $\zeta < \zeta^*$, a decrease in ϕ raises the size of campaign contributions and the corrupt governing party chooses stronger patent protection. Hence, we derive the following proposition.

Proposition 3.3 *If the governing party is benevolent and gives a lot of attention to household utility ($\zeta > \zeta^*$), a higher proportion of campaign contributions discourages innovation and the party chooses weaker patent protection. Conversely, if the party is corrupt and gives a lot of attention to campaign contributions ($\zeta < \zeta^*$), a higher proportion of campaign contributions discourages innovation while the party chooses stronger patent protection.*

3.4 Concluding Remarks

Using an endogenous growth model that incorporates only innovation as a source of economic growth, this study has examined how firms' contributions to the governing party affects IPR policy, innovation, and welfare in an economy. First, as a benchmark, we have assumed that the governing party considers only household utility and there is no campaign contribution. In this model, we have shown that welfare-maximizing patent protection is weaker than growth-maximizing protection because strengthening patent protection hurts households by increasing dead weight loss and decreasing expenditure level.

Next, we have considered the case of campaign contributions. In this case, when the governing party is corrupt and gives a lot of attention to campaign contributions, it is likely to have a strong incentive to strengthen patent protection in order to receive more contributions from the firms. Therefore, the party strengthens patent protection and increases innovation. This explains why patent policies tend to be stronger in countries where campaign contribution is important for politicians, such as the United States.

On the other hand, we observe two different effects when the fraction of campaign contributions in firm profits is higher. When the governing party is benevolent and gives a lot of attention to household utility, a higher fraction of campaign contributions discourages innovation and the party chooses weaker patent protection. This is because higher campaign contributions weaken the incentive to engage in R&D. As a consequence, this effect serves to decrease the marginal benefit of strengthening patent protection. However, when the party is corrupt, a higher fraction of campaign contributions encourages innovation and the party chooses stronger patent protection. This is because higher campaign contributions serve to enhance the marginal benefit of strengthening patent protection. The latter result is particularly interesting because a higher fraction of campaign contributions leads to a stronger IPR policy and higher rate of economic growth in countries where campaign contributions are important for politicians. However, IPR policy in this case may be too strong when we consider household utility. In other words, when campaign contributions are important for policy-making, the governing party may have an incentive to choose undesirably strong patent protection for households.

Appendix

A.1 Derivation of Household Utility in Eq. (3.28)

In this appendix, we derive the household's utility as a function of the patent policy. Combining $D(t) = \beta E(\omega)/P_{\mathrm{D}}(t)$, Eq. (3.6), $p_c = 1$ and $p_m = 1/\alpha$ yields

$$D(t) = \alpha\beta E(\omega) n(t)^{\frac{1-\alpha}{\alpha}} \left[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right]^{\frac{1-\alpha}{\alpha}}. \tag{3.45}$$

Substituting Eq. (3.45) and $Y(t) = (1-\beta)E(\omega)$ into $\beta \log D(t) + (1-\beta) \log Y(t)$ yields

$$\beta^{\beta}(1-\beta)^{\beta} + \beta\left(\frac{1-\alpha}{\alpha}\right) \log\left\{n(t)\left[\omega + (1-\omega)\alpha^{-\frac{\alpha}{1-\alpha}}\right]\right\} + \log E(t). \tag{3.46}$$

Integrating Eq. (3.46) from 0 to infinity with $n(t) = n(0)e^{g(\omega)t}$, we derive Eq. (3.28).

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Chapter 4
Abundant Resource Endowments, Institutions and Economic Growth: A Theoretical Framework and Validation Using China's Provincial Data

Gu Jinghong

Abstract What is the significance of abundant natural resources to the economic development of a country? In this study, we explain the effects of institutions on the economic growth patterns of resource-rich countries theoretically from a macrodynamic perspective by explicitly introducing a capital accumulation and institutions approach to the Big Push model. Our analysis results clarify that two natural-resource-owning countries implementing different institutions converge on two different steady states (high and low). Furthermore, we validate the relationship between the extent of natural resource endowment, the quality of institutions and economic growth using China's provincial data. The results suggest that the resource curse is present in China's provincial data; however, it is possible to convert natural resources from a curse to a grace by improving the quality of institutions.

Keywords Resource endowment · Institutional quality · Rent-seeking · Economic growth · Resource curse

4.1 Introduction

What is the significance of abundant natural resources to the economic development of a country? Ascher (1999) highlights that abundant natural resources are potentially useful for developing a country's economy. This is because countries rich in natural resources such as oil and minerals can raise their growth potential by investing resource rents into the accumulation of physical and human capital.

However, the economies of developing countries that actually possess abundant natural resources have a tendency to fall into low growth and expanding poverty. In

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an analysis based on data from 95 developing countries, Sachs and Warner (1995) find that countries with high rates of natural resource exports in their GDPs have low economic growth rates. Ross (1999) shows that countries with high rates of mineral resource exports in their GDPs tend to have expanding poverty. The relationship between abundant natural resources and the slowdown of growth and the expansion of poverty was termed 'resource curse' in the late 1990s. The term resource curse has in recent years become synonymous with the economic decline that occurred since the development of natural resources such as minerals and oil began in Nigeria and sub-Saharan African countries.

However, this is not to say the resource curse phenomenon applies to all resource-endowed countries. Countries such as Botswana, Norway, Australia and Canada possess abundant natural resources while achieving high economic growth. In particular, the diamond industry in Botswana boasts the second-largest output in the world, accounting for approximately 20% of GDP and approximately 30% of government revenue, while the country's average economic growth rate is approximately 9% over the past 30 years, one of the highest rates of economic growth in the world. Thus, why do differences occur in the economic growth patterns of countries with abundant natural resources? In this study, we explain the reason theoretically from a macrodynamic viewpoint and verify this using China's provincial data.

Many researchers debate the hypotheses surrounding the paradox of possessing abundant natural resources and economic development. There are broadly two theory systems that prove that levels of output decline in resource-rich countries. One is Dutch disease theory developed by Sachs and Warner (1999) and Gylfason et al. (1999). Dutch disease theory intends to provide a theoretical explanation that a resource boom has the effect of reducing production in the industrial sector, further reducing economic growth. Sachs and Warner (1999) explain the characteristic that countries, believing industrialisation based on abundant natural resources to be possible, conversely experience delayed industrialisation using the Big Push model advocated by Murphy et al. (1989). The model is a one-factor model with natural resources, two industrial sectors that can change from constant-yield technology to increasing-yield technology and labour. The Dush disease theory clarifies that the continuous expansion of the resource sector reduces labour input into the industrial sector, resulting in the failure of technology to change and so delaying industrialisation. Gylfason et al. (1999) analyse the effects of abundant resource endowments on production levels, assuming that learning effects from experience occur only in the industrial sector. Their results show that increases in supply capacity through the discovery and development of resources diminish the industrial sector, but the stagnation of industry, which has large external effects on other industries, reduces the production level.

Another theory that explains the phenomenon of the negative effects of abundant natural resource endowments on economic development is rent-seeking theory proposed by Torvik (2002).[1] Rent-seeking theory seeks to explain theoretically that the reason an economy with abundant natural resources is inefficient is due to the opportunity costs created by the improper allocation of resources by the government and the pursuit of resource rent. Torvik (2002) argues that the possession of abundant natural resources reduces levels of production and welfare. To show this, his analysis relies on the Big Push model, in which the industrial sector uses constant and increasing-yield technologies. He shows that, because income from rent-seeking activities in resource-rich countries is high, economic actors do not adopt yield-increasing technologies and instead carry out rent-seeking activities, and consequently, production and welfare levels deteriorate.

In both of the above studies, the possession of abundant natural resources leads to a decrease in the level of production and a deterioration in welfare. However, this phenomenon does not apply to resource-rich countries like Botswana, Canada and so on. Acemoglu et al. (2003) indicate that the difference in the patterns of economic development in resource-rich countries is due to the differing institutions. Robinson et al. (2006) focus on the relationship between institutions and the resource curse. They define a 'good institution' as an environment of institutions with a high-quality bureaucracy and low corruption surrounding the allocation of natural resources, and show that abundant natural resources will lead to increased national income with good institutions, but will cause national income to fall with bad institutions.

Mehlum et al. (2006) introduce the concept of rent-seeking to the Big Push model and analyse the effects of institutions on the resource curse phenomenon. They construct a model that allows economic agents to choose between rent-seeking activities for resource rents or to adopt yield-increasing technologies. In addition, Mehlum et al. (2006) define institutions by the extent to which they prevent rent-seeking. The analysis results show that under institutions with a low degree of rent-seeking prevention, there is a non-productive equilibrium (agents perform rent-seeking activities), while under institutions with a high degree of rent-seeking prevention, there is a productive equilibrium (agents do not perform rent-seeking activities). Furthermore, Mehlum et al. (2006) also present a discussion of the relationship between good institutions and abundant resource endowments. They take the view that achieving a productive equilibrium requires institutions in which the degree of rent-seeking prevention increases with the abundance of natural resources.[2]

In this way, existing theoretical studies show that the implementation of good institutions enables economies with natural resources to avoid the resource curse. However, the static context of the above literature does not discuss the relationship

[1]Rent-seeking is an action in which an economic agent acts to change their working environment to their own benefit at the expense of others by working with governments and bureaucrats. Resource rents in this case refer to the excess profit remaining after subtracting the cost of extraction and processing from the final price of the natural resource.

[2]Kolstad (2009) classifies the institutions defined by Mehlum et al. (2006) as private sector institutions and those defined by Robinson et al. (2006) as public sector institutions.

between abundant natural resource endowments, institutions and economic growth from a long-term perspective.

In this study, we therefore create a dynamic model that can reveal the characteristics by which resource-rich countries follow different growth paths by implementing different institutions. We define institutions by the extent to which they prevent rent-seeking in the model. Specifically, by explicitly introducing a capital accumulation and institutions approach to the Big Push model, we explain the effects of institutions on the economic growth patterns of resource-rich countries theoretically from a macrodynamic perspective and validate it using Chinese province-level panel data. Our analysis results clarify that two natural-resource-owning countries implementing different institutions converge on two different steady states (high and low). In the data analysis, we obtain the empirical result that China has a resource curse, but that by improving the quality of institutions, the government can convert these abundant natural resources from a curse to a grace.

The rest of this chapter proceeds as follows. Section 4.2 establishes the model, while Sect 4.3 considers the economic equilibrium. Then, Sect. 4.4 investigates the relationship between resource ownership, institutions and economic growth using province-level data in China. Lastly, Sect. 4.5 discusses the conclusions and future research topics.

4.2 Setup of the Model

We perform the analysis using a generational overlap model in which households live for two periods (young and old). Actors born in period t are termed the tth generation. We assume that each generation consists of unskilled and skilled workers, which we can express respectively using a continuous index from 0 to 1. Unskilled workers own only one unit of labour in their young period, and supply it inelastically to intermediate goods companies to obtain wages. Skilled workers have one unit of time in their young period, and may choose to use this for rent-seeking activities for resource rents or for innovation activities. We refer to skilled workers who carry out rent-seeking for resource rents grabbers and those who perform innovation as entrepreneurs. In addition, the economy consists of a resources sector, a production sector and households.

4.2.1 The Resources Sector

In the resources sector, we assume that only resource rent R is generated in each period. Resource rent R represents the amount of finished goods that can be procured from the international market from the export of resources. The costs of extracting and processing natural resources are abstracted to simplify the discussion. In addition, skilled workers have the opportunity to perform rent-seeking for resource rents, and the results depend on the institutions. In line with Mehlum et al. (2006), we define institutions by the extent to which they prevent rent-seeking. Expressing the quality of institutions as parameter $v \in (0, 1)$, v expresses the share of rents acquired by entrepreneurs against the resource rents obtained by grabbers. Therefore, where v is small, the degree of rent-seeking prevention is low, meaning that institutions are friendly towards grabbers. Conversely, where v is large, the degree of rent-seeking prevention is high, meaning that institutions are friendly towards entrepreneurs. Resource rents obtained by entrepreneurs and grabbers are formulated thus:

$$\pi_t^g = sR, \tag{4.1}$$

$$\pi_t^e = vsR, \tag{4.2}$$

where π_t^g and π_t^e are the resource rents obtained by the grabbers and entrepreneurs, respectively. s represents the share of resource rents obtained per grabber. In addition, since the sum of the resource rents obtained by grabbers and entrepreneurs is s, we establish the following equation,

$$vsRn_t + sR(1 - n_t) = R, \tag{4.3}$$

where n_t expresses the entrepreneurs' share and $1 - n_t$ the grabbers' share. Solving Eq. (4.3) for s, s is given by the following equation,

$$s = s(n_t, v) = \frac{1}{1 - n_t + vn_t}, \tag{4.4}$$

where, because $\frac{\mathrm{d}s(n_t, v)}{\mathrm{d}n_t} = \frac{1-v}{(1-n_t+vn_t)^2} > 0$ and $\frac{\mathrm{d}s(n_t, v)}{\mathrm{d}v} = \frac{-n_t}{(1-n_t+vn_t)^2} < 0$ are established, $s(n_t, v)$ increases as n_t increases and decreases as v increases.

4.2.2 *The Manufacturing Sector*

The economy has only one finished good of homogenous quality, and various types of intermediate goods. Finished goods are numéraire goods. We assume that intermediate goods can be expressed on a continuous index from 0 to 1. Moreover, we assume that the total number of intermediate goods types is constant.

Production of Finished Goods

The finished goods market is perfectly competitive, and firms continuously inject intermediate goods in the range [0,1] to produce goods. The production function for finished goods is

$$Y_t = \exp\left[\int_0^1 \ln X_t(i)\mathrm{d}i\right]. \tag{4.5}$$

However, Y_t and $X_t(i)$ express the quantity of finished goods produced and the amount of intermediate goods t input in period i, respectively. Finished goods companies determine the amount of intermediate goods input $X_t(i)(i \in [0,1])$ to maximise profits, taking the price of intermediate goods $p_t(i)(i \in [0,1])$ as given. Thus, the conditional factor demand function for intermediate goods i is

$$X_t(i) = \frac{\exp\left[\int_0^1 \log p_t(i)\mathrm{d}i\right]}{p_t(i)} Y_t. \tag{4.6}$$

The cost function for finished goods companies is therefore $\frac{\exp\left[\int_0^1 \log p_t(i)\mathrm{d}i\right]}{p_t(i)} Y_t$. Furthermore, since the market for finished goods is perfectly competitive, in equilibrium, the marginal cost for finished goods companies is equal to the price of finished goods.

$$\exp\left[\int_0^1 \log p_t(i)\mathrm{d}i\right] = 1. \tag{4.7}$$

From Eqs. (4.6) to (4.7), the equation for finished goods' companies demand for intermediate goods i is

$$X_t(i) = \frac{Y_t}{p_t(i)}. \tag{4.8}$$

Intermediate Goods Production and Innovation

Each intermediate good is produced by investing labour and capital. Where innovation does not occur, all companies will produce goods using traditional Cobb-Douglas technology. Where skilled workers choose to become entrepreneurs and carry out innovation in one industry, the cost of production in their chosen industry decreases. However, only goods-producing companies established by entrepreneurs carrying out innovation can use that technology.

Where using traditional technology to produce goods, the cost function is[3]

[3]In the rest of this paper, we will use cost functions to express production technology.

$$C_t^{pr}(X_t(i), r_t, w_t) = A(i) r_t^{\alpha} w_t^{1-\alpha} X_t(i), \tag{4.9}$$

where r_t is the rent price of capital and w_t is the wage rate. α is a parameter taken as $0 < \alpha < 1$. $A(i)$ shows the technology level in the intermediate goods i industry, and we assume that it is constant over time. Under Cobb-Douglas technology, companies' profits are zero in equilibrium, so the price of intermediate goods i is

$$p_t(i) = A(i) r_t^{\alpha} w_t^{1-\alpha}. \tag{4.10}$$

On the other hand, we assume that innovation occurs via fixed investment F ($F > 0$) only. Where innovation occurs in the production of intermediate goods i in period t, we assume that the ratio $1 - \mu$ reduces the cost to produce one unit of goods. Here, μ is the parameter taken as $0 < \mu < 1$. Therefore, when a given entrepreneur engages in innovation in period t for the production of intermediate goods i, the cost function is

$$C_t^{ad}(X_t(i), r_t, w_t) = (1 - \mu) A(i) r_t^{\alpha} w_t^{1-\alpha} X_t(i) + F. \tag{4.11}$$

Due to Bertrand competition, intermediate goods i companies that innovated set a price of $A(i) r_t^{\alpha} w_t^{1-\alpha}$. Therefore, we express the profit of intermediate goods i companies that innovated using the following equation,

$$\begin{aligned} \pi_t^{ad}(i) &= A(i) r_t^{\alpha} w_t^{1-\alpha} X_t(i) - (1 - \mu) A(i) r_t^{\alpha} w_t^{1-\alpha} X_t(i) - F \\ &= \mu A(i) r_t^{\alpha} w_t^{1-\alpha} X_t(i) - F. \end{aligned} \tag{4.12}$$

To simplify the discussion below, we assume that the technology level is the same in all intermediate goods industries under traditional technology. In other words, we assume $A(i) = A$. Therefore, for any arbitrary i, we assume $p_t(i) = A r_t^{\alpha} w_t^{1-\alpha}$. Substituting this into Eq. (4.7) gives the following equation.

$$p_t(i) = A r_t^{\alpha} w_t^{1-\alpha} = 1. \tag{4.13}$$

Substituting Eq. (4.13) into the intermediate goods demand function Eq. (4.8), the quantity of intermediate goods i produced is given as follows:

$$X_t(i) = Y_t. \tag{4.14}$$

Substituting Eqs. (4.13)–(4.14) into Eq. (4.12) and rearranging the terms, the profit of companies that innovated manufacturing intermediate goods i is

$$\pi_t^{ad}(i) = \mu Y_t - F. \tag{4.15}$$

Below, we will consider the factor demand for each intermediate goods company. From the cost function in Eq. (4.9), and using Shepherd's lemma, the demand for the capital and labour of companies using traditional technologies is, respectively,

$$K_t^{pr} = \frac{\alpha}{r_t} Y_t, \tag{4.16}$$

$$L_t^{pr} = \frac{1-\alpha}{w_t} Y_t. \tag{4.17}$$

Similarly, the factor demand for capital and labour from companies that innovated is, respectively,

$$K_t^{ad} = \frac{\alpha}{r_t} [(1-\mu)Y_t + F], \tag{4.18}$$

$$L_t^{ad} = \frac{1-\alpha}{w_t} [(1-\mu)Y_t + F]. \tag{4.19}$$

Due to Bertrand competition, entrepreneurs are able to innovate in different industries. Consequently, the number of companies that innovate in period t is equal to the number of entrepreneurs involved in innovation n_t. Therefore, where the share of skilled workers n_t becomes entrepreneurs, from Eqs. (4.16) and (4.18) to Eqs. (4.17) and (4.19), the equilibrium supply and demand conditions for capital and labour are as follows, respectively.

$$\frac{\alpha}{r_t} [(1-n_t)Y_t + n_t[(1-\mu)Y_t + F]] = K_t, \tag{4.20}$$

$$\frac{1-\alpha}{w_t} [(1-n_t)Y_t + n_t[(1-\mu)Y_t + F]] = L_t \equiv 1. \tag{4.21}$$

From Eqs. (4.20) to (4.21), we establish the following equation.

$$\frac{\alpha}{1-\alpha} \frac{w_t}{r_t} = K_t. \tag{4.22}$$

Furthermore, using Eqs. (4.10) and (4.22), we determine the rent prices of capital and labour as follows.

$$r_t = \alpha K_t^{\alpha-1}, \tag{4.23}$$

$$w_t = (1-\alpha)K_t^{\alpha}. \tag{4.24}$$

Therefore, capital income and wage income become αK_t^{α} and $(1-\alpha)K_t^{\alpha}$ respectively. Since income Y_t in the manufacturing sector is allocated to capital income, labour income and profit, where the share of the intermediate goods sector performing innovation is n_t, the total output satisfies the following equation.

$$Y_t = \alpha K_t^\alpha + (1-\alpha)K_t^\alpha + n_t(\mu Y_t - F) = K_t^\alpha + n_t(\mu Y_t - F). \tag{4.25}$$

Solving Eq. (4.25) for Y_t, we obtain the following for the total output Y_t.

$$Y_t = \frac{K_t^\alpha - n_t F}{1 - n_t \mu}. \tag{4.26}$$

By substituting Eq. (4.26) into Eq. (4.15), the profit of innovating companies is given by the following equation.

$$\pi_t(i) = \frac{\mu K_t^\alpha - n_t F}{1 - n_t \mu}. \tag{4.27}$$

4.2.3 The Household Sector

The tth generation households $j(j = l, g, p)$ have the following utility function.[4]

$$u_t^j = \log C_t^j + \rho \log C_{t+1}^j, \tag{4.28}$$

where u_t^j is the utility of household j in generation t, C_t^j is consumption in the youth period and C_{t+1}^j is consumption in the old age period. ρ is the time preference rate. The tth generation households allocate young period income to consumption and savings S_t^j, and turn total savings and interest into consumption in their old age period. Expressing the income of the tth generation households in their young period as I_t^j, the budget constraint facing tth generation households in their young and old age periods is, respectively,

$$I_t^j = C_t^j + S_t^j, \tag{4.29}$$

$$(1 + r_{t+1})S_t^j = C_{t+1}^j. \tag{4.30}$$

Eliminating S_t^j from Eqs. (4.29) to (4.30), the lifetime budget constraint of tth generation households is

$$C_t^j + \frac{C_{t+1}^j}{1 + r_{t+1}} = I_t^j, \tag{4.31}$$

[4]In this chapter, the subscript l expresses laborers, g grabbers and e entrepreneurs.

where l expresses labourers, g grabbers and e entrepreneurs. The tth generation households maximise their utilities in Eq. (4.28) based on Eq. (4.31). Thus, the consumption and savings functions of the t generation households are

$$C_t^j = \frac{1}{1+\rho} I_t^j, \quad S_t^j = \frac{\rho}{1+\rho} I_t^j, \quad C_{t+1}^j = \left(1 + r_{t+1}\right) \frac{\rho}{1+\rho} I_t^j. \tag{4.32}$$

4.3 Equilibrium Analysis

4.3.1 The Selection of Skilled Workers in Equilibrium

Taking capital stock as given, skilled workers decide whether to rent-seek or to innovate by comparing the income earned as a grabber π_t^g to income earned as an entrepreneur $\left(\pi_t^p + \pi_t(i)\right)$. Below, we will consider that the decisions of skilled workers depend on capital stock. In equilibrium, the following equation is established,

$$\frac{\mu K_t^\alpha - F}{1 - n_t \mu} + vsR = sR. \tag{4.33}$$

The first term on the left side of Eq. (4.33) is the profit obtained by entrepreneurs through innovation, while term 2 is the resource rent obtained by entrepreneurs. The right-hand side represents the resource rent obtained by grabbers through rent-seeking. Substituting Eq. (4.4) into (4.33) and solving for n_t, the share of skilled workers choosing to live as entrepreneurs, namely the ratio of intermediate goods companies that perform innovation n_t, is

$$n_t = \frac{\mu K_t^\alpha - F - R + Rv}{(1-v)\left(\mu K_t^\alpha - F - \mu R\right)}. \tag{4.34}$$

When all industries use traditional technology, namely when the capital stock in $n_t = 0$ is K^R, we can express K^R using the following equation.

$$K^R = \left[\frac{(1-v)R + F}{\mu}\right]^{1/\alpha}. \tag{4.35}$$

In addition, when innovation occurs in all industries, namely when capital stock in $n_t = 1$ is K^I, we obtain K^I as follows.

$$K^I = \left[\frac{(1/v-1)(1-\mu)R+F}{\mu}\right]^{1/\alpha}. \tag{4.36}$$

In order to proceed with the discussion below, we make the following assumptions.

Assumption The parameter satisfies $1 - v > \mu$.

Based on this assumption, capital stock when innovation occurs in all industries, K^I, is greater than capital stock when all industries use traditional technology, K^R. Furthermore, differentiating n_t with respect to K_t yields the following equation.

$$\frac{\mathrm{d}n_t}{\mathrm{d}K_t} = \frac{\alpha\mu K_t^{\alpha-1}(1-v-\mu)R}{(1-v)^2\left(\mu K_t^\alpha - F - \mu R\right)^2} > 0, \tag{4.37}$$

where $K_t > K^R$, $\frac{\mathrm{d}n_t}{\mathrm{d}K_t} > 0$ can be confirmed from the above assumption.[5] In other words, the share of entrepreneurs among skilled workers n_t increases as capital stock increases. As capital stock increases, the profit obtained from carrying out innovation increases, making innovation attractive to skilled workers. Thus, the number of skilled workers choosing to live as entrepreneurs increases, and the range of industries in which innovation occurs expands. Therefore, when $K_t > K^R$, some workers choose to live as entrepreneurs, and an equilibrium in which innovation occurs is achieved.[6]

In addition, since $\frac{\mathrm{d}K^R}{\mathrm{d}R} = \left(\frac{1}{\mu}\right)^{\frac{1}{\alpha}}(1-v)\frac{1}{\alpha}[(1-v)R+F]^{1/\alpha-1} > 0$ is established, K^R increases as R increases. An increase in the size of the resource endowment means that resource rents obtained by rent-seeking become comparatively larger, and profits obtained from innovation become comparatively smaller. Consequently, the number of skilled workers choosing to live as grabbers increases, and the number of skilled workers choosing to live as entrepreneurs decreases. Therefore, the abundance of natural resources is a curse in the sense that it diminishes the range of industries in which innovation occurs.

We can summarise the results of analysis thus far in the following proposition (Fig. 4.1 illustrates the contents of Proposition 4.1).

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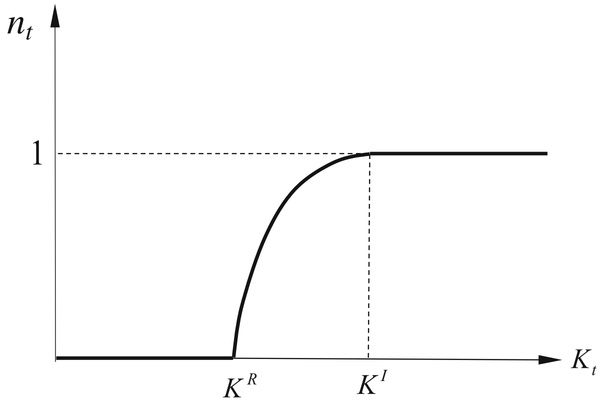
[5]From Eq. (4.37), we establish $\frac{\mathrm{d}^2 n_t}{\mathrm{d}K_t^2} = \alpha\mu K_t^{\alpha-1}(1-v-\mu)R(1-v)\left(\mu K_t^\alpha - F - \mu R\right)\cdot \frac{(1-\alpha)(F+\mu R)-(\mu+\alpha)K_t^\alpha}{K_t}$, where $K_t > \left[\frac{(1-\alpha)(F+\mu R)}{\mu+\alpha}\right]^{1/\alpha}$ and we establish $\frac{\mathrm{d}^2 n_t}{\mathrm{d}K_t^2} < 0$. Under assumption $1 - v > \mu$, this becomes $\left[\frac{(1-\alpha)(F+\mu R)}{\mu+\alpha}\right]^{1/\alpha} < K^R$. Therefore, where $K_t > K^R$, n_t is positive with respect to K_t but diminishing.

[6]$\mu K_t^\alpha < F$, namely, when $K_t < \left(\frac{F}{\mu}\right)^{1/\alpha}$, the profit when carrying out innovation becomes negative, and income decreases. From $0 < v < 1$, we can confirm that $\left(\frac{F}{\mu}\right)^{1/\alpha} < K^R$.

Fig. 4.1 Innovation levels and capital stock



Proposition 4.1. *Given $K_t \leq K^R$, all skilled workers choose to live as grabbers. When $K_t \geq K^I$, all skilled workers choose to live as entrepreneurs. When $K^R < K_t < K^I$, entrepreneurs and grabbers coexist.*

4.3.2 Dynamic Equilibrium

We assume that the investment of technology into capital goods can convert one unit of finished goods into one unit of capital goods. In addition, the capital depletion rate is assumed to be 1. Therefore, in capital market equilibrium, households' total savings S_t matches the capital stock in the next period, K_{t+1}. From Eq. (4.32), the total savings among household agents is as follows.

$$S_t = \frac{\rho}{1+\rho}\left(I_t^l + I_t^g + I_t^e\right), \tag{4.38}$$

where I_t^l is workers' income, and workers' income in manufacturing corresponds to $(1-\alpha)K_t^\alpha$. $I_t^g + I_t^e$ is the sum of grabber and entrepreneurs' income, and corresponds to the total income of intermediate goods companies carrying out resource rents and innovation. Therefore, we establish the following equation.

$$S_t = \frac{\rho}{1+\rho}\left[(1-\alpha)K_t^\alpha + R + \frac{n_t}{1-n_t\mu}\left(\mu K_t^\alpha - F\right)\right]. \tag{4.39}$$

Furthermore, from Proposition 4.1 and Eqs. (4.34) and (4.39), we can express the change in capital stock under equilibrium by the following dynamic equation.

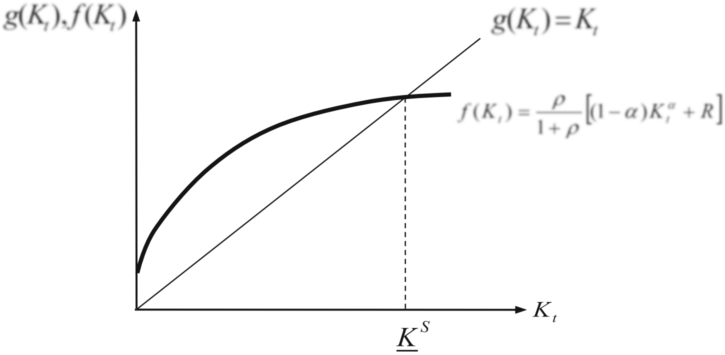


Fig. 4.2 Capital stock in the steady state under traditional technology

$$K_{t+1} = \begin{cases} \dfrac{\rho}{1+\rho}\left[(1-\alpha)K_t^{\alpha} + R\right] \text{ (where } K_t \le K^R) \\ \dfrac{\rho}{1+\rho}\left[\left(1-\alpha+\dfrac{\mu}{1-v+\mu}\right)K_t^{\alpha} + \dfrac{\mu}{1-v+\mu}R - \dfrac{F}{1-v+\mu}\right] \text{ (where } K_t > K^R) \end{cases}. \tag{4.40}$$

Below, we compare the dynamic equilibrium in which firms use traditional technology and perform innovation. We first consider the growth path when traditional techniques continue to be used. Substituting $K_{t+1} = K_t$ into the dynamic equilibrium, where $K_t \le K^R$ in Eq. (4.40) obtains the capital stock $\underline{K}^S$ under the equilibrium in which traditional technology is used. We illustrate this outcome in Fig. 4.2. Therefore, when traditional technologies are used, capital stock converges uniformly towards the steady-state value $K_{t+1} = \frac{\rho}{1+\rho}\left[(1-\alpha)K_t^{\alpha} + R\right]$ according to the dynamic equation $\underline{K}^S$. At this time, innovation does not take place, and the economy is unable to transition to a higher steady state. Therefore, innovation must be performed at some point before $\underline{K}^S$ in order to shift the economy to a higher steady state.

From Eq. (4.35) and Fig. 4.2, it is understood that while $\underline{K}^S$ is independent of v, K^R decreases as v increases. Assuming v to be $v*$, which establishes $\underline{K}^S = K^R$, there are two patterns in which the economy converges to either a low steady state or a high steady state due to the size relation between v and $v*$. Figure 4.3 shows these two patterns of economic growth. We can summarise the results of the analysis above in Proposition 4.2.

Proposition 4.2. *Consider two economies that begin with a sufficiently small capital stock,*K_0. *However, the two economies have different institutions. At this time, in the economy that implements institutions such that* $v < v*$, *innovation is not performed in the intermediate goods industries, and the economy converges to the low steady state. On the other hand, in the economy that implements institutions*

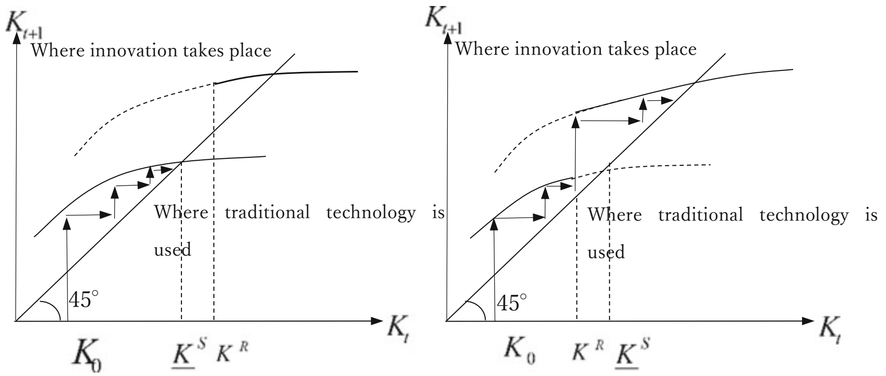


Fig. 4.3 Capital stock accumulation paths

such that $v \geq v*$, innovation is performed and the economy converges to the high steady state.

4.4 Data Analysis

In this section, we examine the relationship between the size of resource endowment, the quality of institutions and economic growth using China's provincial data. Speaking from the conclusion, from the China's provincial data the resource curse is present. However, the economy can escape the resource curse by implementing good institutions.

4.4.1 Estimate Equation

$$\begin{aligned}\boldsymbol{EG_{it}} = \boldsymbol{C} + \boldsymbol{a}_1\boldsymbol{RQ_{it}} + \boldsymbol{a}_2\boldsymbol{INVE_{it}} + \boldsymbol{a}_3\boldsymbol{EDU_{it}} + \boldsymbol{a}_4\boldsymbol{OPEN_{it}} + \boldsymbol{a}_5\boldsymbol{U_{it}} \\ + \boldsymbol{a}_6\boldsymbol{R_{it}}\boldsymbol{U_{it}} + \boldsymbol{u_{it}},\end{aligned} \tag{4.41}$$

where the subscript ***i*** expresses the province, autonomous region or direct-administered municipality, and ***t*** the time period. ***C*** is a constant term. In addition, ***EG*, *R*, *INVE***, ***EDU*, *OPEN*** and ***U*** represent the economic growth rate, extent of natural resource endowment, investment rate, education level, degree of openness and quality of institutions, respectively. As in Sachs and Warner (1999) and James and Adland (2011), we use the share of the natural resource extraction industry against total industrial production as a proxy variable for the extent of resource endowment ***RQ***. Furthermore, we evaluate the quality of institutions ***U*** using three indicators: the degree of product market development, the degree of development of the factor market and the legal environment. When the supply and price of a product are determined by the market without interference from local government or protection and better products are being constantly introduced, the degree of development of the product market is high. On the other hand, when the financial and labour markets are more competitive and the ability to put new technology into practical use is high, the degree of development of the factor market is high.

4.4.2 Data Source, Variable Description and Statistics

Considering the lack of data on the quality of institutions in the Tibet Autonomous Region, we analyse 30 provinces, autonomous regions and directly administered cities in mainland China between 1998 and 2014. We use data from the *Chinese Statistical Yearbook* for all variables (Table 4.1).

4.4.3 Estimation Results

Table 4.2 presents the estimation results, and there are three points to note. First, in the analysis using models 1–6, the coefficients for the extent of natural resource endowment ***QR*** are all negative and significant. This result shows that there is clearly a resource curse in the China's provincial data. Second, the three proxy variables for the quality of institutions, *MS*, *MF* and *LA*, are all negative. In other words, the low marketisation of the manufacturing factor market and product market, together with the weak rule of law, lowers the economic growth rate. Third, the coefficients of the cross terms for the quality of institutions and the extent of natural resource endowment are 0.0212, 0.0156 and 0.0337 in the model 5, respectively. This result suggests that by improving the quality of institutions, abundant natural resources can be converted from a curse to a benefit. Therefore, the following policies are effective in increasing economic growth in regions rich in natural resources. The first is to ensure that the rule of law functions adequately while suppressing rent-seeking. Next is for the government to greatly reduce the direct allocation of the factors of production, and to promote their allocation based on market rules, market prices and market competition. Furthermore, high-pressure sales and fraud should be eliminated, and high-quality markets developed where good products are traded for fair prices and better products are constantly being introduced.

Table 4.1 Variable definitions and descriptive statistics

Variable	Definition	Method of calculation	Average value	Standard deviation	Minimum value	Maximum value
EG	Economic growth rate	(Current period GDP—previous period GDP)/current GDP	0.1331	0.0602	0.0007	0.4311
QR	Extent of natural resource endowment	Value of output from the natural resource extraction industry/ gross industrial product	0.0982	0.0915	0.0011	0.4103
INVE	Investment rate	Value of investment/GDP	0.4035	0.9811	0.2102	0.8091
EDU	Human capital level	Number of graduates from university or junior college/total population	0.0721	0.0652	0.007799	0.3921
OPEN	Degree of openness to the outside world	Total value of imports and exports/GDP	0.01	0.01	0.00044	0.05
MS	Extent of development of the factor market	See Wang et al. (2016)	4.27	2.25	0.37	12.23
MF	Degree of development of the product market	See Wang et al. (2016)	7.43	1.63	1.46	10.61
LA	Legal environment	See Wang et al. (2016)	1.5216	1.0852	-0.45	2.8341

4.5 Concluding Remarks

Existing literature on the ownership of abundant natural resources and institutions is limited to static analysis. Contrary to these prior studies, in this study, we perform dynamic adjustment and present policy implications to promote economic growth in resource-rich countries. We analysed the effects of institutions as a means of preventing rent-seeking on the economic development of resource-rich countries using a Big Push model incorporating the rent-seeking activities of economic actors. The results show that economies follow different economic growth paths under different institutions. Under institutions with a high degree of rent-seeking prevention, some skilled workers choose to live as entrepreneurs who carry out innovation in the intermediate goods sector and the economy converges to the high steady state. On the other hand, under institutions with a low degree of rent-seeking prevention,

Table 4.2 Estimation results

Independent variables	Model 1	Model 2	Model 3	Model 4	Model 5
C	0.0824* (1.7348)	0.0675* (1.6346)	0.0654** (2.1589)	0.0706* (2.0574)	0.6493* (1.8114)
R	−0.0597* (−2.0844)	−0.0618* (−2.0749)	−0.0605* (−1.9846)	−0.0536** (−2.1096)	−0.1632* (−2.4311)
MS	−0.00425* (−2.0298)	−0.0051* (−2.4047)	−0.00359* (−2.1098)	−0.0096* (−1.9648)	−0.0104** (−1.8263)
MF	−0.0086* (−1.8208*)	−0.0109** (−3.0243)	−0.0353 (−1.9704)	−0.0932* (−2.8421)	−0.0831* (−3.1927)
LA	−0.0058** (−1.6506)	−0.0083* (−1.8720)	−0.0105* (−1.7834)	−0.1384* (−2.1927)	−0.1542** (−1.8941)
INVE		0.0037* (0.6602)	0.0038* (0.7023)	0.0035 (0.6732)	0.0032 (0.7317)
EDU		0.3821* (1.8277)	0.3888* (1.9746)	0.3794** (1.7543)	0.3981** (2.1002)
OPEN		0.0073 (0.7050)	0.0069 (0.7425)	0.0082 (0.7521)	0.0075 (0.7213)
$\boldsymbol{R * MS}$			0.0286** (1.5332)	0.0178 (1.6472)	0.0212* (1.5998)
$\boldsymbol{R * MF}$				0.0183* (1.2683)	0.0156* (1.4525)
$\boldsymbol{R * LA}$					0.0337* (1.6902)
F	23.76	22.43	24.63	25.43	20.88
R-squared	0.4062	0.6483	0.4656	0.5720	0.4934
N	510	510	510	510	510

Note: * indicates significance at the 10% level; ** indicates significance at the 5% level

no skilled workers choose to live as entrepreneurs and the intermediate goods sector uses traditional technologies, and the economy converges to the low steady state.

Furthermore, we validate the relationship between the extent of natural resource endowment, the quality of institutions and economic growth using China's provincial data. The results suggest that the resource curse is present in China's provincial data; however, it is possible to convert natural resources from a curse to a grace by improving the quality of institutions.

The conclusion relating to the interdependence of owning abundant natural resources, institutions and economic growth obtained from macrodynamic perspectives and empirical analysis suggests the importance of implementing good institutions when choosing development strategies for developing countries possessing abundant natural resources.

We suggest the following as topics for the future. The theoretical framework in this study treats institutions as exogenous. However, North (1990) views institutions as rules that govern the behaviours of economic agents, and points out that these not only affect economic performance, but are also subject to the effects of economic performance. There is therefore a need to present a theoretical model for the endogeneity of institutions.

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Part II
Macroeconomics of Family

Chapter 5
Endogenous Fertility, Childlessness, and Economic Growth

Daisuke Ikazaki

Abstract Many economically developed countries are confronting a declining birth rate and an aging population. Although many economists have attempted to explain these phenomena, they have not devoted sufficient attention to the existence of households without children. One important factor affecting the low fertility rate is high levels of childlessness. As described herein, we extend a simple growth model to consider the economic effects of rising childlessness rates on the utility of households and on the dynamic behaviors of an economy. Households are assumed to have different preferences for children. As a result of utility maximization, some households choose to have no children. Subsequently, we analyze the economic effect of an increase in the childlessness rate. We show that such a change tends to have adverse effects on household utility. Government policy, specifically a child allowance policy, is also considered. Results show that the marginal effects of such a policy might positively affect the utility of all households: households without children can benefit from such a policy. Furthermore, we provide a simple numerical example and infer the optimal level of a child allowance policy.

Keywords Childlessness, Child allowance, Heterogeneous households, Economic growth, Overlapping generations model

5.1 Introduction

In many countries, the total fertility rate (TFR) has long been declining. In many economically developed countries, the TFR is below the replacement fertility rate (about 2.1). Economic analyses of fertility rates have been developed by many researchers. One pioneer work is that of Becker (1960), who regarded children as a consumption good. Utility in such an economy depends on consumption and on the

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number of children. Households choose a family size to maximize their utility given the relative prices of commodities, as explained by Bagozzi and van Loo (1978), Galor and Weil (1996), and others. Other important works include those reported by Becker and Barro (1988) and Barro and Becker (1989). In their model, household utility depends on the level of consumption and the utility of the children. Choices of fertility and consumption derive from the maximization of a dynastic utility function. Another type of setting is that the parents expect their children's support when they become old (Zhang and Nishimura 1989).

Although many works rely on the assumption that all households have children, an increase in the childlessness rate has been observed (Fig. 5.1). Countries with high levels of childlessness often also have lower fertility rates (Fig. 5.2): childlessness is a key factor of low fertility rates. Figure 5.3 presents the predicted childlessness rate in Japan. According to these data, nearly 40% of women born in 1990 are expected to bear no children during their lifetime. Therefore, devoting attention to difficulties related to childlessness cannot be avoided if one wishes to analyze the relation between low fertility rates and dynamic economic behavior.

Gobbi (2013) provided an endogenous fertility model in which individuals have different preferences for bearing and rearing children. The results demonstrated that reduction in the gender wage gap or increased fixed costs of becoming a parent adversely affects both fertility and childlessness. Baudin et al. (2020) classify childlessness into two main types: poverty-related and high opportunity cost-related. Poverty-driven childlessness decreases as average education increases, but the main reason for childlessness shifts to high opportunity cost. Their papers include only inadequate discussion of the production sector and the role of saving (i.e., investment and physical capital).

As described in this chapter, childlessness-related difficulties are integrated into a standard overlapping generations model. Firms produce final goods using physical

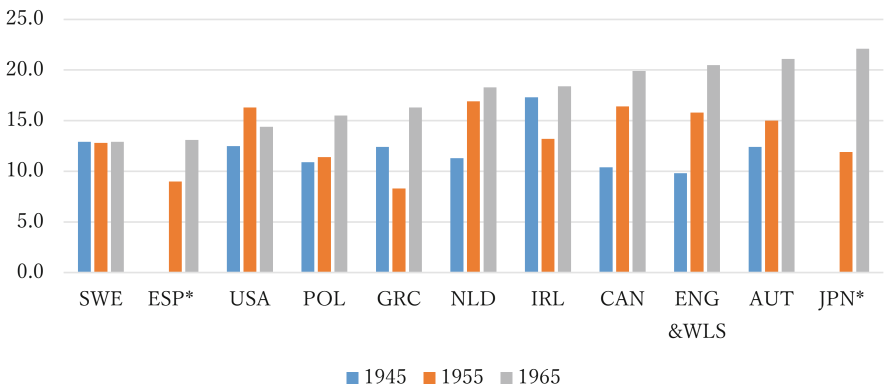


Fig. 5.1 *Definitive childlessness* (proportion (%) of definitive childless women per cohort). Note: Definitive childlessness rate is defined as the proportion of childlessness among women at the end of the reproductive period. 1965 means women born in 1965. Asterisk indicates no data found in Japan and Spain in 1945. (Source: OECD, human family database)

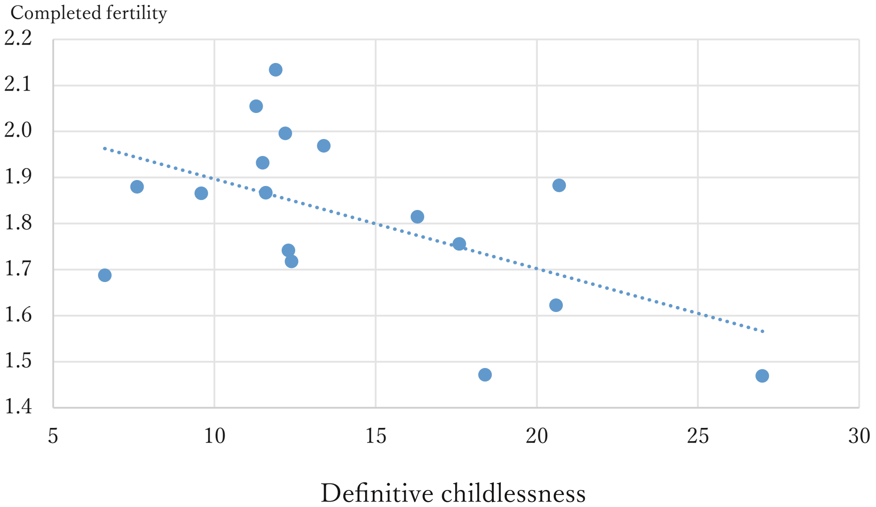


Fig. 5.2 *Definitive childlessness and completed fertility rates* (proportion (%) of cohort definitive childless and completed fertility rates of women born in 1970). (Source: OECD, human family database)

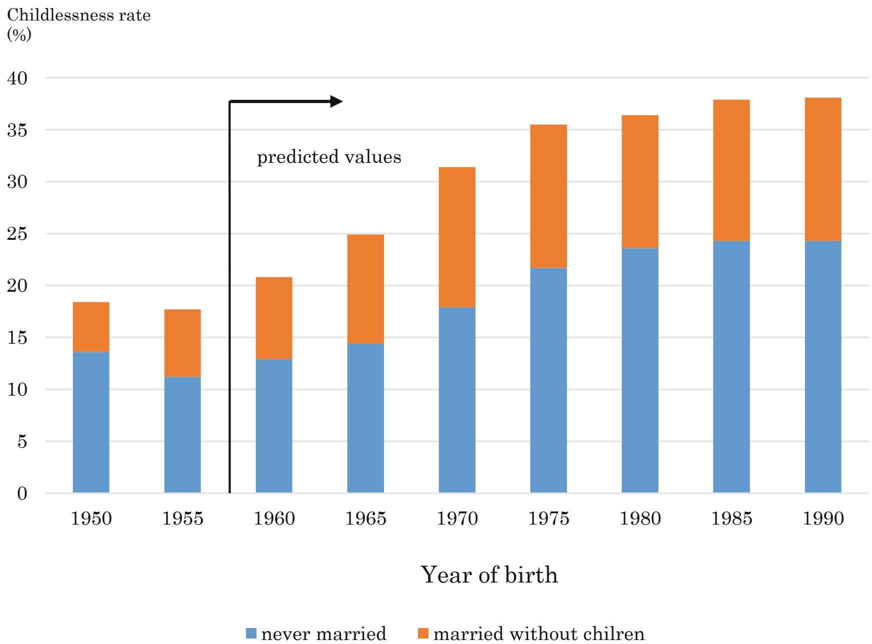


Fig. 5.3 *Childlessness rate in Japan*. Note: Childlessness rate is the proportion (%) of definitive childless women per cohort. (Source: National Institute of Population and Social Security Research, Japan)

capital and labor. As in Becker (1960) and Galor and Weil (1996), the utility of households depends on consumption and on the number of children. However, for these analyses, we assume that preferences for bearing and rearing children differ among households. Households divide their time between labor and caring for their children. Households determine the number of children to maximize their utility: if one uses time to rear children, then consumption decreases because the time spent on work declines. Preferences related to children differ among households. Therefore, the number of children and the time spent for work also differ among households. We examine the effects of an increase in the childlessness rate on economic dynamics. We also demonstrate the effects of a child allowance policy on household utility.

This chapter is organized as follows. Section 5.2 introduces a basic model discussed in this chapter. Section 5.3 derives the dynamic behavior of the economy. Section 5.4 discusses the economic effects of an increase in the childlessness rate. Section 5.5 analyzes how child care policy alters households' incentives for having children and whether or not such a policy can improve household utility. Section 5.7 presents a description of the conclusions of this chapter.

5.2 The Model

This section presents a description of the model considered herein. The final good is homogeneous. Its production function is specified as

$$Y_t = AK_t^{\alpha}L_{Yt}^{1-\alpha}, \tag{5.1}$$

where Y_t represents the aggregate output,[1] A denotes the productivity parameter, K stands for the capital stock, and L_Y signifies the labor input in this sector. We also assume that $0 < \alpha < 1$. Capital depreciates fully during the production process.

Firms maximize their profits at each date, taking interest rate r_t and wage rate w_t as given. From the firms' profit maximization (evaluated market equilibrium), one can infer that

$$(1 + r_t) = A\alpha K_t^{\alpha-1}L_{Yt}^{1-\alpha} \tag{5.2}$$

and

$$w_t = A(1-\alpha)K_t^{\alpha}L_{Yt}^{-\alpha}. \tag{5.3}$$

For a description of consumers (households), generation t is defined as people who work in period t. Individuals live for three periods. During the first period

[1] Subscript t represents the level in period t throughout this chapter.

(childhood), they make no decision. During the second period (young), they work and rear their children, if any. In the second period, individuals choose not to consume some part of their total income, but save it instead. In the final period (old), they retire and consume the savings accumulated during the second period. They also consume government services, as explained later, during the final period.

The utility of an individual in generation t (U_{it}) is defined as

$$U_{it} = \log c_{it+1} + \beta \log g_{t+1} + \gamma_i \log(1 + n_{it}), \tag{5.4}$$

where c_{it+1}, g_{t+1}, and n_{it} respectively represent consumption during the old period, the level of government service that only elderly people can consume, and the number of children of generation t of type i.[2] Also $\beta > 0$ and $\gamma_i > 0$ are assumed. We follow the standard demand model of household fertility behavior (Becker 1960; Galor and Weil 1996). These analyses ignore integer constraints on the number of children. Therefore, n_{it} can be a fractional value.

Households might have a different value of γ_i: the preference for children is not the same among households. Type i households are defined as those for which the preference for children is γ_i. We assume that γ_i is distributed uniformly between 0 and γ_a. Therefore, the ratio of the household of type i is given as $1/\gamma_a$.

During the second period, each household has one unit of time. Time spent for work by the individuals is $1 - zn_{it}$, where z denotes the time cost to rear one child. From our assumption, we obtain

$$c_{it+1} = (1 + r_{t+1})w_t(1-zn_{it})(1-\tau_t), \tag{5.5}$$

where τ_t denotes the tax rate.

A tax on wage income at a rate τ_t is imposed by government to hire labor and provide public services for elderly people. It is assumed that only old people can enjoy this service. For simplicity, one unit of labor is converted to one unit of service as $G_t = L_{Gt}$, where G_t represents the output of public service and L_{Gt} denotes the labor input in the public sector. This service can be regarded as a medical service or nursing care for elderly people. Although this service can be regarded as part of a social security system, it is not considered as a public pension or other government service to compensate income because it is not a cash benefit.

From the budget constraint of the government, $\tau_t\ w_t\ (L_{Yt} + L_{GT}) = w_t L_{Gt}$ must hold. Consequently, one can show that $\tau_t L_t = G_t$, where L_t represents the labor supply. Here, we use the labor market clearing condition: $L_{Yt} + L_{Gt} = L_t$.

The number of generation t is denoted as N_t. In period t, the old people (generation $t - 1$) consume government services. Therefore, the number of beneficiaries of that service is given as N_{t-1}. Consequently, we can show that

[2]Subscript i denotes the type of households throughout this chapter.

$$g_t = G_t/N_{t-1} = \tau_t(L_t/N_t)(N_t/N_{t-1}) = \tau_t l_t n_{t-1}, \tag{5.6}$$

where $l_t \equiv (L_t/N_t)$ denotes the average working time and $n_t \equiv (N_t/N_{t-1})$ represents the total fertility rate in period $t-1$.

A government sets the tax rate to maximize its objective function. It is assumed that government is short-lived and that its objective function is defined as $U_{t-1} + U_t$. These analyses assume rational expectations and myopic decision-making. Rational expectations mean that the short-lived government can estimate the tax rate accurately in the subsequent period. Myopic decision-making implies that the government does not consider effects of current policies on future political decisions. These assumptions imply that the government chooses tax rate τ_t taking τ_{t+1} (the tax rate in the subsequent period) as given (Verbon and Verhoeven 1992; Meijdam and Verbon 1996). We can derive the tax rate as

$$\tau_t = \frac{\beta}{1+\beta} \equiv \tau. \tag{5.7}$$

Equation (5.7) implies that the tax rate is constant over time. It is independent of the fertility rate or capital stock in the economy.

Consumers decide how much they will work and have children to maximize their utilities given as Eq. (5.4), taking w_{it}, τ_t, g_{t+1}, and r_{t+1} as given. From utility maximization, one can show that

$$n_{it} = \begin{cases} 0 & (\text{when } \gamma_i - z \le 0), \\ \dfrac{\gamma_i - z}{z(1+\gamma_i)} & (\text{when } \gamma_i - z > 0). \end{cases} \tag{5.8}$$

From Eq. (5.8), we can obtain the following proposition.

Proposition 5.1. *There exists a unique value of $\gamma_i = z$ for which households are indifferent between being childless or not.*

The number of children (n_{it}) is positive if and only if $\gamma_i - z > 0$. Therefore, $(z/\gamma_a) \times 100\%$ of households decide to have no children. From Eq. (5.8), we can calculate the fertility rate in period t (which is defined as n_t) as

$$\begin{aligned} n_t &= \int_z^{\gamma_a} \frac{1}{\gamma_a} \frac{\gamma_i - z}{z(1+\gamma_i)} \mathrm{d}\gamma_i \\ &= \frac{1}{z\gamma_a} \left\{ \gamma_a - z - (1+z) \log\left(\frac{1+\gamma_a}{1+z}\right) \right\} \equiv n. \end{aligned} \tag{5.9}$$

It is noteworthy that the definition of total fertility rate here differs from the one used generally. For these analyses, we do not distinguish between men and women. Therefore, n denotes the number of children per adult rather than the number per woman. For that reason, $2n$ might be regarded as the natural definition of the total fertility rate. From Eq. (5.9), we obtain the following Lemma.

Lemma 5.1. *An increase in the childlessness rate defined as a decrease in γ_a has a negative effect on the total fertility rate.*

Proof From Eq. (5.9), it can be verified that

$$\frac{\partial n}{\gamma_a} = \frac{1}{z\gamma_a^2(1+\gamma_a)}\left\{z - \gamma_a + (1+\gamma_a)(1+z)\log\left(\frac{1+\gamma_a}{1+z}\right)\right\}.$$

Let us define

$$f(\gamma_a) = z - \gamma_a + (1+\gamma_a)(1+z)\log\left(\frac{1+\gamma_a}{1+z}\right).$$

$f(z) = 0$. Furthermore, one can show that

$$f'(\gamma_a) = z + (1+z)\log\left(\frac{1+\gamma_a}{1+z}\right) > 0.$$

Consequently, $f(\gamma_a) > 0$ when $\gamma_a > z$. From a definition of $f(\gamma_a)$, it can be shown that $\frac{\partial n}{\gamma_a} > 0$ if $\gamma_a > z$. Therefore, an increase in the childlessness rate defined as a decrease in γ_a negatively affects the fertility rate. Q.E.D.

From Lemma 5.1, a decline in γ_a is known to reduce n. An increase in the childlessness rate will decrease the fertility rate. This result is supported by the empirical data (Fig. 5.2). From Eq. (5.9), the total labor supply in the economy can be expressed as shown below.

$$\begin{aligned} L_t &= \int_0^{\gamma_a} \frac{N_t}{\gamma_a}(1 - zn_{it})\mathrm{d}i \\ &= \frac{N_t}{\gamma_a}\left\{z + (1+z)\log\left(\frac{1+\gamma_a}{1+z}\right)\right\} \equiv N_t l = N_t(1 - zn). \end{aligned} \tag{5.10}$$

From Eqs. (5.9) and (5.10), the relation between total fertility rate and the average working time is given as follows.

$$n = \frac{1-l}{z} \tag{5.11}$$

5.3 Dynamic Behavior of the Economy

This section presents analysis of dynamic behavior of the economy. From Eq. (5.11), it is shown that

$$N_{t+1} = N_t n = N_t(1-l)/z. \tag{5.12}$$

Regarding capital, one can demonstrate that

$$\begin{aligned} K_{t+1} &= w_t L_t(1-\tau) \\ &= A(1-\alpha)K_t^{\alpha}[(1-\tau)L_t]^{-\alpha}(1-\tau)L_t \\ &= A(1-\alpha)[(1-\tau)]^{1-\alpha}l^{1-\alpha}N_t k_t^{\alpha}, \end{aligned} \tag{5.13}$$

where $k_t \equiv K_t/N_t$. From Eqs. (5.12) and (5.13), it can be shown that

$$k_{t+1} \equiv \frac{Az(1-\alpha)[(1-\tau)l]^{1-\alpha}}{1-l} k_t^{\alpha}. \tag{5.14}$$

One can then readily demonstrate that k_t converges to a unique steady state monotonically. From Eq. (5.14),

$$k^* \equiv \left(\frac{A(1-\alpha)}{n}\right)^{\frac{1}{1-\alpha}}(1-\tau)(1-zn) \tag{5.15}$$

in the steady state, where k^* denotes the steady-state value of k_t.

5.4 Effects of Declining Birth Rate on Utility

This section presents specific examination of the steady state and presents discussion of how demographic changes affect household utility. From Eqs. (5.5), (5.7), (5.8), and (5.15) and by use of the fact that $k_t = k_{t+1} = k^*$ in the steady state, one can rewrite c_{it+1}, $1+n_{it}$, and g_{it+1} as shown below.

$$\begin{aligned} c_{it+1} &= (1+r_{t+1})w_t(1-zn_{it})(1-\tau) \\ &= A^{\frac{1}{1-\alpha}}\alpha(1-\alpha)^{\frac{\alpha}{1-\alpha}}\left(\frac{1-l}{z}\right)^{\frac{1-2\alpha}{1-\alpha}}\left(\frac{1+z}{1+\gamma_i}\right)\left(\frac{1}{1+\beta}\right), \end{aligned} \tag{5.16}$$

$$1+n_{it} = \frac{\gamma_i(1+z)}{z(1+\gamma_i)}, \tag{5.17}$$

$$g_{t+1} = \frac{\beta l(1-l)}{z(1+\beta)}. \tag{5.18}$$

We denote U_i instead of U_{it} because the utility of each generation remains unchanged in the steady state. Here, we will consider the effect of an increase in the childlessness rate (a decrease in γ_a) on household utility. Although some other definition of "an increase in the childlessness rate" might be given, we specifically examine the case of a decline in γ_a herein. We can show that

$$\frac{\partial U_t}{\gamma_a} = \frac{\partial U_t}{l}\frac{\partial l}{n}\frac{\partial n}{\gamma_a}, \tag{5.19}$$

and

$$\frac{\partial U_t}{l} = -\frac{1-2\alpha}{1-\alpha}\frac{1}{1-l} - \frac{\beta}{1-l} + \frac{\beta}{l} = \frac{\beta(1-\alpha) - l(1-2\alpha)}{(1-\alpha)l(1-l)}. \tag{5.20}$$

Equation (5.20) implies that $\partial U_t/\partial l < 0$ if and only if

$$l > \frac{\beta(1-\alpha)}{1-2\alpha}. \tag{5.21}$$

Consequently, we can obtain the following proposition.

Proposition 5.2. *An increase in the childlessness rate, defined as a decrease in γ_a, has a negative effect on the utilities of households if and only if $l > \frac{\beta(1-\alpha)}{1-2\alpha}$.*

Proof: $\partial U_i/\partial \gamma_a > 0$ if and only if $\partial U_i/\partial l < 0$ because $\frac{\partial U_{it}}{\partial \gamma_a} = \frac{\partial U_{it}}{\partial l}\frac{\partial l}{\partial n}\frac{\partial n}{\partial \gamma_a}$, $\partial l/\partial n < 0$ (see Eq. 5.11), and $\partial n/\partial \gamma_a > 0$ (see Lemma 5.1). Therefore, $\partial U_i/\partial \gamma_a > 0$ when inequality Eq. (5.21) holds. Q.E.D.

We next consider details of Proposition 5.2. Here, α is the share of capital in the production. Therefore, α is regarded as approximately 0.3. Also, β is the weight in the utility function on government service for old people. Estimating this is not a simple matter. We infer the value of β using the fact that the tax rate for this service is $\beta/(1 + \beta)$. Table 5.1 presents the relation between the tax rate and the value of β. Table 5.2 presents the case in which α is 0.3–0.35 and β is 0.1–0.3 (the tax rate is 9.1–23.1%). This range of β might be acceptable if one considers the fact that

Table 5.1 Relation between the tax rate and the value of β

β	0.1	0.15	0.2	0.25	0.3
Tax rate (%) ($\beta/(1+\beta)$)	9.1	13.0	16.7	20.0	23.1

Table 5.2 Value of $\frac{\beta(1-\alpha)}{1-2\alpha}$

		β				
		0.1	**0.15**	**0.2**	**0.25**	**0.3**
α	**0.3**	0.175	0.2625	0.35	0.4375	0.525
	0.33(=1/3)	0.2	0.3	0.4	0.5	0.6
	0.35	0.216	0.325	0.4333	0.5417	0.65

government service here does not include public pensions and that it is provided mainly to elderly people.

Baudin et al. (2020) estimated the time cost for one child as 0.188. Prettner and Werner (2016) used 0.19 for their simulation. However, it is noteworthy that the preconditions of their model and ours differ. Baudin et al. (2020) also considered the fixed cost for rearing children and the wage gap separating genders. Furthermore, decision-making is done by a couple in their model rather than by an individual. Prettner and Werner (2016) assumed other costs for education. Therefore, it might be a good idea to use a higher value of z than they used for their studies. Presuming for the moment that z is 0.2–0.3 and that n is 0.6–1.0,[3] then, roughly speaking, l can be regarded as 0.7–0.9. Therefore, inequality 21 is likely to hold if one assumes the value of l as 0.7–0.9 (see Table 5.2).

Therefore, our model predicts that an increase in the childlessness rate tends to affect the utilities of all households adversely because an increase in the childlessness rate will negatively affect the lifetime real income. Furthermore, government service that each household can consume will decrease because the number of young people providing services declines. These effects will reduce the utility of households.

5.5 Child Allowance Policy

This section presents consideration of the effects of child allowance policy. We assume that the capital income of elderly people is subject to a tax imposed by government. We have already assumed that a part of a wage income of the young generation is collected to provide a government service. Therefore, this policy can be regarded as income redistribution between young and old generations. Defining a tax rate as ξ_t, we assume that the household can receive $m_t w_t(1 - \tau_t)$ per child. The budget constraint of the government is

$$\xi_t A\alpha K_t^{\alpha} L_{Yt}^{1-\alpha} = m_t w_t (1 - \tau_t) \int_0^{\gamma_a} \frac{N_t}{\gamma_a} n_i \mathrm{d}i. \tag{5.22}$$

As for government services for elderly people, we can show again that $\tau_t = \frac{\beta}{1+\beta} \equiv \tau$.

Let us consider the households. The child allowance policy alters the constraint of households, Eq. (5.5), as

[3]We specifically examine the economies of economically developed countries. Family support policies are enforced strongly in countries where total fertility rate is high. For the moment, we discuss a case without such policies. Therefore, the upper limit of n we suppose here might be too high. The probability that inequality 21 holds increases if the upper limit of n is lower.

$$c_{it+1} = (1 + r_{t+1})(1 - \xi_{t+1})w_t(1 - \tau_t)(1 - zn_{it} + m_t n_{it}). \tag{5.23}$$

The problem of each household is to maximize utility given as Eq. (5.4), subject the constraint (Eq. 5.23), taking w_{it}, τ_t, m_t, ξ_{t+1}, g_{t+1} and r_{t+1} as given. From utility maximization, one can obtain

$$n_{it} = \begin{cases} 0 & (\text{when } \ \gamma_i \leq z - m_t), \\ \dfrac{\gamma_i - (z - m_t)}{(z - m_t)(1 + \gamma_i)} & (\text{when } \ \gamma_i > z - m_t). \end{cases} \tag{5.24}$$

From Eq. (5.24), the number of children is known to increase with m_t. Furthermore, a child allowance policy decreases the childlessness rate in the economy. Marginal households (type-z households) decide to have children as a result of such a policy. The childlessness rate becomes $(z - m_t)/\gamma_a$. Regarding type $[z - m_t, z]$ households, they have no children before the policy is introduced. However, their marginal benefit of having children becomes predominant over the cost if a child allowance policy is introduced. The number of their children becomes positive when they can receive a child allowance.

From Eq. (5.24), one can calculate the fertility rate and labor supply as

$$\begin{aligned} n_t &= \int_{z-m_t}^{\gamma_a} \frac{1}{\gamma_a} \frac{\gamma_i - (z - m_t)}{(z - m_t)(1 + \gamma_i)} \mathrm{d}\gamma_i \\ &= \frac{1}{(z - m_t)\gamma_a} \left\{ \gamma_a - z + m_t - (1 + z - m_t) \log \left(\frac{1 + \gamma_a}{1 + z - m_t} \right) \right\} \equiv n_{mt} \end{aligned} \tag{5.25}$$

and

$$\begin{aligned} L_t \equiv N_t l_{mt} &= \int_0^{\gamma_a} \frac{N_t}{\gamma_a} (1 - zn_{mt}) \mathrm{d}i \\ &= N_t(1 - zn_{mt}). \end{aligned} \tag{5.26}$$

Next, the dynamic behavior of the economy is analyzed. Regarding capital, it can be shown that

$$\begin{aligned} K_{t+1} &= w_t L_t (1-\tau) + w_t (1-\tau) m_t N_t n_{mt} \\ &= A(1-\alpha)(1-\tau)^{1-\alpha} l_m^{-\alpha} N_t (1-zn_{mt} + m_t n_{mt}) k_t^{\alpha}. \end{aligned}$$

Then, using the fact that $N_{t+1} = N_t n_{mt}$, the dynamic behavior of k_t can be shown as follows.

$$k_{t+1} \equiv \frac{A(1-\alpha)(1-\tau)^{1-\alpha} l_m^{-\alpha}}{n_{mt}} (1 - zn_{mt} + m_t n_{mt}) k_t^{\alpha}. \tag{5.27}$$

We assume that m_t is constant: $m_t = m$. We also denote n_{mt} as n_m because the total fertility rate is independent of t if m_t is constant over time. We can show that

$$k^{m*} \equiv \left(\frac{A(1-\alpha)}{n_m}\right)^{\frac{1}{1-\alpha}} (1-\tau)(1 - zn_m + mn_m)^{\frac{1}{1-\alpha}} (1 - zn_m)^{\frac{-\alpha}{1-\alpha}} \tag{5.28}$$

in the steady state. Hereinafter, we specifically examine the steady state as before.

Let us consider next the manner in which child allowance policy affects household utility. From Eq. (5.22), we obtain

$$\xi_t = \xi = \frac{(1-\alpha)mn_m}{\alpha(1 - zn_m)}. \tag{5.29}$$

Because $k_t = k_{t+1} = k^{m*}$ in the steady state, c_{it+1}, 1 + n_{it}, and g_{t+1} are derived as

$$\begin{aligned} c_{it+1} &= (1 + r_{t+1}) w_t (1 - zn_{it})(1-\tau)(1-\xi) \\ &= A^2 \alpha (1-\alpha) \left(\frac{n_m}{A(1-\alpha)}\right)^{\frac{1-2\alpha}{1-\alpha}} \left(\frac{1 - zn_m + mn_{mt} 1 + z}{1 - zn_m}\right)^{\frac{2\alpha - 1}{1-\alpha}} \left(\frac{1 + z - m}{1 + \gamma_i}\right) \\ &\quad \left(\frac{1}{1+\beta}\right)\left(1 - \frac{(1-\alpha)mn_m}{\alpha(1 - zn_m)}\right), \end{aligned} \tag{5.30}$$

$$1 + n_{it} = \frac{\gamma_i (1 + z - m)}{(z - m)(1 + \gamma_i)}, \tag{5.31}$$

$$g_{t+1} = \frac{\beta l_m (1 - l_m)}{z(1+\beta)} \quad . \tag{5.32}$$

One can then consider child allowance policy effects on household welfare. First, we show that the following relations hold in equilibrium:

$$\gamma_a \frac{\partial (z - m) n_m}{\partial m} = \log \frac{1 + \gamma_a}{1 + z - m}, \tag{5.33}$$

and

$$\gamma_a \frac{\partial n_m}{\partial m} = \frac{1}{(z - m)^2} \left(\gamma_a - z + m - \log \frac{1 + z - m}{1 + \gamma_i}\right). \tag{5.34}$$

From Eqs. (5.4), (5.7), (5.30)–(5.34), it can be verified that

$$\frac{\partial U_i}{\partial m}=\left(\frac{(1-2\alpha)}{(1-\alpha)}+\beta\right)\frac{\partial n_m}{\partial m}\frac{1}{n_m}+\frac{(1-2\alpha)}{(1-\alpha)}\frac{1}{(1-zn_m+mn_m)}\log\frac{1+\gamma_a}{1+z-m}$$
$$-\left(\frac{(1-2\alpha)}{(1-\alpha)}\right)\frac{z}{(1-zn_m)}\frac{\partial n_m}{\partial m}+\frac{1}{(1-z+m)}-\frac{(\alpha z+(1-\alpha)m)\frac{\partial n_m}{\partial m}+(1-\alpha)n_m}{\alpha(1-zn_m)-(1-\alpha)mn_m}$$
$$+\frac{z\frac{\partial n_m}{\partial m}}{(1-zn_m)}+\gamma_i\left(\frac{1}{z-m}-\frac{1}{1+z-m}\right)\frac{1}{(1-z+m)}-\beta\frac{z\frac{\partial n_m}{\partial m}}{1-zn_m}, \tag{5.35}$$

if $\gamma_i > z - m_t$. For households with no children, one can show that

$$\frac{\partial U_i}{\partial m}=\left(\frac{(1-2\alpha)}{(1-\alpha)}+\beta\right)\frac{\partial n_m}{\partial m}\frac{1}{n_m}+\frac{(1-2\alpha)}{(1-\alpha)}\frac{1}{(1-zn_m+mn_m)}\log\frac{1+\gamma_a}{1+z-m}$$
$$-\left(\frac{(1-2\alpha)}{(1-\alpha)}\right)\frac{z}{(1-zn_m)}\frac{\partial n_m}{\partial m}-\frac{(\alpha z+(1-\alpha)m)\frac{\partial n_m}{\partial m}+(1-\alpha)n_m}{\alpha(1-zn_m)-(1-\alpha)mn_m}$$
$$+\frac{z\frac{\partial n_m}{\partial m}}{(1-zn_m)}-\beta\frac{z\frac{\partial n_m}{\partial m}}{1-zn_m}. \tag{5.36}$$

Here, we consider the marginal effects of the child allowance policy: we evaluate $\frac{\partial U_i}{\partial m}$ at $m = 0$. From Eqs. (5.33)–(5.36), we obtain

$$\left.\frac{\partial U_i}{\partial m}\right|_{m=0}=\left(\frac{(1-2\alpha)}{(1-\alpha)}+\beta\right)\left(\frac{1}{n_m}-\frac{z}{(1-zn_m)}\right)\frac{1}{z^2}\left(\gamma_a-z-\log\frac{1+z}{1+\gamma_a}\right)$$
$$+\frac{(1-2\alpha)}{(1-\alpha)}\frac{1}{(1-zn_m)}\log\frac{1+\gamma_a}{1+z}+\frac{1}{(1-z)}-\frac{1-\alpha}{\alpha}\frac{n_m}{(1-zn_m)}+\gamma_i\frac{z}{z(1+z)}, \tag{5.37}$$

if households have children. However, the following equation must hold when households have no children:

$$\left.\frac{\partial U_i}{\partial m}\right|_{m=0}=\left(\frac{(1-2\alpha)}{(1-\alpha)}+\beta\right)\left(\frac{1}{n_m}-\frac{z}{(1-zn_m)}\right)\frac{1}{z^2}\left(\gamma_a-z-\log\frac{1+z}{1+\gamma_a}\right)$$
$$+\frac{(1-2\alpha)}{(1-\alpha)}\frac{1}{(1-zn_m)}\log\frac{1+\gamma_a}{1+z}-\frac{1-\alpha}{\alpha}\frac{n_m}{(1-zn_m)}. \tag{5.38}$$

Then we can present the following proposition.

Proposition 5.3.

1. *Assume that households have children before the child allowance policy is introduced. Then a marginal effect of such a policy on the utility of each household becomes positive if inequality* Eq. (5.37) *holds.*
2. *Assume that households have no children before the child allowance policy is introduced. Then a marginal effect of such a policy on the utility of each household becomes positive if inequality* Eq. (5.38) *holds.*

5.6 Numerical Example

5.6.1 Marginal Effect of Child Allowance Policy

Sections 5.4 and 5.5 present the theoretical framework of the model. This section provides numerical examples and presents derivation of the economic effects of a child allowance policy. First, we analyze the marginal effects of such policies.

Figure 5.4 presents the marginal effects of child allowance policy on household utility. In Fig. 5.4a, the vertical line shows the value of $\left.\frac{\partial U_i}{\partial m}\right|_{m=0}$; the horizontal line represents γ_a. In Fig. 5.4a, we assume that $\alpha = 0.3$, $\beta = 0.2$, and $z = 0.2$ and take the interval of γ_a as [0.6, 1.4] ($2n$ is 0.91–2.63). The bold and the solid lines respectively show the marginal effects of a policy on the type-γ_a and the type-z households. The dotted line shows marginal effect on households without children. In these parameter restrictions, such a policy has a positive effect on the utility of households with children. For households without children, such a policy has a positive effect if γ_a is less than 1.37 (the corresponding total fertility rate, $2n$, is less than 2.58). Therefore, the marginal effects of this policy might become positive effects on the utilities of *all* households for a wide parameter range.

Figure 5.4b presents the case in which $\alpha = 0.35$, $\beta = 0.1$, and $z = 0.25$ and takes the interval of γ_a as [0.7, 1.7] ($2n$ is 0.75–2.29). All changes in the parameter values tend to have negative effects on utility. A similar tendency is apparent even if these parameter restrictions are imposed. Households with children can enjoy benefits from the policy. Such a policy has a positive effect on the utility of households without children if the total fertility rate is less than some critical value (in this case, the corresponding critical value of $2n$ is 2.05). Therefore, the child allowance policy seems to have a positive effect on the utility of all households if we specifically examine the case most economically developed countries are confronting today.[4]

[4] Actually, total fertility rates in some economically developed countries are around 2. However, such countries tend to employ a strong family support policy. The value 2.05 here is the total fertility rate without such a policy. Therefore, we might say that this restriction tends to hold in many economically developed countries. The U.S., in which many immigrants contribute to high fertility rates, might be regarded as an exception.

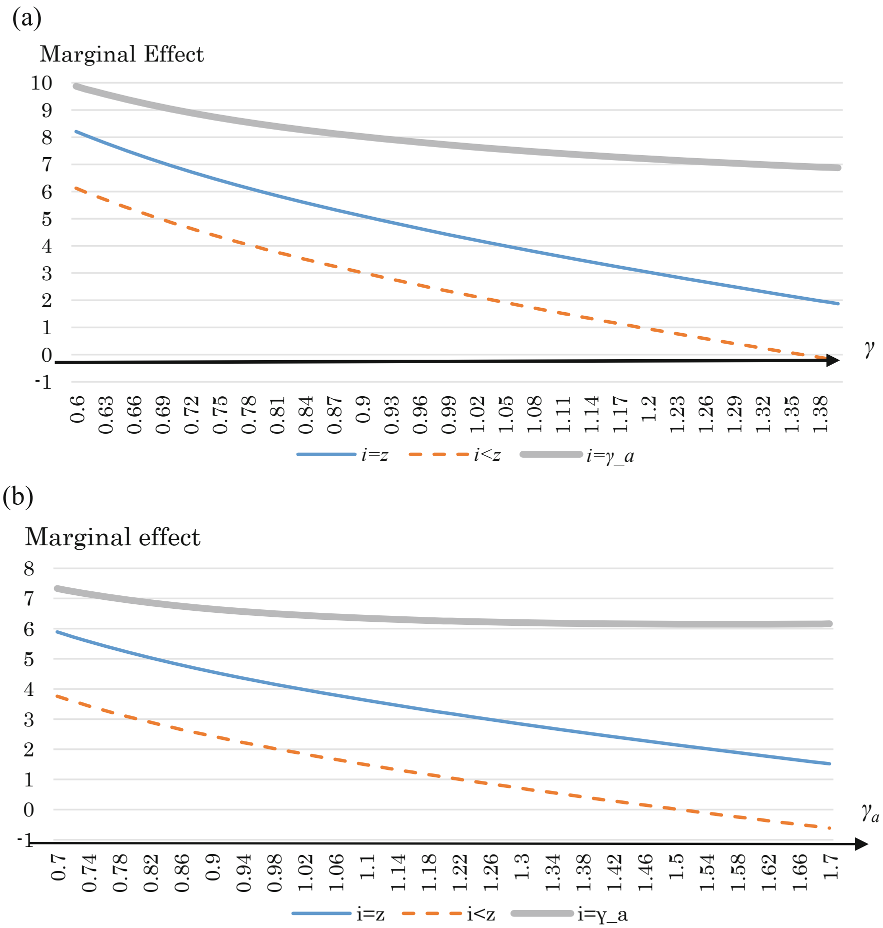


Fig. 5.4 (**a**) Marginal effect of childcare policy on three types of households ($\alpha = 0.3$, $\beta = 0.2$, $z = 0.2$). (**b**) Marginal effect of childcare policy on three types of households ($\alpha = 0.35$, $\beta = 0.1$, $z = 0.25$)

It is particularly interesting that households without children tend to benefit from such policies. This is true even though they must pay additional taxes for children of other households and do not gain the monetary benefits directly. The intuition for this result is the following. First, the households without children can earn higher wages when they are young because the labor supply is decreased. Second, they can earn higher capital income when they become old because total saving decreases. Third, they can also obtain more government services when they become old because the number of children increases as a result of the child allowance policy.

These benefits are predominant over the costs. Therefore, the child allowance policy might increase the utility of households without children as well as households that receive a subsidy to rear the children.

5.6.2 Relation Between the Tax Rate and Utility

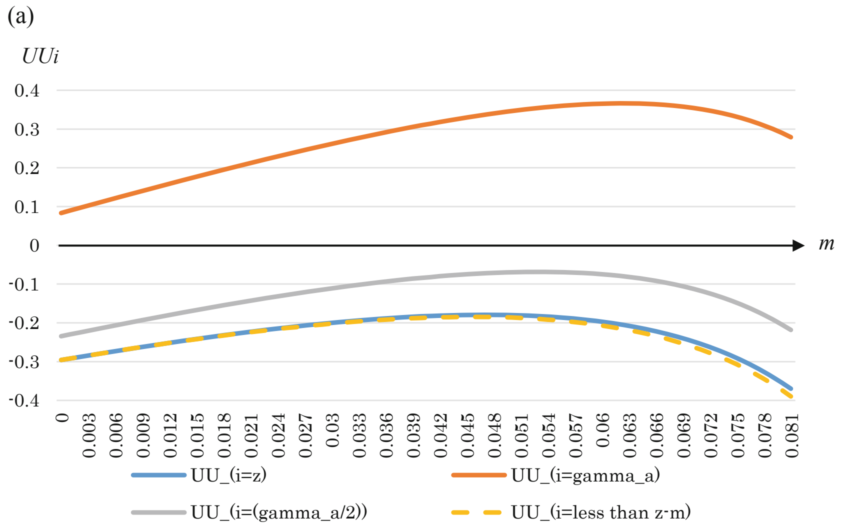
This subsection presents an examination of the relation between U_i and m. We compare households of four types for which the values of γ_i are γ_a, $\gamma_a/2$, z and less than $z - m$. First, γ_a is the highest value of γ_i (households that have the highest preference for children); $\gamma_a/2$ is the median. The government maximizes the utility of ($\gamma_a/2$)-type households if we assume that the policy is determined by political processes and employ the median voter theorem. Type-z refers to the marginal households that began to rear children after the policy was introduced. Type-j households, those for which j is less than $z - m$, do not choose to rear children even if such a policy is introduced.

Figure 5.5 shows the relation between UU_i and m, where UU_i is defined as Utility minus constant terms of the utility. Tables 5.3 and 5.4 show the optimal subsidy rate for each household. Parameter values are assumed as $\alpha = 0.3$, $\beta = 0.2$, $z = 0.2$, and $\gamma_a = 0.8$ in Fig. 5.5a and Table 5.3. An alternative case is shown in Fig. 5.5b and Table 5.4, where $\alpha = 0.35$, $\beta = 0.1$, $z = 0.25$, and $\gamma_a = 0.8$.

The effect of a child allowance policy on the utility differs among households. Households with a higher value of γ_i can obtain more benefit from that policy. We can also confirm that the optimal subsidy rate differs among households. The higher the value of γ_i, the higher is the optimal tax rate.

One can consider the case where the parameters are $\alpha = 0.3$, $\beta = 0.2$, $z = 0.2$, and $\gamma_a = 0.8$. For households with no children, the optimal value of m is about 0.045. Optimal values are about 0.053, 0.057, and 0.062, respectively, for type-z, type-(γ_a/2), and type-γ_a. Optimal subsidy rates are 22.5% for households without children and 31.0% for type-γ_a households if one interprets m/z as the subsidy rate (see Table 5.3). The subsidy rate determined by the government is 26.5% if one uses the median voter theory. The utilities of all households increase if such a policy is employed. The total fertility rate (per person) increases to 1.16 from 0.71. The childlessness rate decreases to 18.4% from 25.0%.

In Table 5.4, where $\alpha = 0.35$, $\beta = 0.1$, and $z = 0.25$, the optimal values of m are 0.067 (26.8%) for the household without children and 0.091 (36.4%) for type-γ_a. The subsidy rate determined by the government is 31.2% if we assume the median voter theory. The total fertility rate (per person) increases to 0.91 from 0.47. The childlessness rate decreases to 21.5% from 31.2%. We can also demonstrate that such a tax rate increases the utility of all households.



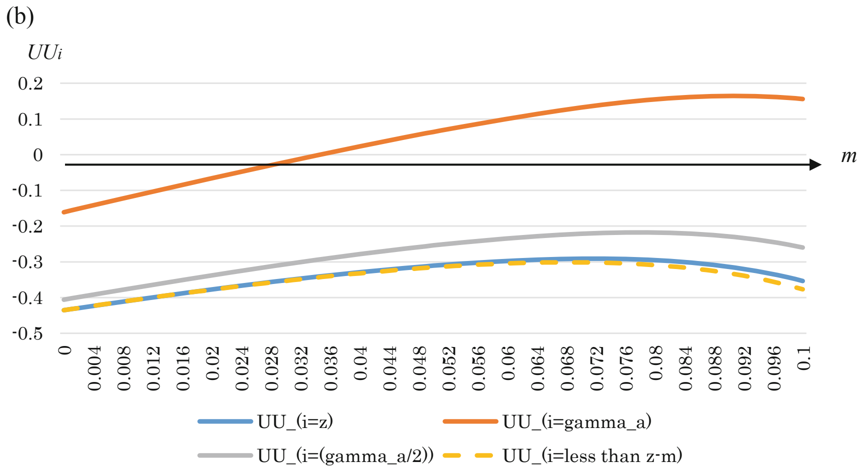


Fig. 5.5 (**a**) Relation between (adjusted) utility and m ($\alpha = 0.3$, $\beta = 0.2$, $z = 0.2$, $\gamma_a = 0.8$). (**b**) Relation between (adjusted) utility and m ($\alpha = 0.35$, $\beta = 0.1$, $z = 0.225$, $\gamma_a = 0.8$)

Table 5.3 Optimal subsidy rate for each household: $\alpha = 0.3$, $\beta = 0.2$, $z = 0.2$, and $\gamma_a = 0.8$

	Type of household			
Variable	Without children	z	$\gamma_a/2$	γ_a
Optimal value of m	0.045	0.047	0.053	0.062
Optimal value of subsidy rate (m/z) (%)	22.5	23.5	26.5	31.0

Table 5.4 Optimal subsidy rate for each household: $\alpha = 0.35$, $\beta = 0.1$, $z = 0.25$, and $\gamma_a = 0.8$

Variable	Type of household			
	Without children	z	$\gamma_a/2$	γ_a
Optimal value of m	0.067	0.072	0.078	0.091
Optimal value of subsidy rate (m/z) (%)	26.8	28.8	31.2	36.4

5.7 Concluding Remarks

As described herein, we construct a simple growth model and examine how an increase in the childlessness rate affects economic dynamics and the household utility level. Results show that a rise in the childlessness rate decreases the total fertility rate in the economy and tends to have negative effects on the utility of all households in the steady state.

Next, we discussed the effects of child allowance policy on the utility of households. Results verify that the marginal effect of such policy tends to be positive for all households if we impose reasonable parameter restrictions: households without children can also be a beneficiary of such a policy. An increase in the total fertility rate will enrich government services that older households can enjoy. The wage rate increases because such policies decrease the labor supply. These positive effects tend to predominate over the associated cost: increased taxes for child allowance policy. We consider the marginal effect. Therefore, the utility of households without children can also increase even if their tax burden increases and they cannot receive a child allowance. We also demonstrated that the utilities of all households increase if we assume a median voter theory: child care policy might be a Pareto-improving policy.

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Chapter 6
Economic Growth and Work–Life Balance

Hideo Noda

Abstract A number of developed countries have recently been paying considerable attention to labor policies that take work–life balance into consideration. Needless to say, the purpose of work–life balance policies is to help increase people's life satisfaction through an improved balance between their work and private lives. However, whether conventional policies are appropriate for this purpose has yet to be adequately verified based on internationally comparable macroeconomic data. The primary purpose of this study is to answer the following questions: (1) how does a change in work–life balance affect the life satisfaction of men and women? and (2) are there any region-specific factors in various global regions that affect life satisfaction? We use data from the Better Life Index published by the Organization for Economic Co-operation and Development (OECD) to examine the relationship between the work–life balance indicator based on the percentage of employees paid to work more than 50 h per week on a regular basis and life satisfaction. The results of our cross-sectional analysis do not support the hypothesis that the work–life balance indicator based on the percentage of employees paid to work more than 50 h per week on a regular basis affects the life satisfaction of either men or women. In addition, analysis of the regional characteristics of the 34 countries comprising the OECD found no indication of regional differences affecting the life satisfaction of either men or women.

Keywords Economic growth · Health · Life satisfaction · Unemployment · Work–life balance

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6.1 Introduction

Since the 2000s, the governments of many developed countries have emphasized labor policies that incorporate consideration of the balance between workers' work and personal lives. Meanwhile, the work–life balance concept has been widely disseminated among the general public. For instance, a search of Amazon.com in September 2019 using the term "work–life balance" returned more than 3000 hits. Work–life balance policies, as one might expect, are aimed at improving people's work–life balance with the aim of increasing their life satisfaction. To date, there have been no adequate studies of the effects of work–life balance policies on people's life satisfaction, with no studies conducted using internationally comparable macroeconomic data. This study analyzes data from the 34 countries that were members of the OECD as of January 2016 in an attempt to answer the following questions: What effect does a change in work–life balance have on the life satisfaction of men and women? Are there region-specific factors in various global regions that affect life satisfaction?

Many previous studies of work–life balance have addressed issues such as developing a work environment that complies with work–life balance policies, workers' comfort in their jobs, and effects on worker productivity from the firm's perspective in particular countries. For example, Abendroth and den Dulk (2011) focused on the role of some types of support in relation to work–life balance. Based on a survey of 7867 service-sector workers in eight European countries, the results showed that support for employees' work–life balance has a direct and positive effect. In addition, emotional support and instrumental support in the workplace were found to have a complementary relationship. Emotional support from the family had a positive impact on employees' work–life balance, while instrumental support from the family did not. Using data from 87 Spanish small and medium enterprises (SMEs), Adame et al. (2016) analyzed the relationship between women's presence in the workplace and the implementation of work–life balance policies. Using a fuzzy-set approach, they concluded that the number of women in the workplace did not determine the level of implementation of work–life balance policies, while the absence of women did appear to be related to the absence of such policies. In addition, the absence of organizational commitment to work–life balance was found to lead to an absence of work–life balance policies. Adame-Sánchez et al. (2016) examined whether SMEs display a common pattern of behavior when implementing work–life balance policies. Using fuzzy-set qualitative comparative analysis, they found no association between a particular combination of factors and the implementation of work–life balance policies, and identified two key points. First, potential market-based benefits were the main factor determining whether the firm implemented work–life balance policies. Second, a greater degree of perceived benefits (i.e., organizational performance) made the implementation of work–life balance policies more attractive to human resources managers.

Nonetheless, the implementation of work–life balance policies is now a common trend in many economically developed countries. In other words, active efforts to improve employees' work–life balance are not limited to particular countries. Using internationally comparable data to identify certain features related to work–life balance policies that are shared by developed countries makes it possible to propose a policy that is beneficial across numerous countries rather than only being applicable to a few countries. In this sense, an empirical study of work–life balance based on internationally comparable data has academic significance. Although the recent promotion of work–life balance policies is primarily aimed at increasing people's life satisfaction, as noted earlier, few studies have empirically demonstrated a positive correlation based on internationally comparable data. Noda (2020) is one of the few studies to have attempted to explain the relationship between life satisfaction and work–life balance through macroeconometric analysis using cross-national data. Noda (2020) used data from the 2014 edition of the OECD Better Life Index and emphasized the percentage of time devoted to leisure and personal care as the primary indicator of work–life balance. The result of this analysis suggested that in terms of the percentage of time devoted to leisure and personal care, an increase in work–life balance would help improve life satisfaction for both men and women.

Data on work–life balance from the OECD Better Life Index include an indicator of the percentage of employees who usually work 50 h or more per week. However, Noda (2020) did not use this indicator to examine the relationship between work–life balance and life satisfaction. When performing cross-sectional analysis using internationally comparable data, any region-specific factors that might affect life satisfaction should be controlled for in advance. However, Noda (2020) did not investigate the possible presence of such region-specific factors, thereby ignoring an important analytical issue. This study addresses the above-mentioned issues using data from the 2015 edition of the OECD's Better Life Index. Therefore, this study falls within the research field of approaches based on macroeconometric analysis and complements the study of Noda (2020). The cross-sectional analysis undertaken in this study found no relationship between the work–life balance indicator based on the percentage of employees who usually work 50 h or more per week and life satisfaction. Further, an examination of the possible effects of region-specific factors in OECD member countries in regions such as Asia, North America, and Latin America found no evidence of such factors. These findings applied to both men and women.

The rest of this chapter is organized as follows. Section 6.2 presents a theoretical examination of the mechanism underlying the observation that the average number of working hours decreases as an economy develops as a preliminary examination for the empirical analysis. Section 6.3 examines an indicator of work–life balance (the percentage of employees who usually work 50 h or more per week), an indicator of health (self-reported health), and an indicator of employment (long-term unemployment rate) as determinants of life satisfaction using data from the 2015 edition of the OECD's Better Life Index. This study also aims to confirm the presence or absence of region-specific factors that affect life satisfaction using region dummy variables. Section 6.4 presents a summary of the main results and proposes areas for future research.

6.2 Barro and Sala-i-Martin's Labor/Leisure Choice Model

Work–life balance literally represents a balance between people's jobs outside the home and their personal lives, which, in economics, is closely related to the problem of selection between work and leisure in terms of household finances. Figure 6.1 shows the average annual hours worked per person in employment during the period 2000–2016 based on labor statistics from the OECD published by the Japan Institute for Labour Policy and Training (2018).

As can be seen from Fig. 6.1, there is a flat or declining trend in the average total annual working hours per worker in Japan, the US, Germany, France, Italy, and South Korea. Such a trend in time-series data is consistent with rational household behavior aiming to maximize lifetime utility. In other words, the selection of working hours and leisure time by households attempting to maximize their lifetime utility under given conditions results in a decrease in the average working hours of each household during the process of transition to the steady state, until they become mostly constant in the neighborhood of the steady state. The changes in the average annual hours worked per person in employment shown in Fig. 6.1 conform to the implications of the relevant economic models. Based on the labor–leisure choice model of Barro and Sala-i-Martin (2004, Ch. 9), this section theoretically explains that working hours gradually decrease as the economy approaches a steady state.

Let $N(t)$ be the total population of a country at time t. We assume that the total population grows exogenously at a constant rate n. The labor input of the country is expressed as $L(t)$. Barro and Sala-i-Martin (2004, Ch. 9) express the intensity of the

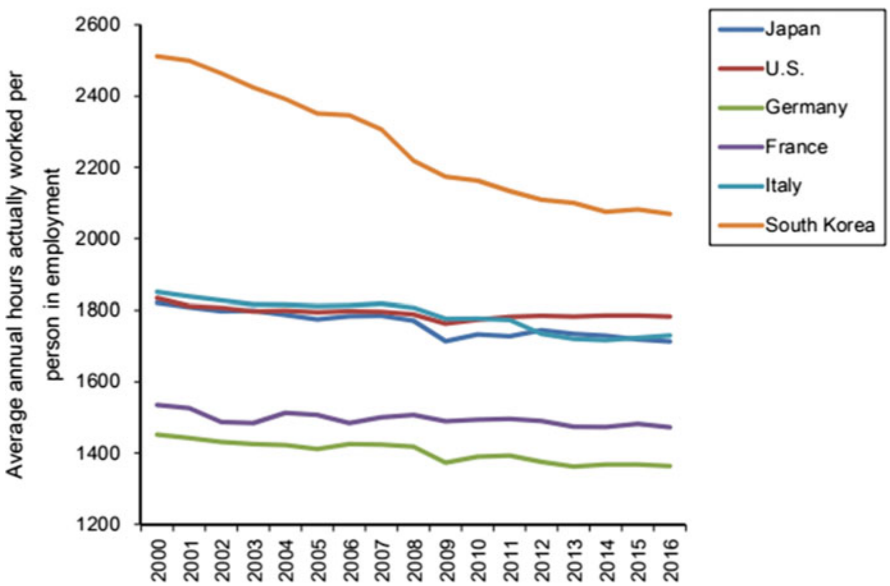

Fig. 6.1 Average annual hours worked per person in employment (2000–2016)

labor effort as $\ell(t)$ and define it as $L(t) \equiv \ell(t)\, N(t)$. In effect, $\ell(t)$ can be interpreted as the percentage of time spent on labor.

When we express the total assets of the household sector as $A(t)$, the increase in total assets of household sector, $\dot{A}(t) \equiv dA(t)/dt$, is determined by

$$\begin{aligned} \dot{A}(t) &= w(t)L(t) + r(t)A(t) - C(t) \\ &= w(t)\,\ell(t)\,N(t) + r(t)A(t) - C(t), \end{aligned} \tag{6.1}$$

where $C(t)$ represents aggregate consumption, $r(t)$ represents the interest rate, and $w(t)$ represents the wage rate. Defining $c(t) \equiv C(t)/N(t)$ and $a(t) \equiv A(t)/N(t)$ and dividing both sides of Eq. (6.1) by $N(t)$ yields

$$c(t) + \frac{\dot{A}(t)}{N(t)} = w(t)\ell(t) + r(t)k(t). \tag{6.2}$$

Differentiating $a(t) = A(t)/N(t)$ with respect to time implies that

$$\frac{\dot{A}(t)}{N(t)} = \dot{a}(t) + na(t). \tag{6.3}$$

Using Eqs. (6.2) and (6.3), we obtain the budget constraint of the household as follows:

$$c(t) + \dot{a}(t) + na(t) = w(t)\ell(t) + r(t)a(t). \tag{6.4}$$

Therefore, Eq. (6.4) can be rewritten as

$$\dot{a}(t) = w(t)\ell(t) + [r(t) - n]a(t) - c(t). \tag{6.5}$$

The discounted present value of lifetime utility, U, is given by

$$U = \int_0^\infty u[c(t), \ell(t)\,]e^{-(\rho-n)t}dt. \tag{6.6}$$

In Eq. (6.6), ρ represents the rate of time preference. From Eqs. (6.5) and (6.6), the present value Hamiltonian is set up as follows:

$$\mathcal{H} = u(c, \ell)e^{-(\rho-n)t} + \nu[w\ell + (r-n)a - c],$$

where ν is a co-state variable. When the solution to this constrained optimization problem exists, the necessary conditions to be satisfied by the solution are given by

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \quad \Rightarrow \quad \nu = \frac{\partial u}{\partial c} e^{-(\rho - n)t}, \tag{6.7}$$

$$\frac{\partial \mathcal{H}}{\partial \ell} = 0 \quad \Rightarrow \quad \nu = -\frac{\frac{\partial u}{\partial \ell} e^{-(\rho - n)t}}{w}, \tag{6.8}$$

$$\dot{\nu} = -\frac{\partial \mathcal{H}}{\partial a} \quad \Rightarrow \quad \dot{\nu} = -(r - n)\nu, \tag{6.9}$$

and the transversality condition

$$\lim_{t \to \infty} [\nu(t) a(t)] = 0. \tag{6.10}$$

From Eq. (6.7), we get

$$\frac{d\nu}{dt} = \frac{\partial^2 u}{\partial c^2} e^{-(\rho - n)t} \dot{c} + \frac{\partial^2 u}{\partial \ell \partial c} e^{-(\rho - n)t} \dot{\ell} - \frac{\partial u}{\partial c} (\rho - n) e^{-(\rho - n)t}. \tag{6.11}$$

In addition, substitution from (6.7) and (6.9) into (6.11) leads to

$$r = \rho - \frac{\left(\frac{\partial^2 u}{\partial c^2} c\right)}{\left(\frac{\partial u}{\partial c}\right)} \cdot \left(\frac{\dot{c}}{c}\right) - \frac{\left(\frac{\partial^2 u}{\partial \ell \partial c} \ell\right)}{\left(\frac{\partial u}{\partial c}\right)} \cdot \left(\frac{\dot{\ell}}{\ell}\right). \tag{6.12}$$

Furthermore, Eqs. (6.7) and (6.8) imply that

$$-\frac{\left(\frac{\partial u}{\partial \ell}\right)}{\left(\frac{\partial u}{\partial c}\right)} = w\,. \tag{6.13}$$

Barro and Sala-i-Martin (2004, Ch. 9) formulate the utility function, $u(c, \ell)$, as follows:

$$u(c, \ell) = \frac{c^{1-\theta} \exp\left[(1 - \theta)\omega(\ell)\right] - 1}{1 - \theta}. \tag{6.14}$$

where θ in Eq. (6.14) is a positive parameter indicating the inverse of the elasticity of intertemporal substitution for consumption. We also assume that $\omega'(\ell) < 0$ and $\omega''(\ell) \leq 0$. From Eq. (6.14),

$$\frac{\partial^2 u}{\partial \ell \partial c} = (1-\theta)c^{-\theta}e^{(1-\theta)\omega(\ell)}\omega'(\ell). \tag{6.15}$$

Because $\omega'(\ell) < 0$ based on the assumption, the following relationships hold:

$$\theta < 1 \quad \Rightarrow \quad \frac{\partial^2 u}{\partial \ell \partial c} < 0,$$
$$\theta = 1 \quad \Rightarrow \quad \frac{\partial^2 u}{\partial \ell \partial c} = 0,$$
$$\theta > 1 \quad \Rightarrow \quad \frac{\partial^2 u}{\partial \ell \partial c} > 0.$$

When $u(c, \ell)$ is given by Eq. (6.14), Eq. (6.13) can be rewritten as

$$-\omega(\ell) = \frac{w}{c} \tag{6.16}$$

In the following discussion, we focus on the specific case $\theta \to 1$. Because Eq. (6.14) becomes an indeterminate form when $\theta \to 1$, by application of L'Hospital's rule, we obtain

$$u(c, \ell) = \log c + \omega(\ell) \tag{6.17}$$

When the utility function is expressed by Eq. (6.17), $u_{c\ell} = 0$, $u_{cc} - 1/c^2$, and $u_c = 1/c$ hold. Substitution of these into Eq. (6.12) yields

$$r = \rho - \frac{\dot{c}}{c}. \tag{6.18}$$

Following Barro and Sala-i-Martin (2004), the growth rate of the variable z is expressed as γ_z. Then, Eq. (6.18) can be rewritten as

$$\gamma_c = r - \rho \ . \tag{6.19}$$

Let $B(t)$ be the technological level of a country. We assume that $B(0) = 1$. When the technological advancement rate is expressed as x, $B(t) = e^{xt}$ holds.

The profit Π of a firm of level $\widehat{L} \equiv B \times L = e^{xt}L$ is given by the following equation (see Barro and Sala-i-Martin, 2004, Ch. 2):

$$\Pi = \widehat{L}\left[f\left(\widehat{k}\right) - (r+\delta)\widehat{k} - we^{-xt}\right], \tag{6.20}$$

where δ represents the depreciation rate.

Competitive firms that are price-takers regard the interest rate r and the wage rate w as given. The following condition must be satisfied if a firm is to maximize its profit in Eq. (6.20) for a given $\widehat{L}$:

$$f'\left(\widehat{k}\right) = r + \delta. \tag{6.21}$$

That is, the firm selects the capital–labor ratio $k \equiv K/N$ to match the marginal productivity of capital and the rental price $r + \delta$.

The profit gained as a result will either be zero or have a positive or negative sign. When the profit is positive, the firm can increase profit to an unlimited degree by increasing labor input. Therefore, it is likely that it will attempt to input an infinite amount of labor. However, this would not hold in an equilibrium condition. When profit is negative, the firm should reduce its labor input to zero. The reason is that zero labor input would result in zero profit. Thus, the firm would be able to increase its profit. However, because nothing is produced when labor input is zero, such an equilibrium is excluded from our analysis. Therefore, labor input in an equilibrium becomes a positive finite value only when profit is zero. Then, we obtain

$$w = \left[f\left(\widehat{k}\right) - \widehat{k}f'\left(\widehat{k}\right)\right]e^{xt}. \tag{6.22}$$

We define $\widehat{k} \equiv K/(\ell N e^{xt}) = k/(\ell e^{xt})$ and $\widehat{c} \equiv C/(\ell N e^{xt}) = c/(\ell e^{xt})$. Moreover, $\widehat{c} = c/(\ell e^{xt})$ implies that

$$\gamma_{\widehat{c}} = \gamma_c - \gamma_\ell - x\,. \tag{6.23}$$

Using Eqs. (6.19) and (6.23), we obtain

$$\gamma_{\widehat{c}} = r - \rho - \gamma_\ell - x. \tag{6.24}$$

Moreover, Eq. (6.21) and (6.24) imply that

$$\gamma_{\widehat{c}} = f'\left(\widehat{k}\right) - (\delta + \rho + x) - \gamma_\ell. \tag{6.25}$$

From $\widehat{k} = k/(\ell e^{xt})$, we obtain

$$\gamma_{\widehat{k}} = \gamma_k - \gamma_\ell - x\,. \tag{6.26}$$

Furthermore, Eq. (6.4) leads to

$$\frac{\dot{k}}{k} = \frac{w\ell}{k} + r - n - \frac{c}{k}. \tag{6.27}$$

Substitution from Eq. (6.27) into (6.26) leads to

$$\gamma_{\widehat{k}} = \frac{we^{-xt}}{\widehat{k}} - r - n - x - \frac{\widehat{c}}{\widehat{k}} - \gamma_{\ell}. \tag{6.28}$$

Additionally, substitution from Eqs. (6.21) and (6.22) into (6.28) leads to

$$\gamma_{\widehat{k}} = \frac{f\left(\widehat{k}\right)}{\widehat{k}} - (x + n + \delta) - \frac{\widehat{c}}{\widehat{k}} - \gamma_{\ell}. \tag{6.29}$$

The following section develops an argument in the case of the Cobb–Douglas production function.

$$Y = F(K, AL) = K^{\alpha}(AL)^{1-\alpha}, \tag{6.30}$$

where α is a parameter that satisfies $0 < \alpha < 1$. It can easily be confirmed that α denotes the capital share and $1 - \alpha$ represents the labor share. From Eq. (6.30), we obtain

$$\widehat{y} = f\left(\widehat{k}\right) = \widehat{k}^{\alpha}, \tag{6.31}$$

where $\widehat{y} \equiv Y/(AL)$.

To examine the case in which the elasticity of the marginal disutility of labor is constant, we assume that

$$\omega(l) = -\zeta l^{1+\sigma}, \tag{6.32}$$

where ζ and σ are parameters that satisfy $\zeta > 0$ and $\sigma \geq 0$. In this case, the following relationship holds if the elasticity of the marginal disutility of labor is expressed as η:

$$\eta = \frac{\left(\frac{\partial^2 u}{\partial \ell^2}\right)\ell}{\left(\frac{\partial u}{\partial \ell}\right)} = \sigma.$$

Based on the Cobb–Douglas production function in Eq. (6.31), we obtain

$$w(t) = (1 - \alpha)\widehat{k}^{\alpha} e^{xt}. \tag{6.33}$$

Substituting Eq. (6.33) into (6.16) and considering Eq. (6.32) yields

$$\ell = \left[\frac{(1-\alpha)}{\zeta(1+\sigma)}\frac{\widehat{y}}{\widehat{c}}\right]^{\frac{1}{1+\sigma}}. \tag{6.34}$$

As can be seen from Eq. (6.34), when ℓ rises, $\widehat{c}/\widehat{y}$ falls. Furthermore, differentiating the natural logarithms of both sides of Eq. (6.34) with respect to time, we have

$$\gamma_\ell = \left(\frac{\alpha}{1+\sigma}\right)\gamma_{\widehat{k}} - \left(\frac{1}{1+\sigma}\right)\gamma_{\widehat{c}}. \tag{6.35}$$

We obtain $\gamma_{\widehat{k}}$ and $\gamma_{\widehat{c}}$ in the case of the Cobb–Douglas production function in Eq. (6.31). First, Eq. (6.35) can be rewritten for γ_ℓ using Eqs. (6.25) and (6.29) as follows:

$$\gamma_\ell = \left(\frac{1-\alpha}{\alpha+\sigma}\right)(x+\delta) - \left(\frac{\alpha}{\alpha+\sigma}\right)n - \left(\frac{\alpha}{\alpha+\sigma}\right)\frac{\widehat{c}}{\widehat{k}} + \left(\frac{1}{\alpha+\sigma}\right)\rho. \tag{6.36}$$

Therefore, combining Eqs. (6.29), (6.31), and (6.36) yields

$$\gamma_{\widehat{k}} = \widehat{k}^{\alpha-1} - \left(\frac{1}{\alpha+\sigma}\right)\left[\sigma\left(\frac{\widehat{c}}{\widehat{k}}\right) + (1+\sigma)(x+\delta) + \rho + \sigma n\right]. \tag{6.37}$$

In addition, using Eqs. (6.25), (6.31), and (6.36), we obtain

$$\gamma_{\widehat{c}} = \alpha\widehat{k}^{\alpha-1} + \left(\frac{1}{\alpha+\sigma}\right)\left[\alpha\left(\frac{\widehat{c}}{\widehat{k}}\right) - (1+\sigma)(x+\delta) - (1+\alpha+\sigma)\rho + \alpha n\right]. \tag{6.38}$$

We find from Eq. (6.25) that $\widehat{k}$ is constant in the steady state. When $\widehat{k}$ is constant, Eq. (6.2) implies that $\widehat{c}$ must also be constant in the steady state. Therefore, based on Eq. (6.34), the relationship $\left(\gamma_{\widehat{c}}\right)^* = \left(\gamma_{\widehat{k}}\right)^* = (\gamma_\ell)^* = 0$ holds in the steady state. Consequently, combining Eqs. (6.25) and (6.31) leads to

$$\widehat{k}^{\alpha-1} = \frac{\rho+\delta+x}{\alpha}. \tag{6.39}$$

In addition, from Eqs. (6.29) and (6.31), we have

$$\widehat{k}^{\alpha-1} = (x+n+\delta) + \frac{\widehat{c}}{\widehat{k}}. \tag{6.40}$$

Therefore, Eqs. (6.39) and (6.40) imply that

$$\widehat{k}^{\alpha-1}\frac{\widehat{k}}{\widehat{c}} = \frac{\delta+\rho+x}{\delta+\rho+x-\alpha(n+x+\delta)}. \tag{6.41}$$

Using Eqs. (6.34) and (6.41), labor intensity in the steady state can be written as follows:

$$\ell^* = \left[\frac{1-\alpha}{\zeta(1+\sigma)}\cdot\frac{\delta+\rho+x}{\delta+\rho+x-\alpha(n+x+\delta)}\right]^{\frac{1}{1+\sigma}}. \tag{6.42}$$

If $n > \rho$, Eq. (6.42) implies that the steady-state value of labor intensity decreases when the rate of technological progress increases.

Next, we derive the $\dot{\widehat{k}} = 0$ locus and the $\dot{\widehat{c}} = 0$ locus. $\gamma_{\widehat{k}}$ is given by

$$\gamma_{\widehat{k}} = \widehat{k}^{\alpha-1} - \left(\frac{1}{\alpha+\sigma}\right)\cdot\left[\sigma\left(\frac{\widehat{c}}{\widehat{k}}\right) + (1+\sigma)(x+\delta)+\rho+\sigma n\right]$$

and $\gamma_{\widehat{c}}$ is given by

$$\gamma_{\widehat{c}} = \alpha\widehat{k}^{\alpha-1} + \left(\frac{1}{\alpha+\sigma}\right)\cdot\left[\alpha\left(\frac{\widehat{c}}{\widehat{k}}\right) - (1+\sigma)(x+\delta) - (1+\alpha+\sigma)\rho+\alpha n\right].$$

The $\dot{\widehat{k}} = 0$ locus is obtained from Eq. (6.37) as follows:

$$\widehat{c} = \left(\frac{\alpha+\sigma}{\sigma}\right)\widehat{k}^{\alpha} - \frac{1}{\sigma}[(1+\sigma)(x+\delta)+\rho+\sigma n]\widehat{k}. \tag{6.43}$$

The $\dot{\widehat{c}} = 0$ locus is obtained from (6.38) as follows:

$$\widehat{c} = -(\alpha+\sigma)\widehat{k}^{\alpha} + \frac{1}{\alpha}[(1+\sigma)(x+\delta)+(1+\alpha+\sigma)\rho-\alpha n]\widehat{k}. \tag{6.44}$$

For the $\dot{\widehat{k}} = 0$ locus, Eq. (6.43) implies that

$$\frac{d\widehat{c}}{d\widehat{k}} = \alpha\left(\frac{\alpha+\sigma}{\sigma}\right)\widehat{k}^{-(1-\alpha)} - \frac{1}{\sigma}[(1+\sigma)(x+\delta)\rho+\sigma n]$$

and

$$\frac{d^2\widehat{c}}{d\widehat{k}^2} = -\alpha(1-\alpha)\left(\frac{\alpha+\sigma}{\sigma}\right)\widehat{k}^{-(2-\alpha)} < 0.$$

Consequently, the $\dot{\widehat{k}} = 0$ locus is concave in the interval $(0, \infty)$. In addition, we find that when $\widehat{k} \to 0$, $\left(d\widehat{c}/d\widehat{k}\right) \to \infty$, and when $\widehat{k} \to \infty$, $\left(d\widehat{c}/d\widehat{k}\right) \to -(1/\sigma) \cdot [(1+\sigma)(x+\delta)\rho + \sigma n]$.

Conversely, regarding the $\dot{\widehat{c}} = 0$ locus,

$$\frac{d\widehat{c}}{d\widehat{k}} = -\alpha(\alpha+\sigma)\widehat{k}^{\alpha-1} + \frac{1}{\alpha}[(1+\sigma)(x+\delta) + (1+\alpha+\sigma)\rho - \alpha n]$$

and

$$\frac{d^2\widehat{c}}{d\widehat{k}^2} = \alpha(1-\alpha)(\alpha+\sigma)\widehat{k}^{-(2-\alpha)} > 0.$$

It follows that the $\dot{\widehat{c}} = 0$ locus is convex in the interval $(0, \infty)$. Moreover, when $\widehat{k} \to 0$, $\left(d\widehat{c}/d\widehat{k}\right) \to -\infty$, and when $\widehat{k} \to \infty$, $\left(d\widehat{c}/d\widehat{k}\right) \to -(1/\alpha) \cdot [(1+\sigma)(x+\delta) + (1+\rho+\sigma) - \alpha n]$.

Here, we demonstrate that the $\dot{\widehat{k}} = 0$ locus and the $\dot{\widehat{c}} = 0$ locus intersect in the first quadrant of plane $\left(\widehat{k}, \widehat{c}\right)$. Assume that in Eq. (6.43), the value of $\widehat{k}$ is $\widehat{k}_1$ when $\widehat{c} = 0$. Then,

$$\left(\frac{\alpha+\sigma}{\sigma}\right)\widehat{k}_1^{\alpha} - \frac{1}{\sigma}[(1+\sigma)(x+\delta) + \rho + \sigma n]\widehat{k}_1 = 0.$$

Solving for $\widehat{k}_1$ yields

$$\widehat{k}_1 = \left[\frac{(1+\sigma)(x+\delta) + \rho + \sigma n}{(\alpha+\sigma)}\right]^{-\frac{1}{(1-\alpha)}}. \tag{6.45}$$

In Eq. (6.44), we express the value of $\widehat{k}$ as $\widehat{k}_2$ when $\widehat{c} = 0$. Therefore, we have

$$-(\alpha+\sigma)\widehat{k}_2^{\alpha} + \frac{1}{\alpha}[(1+\sigma)(x+\delta) + (1+\alpha+\sigma)\rho - \alpha n]\widehat{k}_2 = 0.$$

Solving for $\widehat{k}_2$ gives

$$\widehat{k}_2 = \left\{\frac{\left(\frac{1}{\alpha}\right) \cdot [(1+\sigma)(x+\delta) + (1+\alpha+\sigma)\rho - \alpha n]}{(\alpha+\sigma)}\right\}^{-\frac{1}{(1-\alpha)}}. \tag{6.46}$$

We now define Γ and Δ as follows:

$$\Gamma = (1+\sigma)(x+\delta) + \rho + \sigma n$$

and

$$\Delta = \left(\frac{1}{\alpha}\right) \cdot [(1+\sigma)(x+\delta) + (1+\alpha+\sigma)\rho - \alpha n].$$

Therefore, Eqs. (6.45) and (6.46) can be rewritten as follows:

$$\widehat{k}_1 = \left(\tfrac{\Gamma}{\alpha+\sigma}\right)^{-\frac{1}{(1-\alpha)}} \tag{6.47}$$

and

$$\widehat{k}_2 = \left(\tfrac{\Delta}{\alpha+\sigma}\right)^{-\frac{1}{(1-\alpha)}}. \tag{6.48}$$

From Eqs. (6.47) and (6.48), if $\Delta > \Gamma$, $\widehat{k}_1 > \widehat{k}_2$. In the steady state, when the transversality condition in Eq. (6.10) is satisfied, $\rho - \alpha n > 0$ holds. Consequently, we have

$$\Delta - \Gamma = \frac{(1-\alpha)(1+\sigma)(x+\delta)}{\alpha} + \frac{(1+\sigma)(\rho - \alpha n)}{\alpha} > 0\ \cdot$$

Therefore, the relationship $\widehat{k}_1 > \widehat{k}_2$ is verified because $\Delta > \Gamma$ holds. Hence, the $\dot{\widehat{k}} = 0$ and $\dot{\widehat{c}} = 0$ loci have a unique intersection in the first quadrant on the $\left(\widehat{k}, \widehat{c}\right)$ plane. Figure 6.2 shows a phase diagram based on the above analytical results.

It can be seen from Fig. 6.2 that the system represented by the model of Barro and Sala-i-Martin (2004, Ch. 9) is saddle-path stable. In this model, the intensity of labor effort ℓ decreases monotonously from the initial value $\ell(0)$ to the steady-state value ℓ^* throughout the transition process. The following section demonstrates this proposition. We now define $\chi \equiv \left(\widehat{c}/\widehat{k}\right)$. Moreover, substitution from Eqs. (6.37) and (6.38) into (6.35) leads to

$$\begin{aligned}\gamma_\ell = \left(\frac{\alpha}{\alpha+\sigma}\right)\Bigg[&-\chi + \frac{(\rho+\delta+x)}{\alpha} - (n+x+\delta) \\ &\times + \frac{-\alpha(\rho+\sigma) + (1+\alpha+\sigma)\rho - (1+\sigma)\rho + \sigma\alpha n}{\alpha(1+\sigma)}\Bigg].\end{aligned} \tag{6.49}$$

From Eq. (6.50), the value of χ^*, that is, χ in the steady state, is given by

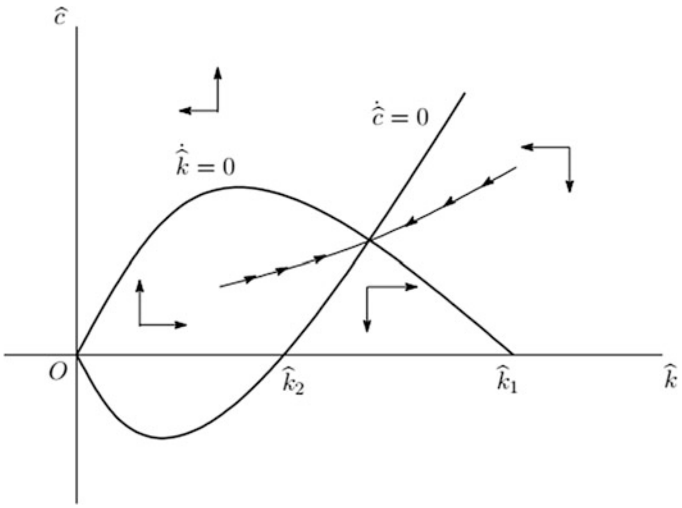

Fig. 6.2 Phase diagram of the labor/leisure choice model

$$\chi^* = \frac{(\rho + \delta + x)}{\alpha} - (n + x + \delta). \tag{6.50}$$

Moreover, considering the relationship

$$\frac{-\alpha(\rho + \sigma) + (1 + \alpha + \sigma)\rho - (1 + \sigma)\rho + \sigma\alpha n}{\alpha(1 + \sigma)} = 0.$$

Eq. (6.49) can be rewritten as follows:

$$\gamma_\ell = \left(\frac{\alpha}{\alpha + \sigma}\right)(\chi^* - \chi). \tag{6.51}$$

Differentiating $\chi = \widehat{c}/\widehat{k}$ with respect to time leads to

$$\gamma_\chi = \gamma_{\widehat{c}} - \gamma_{\widehat{k}}. \tag{6.52}$$

Substitution from Eqs. (6.37) and (6.38) into (6.52) leads to

$$\gamma_\chi = -(1 - \alpha)\widehat{k}^{\alpha - 1} + \chi - (\rho - n). \tag{6.53}$$

Moreover, differentiating both sides of Eq. (6.53) with respect to time yields

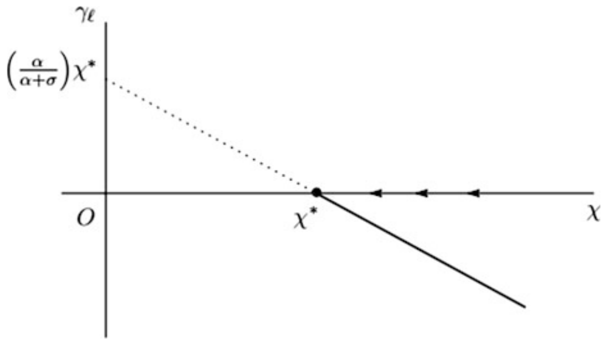

Fig. 6.3 The relationship between χ and γ_ℓ

$$\dot{\gamma}_\chi = (1-\alpha)^2 \widehat{k}^{-(1-\alpha)} \gamma_{\widehat{k}} + \dot{\chi}. \tag{6.54}$$

We now consider the case of $\widehat{k}(0) < \widehat{k}^*$. Because $\gamma_{\widehat{k}} > 0$ holds, from Eq. (6.54), we obtain the following proposition:

$$\exists t' \in [0, \infty), \;\; \dot{\chi} \geq 0 \Rightarrow \forall t \geq t', \;\; \dot{\gamma}_\chi > 0.$$

However, this proposition contradicts the fact that χ approaches the steady-state value χ^* in Eq. (6.50) over time. If χ converges to χ^*, $\dot{\gamma}_\chi < 0$ must be satisfied. Therefore, χ approaches the steady-state value χ^* over time only when $\forall t \in [0, \infty), \dot{\chi} < 0$ is satisfied. In other words, when $\widehat{k}(0) < \widehat{k}^*$, $\chi > \chi^*$ holds at any point in time. Therefore, Eq. (6.51) implies that

$$\gamma_\ell = \left(\frac{\alpha}{\alpha + \sigma}\right) \cdot (\chi^* - \chi) < 0.$$

This suggests that labor intensity ℓ decreases monotonously throughout the transition process, as depicted in Fig. 6.3.

6.3 Empirical Analysis of Work–Life Balance

6.3.1 Overview of Data

The empirical analysis of work–life balance in this study uses the OECD Better Life Index. The reasons are the same as those reported by Noda (2020). In other words, first, the Better Life Index, as suggested by its name, includes wide-ranging indicators related to a better life such as work–life balance and life satisfaction. Second, the Better Life Index covers the largest number of countries compared with other international databases related to work–life balance and its determinants. Finally, the OECD (2015) provides detailed explanations of the data used for the Better Life Index, indicating a high level of data reliability. This study uses data from the 2015 edition and undertakes empirical analyses using cross-national data from the 34 countries that were OECD members as of January 2016. In general, cross-national analysis is based on the assumption that the countries analyzed are fundamentally homogeneous and that they follow similar paths of economic development.

The 2015 edition of the Better Life Index comprises 11 segments: housing, income, jobs, community, education, environment, governance, health, life satisfaction, safety, and work–life balance. Of these 11 segments, this study specifically examines four, including work–life balance, health, jobs, and life satisfaction. The reasons for using the health and jobs variables are that being healthy is an important aspect of people's lives that affects whether one is able to have a job, earn adequate income, and participate in various social activities. A survey conducted by the OECD member countries suggests that health is always considered to be among the most valuable elements of life. Data related to jobs were included after considering the analytical results of previous studies (Ohtake 2012; Mitani 2015) indicating that long-term unemployment significantly reduced the happiness levels of individuals. The work–life balance, health, and jobs sections in the OECD Better Life Index include multiple variables. The data used for this study include a variable labeled "employees working very long hours" as an indicator of work–life balance, a variable labeled "self-reported health" as an indicator of health, and a variable labeled "long-term unemployment rate" as an indicator of employment. "Employees working very long hours" is defined as the proportion of employees who usually work 50 h or more per week, "self-reported health" is defined as the health condition based on employees' reports of their own health, and "long-term unemployment rate" is defined as the proportion of the overall labor force that was unemployed for 1 year or more. The variable for life satisfaction was labeled "life satisfaction." The respondents were asked to rate their current life satisfaction on a scale from 0, the worst conceivable life, to 10, the best conceivable life. The average score was calculated from the weighted sum in each answer category (OECD 2015).

6.3.2 Regression Models

First, we construct a regression model that incorporates the work–life balance indicator, health indicator, and employment indicator as the determinants of life satisfaction (designated as the "basic model"). The settings of the basic model conform to those reported by Noda (2020). The work–life balance indicator is expressed as x_1, the health indicator as x_2, the employment indicator as x_3, and the life satisfaction indicator as y. In addition, the set of the i-th observed values of these indicators is expressed as $(x_{1i}, x_{2i}, x_{3i}, y_i)$. When $\log x_1$, $\log x_2$, and $\log x_3$ are specified as explanatory variables and $\log y$ is set as the explained variable for a sample of size N, the basic model is given by

$$\log y_i = \alpha + \beta_1 \log x_{1i} + \beta_2 \log x_{2i} + \beta_3 \log x_{3i} + u_i. \quad (i = 1, 2, \cdots, N) \tag{6.55}$$

In Eq. (6.55), α, β_1, β_2, and β_3 are unknown constant parameters and u_i is an error term. Because this study analyzes the 34 OECD member countries, $N = 34$. The estimated coefficients of $\log x_{1i}$, $\log x_{2i}$, and $\log x_{3i}$ have important meanings in economics. For instance, the absolute value of the estimated coefficient of $\log x_{1i}$ represents the number of percentage points by which life satisfaction decreases as a result of a one-percent increase in the number of employees reporting that they usually work 50 h or more per week (work–life balance elasticity of life satisfaction). The estimated coefficient of $\log x_{2i}$ represents the number of percentage points by which life satisfaction increases as a result of a one-percent increase in the number of respondents who stated that their health condition was good or very good (health elasticity of life satisfaction). The absolute value of the estimated coefficient of $\log x_{3i}$ expresses the number of percentage points by which life satisfaction decreases as a result of a one-percent increase in long-term unemployment (unemployment elasticity of life satisfaction).

The OECD members comprise many European countries and also countries in Asia, the Middle East, North America, and Oceania. Considering the geographical diversity of the member countries, it is possible that factors affecting life satisfaction that are specific to particular regions might exist. This study uses constant term dummies as region dummy variables and designates the basic model with the addition of the constant term dummies as the extended model. More specifically, the extended model is given by

$$\begin{aligned}\log y_i = {} & \alpha + \beta_1 \log x_{1i} + \beta_2 \log x_{2i} + \beta_3 \log x_{3i} + \beta_4 D_{1i} + \beta_5 D_{2i} + \beta_6 D_{3i} \\ & + \beta_7 D_{4i} + \beta_8 D_{5i} + v_i. \quad (i = 1, 2, \cdots, N)\end{aligned} \tag{6.56}$$

In Eq. (6.56), α, β_1, β_2, β_3, β_4, β_5, β_6, β_7, and β_8 are unknown constant parameters and v_i is an error term. In addition, D_{1i} is the Asia region dummy variable, D_{2i} is the Middle East region dummy variable, D_{3i} is the Latin America region dummy variable, D_{4i} is the North America region dummy variable, and D_{5i} is the Oceania region dummy variable.

The error term u_i for the basic model and the error term v_i for the extended model are assumed to satisfy the conditions that allow the application of the classical least squares method.

6.3.3 *Results and Discussion*

First, we examine the case using data for women in the OECD member countries. The ordinary least squares (OLS) method is used to estimate the parameters. The OLS estimation of the basic model parameters using data for women in the OECD member countries results in.

$$
\begin{aligned}
\log \widehat{y}_i = 0.629 - -0.017 \log x_{1i} + 0.313 \log x_{2i} - 0.046 \log x_{3i}.& \\
(0.265) \qquad (0.015) \qquad (0.062) \qquad (0.011)& \\
\overline{R}^2 = 0.543, SER = 0.087, AIC = -1.929&
\end{aligned} \tag{6.57}
$$

In Eq. (6.57), the values in parentheses located directly below the estimated regression coefficients are the standard errors of the regression coefficients. $\overline{R}^2$ is an adjusted coefficient of determination, *SER* is the standard error of the regression, and *AIC* is the Akaike information criterion. The same expressions are used to present the results of the other regression analyses.

In addition, the OLS estimation of the extended model parameters using data for women in the OECD member countries yields.

$$
\begin{aligned}
\log \widehat{y}_i = -0.039 - 0.018 \log x_{1i} - 0.469 \log x_{2i} - 0.032 \log x_{3i}& \\
(0.552) \quad (0.022) \qquad (0.129) \qquad (0.016)& \\
+0.116 D_{1i} + 0.035 D_{2i} + 0.085 D_{3i} + 0.008 D_{4i} - 0.026 D_{5i}.& \\
(0.152) \quad (0.082) \quad (0.091) \qquad (0.078) \qquad (0.083)& \\
\overline{R}^2 = 0.496, SER = 0.092, AIC = -1.718&
\end{aligned} \tag{6.58}
$$

In Eq. (6.58), none of the regression coefficients of the region dummy variables are statistically significant at the 5% significance level. Thus, there is no evidence of inherent regional factors affecting life satisfaction. In terms of the AIC, the basic model's AIC (−1.929) is less than that of the extended model (−1.718). That is, the basic model is considered a better model. Therefore, we interpret the estimation results using Eq. (6.57).

Here, we examine the significance tests of the regression coefficients based on the estimation results using Eq. (6.57). The constant term is statistically significant at the 5% significance level. However, the regression coefficient of $\log x_{1i}$ is not statistically significant at the 5% significance level. Therefore, work–life balance in terms of the percentage of employees who usually work 50 h or more per week cannot be regarded as affecting life satisfaction. The regression coefficients of $\log x_{2i}$ and $\log x_{3i}$ are both statistically significant at the 1% significance level. The results that improvement in self-reported health has a positive effect on life satisfaction and that an increase in the long-term unemployment rate has a negative effect on life satisfaction were also observed in the analysis performed by Noda (2020) using the 2014 Better Life Index. The results of this study, which used the 2015 Better Life Index, estimated that the health elasticity of life satisfaction was 0.313, while the unemployment elasticity of life satisfaction was 0.046.

Next, we analyze the data for men in the OECD member countries. The OLS estimation of the basic model parameters using data for men in the OECD member countries leads to.

$$\log \widehat{y}_i = \underset{(0.359)}{0.402} - \underset{(0.022)}{0.027} \log x_{1i} - \underset{(0.081)}{0.367} \log x_{2i} - \underset{(0.073)}{0.044} \log x_{3i}. \tag{6.59}$$

$$\overline{R}^2 = 0.458, SER = 0.097, AIC = -1.710$$

In addition, the OLS estimation of the extended model parameters using the data for men in the OECD member countries yields

$$\begin{aligned} \log \widehat{y}_i = & \underset{(0.798)}{-0.220} + \underset{(0.031)}{0.017} \log x_{1i} + \underset{(0.182)}{0.057} \log x_{2i} - \underset{(0.012)}{0.037} \log x_{3i} \\ & \underset{(0.166)}{+0.130} D_{1i} + \underset{(0.095)}{0.065} D_{2i} + \underset{(0.099)}{0.004} D_{3i} + \underset{(0.088)}{0.029} D_{4i} - \underset{(0.096)}{0.019} D_{5i}. \end{aligned} \tag{6.60}$$

$$\overline{R}^2 = 0.379, SER = 0.104, AIC = -1.463$$

The estimation results based on the data for men are similar to those based on the data for women. As shown in Eq. (6.60), none of the regression coefficients of the region dummy variables are statistically significant at the 5% significance level, suggesting the absence of region-specific factors that affect life satisfaction. When the data for men were used, the basic model AIC was −1.710 and the extended model AIC was −1.463, which reveals the small size of the basic model AIC. Therefore, the basic model is regarded as the better model. Thus, we interpret the estimation results using Eq. (6.59).

Based on the estimation results using Eq. (6.59), the regression coefficients of the constant terms and $\log x_{1i}$ are not statistically significant at the 5% significance level. Therefore, in the case of men, as with women, work–life balance in terms of the percentage of employees who usually work 50 h or more per week cannot be regarded as affecting life satisfaction. In addition, the regression coefficients of both $\log x_{2i}$ and $\log x_{3i}$ are found to be significant at the 1% significance level. In other words, as with the women, improvement in self-reported health positively affects life satisfaction and an increase in the long-term unemployment rate negatively affects life satisfaction. The health elasticity of life satisfaction for men is 0.313, while the unemployment elasticity of life satisfaction is 0.046.

Noda (2020) discovered that work–life balance in view of the percentage of time devoted to leisure and personal care had a positive effect on life satisfaction. However, the results of this study indicated no clear relationship between work–life balance and life satisfaction based on the percentage of employees who usually worked 50 h or more per week. The results of Noda (2020) and this study provide important suggestions for work–life balance policies. An increase in life satisfaction cannot be expected from the implementation of a work–life balance policy that limits long working hours. However, an increase in life satisfaction can be expected from work–life balance policies aimed at increasing the time allocated to leisure and personal activities. Therefore, we must consider how we could increase the amount of time available for leisure and personal care while maintaining a reasonable number of working hours. For instance, the recent increase in the numbers of children on waiting lists for nurseries and elderly people on waiting lists for long-term care facilities has become a significant problem in Japan. Solving this problem is likely to help workers increase their time used for leisure and personal care. Therefore, overcoming the shortages of nursing care professionals and preschool teachers is an urgent task that must be addressed not only for the people engaging in nursing care and childcare, but from the macroscopic perspective of improving people's work–life balance.

6.4 Conclusion

There is a flat or declining trend in the average total annual working hours per worker in some OECD countries. Such a trend is consistent with rational household behavior. More specifically, the selection of working hours and leisure time by households attempting to maximize their lifetime utility under given conditions results in a decrease in the average working hours of each household during the process of transition to the steady state, until they become mostly constant in the neighborhood of the steady state. The labor–leisure choice model of Barro and Sala-i-Martin (2004, Ch.9) suggests that Japan is located in the neighborhood of the steady state, while South Korea is in the process of transition.

Moreover, this study undertook an empirical analysis using the 2015 edition of the OECD Better Life Index in an attempt to identify the effects that changes in work–life balance have on the life satisfaction of men and women, and whether there are region-specific factors affecting life satisfaction.

The relationship between the work–life balance indicator based on the percentage of employees who usually work 50 h or more per week and life satisfaction was examined using a cross-sectional analysis of 34 OECD member countries. The results indicated that there was no clear relationship for either men or women. This suggests that reducing the working hours of workers working long hours would not necessarily increase their life satisfaction. In addition, the presence or otherwise of region-specific factors in OECD member countries in regions such as Asia, North America, and Latin America that affected people's life satisfaction was examined. No evidence of the existence of region-specific factors affecting people's life satisfaction was found. Therefore, differences in national characteristics and cultures are not key factors affecting people's life satisfaction. The results of this study suggest that people's life satisfaction depends largely on their health and employment status.

Finally, this study has some limitations. The empirical analyses in this study were performed on samples of males and females from the OECD member countries regardless of age. However, perceptions regarding life satisfaction are not necessarily the same across various age groups among the working population. For instance, differences in terms of physical strength exist between people in their twenties and those in their fifties. Further, expenditure in areas such as education differ between these two age groups. Incorporating these differences into our analyses might result in varying perceptions of the effect of long working hours. Therefore, new findings that result in improved work–life balance policies can be anticipated from detailed analyses of the effects of work on different age groups.

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Part III
Knowledge-Based Economy, Education, and Inequality

Chapter 7
Uncertainty of Educational Outcome, Demographic Transition, and Income Distribution

Tomoya Sakagami and Miki Matsuo

Abstract This chapter presents an investigation of how class-size policy affects income differences. Agents who live for two periods decide whether to enroll in school. We introduce uncertainty of educational results: the probability that the individual becomes a skilled laborer is less than one, given that they have chosen schooling. Results show that there might exist two steady-state equilibria with different income distributions. If the steady-state equilibrium corresponds to a narrower income distribution, then the larger the initial class size, the more likely it is to produce a smaller reduction in income differentials.

Keywords Uncertain educational outcome · Class size · Steady-state equilibrium · Income distribution

7.1 Introduction

This study investigates how class-size policy affects income distribution. In an overlapping-generational economy in which agents live for two periods, agents decide whether to enroll in school in the first period. Educational outcomes and the possibility of becoming a skilled laborer depend on the class size (teacher–student ratio). Two steady-state equilibria might exist with different income

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distribution and fertility. Such an argument is possible because the model incorporates uncertainty of educational results.

Numerous studies have examined class size from an economical viewpoint. Most of those studies specifically examined the relationship between class size and per capita educational expense.

Dahan and Tsiddon (1998) show the relationship between education and income growth in a case in which parents have income distribution. Given that a child's education is paid in the form of a bequest, a child who was born into a high-income household can choose education without borrowing; in contrast, a child born into a low-income household cannot. No uncertainty of educational outcome pertains. Children who choose education definitely become skilled laborers in the second period and earn high wages. Dahan and Tsiddon show the existence and uniqueness of the steady-state equilibrium and that fertility and income distribution follow an inverted U-shaped dynamic in the process of economic development.

Lazear (2001) presents a static disruption model of educational production. He introduces a probability p, which is the probability that a given student does not impede his own or other's learning at any moment in time. If a classroom includes n students, then the probability of disruption is $1 - p^n$. Based on that model, the optimal class size increases the probability that students behave well and increases the educational expense (the teacher's wage) because no relationship links educational expense and discipline of students.

Eicher (1996) introduces the teacher/student ratio into a dynamic model. However, like Lazear (2001), he merely addresses the cost of education: smaller class size corresponds only to higher educational expense. This conception differs from that of Dahan and Tsiddon. In their model, each child is financially equal at the beginning of the first period because parents do not give their children a bequest. He defines the equilibrium as a situation where the educational choice-maker is indifferent.

The class-size modeling described in this article differs from approaches used in previous research in two respects. The first difference is that we consider not only the cost of education, but also the benefits derived from a smaller class size. The second difference is that we introduce educational uncertainty into Dahan and Tsiddon's framework. That is, the probability of becoming a skilled laborer is p, and the probability to be an unskilled laborer is $1 - p$. Because of the uncertainty of educational outcomes, five different life patterns exist in the model. Each life pattern corresponds to a different lifetime income and fertility. Fertility differences play a pivotal role in proving the existence of a steady-state equilibrium in our model.

Regarding the first difference, we introduce an Eicher-type (1996) teacher/student ratio ϕ into our model. We assume that an increase in the value ϕ, which corresponds to a smaller class size, also increases the probability of becoming a skilled laborer. Although Lazear (2001) also introduced class size into his model, it was a static analysis. In contrast, our model has a dynamic structure. Using this model, we examine the rate of relative wages and fertility in steady-state equilibria.

This model is intended to demonstrate the possible existence of two different steady-state equilibria with different income distribution. The first steady-state equilibrium is identical to that presented by Dahan and Tsiddon (1998). For a child born into a low-income household, the utility of expected income from choosing schooling equals utility from foregone schooling. For a child who was born into a high-income household, the utility of expected income from choosing schooling is greater than the utility from foregone schooling. The second steady-state equilibrium will exist in the case that all children choose education irrespective of their parents' income. When all choose education, however, those who do not become skilled laborers will occur with probability $1 - p$. Multiple steady-state equilibria might exist when probability p is sufficiently small. In such a case, it is possible that the ratio between skilled laborers and unskilled laborers in the next period will be constant in the current period. We can show such equilibrium by introducing the uncertainty of educational outcome.

In our model, a corresponding rate of relative wages exists for each steady-state equilibrium. The rate of relative wages is useful as an index of income differentials in the economy. Therefore, if two different steady-state equilibria exist, the degree of income differentials and fertility are also different.

We then analyze the effects of class-size policy on steady-state equilibria. If the steady-state equilibrium corresponds to smaller income differentials, then we will show that smaller class-size policy reduces income differentials when pre-class size is sufficiently large.

The chapter is organized as follows. Section 7.2 develops the model. Section 7.3 shows there are five different life patterns, and each corresponds to a different lifetime income. In Sect. 7.4, we calculate the fertility in each life pattern. Section 7.5 defines the steady-state equilibrium and presents some lemmas generated in the steady-state equilibrium. Section 7.6 brings the existence of the equilibrium and shows the condition of multiple equilibria. Section 7.7 examines the effects of class-size policy on steady-state equilibria.

7.2 The Model

7.2.1 Firm

Production occurs in one sector with constant returns to scale. Three factors of production exist: skilled laborers, unskilled laborers, and physical capital. The production function is

$$Y_t = F\left(K_t, H_t^Y, L_t\right) = AK_t{}^d \left(H_t^Y\right)^e L_t{}^{1-d-e}, \tag{7.1}$$

where Y_t is the output in period t, A is the level of technology, which is constant. In addition, L_t is the number of unskilled laborers (called L-type) in period t, and H_t^Y

represents the number of skilled laborers (called H-type) in period t.[1] The economy is open, and the international interest rate r is exogenous. The firm faces perfect competition, and $w_{H,\,t}$ and $w_{L,\,t}$ respectively denote the wages of skilled laborers and unskilled laborers in period t. Profit maximization yields the first order conditions:

$$r = F_{K,t} = dA\left(\frac{L_t}{K_t}\right)^{1-d}\left(\frac{H_t^Y}{L_t}\right)^{1-d}, \tag{7.2}$$

$$w_{H,t} = F_{H_t^Y,t} = eA\left(\frac{L_t}{K_t}\right)^{-d}\left(\frac{L_t}{H_t^Y}\right)^{1-e} = eA\left(\frac{dA}{r}\right)^{\frac{d}{1-d}}\left(\frac{L_t}{H_t^Y}\right)^{\frac{1-d-e}{1-d}}, \tag{7.3}$$

$$\begin{aligned} w_{L,t} = F_{L_t,t} &= (1-d-e)A\left(\frac{L_t}{K_t}\right)^{-d}\left(\frac{L_t}{H_t^Y}\right)^{-e} \\ &= (1-d-e)A\left(\frac{dA}{r}\right)^{\frac{d}{1-d}}\left(\frac{L_t}{H_t^Y}\right)^{\frac{-e}{1-d}} \end{aligned} \tag{7.4}$$

From Eqs. (7.3) and (7.4), the relative wages of H-type and L-type workers in each period are

$$\frac{w_{H,t}}{w_{L,t}} = \left(\frac{e}{1-d-e}\right)\frac{L_t}{H_t^Y}. \tag{7.5}$$

We assume that v_t is the ratio of H-type workers who work in the production sector, and $1 - v_t$ is the ratio of H-type workers who work as teachers in the educational sector ($0 < v_t < 1$). Therefore $H_t^Y = v_t H_t$, we can rewrite Eq. (7.5) as the following.

$$\frac{w_{H,t}}{w_{L,t}} = \left(\frac{e}{1-d-e}\right)\frac{L_t}{v_t H_t}. \tag{7.5'}$$

According to Eq. (7.5'), the relative wage of each type of laborer is a linear function of the ratios of H-type and L-type.

7.2.2 Households

Agents live for two periods: agents born at time t inherit a bequest from parents. In the first period, the individual decides whether to choose education. We assume that whether or not her parents belong to H-type, an individual who chooses to get her

[1] The number of unskilled laborers in period t is the sum of children in period t who do not choose education, and also adults in period t who work as unskilled laborers. This will be described in greater detail in Sect. 7.2.2.

education can become a skilled laborer with probability p. In the second period, the individual works as a skilled laborer or as an unskilled laborer and earns income. He then decides the consumption, the number of children, and the bequest.

Each individual derives utility from consumption in the second period, from the number of children, and from leaving a bequest. The utility function of an individual born at time t is

$$U_t = \alpha \log C_{t+1} + \beta \log N_{t+1} + \gamma \log B_{t+1}, \quad \alpha + \beta + \gamma = 1, \quad \alpha, \beta, \gamma > 0, \tag{7.6}$$

where C_{t+1}, N_{t+1}, and B_{t+1} respectively denote second-period consumption, the number of children, and the total bequeathed estate.

An individual's lifetime income in terms of second-period consumption, I_{t+1}, is spent on consumption, child rearing, and bequests. The cost of rearing children is measured in terms of foregone work time, at δ per child. Consequently, lifetime income represents the following:

$$I_{j,t+1} = C_{j,t+1} + \delta N_{j,t+1} w_{j,t+1} + B_{j,t+1}, \quad j = H, L \tag{7.7}$$

where $C_{j,\ t+1}$ is second-period consumption of the individual type of type j, the second term is the opportunity cost of rearing children, $B_{j,t+1}$ is the bequest that type j leaves, and $I_{j,\ t+1}$ is type j lifetime income in terms of the second period.

Each individual determines their own consumption, number of children, and bequests to maximize utility subject to a personal budget constraint. For each generation t, we have the first order conditions:

$$C_{j,t+1} = \alpha I_{j,t+1}, \quad N_{j,t+1} = \frac{\beta}{\delta w_{j,t+1}} I_{j,t+1}, \quad B_{j,t+1} = \gamma I_{j,t+1}, \quad \text{for} \quad j = H, L \tag{7.8}$$

Substituting Eq. (7.8) for (7.6), the indirect utility at the optimum is

$$U_{j,t} = \log I_{t+1} + \varepsilon_{j,t+1}, \tag{7.9}$$

where

$$\varepsilon_{j,t+1} = \alpha \log \alpha + \beta \log \beta + \gamma \log \gamma - \beta \log \delta w_{j,t+1}, \quad j = H, L.$$

From Eq. (7.9), the indirect utility is shown to depend on lifetime income and the second-period wage.

Assuming that parents divide the bequest equally among heirs, the bequest per child is a function of second-period income only:

$$b_{j,t+1} \equiv \frac{B_{j,t+1}}{N_{j,t+1}} = \frac{\gamma\delta}{\beta} w_{j,t+1}, \quad j = H, L \tag{7.10}$$

Parents leave a fraction of the wage that they earn in the second period. Therefore, we assume $\frac{\gamma\delta}{\beta}$ is larger than 0 and smaller than 1. According to Eq. (7.10), the bequest that each generation t accepts from his parents is $b_{j,t} = \frac{\gamma\delta}{\beta} w_{j,t}$. With that bequest, he then decides whether to seek an education.

7.3 Educational Choice and the Number of Children

Each individual has one unit of time in each period of life. As described above, the individual receives his share of his parents' bequest and decides whether to be educated in the first period. An individual who does not choose an education engages in L-type in both periods of his life and earns w_L each period. Previously, we introduced uncertainty into the educational result. That is, an educated individual does not necessarily become an H-type laborer. We assume that the individual who is educated becomes an H-type laborer with probability p, or becomes an L-type laborer with probability $1 - p$. Being educated in the first period, the laborers who become H-type can earn $w_{H,\ t+1}$ in the second period. However, individuals who have probability of $1 - p$ end up working as L-type laborers in spite of having received education. Therefore, the wage of L-type laborers is the same wage, $w_{L,\ t+1}$, as that of laborers who received no education in the first period.

In this section, we investigate the static choice of an individual born on a random date with a random bequest amount. We assume the following:

[A1]. When the individual born at time t chooses to be educated, the person must pay tuition. The tuition per student, $\phi w_{H,\ t}$, equals the teacher–student ratio, $\phi = \frac{T_t}{S_t}$, times the teacher's wage in term t. T_t is the number of teachers in term t, and S_t is the number of students in term t. We assume the teacher–student ratio is constant because of the government's determination of the class size.

[A2]. We assume that the probability is $p(0) = 0$, $p'(\phi) > 0$, $p''(\phi) < 0$. This assumption means that, although smaller class size raises the probability, the rising decreases.[2]

[A3]. .

(a) A child who is born of rich parents (H-type) receives a bequest from his parents that matches the tuition: $\frac{\gamma\delta}{\beta} w_{H,t} - \phi w_{H,t} > 0$.

(b) A child who is born of poor parents (L-type) receives no bequest to pay tuition. Therefore, he borrows the remaining amount $\phi w_{H,\ t} - b_{L,\ t}$ at the rate of interest i of the first period; he must pay back the loan in the second

[2]We estimate $p(\phi)$ in the Japanese data. See Appendix 1.

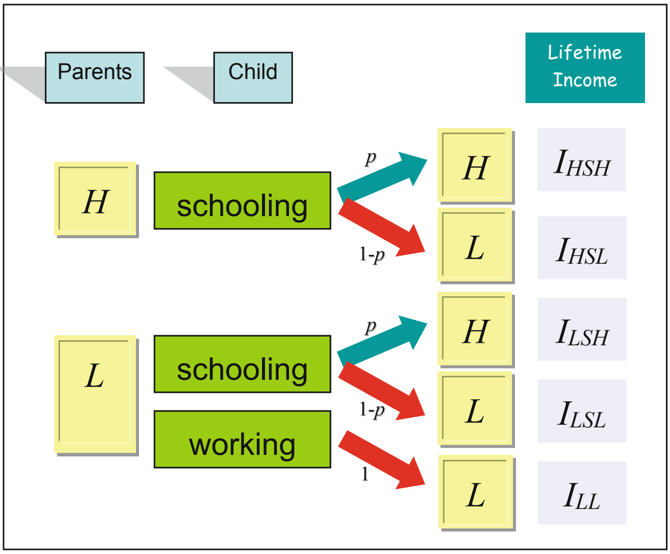


Fig. 7.1 Five different life patterns

period. We assume that the rate of interest to lenders r is less than that to borrowers i.

[A4]. We assume that all children who are born of rich parents choose schooling.

For someone born at time t, both the life pattern and lifetime income are of the following five forms (Fig. 7.1): (1) a person who is born into a high-income household chooses education and becomes a skilled laborer in the next period; (2) a person who is born into a high-income household chooses education, but becomes an unskilled laborer in the next period; (3) a person who is born into a low-income household chooses education with borrowing, and becomes a skilled laborer in the next period; (4) a person who is born into a low-income household chooses education with borrowing, but becomes an unskilled laborer in the next period; (5) a person who is born into a low-income household chooses work as an unskilled laborer, and continues to be an unskilled laborer in the next period.

To depict those five lives simply, we introduce the following notation. LL denotes an individual who is born of L-type parents, chooses not to be educated, and becomes L-type. LSL denotes an individual who is born of L-type parents and becomes L-type with probability $1 - p$ in spite of choosing to be educated. LSH denotes an individual who is born of L-type parents, gets an education, and becomes H-type with probability p. HSL denotes an individual who is born of H-type parents, and becomes L-type with probability $1 - p$ in spite of choosing to be educated. Lastly, HSH denotes an individual who is born of H-type parents, gets an education, and becomes H-type with probability p.

Next, we show the five forms of lifetime income. First, a lifetime income of an individual LL is

$$I_{\mathrm{LL},t+1} = (1+r)(b_{L,t} + w_{L,t}) + w_{L,t+1}.$$

From Eq. (7.10), substituting a bequest $b_{L,t} = \frac{\gamma\delta}{\beta} w_{L,t}$ into the previous equation, we obtain.

$$I_{\mathrm{LL},t+1} = (1+r)\left(\frac{\gamma\delta}{\beta} w_{L,t} + w_{L,t}\right) + w_{L,t+1}. \tag{7.11a}$$

Secondly, lifetime incomes of LSL and LSH are respectively represented as

$$I_{\mathrm{LSL},t+1} = (1+i)\left(\frac{\gamma\delta}{\beta} w_{L,t} - \phi w_{H,t}\right) + w_{L,t+1},$$

$$I_{\mathrm{LSH},t+1} = (1+i)\left(\frac{\gamma\delta}{\beta} w_{L,t} - \phi w_{H,t}\right) + w_{H,t+1}.$$

Because p is the probability that the educated individual becomes an H-type worker after schooling, she will get an expected lifetime income $pI_{\mathrm{LSH},\ t\ +\ 1} + (1-p)I_{\mathrm{LSL},\ t\ +\ 1}$ in the second period. The above equations show that the expected lifetime income of an individual born to L-type parents is

$$I_{\mathrm{LS},t+1} = (1+i)\left(\frac{\gamma\delta}{\beta} w_{L,t} - \phi w_{H,t}\right) + (1-p)w_{L,t+1} + pw_{H,t+1}. \tag{7.11b}$$

Thirdly, a child who is born of rich parents chooses to be educated only if the expected utility of getting education is higher than the utility of not doing so. From A.4, the respective lifetime incomes of HSL and HSH are

$$I_{\mathrm{HSL},t+1} = (1+r)\left(\frac{\gamma\delta}{\beta} w_{H,t} - \phi w_{H,t}\right) + w_{L,t+1},$$

$$I_{\mathrm{HSH},t+1} = (1+r)\left(\frac{\gamma\delta}{\beta} w_{H,t} - \phi w_{H,t}\right) + w_{H,t+1}.$$

Using p along with the above equations, the expected lifetime income of an individual born to H-type parents is

$$I_{\mathrm{HS},t+1} = (1+r)\left(\frac{\gamma\delta}{\beta} w_{H,t} - \phi w_{H,t}\right) + (1-p)w_{L,t+1} + pw_{H,t+1}. \tag{7.11c}$$

As with expected lifetime income for all L-types (Eq. 7.11b), Eq. (7.11c) behaves similarly. Substituting Eqs. (7.11a)–(7.11c) for (7.9), we can compare the utility of expected income getting an education and the utility of not doing so. Therefore, we will see a utility level for every case.

[Case 1]: The utility of the expected income of a child born to L-type parents in time t and who does not choose education is

$$U_{\mathrm{LL},t} = \log\left[(1+r)\left(\frac{\gamma\delta}{\beta}+1\right)w_{L,t} + w_{L,t+1}\right] + \varepsilon_{L,t+1}. \tag{7.12}$$

[Case 2]: The utility of expected income of a child born to L-type parents in time t and who chooses education is

$$U_{\mathrm{LS},t} = \log\left[(1+i)\left(\frac{\gamma\delta}{\beta}w_{L,t} - \phi w_{H,t}\right) + (1-p)w_{L,t+1} + pw_{H,t+1}\right] + \varepsilon_{\mathrm{LS},t+1}, \tag{7.13}$$

where the wage in $\varepsilon_{\mathrm{LS},\,t+1}$ is expected income $(1-p)w_{L,\,t+1} + pw_{H,\,t+1}$.

[Case 3]: The utility of expected income of a child born to H-type parents in time t and who chooses an education is

$$U_{\mathrm{HS},t} = \log\left[(1+r)\left(\frac{\gamma\delta}{\beta}w_{H,t} - \phi w_{H,t}\right) + (1-p)w_{L,t+1} + pw_{H,t+1}\right] + \varepsilon_{\mathrm{HS},t+1}, \tag{7.14}$$

where the wage in $\varepsilon_{\mathrm{HS},\,t+1}$ is also expected income $(1-p)w_{L,\,t+1} + pw_{H,\,t+1}$.

In every generation, although children of H-type families always choose to get an education, children of L-type families choose to get an education only if the following condition holds.

$$U_{\mathrm{LL},t} \leq U_{\mathrm{LS},t}$$

That is to say:

$$(1+i)\left(\frac{\gamma\delta}{\beta}w_{L,t} - \phi w_{H,t}\right) + (1-p)w_{L,t+1} + pw_{H,t+1} \geq \left[(1+r)\left(\frac{\gamma\delta}{\beta}+1\right)w_{L,t} + w_{L,t+1}\right]\left(\frac{(1-p)w_{L,t+1} + pw_{H,t+1}}{w_{L,t+1}}\right)^{\beta} \tag{7.15}$$

From Eq. (7.15), we can derive the minimum level of bequest necessary for choosing education. Considering for a moment the case of exogenous fertility ($\beta = 0$), the minimum level of bequest is

$$\min b_L = \frac{(1+r)w_{L,t} + w_{L,t+1} - w_{\mathrm{LS}}(\phi) + (1+i)\phi w_{H,t}}{i-r}, \tag{7.16}$$

where $w_{\mathrm{LS}}(\phi)$ represents the expected wage. Using Eq. (7.16), we examine the effect of increasing ϕ on the minimum level of bequest. When ϕ increases, the plus and minus of the numerator in Eq. (7.16) is affected. Each effect is an increase in education expense and expected wage. Given these two effects, it is impossible to determine the effect of an increase of ϕ on the minimum level of bequest. However, assuming $p'(\phi) > 0$, $p''(\phi) < 0$, the increase in expected wages is larger than in the education expense if ϕ is sufficiently small.[3] We point out that this result is different from Dahan and Tsiddon (1998). In their paper, an increase of ϕ (smaller class size) merely engenders an increase in educational expense. Therefore, because an increase of ϕ increases the threshold of bequest for choosing to enroll in school. In our model, however, the smaller the class size, the greater the probability of becoming a skilled laborer. Therefore, the expected wage increases, thereby decreasing the probability of the minimum level of bequest, in contrast with Dahan and Tsiddon (1998).

7.4 Fertility Choice

As shown in Fig. 7.1, we depict five types of lives. Initially of the second period, the individual determines which type he belongs to. Afterward, the lifetime income becomes definite, and each decides the number of children and the bequest to maximize his utility.

In the following, we calculate the fertility rate of each type. Substituting Eq. (7.8) for Eqs. (7.11a)–(7.11c), we have the following equations:

1. Case of LL

$$\begin{aligned} N_{\mathrm{LL},t+1} &= \frac{\beta}{\delta w_{L,t+1}}\left\{(1+r)\left(\frac{\gamma\delta}{\beta}+1\right)w_{L,t} + w_{L,t+1}\right\} \\ &= \frac{\beta}{\delta}\left\{(1+r)\left(\frac{\gamma\delta}{\beta}+1\right)\frac{w_{L,t}}{w_{L,t+1}}+1\right\}. \end{aligned} \tag{7.17}$$

2. Case of LSL

[3]Because of $w_{LS} = pw_H + (1-p)w_L$, we find $\frac{dw_{LS}}{d\phi} = p'(\phi)(w_H - w_L) > 0$. Furthermore, because $p'' < 0$, the smaller ϕ is, the more w_{LS} increases. We infer that an increase of the marginal benefit w_{LS} is larger than that of the marginal cost of education $(1+i)\phi$ when ϕ is sufficiently small.

$$N_{\mathrm{LSL},t+1} = \frac{\beta}{\delta}\left\{(1+i)\left(\frac{\gamma\delta}{\beta}\frac{w_{L,t}}{w_{L,t+1}} - \phi\frac{w_{H,t}}{w_{L,t+1}}\right) + 1\right\}. \tag{7.18}$$

3. Case of LSH

$$N_{\mathrm{LSH},t+1} = \frac{\beta}{\delta}\left\{(1+i)\left(\frac{\gamma\delta}{\beta}\frac{w_{L,t}}{w_{H,t+1}} - \phi\frac{w_{H,t}}{w_{H,t+1}}\right) + 1\right\}. \tag{7.19}$$

4. Case of HSL

$$N_{\mathrm{HSL},t+1} = \frac{\beta}{\delta}\left\{(1+r)\left(\frac{\gamma\delta}{\beta}\frac{w_{H,t}}{w_{L,t+1}} - \phi\frac{w_{H,t}}{w_{L,t+1}}\right) + 1\right\}. \tag{7.20}$$

5. Case of HSH

$$N_{\mathrm{HSH},t+1} = \frac{\beta}{\delta}\left\{(1+r)\left(\frac{\gamma\delta}{\beta}\frac{w_{H,t}}{w_{H,t+1}} - \phi\frac{w_{H,t}}{w_{H,t+1}}\right) + 1\right\}. \tag{7.21}$$

7.5 Steady Equilibrium

In this section, we provide a definition of steady-state equilibrium and present some lemmas that describe a steady-state equilibrium. Following Dahan and Tsiddon (1998), we define a steady-state equilibrium as the following:

Definition *A steady-state equilibrium is a situation in which the growth rate of L-type and H-type are both equal and constant.*

This means that a population composition (in terms of the proportion of *L*-type to *H*-type laborer) is constant.[4] Furthermore, we demonstrate the following lemma in a steady-state equilibrium:

Lemma 1 *Wage rates are constant in a steady-state equilibrium.*

Proof Recalling that ϕ is the teacher–student ratio and that $1 - v_t$ is the ratio of *H*-type workers who work as teachers, we can rewrite $\phi = \frac{T_t}{S_t} = \frac{(1-v_t)H_t}{S_t}$. Then, students in term t become *H*-type laborers with probability p. Therefore, the number of *H*-

[4]As mentioned in footnote 1, L_t is the total of unskilled laborers in term t, consisting of a child born in time t who does not choose schooling, and also an adult who is an unskilled laborer in time t.

type laborers in the subsequent period, H_{t+1}, is expressed as $H_{t+1} = p(\phi)S_t$. From the above, substituting $S_t = \frac{H_{t+1}}{p(\phi)}$ for ϕ, we can rewrite ϕ as the following:

$$\phi = \frac{H_t}{H_{t+1}}(1 - v_t)p(\phi). \tag{7.22}$$

Because the teacher–student ratio ϕ is given as a political decision, $p(\phi)$ is also fixed. In a steady-state equilibrium, $\frac{H_t}{H_{t+1}}$ becomes constant. For that reason, v is settled uniquely, i.e., $v_t = v$. From profit maximization conditions:

$$w_{H,t} = eA\left(\frac{dA}{r}\right)^{\frac{d}{1-d}}\left(\frac{L_t}{H_t^Y}\right)^{\frac{1-d-e}{1-d}}, \tag{7.3}$$

$$w_{L,t} = (1 - d - e)A\left(\frac{dA}{r}\right)^{\frac{d}{1-d}}\left(\frac{L_t}{H_t^Y}\right)^{\frac{-e}{1-d}}. \tag{7.4}$$

Both $w_{H,\,t}$ and $w_{L,\,t}$ are also constant if $\frac{L_t}{H_t^Y}$ is constant. Note that we assume that v is the ratio of H-type who works in a production section. Therefore, we express $H_t^Y = vH_t$. The proportion of L_t to H_t^Y is rewritten in the following equation:

$$\frac{L_t}{H_t^Y} = \frac{L_t}{vH_t}. \tag{7.23}$$

In a steady-state equilibrium, $\frac{L_t}{H_t}$ is constant, so $\frac{L_t}{H_t^Y}$ becomes constant. Therefore, $w_{H,\,t}$ and $w_{L,\,t}$ are constant in a steady-state equilibrium.

Q. E. D.

Lemma 2 *The relative wage is constant in a steady-state equilibrium.*

Proof Lemma 1 shows that the relative wage is constant.

Q. E. D.

We derive fertility in a steady-state equilibrium by adapting Eqs. (7.17)–(7.21) to Lemmas 1 and 2.

$$N_{\mathrm{LL}} = \frac{\beta}{\delta}\left\{(1 + r)\left(\frac{\gamma\delta}{\beta} + 1\right) + 1\right\} \tag{7.24}$$

$$N_{\mathrm{LSL}} = \frac{\beta}{\delta}\left\{(1 + i)\left(\frac{\gamma\delta}{\beta} - \phi\frac{w_H}{w_L}\right) + 1\right\} \tag{7.25}$$

$$N_{\mathrm{LSH}} = \frac{\beta}{\delta}\left\{(1 + i)\left(\frac{\gamma\delta}{\beta}\frac{w_L}{w_H} - \phi\right) + 1\right\} \tag{7.26}$$

$$N_{\mathrm{HSL}} = \frac{\beta}{\delta}\left\{(1+r)\left(\frac{\gamma\delta}{\beta}\frac{w_H}{w_L} - \phi\frac{w_H}{w_L}\right) + 1\right\} \tag{7.27}$$

$$N_{\mathrm{HSH}} = \frac{\beta}{\delta}\left\{(1+r)\left(\frac{\gamma\delta}{\beta} - \phi\right) + 1\right\} \tag{7.28}$$

A size comparison of fertilities among the five life patterns is described by Lemma 3.

Lemma 3 *The relationship among fertilities in the steady-state equilibrium is the following:*

$N_{\mathrm{LSL}} < N_{\mathrm{LSH}} < N_{\mathrm{HSH}} < N_{\mathrm{HSL}}$ or N_{LL}.

Proof See Appendix 2.

Q. E. D.

Existence of a steady-state equilibrium is proven by means of Lemma 3.

7.6 Existence of Steady Equilibrium

In this section, we examine whether a steady-state equilibrium exists or not. There are three possibilities that a steady-state equilibrium exists. The first case is that in which all children born to *H*-type parents choose an education. The second case includes all children born to *H*-type parents who choose an education and some children born to *L*-type parents who choose education. The third case includes all children from both *H*-type parents and *L*-type parents who choose an education.[5]

First Case All children born to *H*-type parents who choose an education.

This case includes three life types: LL, HSL, and HSH. For fertility corresponding to each type, we have the following relations by Lemma 3: $N_{\mathrm{HSH}} < N_{\mathrm{HSL}}$ or N_{LL}. Therefore, the growth rate of L_t is higher than that of H_t, so $\frac{L_t}{H_t}$ increases intertemporally and a steady-state equilibrium does not exist.[6]

Second Case All children born to *H*-type parents who choose an education and some children born to *L*-type parents who choose education.

In this case, it is indifferent for a child who is born of *L*-type parents to be educated or not. From Lemma 3, with the exception of LSL, the fertility of those who became *L*-type is greater than those who became *H*-type ($N_{\mathrm{LSH}} < N_{\mathrm{HSH}} < N_{\mathrm{HSL}}$ or N_{LL}). On the other hand, a transition from *L*-type to *H*-type has taken place in every

[5]For the reason shown in Appendix 1, we exclude the case in which a child who is born of *H*-type parents does not get an education.

[6]See Appendix 3 for details.

term. If the transition is sufficiently large, the possibility exists that a steady-state equilibrium where $\frac{L_t}{H_t}$ is intertemporally constant.

In this case, a child born of L-type parents is indifferent to choosing schooling or not. In a steady-state equilibrium, Eq. (7.15) restates the following:

$$(1+i)\left(\frac{\gamma\delta}{\beta}w_L - \phi w_H\right) + (1-p)w_L + pw_H$$
$$= \left[(1+r)\left(\frac{\gamma\delta}{\beta}+1\right)w_L + w_L\right]\left(\frac{(1-p)w_L + pw_H}{w_L}\right)^{\beta}. \tag{7.29}$$

By dividing both sides of Eq. (7.29) by w_L, and replacing $\frac{w_H}{w_L}$ with x, the left-hand side (LHS) and the right-hand side (RHS) respectively become

$$\text{LHS} = 1 - p + (1+i)\frac{\gamma\delta}{\beta} + [p - (1+i)\phi]x, \tag{7.30}$$

and

$$\text{RHS} = \left[(1+r)\left(1+\frac{\gamma\delta}{\beta}\right) + 1\right][(1-p) + px]^{\beta}. \tag{7.31}$$

The following assumption guarantees the upward slope of the LHS:

[**A4**] $p(\phi) > (1 + i)\phi$.

Under assumption [A4],[7] the LHS of Eq. (7.29) takes the form of a straight line with a positive slope. The RHS is a concave curve that starts at the origin of the positive quadrant. The line then intersects the curve once,[8] so there exists a steady-state equilibrium in this case (see Fig. 7.2).

Third Case All children from both H-type parents and L-type parents choose an education.

In this case, all children getting an education become H-type with probability p or become L-type with probability $1 - p$. The possibility thereby exists that $\frac{L_t}{H_t}$ is intertemporally constant.

Given the number of offspring born in term t, we examine the ratio of L-type to H-type laborers in term $t + 1$, $\frac{L_{t+1}}{H_{t+1}}$. Variable s_t denotes the number of children born to H-type parents in term t, and u_t denotes the number of children born to L-type parents in

[7]This can be rewritten as $p(\phi)w_H - (1 + i)\phi w_H > 0$. There exists $\phi_3 > 0$ such that for any $\phi < \phi_3$ this inequality holds. In this inequality, pw_H implies an expected wage-like value in the next period when a child chooses schooling, and $(1 + i)\phi w_H$ implies the cost of education evaluated by the next period. Consequently, $p(\phi) > (1 + i)\phi$ indicates that the marginal benefit from education exceeds the marginal cost. This is the necessary condition for a child to choose schooling.

[8]We can demonstrate that there exists an intersection on the right of $x = 1$ in this case.

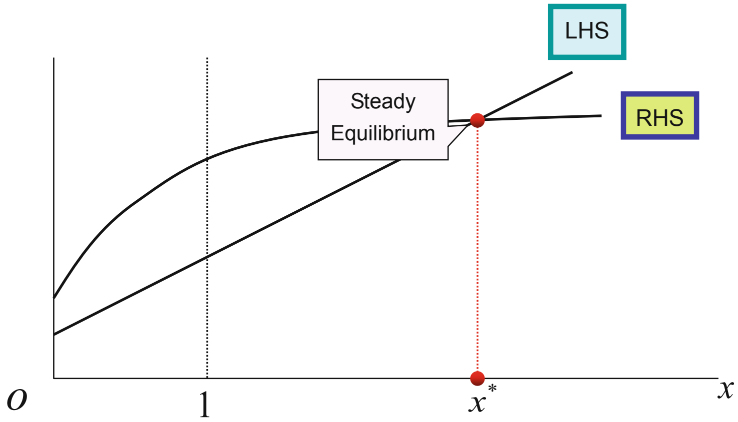


Fig. 7.2 Steady-state equilibrium in the second case

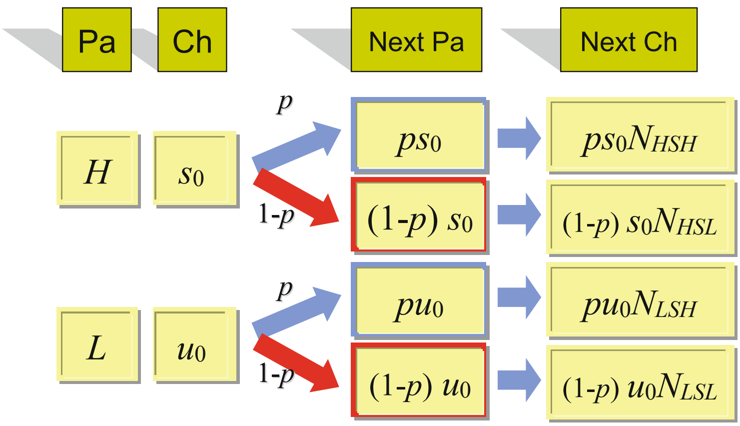


Fig. 7.3 Number of laborers and offspring in the case in which all children choose schooling. *Pa* indicates *parents*, and *Ch* denotes *children*

term t (see Fig. 7.3). After they are educated, the number of children who become H-type laborers with probability p is $ps_t + pu_t$; the number of children who become L-type laborers with probability $1 - p$ is $(1 - p)s_t + (1 - p)u_t$. Therefore, the ratio of L-type to H-type laborers in term $t + 1$ is

$$\frac{L_{t+1}}{H_{t+1}} = \frac{(1-p)s_t + (1-p)u_t}{ps_t + pu_t} = \frac{(1-p)[s_t + u_t]}{p[s_t + u_t]} = \frac{1-p}{p}. \tag{7.32}$$

From Eq. (7.32), we show that $\frac{L_{t+1}}{H_{t+1}}$ is constant: a steady-state equilibrium exists in this case.

7.7 Small Class-Size Policy Effects on Steady-State Equilibria

From Eqs. (7.30) and (7.31), an increase of ϕ affects the LHS and RHS in the graph. Especially, in the range of $x > 1$, the value of RHS increases, where RHS always passes $(1+r)\left(1+\frac{\gamma\delta}{\beta}\right)+1$ when $x = 1$. However, the effect on LHS depends on the value of ϕ before it changes. Because of assumption [A2], increased ϕ increases the slope if ϕ is sufficiently small. On the other hand, if ϕ is large, increased ϕ decreases the slope of LHS. This is also attributed to the slope of LHS being $p(\phi) - (1 + i)\phi$. Especially, in the case in which ϕ is small, increasing ϕ through an appropriate policy will shift the LHS to the left. Consequently, the steady-state equilibrium also shifts to the left (Fig. 7.4). In Fig. 7.4, the dotted line represents the LHS before ϕ changes. The solid line represents LHS after ϕ changes. In this case, the social income distribution decreases because the relative wage in a steady-state equilibrium decreases.

On the other hand, when ϕ is large, increasing ϕ through enactment of a certain policy will shift the LHS to the right. However, RHS behaves similarly as above. In Fig. 7.5, the steady-state equilibrium shifts to the right. The relative wage in the steady-state equilibrium thereby increases. Therefore, we find that the social income distribution expands.

We then have the following proposition:

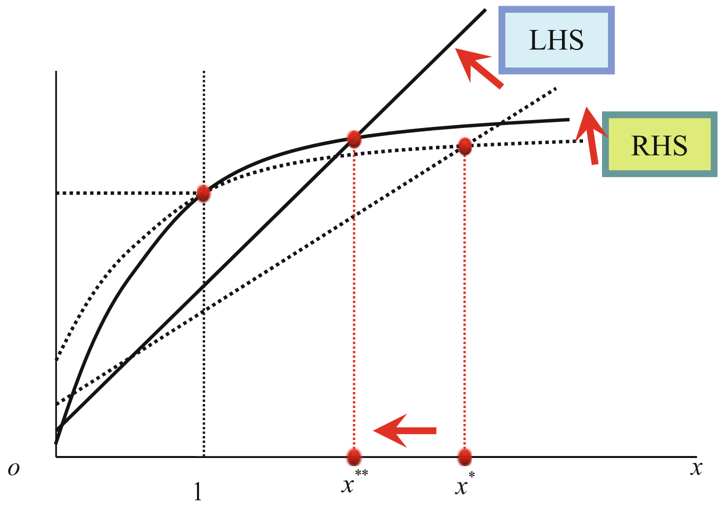


Fig. 7.4 Effects on the steady-state equilibrium of raisingφwhenφis sufficiently small

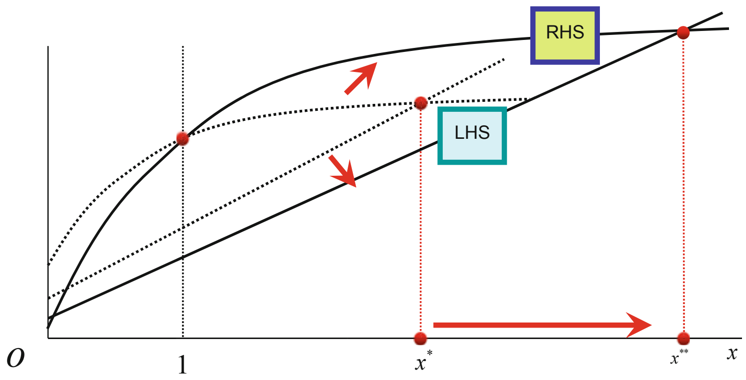


Fig. 7.5 Effect on the steady-state equilibrium of raisingφwhenφis sufficiently large

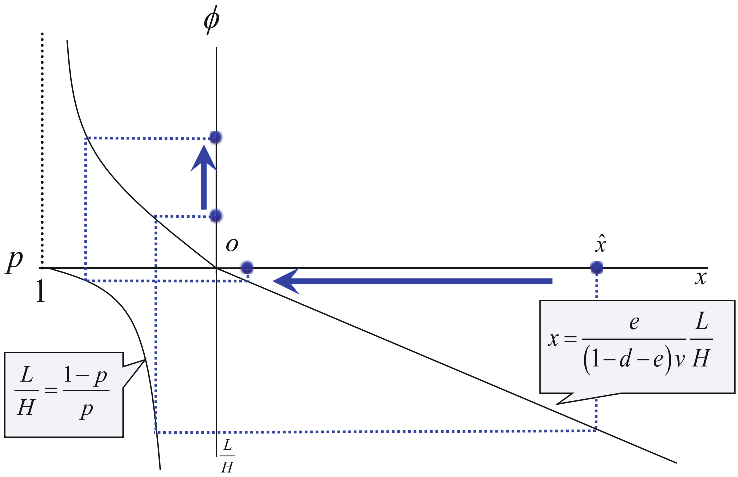


Fig. 7.6 Effect of raising ϕ on the third case steady-state equilibrium

Proposition *There exists ϕ, which minimizes the income distribution.*

The smaller number of ϕ corresponds to a larger class size. Therefore, according to this proposition, a smaller class policy is effective in the sense of reducing the income distribution in an economy when the pre-class size is sufficiently large.

Next, we analyze the relationship between the steady-state equilibrium $\widehat{x}$ and ϕ in the third case. Graphs of Eq. (7.5'), $\frac{L}{H}=\frac{1-p}{p}$, and $p(\phi)$ in the second, third, and fourth quadrants, are shown respectively in Fig. 7.6.

Based on that figure, we find that an increase in ϕ shifts the equilibrium point to the left.

Finally, we consider the effect of raising ϕ on both equilibria (the second case and the third case equilibria). Comparison of Fig. 7.4 with Fig. 7.6 shows that a smaller ϕ

raises the possibility that $x^* < \hat{x}$. In this case, two equilibria exist. On the other hand, comparing Fig. 7.5 with Fig. 7.6, we can infer that a larger ϕ results in $\hat{x} < x^*$. In this case, if $x = \hat{x}$ were chosen by accident, and if the area to the left of $x*$ represents that only a child who is born to H-type parents chooses schooling, then the H-type population grows faster than that of L-type, meaning that x approaches $x*$.[9]

7.8 Conclusion

This paper introduced uncertainty of educational outcomes into Dahan and Tsiddon's overlapping-generations model. We described five life patterns and showed that multiple equilibria exist for which the ratio between skilled and unskilled laborers is constant over time. If two steady-state equilibria exist, then income distribution and fertility would differ.

A proper description of the transitional dynamics of each equilibrium is a remaining task. Dahan and Tsiddon (1998) attempted to explain the stability of the equilibrium. However, showing this stability in our model is complicated. To do so, we must describe the teacher's market clearly, then calculate the population shares among five different life patterns.

Appendix 1: Empirical Analyses of the Graduation Probability

One salient feature of this paper is its introduction of uncertainty in educational outcomes. In this appendix, we analyze the probability of graduation empirically. We assume that the probability depends on teacher-student ratio ϕ in this model. Therefore, we first estimate the value of ϕ under our model. Then, we consider whether the estimate of ϕ is consistent with the real value. Finally, we derive the relation between ϕ and the probability.

Data

We estimate the value of ϕ using the number of students, enrolled students, graduates, teachers, and staff, along with the tuition, enrollment fee, and the wages of teachers and staff of private universities in Japan. The source of those data is the Ministry of Education, Culture, Sports, Science and Technology web pages (The Japanese Ministry of Education, Culture, Sports, Science and Technology n.d.).

[9]This explanation is based on section 2.5.2 in Dahan and Tsiddon (1998).

Estimate of the Value of ϕ

We consider the tuition per private university student to be (the number of students $\times$ a tuition + the number of enrolled students $\times$ an enrollment fee)/the number of students. The total amount of tuition is comparable to the total faculty wage. Therefore, we assume that the school personnel's salaries are balanced by the students' tuition. We have regressed the tuition per capita as: const. Term + ϕ $\times$ (per capita wage of the faculty) + error term. That outcome is the following.

Regression	
Correlation coefficient	0.961235819
R-squared	0.9239743
Adjusted R-squared	0.917638825
S. E. of regression	24396.14516
Number of observations	14

	Correlation coefficient	S. E. of regression	t
Constant	102670.8626	55423.36148	1.852483499
Wage for faculty per capita	0.080694339	0.006681942	12.07647966

The estimate of ϕ is 0.08, but the average ratio of faculty to students during 1988–2001 was 0.095. An estimate of the number of students per faculty member is 12.5 people, whereas the actual number of students per faculty member is 10, on average.

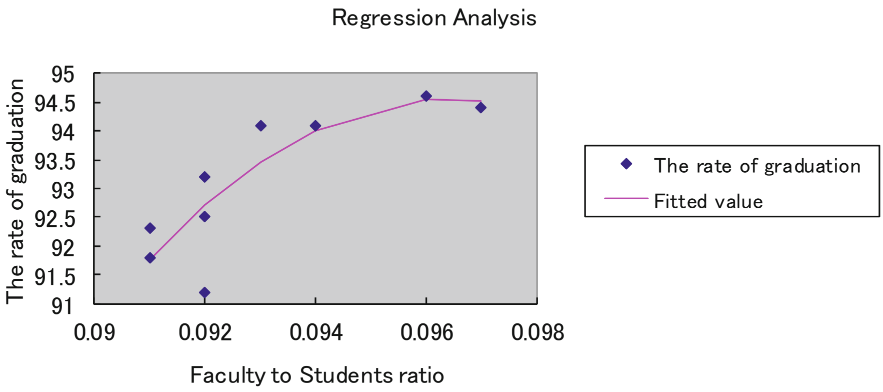
Regression Analysis of the Graduation Probability and ϕ

To regress the probability of graduation and ϕ, we use the number of enrolled students, the number of graduated students, and the number of faculty during 1991–1999. We use the "graduation rate" ((the total number of students who graduated/the total number of enrolled students) $\times$ 100) to approximate the graduation probability.

The regression curve is $p = a\phi^2 + b\phi + k + u$. Quadratic regression analysis between the rate of graduation and the student-to-faculty ratio showed the following.

Coefficients	
a	−97335.4
b	18754.23
k	−808.828
Coefficient of determination R^2	0.731696
Correlation coefficient R	0.855392

Estimate ϕ	Graduation rate	Fitted value
0.091	92.3	91.77226
0.091	91.8	91.77226
0.092	93.2	92.71411
0.092	92.5	92.71411
0.092	91.2	92.71411
0.093	94.1	93.46129
0.094	94.1	94.01379
0.096	94.6	94.5348
0.097	94.4	94.50329



By this regression analysis, we verified that the assumptions $p'(\phi) > 0$, $p''(\phi) < 0$ are plausible.

Appendix 2: Proof of Lemma 3

Lemma 3 *Relationships among fertilities in the steady-state equilibrium are:*

$$N_{\mathrm{LSL}} < N_{\mathrm{LSH}} < N_{\mathrm{HSH}} < N_{\mathrm{HSL}} \quad \text{or} \quad N_{\mathrm{LL}}.$$

Proof The number of offspring in the steady-state equilibrium is indicated by Eqs. (7.24)–(7.28).

(a) Comparison of N_{LSL} with N_{LSH}.

$$N_{\mathrm{LSL}} = \frac{\beta}{\delta}\left\{(1+i)\left(\frac{\gamma\delta}{\beta} - \phi\frac{w_H}{w_L}\right) + 1\right\}. \tag{7.25}$$

$$\begin{aligned} N_{\mathrm{LSH}} &= \frac{\beta}{\delta}\left\{(1+i)\left(\frac{\gamma\delta}{\beta}\frac{w_L}{w_H} - \phi\right) + 1\right\} \\ &= \frac{\beta}{\delta}\left\{(1+i)\frac{w_L}{w_H}\left(\frac{\gamma\delta}{\beta} - \phi\frac{w_H}{w_L}\right) + 1\right\}. \end{aligned} \tag{7.26}$$

Note that we assume $\frac{\gamma\delta}{\beta}w_L - \phi w_H < 0$ in the L-type household and assume $1 < \frac{w_H}{w_L}$. Consequently, we find that $N_{\mathrm{LSL}} < N_{\mathrm{LSH}}$.

(b) Comparison of N_{LSH} with N_{HSH}.

$$N_{\mathrm{HSH}} = \frac{\beta}{\delta}\left\{(1+r)\left(\frac{\gamma\delta}{\beta} - \phi\right) + 1\right\}. \tag{7.28}$$

We have assumed $\frac{\gamma\delta}{\beta}w_H - \phi w_H > 0$ in the H-type household, which means that $\frac{\gamma\delta}{\beta} - \phi > 0$. By this inequality, the part $(1+i)\left(\frac{\gamma\delta}{\beta}\frac{w_L}{w_H} - \phi\right)$ in Eq. (7.26) becomes negative, and the part $(1+r)\left(\frac{\gamma\delta}{\beta} - \phi\right)$ in Eq. (7.27) becomes positive. Therefore, we have $N_{\mathrm{LSH}} < N_{\mathrm{HSH}}$.

(c) Comparison of N_{HSH} with N_{HSL}.

$$\begin{aligned} N_{\mathrm{HSL}} &= \frac{\beta}{\delta}\left\{(1+r)\left(\frac{\gamma\delta}{\beta}\frac{w_H}{w_L} - \phi\frac{w_H}{w_L}\right) + 1\right\} \\ &= \frac{\beta}{\delta}\left\{(1+r)\frac{w_H}{w_L}\left(\frac{\gamma\delta}{\beta} - \phi\right) + 1\right\}. \end{aligned} \tag{7.27}$$

Because we assume that $1 < \frac{w_H}{w_L}$, the inequality $N_{\mathrm{HSH}} < N_{\mathrm{HSL}}$ is trivial. The relation $N_{\mathrm{HSH}} < N_{\mathrm{LL}}$ is also trivial. However, the size comparison of N_{HSL} with N_{LL} remains unclear.

Q. E. D.

Appendix 3: Proof That No Steady-State Equilibrium Exists in the Case of Children Born into *H* Household, Who Choose Only Schooling

In this Appendix, we prove that no steady-state equilibrium pertains in the case of children born into H household who choose only schooling, as mentioned in Sect. 7.6.

Proof We use the notation H_{t-1}^{C} to represent the number of children born into the H-type household at period $t-1$. Superscript C denotes Child; similarly, L_{t-1}^{C} represents the number of children born into the L-type household at period $t-1$. In this case, although all children who were born into the H-type households choose schooling, only a fraction p of them become H-type parents in the next period. Therefore, the number of H-type parents in period t is pH_{t-1}^{C}. They then supply the total H-type labor force in period t, and

$$H_t = pH_{t-1}^{C}. \tag{7.28}$$

The rest, $(1-p)H_{t-1}^{C}$, become L-type parents in the next period. Notice that the children born into the L-type household do not choose schooling. Therefore, the total number of L-type parents in the next period (period t) is $L_{t-1}^{C} + (1-p)H_{t-1}^{C}$. They supply the *L-type* labor force in period t. Moreover, because their children $L_{t-1}^{C}N_{\text{LL}} + (1-p)H_{t-1}^{C}N_{\text{HSL}}\ \left(= L_t^{C}\right)$ do not choose schooling, they also supply the *L-type* labor force. Consequently, the total number of L is

$$L_t = (1+N_{\text{LL}})L_{t-1}^{C} + (1-p)(1+N_{\text{HSL}})H_{t-1}^{C}. \tag{7.29}$$

However, this situation is contradictory to the definition of a steady-state equilibrium because the growth rate of H_t is lower than the growth rate of L_t. To verify this fact, it is sufficient to show that the maximum growth rate of H_t is lower than the growth rate of L_t. Here, the maximum growth rate of H_t is the growth rate attained at $p=1$. This maximization can be verified as follows: The growth rate of H_t is explained as

$$\frac{H_{t+1}-H_t}{H_t} = \frac{p\left(pH_{t-1}^{C}N_{\text{HSH}}\right) - pH_{t-1}^{C}}{pH_{t-1}^{C}} = pN_{\text{HSH}} - 1.$$

From that expression, we find that the maximum growth rate is $N_{\text{HSH}} - 1$. It is attained at $p = 1$. That is,

$$\underset{p}{Max}\frac{H_{t+1}-H_t}{H_t}=N_{\mathrm{HSH}}-1. \tag{7.30}$$

On the other hand, because we have $L_t=(1+N_{\mathrm{LL}})L_{t-1}^C$ at $p=1$, the growth rate of L_t is

$$\frac{L_{t+1}-L_t}{L_t}=\frac{(1+N_{\mathrm{LL}})L_{t-1}^C N_{\mathrm{LL}}-(1+N_{\mathrm{LL}})L_{t-1}^C}{(1+N_{\mathrm{LL}})L_{t-1}^C}=N_{\mathrm{LL}}-1. \tag{7.31}$$

Noting that $N_{\mathrm{HSH}}<N_{\mathrm{LL}}$ is held in the steady-state equilibrium by Lemma 3, and comparing Eq. (7.30) with (7.31), we can verify that the maximum growth rate of H_t is lower than the growth rate of L_t. For all values of p, the growth rate of H_t is therefore always lower than the growth rate of L_t. This statement contradicts the definition of a steady-state equilibrium.

Q. E. D.

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Chapter 8
Investment in Education and Intergenerational Conflicts of Interest within the Family

Miki Matsuo

Abstract The purpose of this study is to analyze whether investment in education is sustainable. The study applies a model of repeated games to investment in child education under an overlapping generations (OLG) setting. It is assumed that a household consists of three generations and each individual lives through three stages of life. If there are no intergenerational altruistic links, an intergenerational conflict will arise among the family members. In such a situation, the main result is that investment in child education is unsustainable in the weak equilibrium. Therefore, it would be extremely difficult for poor households to escape the poverty trap.

Keywords Human capital · Poverty trap · Intergenerational conflicts · Overlapping generations games · Trigger strategy

8.1 Introduction

According to the World Bank, the extreme poor fell below 10% of the world population in 2015 for the first time. However, poverty remains as one of the most serious global issues, and there are still many countries where it is hard to escape

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poverty in our world.[1] The World Bank also states that there are still hurdles to end extreme poverty by 2030.

There has been significant and extensive research and reporting about adversity in underdeveloped countries. Most studies suggest that one reason why poverty does not end in the poorest of countries is that it is most challenging to accumulate human capital through education. In modern society, knowledge and technology are the most important factors to derive economic growth. In the context of economics, these factors are perceived as human capital. There are indeed millions of children who cannot get sufficient education (even primary education) around the world. In fact, in *Fixing the Broken Promise of Education for All: Findings from the Global Initiative on Out-of-School Children*, UNICEF and UIS reported there were about 63 million children who were not allowed the right to get an education in 2015. In 2016, 263 million children were still out of school, of whom 63 million were children of primary school age (about 6 to 11 years old).[2] There are indeed schools in underdeveloped countries where elementary school is at least offered. However, there are many children who are absent from school for family reasons; illness; a lack of willingness; a lack of parental enforcement; and so on. Given these circumstances the United Nations focused on the achievement of primary education in the poorest countries as the most important target in "*The Millennium Development Goals*."

Among the many reasons for those out-of-school, child labor is the most pressing. Putnick and Bornstein (2015) suggested significant negative relationships between child labor and school enrollment in low-and middle-income countries.[3] In fact, according to data from ILO (2017), about 150 million children aged 5 to 15 are subjected to labor due to extreme poverty. Their families rely on their children's income. As shown in Fig. 8.1, the lower the income group, the higher the out-of-school rate. Moreover, parents who did not get an education do not understand the necessity of education for their children.

Despite realizing the significance for a long time, the poverty problem has not been solved, and it will take a long time to eliminate poverty entirely. The reason why is that there are some specific challenges such as education, health, culture, financial systems, politics within the underdeveloped countries, and so on which are intertwined.

Recently; as data about underdeveloped economies have been collected with randomized and controlled trials (such as Banerjee and Duflo); new knowledge and findings about these issues has advanced greatly.

One of the most significant problems is child labor with extreme poverty in underdeveloped countries. Many economists have demonstrated the relationship

[1]In October 2015, the World Bank has revised international poverty line from US $1.25 to $1.90 a day. It takes into account differences in the cost of living across countries.

[2]See UNICEF and UIS (2018) : UIS Fact Sheet No.48. "*One in Five Children, Adolescents and Youth is Out of School*" (The UNESCO Institute for Statics).

[3]Putnick and Bornstein (2015) investigated relationships of child labor with school enrollment in more than 185,000 children (7- to 14-year-old) in 30 countries which are classified low- and middle-income group.

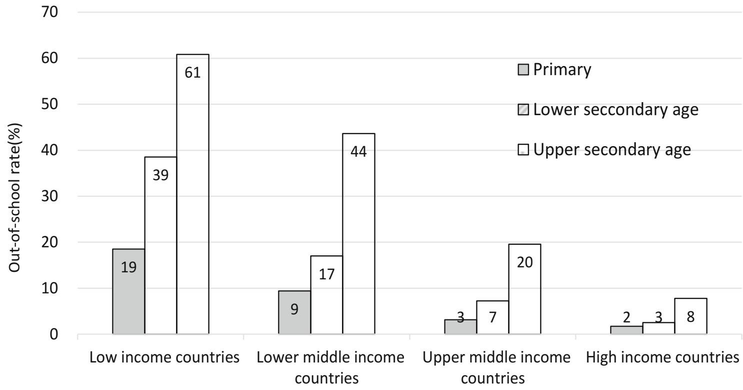

Fig. 8.1 Out-of-school rate by income level and age group, 2018. (Source: UNESCO Institute for Statistics Database (2018), http://data.uis.unesco.org/)

between poverty and child labor and shown the reason why the poor cannot get out of poverty. Basu and Van (1998) and Fan (2004) pointed out that as it is expensive for poor households to get an education, they tend to prioritize daily consumption over the pursuit of education. These studies illustrate extreme poverty prevents children from acquiring the skills and education by forcing them to work. As a result, they are trapped in a vicious cycle of poverty from generation to generation, and cannot escape from it.

There are many studies about the poverty trap. Generally, in these studies, the poverty trap is explained by human capital accumulation. The most representative model with the poverty trap such as Becker and Lewis (1973) takes into account the trade-off between quality and quantity of children. These studies imply that since poor households rely on child labor, they have an incentive to have many children, and these families tend to not invest in education.[4]

Implementing bans on child labor and offering free education ought to decrease or eliminate child labor and improve human capital accumulation in underdeveloped countries. However, from the latest perspective of development economics, such policies are not always effective. For example, Bharadwaj et al. (2013) estimated empirically the effectiveness of India's 1986 national ban on child labor and showed that children's wages fell and child labor actually increased.[5]

The reason for the trend away from education in underdeveloped countries is the incentive of parents to invest in education is different from those in developed countries. Some people consider children as financial instruments in underdeveloped

[4]Poor households prioritize consumption for subsistence, they cannot afford to invest in education. Fan (2004) showed that consumption is a necessary good and education is a luxury good.

[5]Fan (2004), Edmonds (2005), Basu (2005), and Contreras (2008) show theoretically that this policy does not improve economies.

countries.[6] Parents want children to take care of them in their old age. Furthermore, there are challenges in providing education in that there are no effective means of saving in these countries. Their children (who become adults in the next period) will have choose to educate their offspring and take care of their parents in addition to their own consumption. Moreover, after they retire, their children may not support them even if they give their offspring the benefit of education. There is, as such, an intergenerational conflict of interest among the family members. In these situations, parents may not be able to trust that their children will support them in the future. Parents thus have some incentives to forgo investment in education for their children.

This is similar to the prisoner's dilemma structure in game theory. The prisoner's dilemma is well founded that if all players pursue self-interested behavior for the sake of profit it derives a negative outcome for all. It is better for all players to be cooperative, but the cooperative action requires some cost. Under one-shot game, all players are able to get more gain from deviation than cooperation since they will not exist in future. If all players remain in the game after their decision, they would choose another choice. Players predict that depending on one's own current behavior, cooperation is rewarded and deviation is punished in the future. In repeated game theory it is proved that when players expect to engage in a long-term relationship, they may choose cooperative action continuously. In order to analyze investment in education within the household, we have to consider the interaction with each other among the members. This chapter studies applies the OLG repeated game to a problem of investment in education. The structure of the OLG repeated game is that organization/household is infinitely lived, but its members are finitely-lived and old members are replaced with young ones. Cremer (1986), Kandori (1992), and Smith (1992) showed that cooperation can be sustainable as equilibrium in the OLG repeated game through the chain of rewards and penalties over generations. In a repeated game, however, if one player chooses deviation once, the others move toward a deviate action, which is well known as "tit for tat" and "trigger strategy." Using trigger strategy, either player always chooses deviation once the other moves toward noncooperative action. As a result, only an undesirable equilibrium is achieved and an efficient equilibrium would never be achieved.

Let us return to investment in education in underdeveloped countries. Even though some parents choose to invest in education for children instead of saving, there is no guarantee that their children will support them after retirement. As they cannot trust their children, they choose the selfish move, that is, not to invest in education. Therefore, it is difficult for families to invest in education and to escape from the poverty trap. As a result, investment in education remains low in the economy. The structure resembles a theoretical model with the OLG repeated game.

[6] Banerjee and Duflo (2011) pointed out this aspect in "*Poor Economics*"(page 118 in Chap. 5). They referred the reason why there are hardly financial instruments for their retirement in Chaps. 6, 7, and 8.

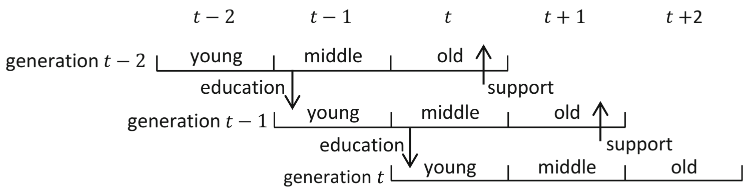

Fig. 8.2 The household structure with OLG

The literature related to this study is Ando and Kobayashi (2008). They constructed the model where intergenerational conflicts of interest arise within an organization which is infinitely lived and consists of three overlapping generations. They demonstrated that the seniority system including the seniority-based task allocation system and the seniority-based profit allocation system solves the conflict. Assuming the current profit is dependent on the current action and the previous action, they propose that the cooperative equilibrium is sustainable.

In families, there is also an intergenerational conflict regarding to whom and how much household income is allocated. The structure is similar to one explored by Ando and Kobayashi (2008). Combining the traditional setup with Ando and Kobayashi (2008), we consider whether a cooperative strategy profile is able to be sustained in equilibrium in the OLG model with the structure shown in Fig. 8.2.

The chapter is organized as follows: the basic model and analysis are presented in Sects. 8.2 and 8.3 respectively. The interpretation of the results is provided in Sect. 8.4. The conclusions are presented in Sect. 8.5.

8.2 Model

We shall consider the overlapping generations' model with a household that consists of three generations at period t. The household itself is an ongoing entity, but its members are short-lived and an old member is replaced with a young one. An individuals who is born in period t are called generation t ($t = 0, 1, 2, \ldots$). At the beginning of period zero, we introduce the individual -2 and -1. He/she is homogeneous and lives for three periods, young, middle, and old age. There is no population growth, and the size is normalized to be unified. He/she is endowed with one unit of time at every period and also is endowed with one unit of human capital in the first period.

In the first period, each individual either spends his/her time at school to acquire human capital or works as child labor. The decision whether his/her time is allocated between schooling and labor-force participation is made by his/her parent. Note that it costs e(constant) to get an education. He/she will be skilled labor with human capital h (>1)in his/her second period through education. On the other hand,

Table 8.1 The payoffs of the basic model

a_{t-1} \ a_t	C	D
C	$wh-e$	$wh+w$
D	$w-e$	$w+w$

w and e denote the wage and education tuition, respectively

working as child labor, he/she does not get additional human capital and he/she is still endowed with one unit of human capital in the second period.

In the second period, each individual works one unit of time inelastically as a high or low skilled labor and receives the wage. We assume that the household income, which is the sum of the wage earned in the period, is allocated among the middle and the old within the family. Denote λ_M and λ_O by the allocation rate of the income for the middle and the old respectively, and these are given exogenously, we assume $\lambda_M + \lambda_O = 1$. We suppose, in addition, that each individual is unable to carry over the household income in the next period. Furthermore, he/she chooses whether to invest in the offspring's education or not.

In the third period, each individual retires and receives a fraction λ_O of the household income generated at the period. Therefore, the amount receivable in the third period is dependent on both the current action which is made by their children and the previous own action. That is, in turn own action reflects the lifetime utility.

Let us consider the following game. We assume that the decision maker is always a menber of the middle generation. The decision maker chooses whether to give his/her child an education or not. Denote the current action at period t that the middle born at period $t-1$ chooses by $a_t \in (C, D)$. The action C denotes cooperation, which means the middle invests in education. The action D denotes deviation, which means he/she has his/her offspring work without investing in education. Then, the middle chooses the action after he/she observes his/her parent's action at period $t-1$. That is, he/she plays sequential games. We consider the household income as payoffs and denote it by $\pi_t(a_{t-1}, a_t)$, which a_{t-1} indicates the action of the middle at period $t-1$, namely, parent's action, and a_t indicates the current action of the middle at period t. Then, given the wage w, the payoff structure is represented by Table 8.1.

When $(a_{t-1}, a_t) = (C, C)$, the middle has gotten an education during the young period, he/she is able to obtain the labor income wh. In this case, he/she chooses to invest in education paying tuition e, the disposal income is represented as $wh - e$. When $(a_{t-1}, a_t) = (C, D)$, while the middle has gotten an education, he/she does not invest in education and makes his/her offspring work as child labor. Then, the total household income is represented as $wh + w$, where w is the wage of low skilled labor. When $(a_{t-1}, a_t) = (D, C)$, the uneducated middle invests in education, then the disposable income in that case is expressed in $w - e$. Lastly, when $(a_{t-1}, a_t) = (D, D)$, both the middle and young do not get an education and they work as low skilled labor. Then the household income is 2w. Each individual chooses his/her optimum response after each version of the game. In this situation, the strategy profile (D, D) is a Nash Equilibrium. The strategy (D, D) is the dominant equilibrium in the one-shot game. The following assumption guarantees that the strategy profile (C, C) is the efficient action profile for each individual.

Assumption 1

$$wh - e > 2w.$$

As we see in Table 8.1, the strategy (D, D) profile is the only equilibrium in the game with one-shot. However, we would like to consider individuals interact repeatedly over time because the household will be ongoing for a long time in the future. The old individual desires to get immediate income gains since he dies at the next period. On the other hand, the middle and young have to consider their income in the next period. This brings about an intergenerational conflict within the family.

Furthermore, when each individual makes a decision, he/she faces information constraints. It is impossible for him/her to observe what happened before he/she is born. We assume that he/she can observe the previous actions that is chosen by his/her parent and he/she makes a decision depending on that observation. We assume there is no additional information about the previous generational actions.

Given the household structure and allocation rule, an individual born at period $t - 1$ chooses his/her action to maximize his/her discounted utility. Suppose that each individual's utility of payoffs is represented by

$$u_{t-1} = \lambda_{\mathrm{M}}(a_{t-1}, a_t) + \delta\lambda_{\mathrm{O}}(a_t, a_{t+1}), \tag{8.1}$$

where $\delta \in [0, 1]$ is a discount factor.

8.3 Analysis

In this section, we will show whether the cooperative action is able to be sustained when the individual plays the same strategy that his/her parent choose at the previous period. In this model, we assume the DM chooses action in accordance with the previous action, which is called the Markov trigger strategy. The DM chooses C if $a_{t-1} = C$, and he/she chooses D if $a_{t-1} = D$. Then, the Markov trigger strategies profile is the Markov perfect equilibrium if it is a Nash equilibrium in every state of the game.

Table 8.2 The normalized payoffs of the basic model

a_{t-1} \ a_t	C	D
C	$wh - 2w - e$	$wh - w$
D	$-w - e$	0

We define the sustainability in the cooperative equilibrium.[7]

Definition *Given the allocation rule exogenously, the household can sustain the cooperative action profile if there exist individuals' strategies that satisfy the following.*

1. *In every period, the realized action is cooperation C.*
2. *The individuals' strategies constitute a sequential equilibrium.*
3. *The DM has incentives to choose C if the previous action is C in equilibrium.*

8.3.1 Case of Basic Model

Now, we consider whether the cooperative action is sustainable in the equilibrium. To calculate easily, normalizing the payoffs in the strategy profile (D, D) to $(0, 0)$, we rewrite the payoffs as Table 8.2. Then, the necessary and sufficient conditions for the Markov trigger strategy to achieve the sustainable equilibrium are

$$\lambda_{\mathrm{M}}(C, C) + \delta\lambda_{\mathrm{O}}(C, C) \geq \lambda_{\mathrm{M}}(C, D) + \delta\lambda_{\mathrm{O}}(D, D), \tag{8.2}$$

$$\lambda_{\mathrm{M}}(D, D) + \delta\lambda_{\mathrm{O}}(D, D) \geq \lambda_{\mathrm{M}}(D, C) + \delta\lambda_{\mathrm{O}}(C, C). \tag{8.3}$$

Substituting the payoffs into inequality Eqs. (8.2) and (8.3), respectively, we derive the following conditions.

$$\delta \geq \frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{O}}} \frac{w + e}{(wh - 2w - e)},$$

[7] Following Ando and Kobayashi (2008), we define the sustainability. Concerning third property, however, Ando and Kobayashi (2008) holds that the DM has strict incentives to choose C if the previous action is C in equilibrium. It means inequalities Eqs. (8.2) and (8.3) do not include equality.

Table 8.3 The payoffs of model with difference in income

a_{t-1} \ a_t	C	D
C	$wh - w - \gamma w - e$	$wh - w$
D	$-\gamma w - e$	0

$$\frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{O}}} \frac{w + e}{(wh - 2w - e)} \geq \delta.$$

From the above inequalities, we obtain

$$\delta = \frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{O}}} \frac{w + e}{(wh - 2w - e)}.$$

Thus, we obtain the following proposition.

Proposition 1 *There exists the weak equilibrium with the Markov trigger strategy if and only if* $\delta = \frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{O}}} \frac{w+e}{(wh-2w-e)}$.

This proposition implies it is extremely difficult to take the cooperative action since there is only one discount rate that satisfies Eqs. (8.2) and (8.3).

8.3.2 Case of Model with Difference in Income

In the described above, there is no difference in the wage between adult and child labor. In most organizations, generally, senior workers receive higher wage than junior workers. The reason why is as labor's productivity is improved through practice and experience; the wage is also higher with increases in labor productivity. Then, we suppose that adult wage is higher than the child's, and we examine whether the cooperative action profile is sustainable in this case. Let the child wage denote γw where $\gamma \in (0, 1)$. Table 8.3 shows the payoffs in the case incorporating wage differences and normalized in the strategy profile (D, D) to $(0, 0)$.

Calculate the necessary and sufficient condition for the Markov trigger equilibrium using the payoffs in Table 8.3.

$$\lambda_{\mathrm{M}}(wh - w - \gamma w - e) + \delta\lambda_{\mathrm{O}}(wh - w - \gamma w - e) \geq \lambda_{\mathrm{M}}(wh - w) \Leftrightarrow \delta \geq \frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{O}}} \frac{\gamma w + e}{(wh - w - \gamma w - e)}, \tag{8.4}$$

and

$$0 \geq \lambda_{\mathrm{M}}(-\gamma w - e) + \delta\lambda_{\mathrm{O}}(wh - w - \gamma w - e) \Leftrightarrow \frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{O}}} \frac{\gamma w + e}{(wh - w - \gamma w - e)} \geq \delta. \tag{8.5}$$

From the above conditions, we obtain as below.

$$\delta = \frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{O}}} \frac{\gamma w + e}{(wh - w - \gamma w - e)}.$$

In this case, there is the sustainable equilibrium if and only if δ is equal to $\frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{O}}} \times \frac{\gamma w + e}{(wh - w - \gamma w - e)}$.

8.3.3 Case of Model with Difference in Allocation Rule

In this subsection, we introduce the difference of the allocation rule into the basic model. In the above, under the allocation rule $\lambda_{\mathrm{M}} + \lambda_{\mathrm{O}} = 1$, the DM chooses to act responding to the parent's action in the preceding period. In this case, we assume that the allocation rule is different whether previous action is cooperative or not and we show that the cooperative action is able to be sustainable in the equilibrium.

Suppose that the allocation ratio of payoffs for the old generation is $\alpha \in (0, 1)$ if $a_{t-1} = C$, and the allocation ratio of that is and $\beta \in (0, 1)$ if $a_{t-1} = D$. Table 8.4 shows the combination of old and middle payoffs obtained under the new allocation rule responding to each strategy profile.

The necessary conditions for the Markov trigger equilibrium are rewritten by,

$$(1-\alpha)(C, C) + \delta\alpha(C, C) \geq (1-\alpha)(C, D) + \delta\beta(D, D), \tag{8.6}$$

$$(1-\beta)(D, D) + \delta\alpha(D, D) \geq (1-\beta)(D, C) + \delta\alpha(C, C). \tag{8.7}$$

From both the above conditions and Table 8.3, we derive the necessary conditions as follows, respectively.

Table 8.4 The combination of payoffs under the allocation rule

a_{t-1} \ a_t	C	D
C	$\alpha(wh - 2w - e),\ (1-\alpha)(wh - 2w - e)$	$\alpha(wh - w),\ (1-\alpha)(wh - w)$
D	$\beta(-w - e),\ (1-\beta)(-w - e)$	$0,\ 0$

$$(1-\alpha)(wh-2w-e)+\delta\alpha(wh-2w-e) \geq (1-\alpha)(wh-w),$$
$$0 \geq (1-\beta)(-w-e)+\delta\alpha(wh-2w-e).$$

Solving with respect to δ, we are able to rewrite the above conditions as below.

$$\frac{(1-\alpha)(w+e)}{\alpha(h-2w-e)} \leq \delta,$$
$$\delta \leq \frac{(1-\beta)(w+e)}{\alpha(h-2w-e)}.$$

Hence, if the value of δ holds the following inequality,

$$\frac{(1-\alpha)(w+e)}{\alpha(wh-2w-e)} \leq \delta \leq \frac{(1-\beta)(w+e)}{\alpha(wh-2w-e)},$$

there exists the equilibrium. Whether it holds or not is or not is dependence of the value of α and β. Then, there exists the value of δ if $\alpha \geq \beta$. On the other hand, if $\alpha < \beta$, this inequality does not satisfy.

Therefore, we obtain the following proposition.

Proposition 2 *The cooperative action profile by using the Markov trigger strategy is sustainable if $\alpha \geq \beta$ and the value of δ is satisfied with,*

$$\frac{(1-\alpha)(w+e)}{\alpha(wh-2w-e)} \leq \delta \leq \frac{(1-\beta)(w+e)}{\alpha(wh-2w-e)}. \tag{8.8}$$

8.4 Discussion

Using the basic setup, we presented it is extremely difficult that the Markov trigger strategy equilibrium is able to be sustainable. In Proposition 1, we showed there exists the weak equilibrium if and only if given allocation rule, δ is equal to a certain variable. Proposition 1 is interpreted as following.

In underdeveloped countries where the basis of the financial system has not been constructed, some parents consider their children as a financial instrument. While they want their children to support them after retirement, there is uncertainty that their children will not support them in the future. With uncertainty, parents will not invest in their children as savings for the future. Proposiotion 1 suggests that in a model where individuals are not altruistic, investment in education for children as savings through a Markov trigger strategy is almost impossible. Therefore, it is extremely unstable that households continue to invest in education in such a situation.

Moreover, we explored the basic model by introducing the difference in the allocation rate. The allocation rate of the middle member and the old member is different between parent's action (previous action); whether investing in education or not. In order to analyze in this case, we assumed that the allocation rate for parents who give their children to education is higher than that of parents not educating their children. Then, we presented there exists the Markov trigger equilibrium under some conditions. As a result, the cooperative action is able to be sustainable, which means that households might continue to invest in education. We can interpret that the difference in the allocation rate is the difference in social norm, and the household is able to sustain investment in education when an educated household's social norm is higher compared with a non-educated household's one.

8.5 Conclusion

We analyzed whether investment in education is sustainable in the case that there is an intergenerational conflict of interest within the family. This study applied a repeated game theory to a problem of investment in education. We showed that there might be the weak equilibrium in which the decision maker has weaker incentives to choose C if the previous action is C. The equilibrium is very restrictive and extremely unstable. It means that investment in education is unable to be sustainable, which may make it difficult for households to escape the poverty trap. Moreover, we introduced the difference in the allocation rate for parents between educated household or not educated and examined whether the cooperative action is able to be sustained as the equilibrium in the OLG repeated game. We showed if the allocation rate for parents who give education to their children is higher than that of parents not educating their children, the cooperative action profile by using the Markov trigger strategy might be sustainable. In this case, the decision maker has strict incentives to choose C if the previous action is C.

Finally, I would like to make comments to extend our model. In this chapter, there is the weak equilibrium in the basic setup. This is referenceable to the basic setup. For simplicity, we assumed all variables are exogenously given and the population is also constant. However, we need to take account the number of children since it must affect the total education cost. We will extend this model by incorporating Becker and Lewis (1973) to the model which the number of children is decided endogenously.

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Chapter 9
Income Inequality and Intergenerational Mobility in an Endogenously Growing Economy

Tamotsu Nakamura

Abstract Incorporating skill-biased technological changes and education (skill-acquisition) costs into a simple intergenerational mobility model, this chapter analyzes the dynamics of income inequality and intergenerational mobility, and the effects of technological progress on inequality in an endogenously growing economy. The education cost plays a crucial role in the model. If it does not increase sharply with technological progress, the economy monotonically converges toward the steady state. In this case, the skill-premium puzzle can be explained if human capital greatly contributes technological progress. In contrast, when the cost increases rapidly with technological progress, then aggregate growth, mobility and inequality exhibit cyclical behavior. In addition, a possibility is pointed out that if human capital greatly influences technological progress, then a one-time increase in exogenous technological progress decreases aggregate growth not only in the transition but also at the steady state.

Keywords Intergenerational mobility · Income inequality · Education choice · Skill-biased technological changes · Skill-premium puzzle

9.1 Introduction

Technological progress, income inequality and intergenerational mobility have attracted growing interests in theoretical and empirical studies in the growth literature. Unless it is purely skill-neural, technological progress has some impact on income inequality between skilled (educated) and unskilled (uneducated) workers. Also, inequality is an important driving force for intergenerational mobility, while mobility itself affects inequality and possibly technological progress through

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changes in human capital or the composition of educated and uneducated workers. Hence, these three factors are intrinsically interrelated.

Maoz and Moav (1999) provide an interesting and tractable framework to analyze the relationship between inequality and mobility. However, economic growth will disappear in the long run in their model because no technological change nor capital accumulation is introduced. Also, as Nakamura and Murayama (2011) point out, the dynamics of the model and the effects of technology shock on intergenerational mobility and income inequality depend crucially on the specific education (skill-acquisition) cost function. Although Maoz and Moav (1999) show that the economy monotonically approaches the steady state with decreasing intergenerational mobility, the cyclical behavior of intergenerational mobility can emerge depending on the behavior of the education cost. In addition to theoretical findings, empirical studies have also shown that various patterns of intergenerational mobility have been observed in developed economies. For example, Nicoletti and Ermisch (2007) have found that Britain has experienced the monotonous decrease in intergenerational mobility, while Bratberg et al. (2007) have shown that the mobility has monotonically increased in Norway. In contrast, Lee and Solon (2009) have found that the mobility has changed cyclically over decades in the USA,

In this chapter, using a dynamic model with human capital, we analyze economic growth and the relationship between intergenerational mobility and inequality. Then, there are at least three important factors to be considered.[1] First, technological progress tends to be skill-biased. For example, Galor and Moav (2000) emphasize "an increase in the rate of technological progress raises the rate of return to skills." (p. 472) Second, human capital contributes technological progress. For this regard, Galor and Moav (2000) state "an increase in the level of human capital increases the rate of technical progress." (p.472) Third, since "new skills are more costly to acquire than skills required by preexisting types of equipment" (Caselli 1999: p. 78), substantial costs are involved in acquiring advanced skills. This chapter introduces technical progress that is embodied into human capital to examine the relationships among technological progress, income inequality and intergenerational mobility in a growing economy.

In concrete, incorporating the above three elements into the Maoz and Moav (1999) framework of intergenerational mobility, this chapter shows that the education (skill-acquisition) cost plays a crucial role not only in characterizing the dynamic behavior of the model but also in determining the effects of technological progress on intergenerational mobility and income equality. As a result, it is shown that the cyclical behavior of mobility and equality can appear when the cost increases rapidly with technical progress, while the monotonic transition to the steady growth takes place when it does not increase sharply.

[1] Although we do not consider population growth, it also plays an important role in determining the relationship between income inequality and intergenerational mobility. See, for example, Aso and Nakamura (2019) for the effects of population growth.

Since 1980s in the United States, although the supply of skilled labor has substantially increased, the skilled labor premium has also increased. This phenomena is known as the skill premium puzzle. In the model presented in this chapter, when the education (skill-acquisition) cost does not increase sharply, then the puzzle can be explained under weaker conditions than in Acemoglu (2002, 2009). Human capital contributes technological progress that raises the rate of return to skills. An exogenous rise in skill-biased technological progress raises the returns to skilled workers, and hence increases the number of skilled workers or human capital. This in turn raises technological progress. As a result, the wage premium for skilled workers increases with the number of skilled workers. In other words, the skill-premium puzzle takes place.

When the education (skill-acquisition) cost increases sharply with the rate of technological progress, then the growth rate of income, intergenerational mobility and income inequality exhibit cyclical behavior. Also, there is the possibility that an exogenous increase in technological progress decreases the long-run growth rate. Even if it does not reduce the long-run growth rate, the exogenous increase may lower the growth rates during the transition because the cyclical behavior of intergenerational mobility takes place.

The rest of the chapter is organized as follows. Section 9.2 sets up the model. Section 9.3 analyzes the model to derive the main results. Section 9.4 concludes the chapter.

9.2 The Model

Consider an overlapping generations economy without population growth in which individuals live for two periods. The model presented in this chapter is based on the Maoz and Moav (1999) model of intergenerational mobility. We extend it to a growth model introducing skill-biased technological progress mainly contributed by human capital accumulation.

9.2.1 Technology and Factor Prices

In order to take skill-biased technological changes into account, differently from the Maoz and Moav (1999) model that uses a Cobb-Douglass production function, we assume a CES production technology. In concrete, technology in the economy at period t is characterized by the following production function:

$$Y_t = \left[(A_t E_t)^{\frac{\sigma-1}{\sigma}} + (A_{t-1} U_t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{9.1}$$

where Y_t is output, A_t and A_{t-1} are the productivity parameters, E_t is the number of educated (skilled) workers, U_t is the number of uneducated (unskilled) workers at period t, and σ is the elasticity of substitution between the educated and the uneducated. The leading-edge technology A_t at period t can be embodied only into the educated workers, while the old technology A_{t-1} is embodied in the unskilled at the beginning of each period. Technological progress is considered skill-biased in the sense that only the skilled acquire it.

To simplify the analysis, suppose that the total number of individuals supplying labor at each period is normalized to unity and each supply one unit of labor. Then, $E_t + U_t = 1$, and therefore the production function Eq. (9.1) can be rewritten as

$$Y_t = \left[(A_t E_t)^{\frac{\sigma-1}{\sigma}} + (A_{t-1}(1-E_t))^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{9.2}$$

Since each factor receives its marginal product in equilibrium, we have the following equilibrium wage rates:

$$w_t^e = A_t^{(\sigma-1)/\sigma} E_t^{-1/\sigma}\left[(A_t E_t)^{\frac{\sigma-1}{\sigma}} + (A_{t-1}(1-E_t))^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}, \tag{9.3}$$

$$w_t^u = A_t^{(\sigma-1)/\sigma} (1-E_t)^{-1/\sigma}\left[(A_t E_t)^{\frac{\sigma-1}{\sigma}} + (A_{t-1}(1-E_t))^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}, \tag{9.4}$$

where superscript e stands for "educated," and superscript u stands for "uneducated." Hence, the resulting wage inequality, ω_t, becomes

$$\omega_t = \frac{w_t^e}{w_t^u} = \left(\frac{A_t}{A_{t-1}}\right)^{(\sigma-1)/\sigma}\left(\frac{1-E_t}{E_t}\right)^{1/\sigma}. \tag{9.5}$$

In what follows, we assume $\omega_t > 1$ and $\sigma > 1$.[2]

9.2.2 Technological Progress

Although we realize that R&D investment is an important driving force of technological progress, for the sake of simplicity, new technology is assumed to appear without R&D expenditure. Instead, aggregate human capital is assumed to have a positive influence on technological progress. Since, in general, existing leading-edge technology together with the educated workers, namely, researchers, creates new

[2]The assumption $\sigma > 1$ is required to guarantee that technological progress raises the returns to the educated workers more than those to the uneducated. See Eq. (9.10c) in what follows.

technology, technical progress is assumed to be an increasing function of the level of leading-edge technology A_t and the number of the educated workers E_t:

$$A_{t+1} = aA_t + \Phi(A_t, E_t) \quad \text{with} \quad \Phi_1 > 0 \quad \text{and} \quad \Phi_2 > 0, \tag{9.6}$$

where $a \geq 1$ is a parameter, and aA_t is an exogenous component of technological progress. To simply the analysis, we assume:

$$A_{t+1} = aA_t + \varphi(E_t)A_t \quad \text{with} \quad a \geq 1, \ \ \varphi(0) = 0 \quad \text{and} \quad \varphi'(E_t) > 0. \tag{9.7}$$

Denoting $z_{t+1} = A_{t+1}/A_t$, namely, the gross rate of technological progress, the above can be rewritten as follows:

$$z_{t+1} = A_{t+1}/A_t = a + \varphi(E_t). \tag{9.8}$$

Here, we assume that if $a = 1$ and $E_t = 0$, no technological progress exists, i.e., $z_{t+1} = 1$. However, technological progress always exists because E_t is positive in equilibrium.

Taking the above into account, the wage inequality Eq. (9.5) becomes:

$$\omega_t = \omega(E_{t-1}, E_t, a) = (a + \varphi(E_{t-1}))^{(\sigma-1)/\sigma}((1 - E_t)/E_t)^{1/\sigma}, \tag{9.9}$$

and its partial derivatives are

$$\omega_1 \equiv \partial\omega_t/\partial E_{t-1} = ((\sigma - 1)\varphi'(E_{t-1})/\sigma(a + \varphi(E_{t-1}))\omega_t > 0, \tag{9.10a}$$

$$\omega_2 \equiv \partial\omega_t/\partial E_t = -\omega_t/\sigma(1 - E_t)E_t < 0, \tag{9.10b}$$

$$\omega_3 \equiv \partial\omega_t/\partial a = (\sigma - 1)\omega_t/\sigma(a + \varphi(E_{t-1})) > 0. \tag{9.10c}$$

While Eq. (9.10a) shows that an increase in the human capital at period $t - 1$ raises technology level at period t and hence the skill premium, Eq. (9.10b) demonstrates that an increase in the number of the educated at period t, E_t, decreases the premium through a decrease in the marginal productivity. As long as $\sigma > 1$, as is shown by Eq. (9.10c), an increase in skill-biased technological progress surely increases the wage premium.

9.2.3 Individual Education Choice

Each individual, who lives for two periods, has a single parent and a single child. In the first period she does not work and receives a transfer from her parent for her consumption and possible education. In the second period she works and divides her income between her own consumption and a transfer to her child.

The preference of individual i, born at period t, is characterized by the following utility function:

$$u_t^i = \log c_t^i + \log c_{t+1}^i + \log x_{t+1}^i, \tag{9.11}$$

where c_t^i is consumption at period t, c_{t+1}^i is consumption at period $t + 1$, and x_{t+1}^i is a transfer to her child born at period $t + 1$.

Let h_t^i and w_{t+1}^i denote the education cost of individual i at period t and her wage at period $t + 1$, respectively. Assuming that there is no capital market in the economy as in the Maoz and Moav (1999) model, if she acquires education, then her budget constraints become

$$c_t^i + h_t^i = x_t^i, \quad c_{t+1}^i + x_{t+1}^i = w_{t+1}^e. \tag{9.12}$$

If she does not acquire education, the constraints are.

$$c_t^i = x_t^i, \quad c_{t+1}^i + x_{t+1}^i = w_{t+1}^u. \tag{9.13}$$

The utility maximization problem can be solved backward in two stages. First, assuming that the education choice at the first period is already done, an individual considers how to allocate the wage between consumption and a transfer at her second period. Next, turning to the first period, she decides whether to acquire education or not taking the second period decision into account. The second period maximization problem is formulated as follows:

$$\max_{c_{t+1}^i, x_{t+1}^i} \left[\log c_{t+1}^i + \log x_{t+1}^i\right] \quad \text{subject to} \quad c_{t+1}^i + x_{t+1}^i = w_{t+1}^i, \quad w_{t+1}^i \in \left\{w_{t+1}^e, w_{t+1}^u\right\}. \tag{9.14}$$

The maximization gives us the following optimal allocation:

$$c_{t+1}^i = x_{t+1}^i = w_{t+1}^i/2, \tag{9.15}$$

and hence the maximum utility, namely, the indirect utility function at the second period becomes

$$\begin{aligned} z\left(w_{t+1}^i\right) &\equiv \max_{c_{t+1}^i, x_{t+1}^i} \left[\log c_{t+1}^i + \log x_{t+1}^i\right] = \log\left(w_{t+1}^i/2\right) + \log\left(w_{t+1}^i/2\right) \\ &= 2\log\left(w_{t+1}^i/2\right). \end{aligned} \tag{9.16}$$

Let us turn to the first period decision problem on education choice. From the above, we can see that if the following holds, then individual i acquires education.

i.e.,

$$\log\left(x_t^i - h_t^i\right) + z\left(w_{t+1}^e\right) \geq \log\left(x_t^i\right) + z\left(w_{t+1}^u\right), \quad i.e., \quad h_t^i \leq x_t^i\left(1 - \left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^2\right). \tag{9.17}$$

Hence, the critical value of education cost for individual i, $\widehat{h}_t^i$, becomes

$$\widehat{h}_t^i = x_t^i\left[1 - \left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^2\right]. \tag{9.18}$$

She acquires education if $h_t^i \leq \widehat{h}_t^i$, and vice versa.

The resulting wage inequality $\omega_{t+1} = w_{t+1}^e / w_{t+1}^u$ is very important in her education choice because it is benefit from acquiring education. Looking at the cost side, it is crucial that the transfer x_t^i is either x_t^e or x_t^u, namely, her parent belongs to either the educated worker group or the uneducated worker group. She receives $w_t^e/2$ in the form of transfer if she is born to the educated, while $w_t^u/2$ if born to the uneducated. Suppose that $\widehat{h}_t^e$ is the critical value of education cost for the individual born to an educated worker to acquire education, and $\widehat{h}_t^u$ is the critical value for the individual born to an uneducated worker. Then, from Eq. (9.18), we have

$$\widehat{h}_t^e = \frac{w_t^e}{2}\left[1 - \left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^2\right], \quad \widehat{h}_t^u = \frac{w_t^u}{2}\left[1 - \left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^2\right]. \tag{9.19}$$

9.2.4 The Distribution of Required Education Costs among Individuals

Undoubtedly, non-negligible costs are needed to be educated in reality. In order for individual i to be educated, the following cost is required at period t:

$$h_t^i = \theta^i C(z_{t+1}, \overline{w}_t) = \theta^i c(z_{t+1})\overline{w}_t \quad \text{with} \quad c'(z_{t+1}) \geq 0 \quad \text{and} \quad c(1) > 0, \tag{9.20}$$

where $\overline{w}_t$ is the average wage in the economy, which implies that the average education cost increases with the average wage, and θ^i is a parameter representing individual i's ability to learn such that the higher is the ability, the lower is θ^i. Here, it is assumed that if the rate of technological progress z_{t+1} increases, then the cost to be educated also increases. To be precise, the cost does not decrease with z_{t+1}, namely, $c'(z_{t+1}) \geq 0$. Also, even without technological progress, individuals must pay positive cost to acquire education, namely, $c(1) > 0$.

According to Maoz and Moav (1999), let us further assume that θ^i is uniformly distributed in the interval $\left[\underline{\theta}, \overline{\theta}\right]$ regardless of their parents' characteristics, educated or uneducated, at any period. Hence, it is evident from Eq. (9.20) that h_t^i is also uniformly distributed in the interval $\left[\underline{h}_t, \overline{h}_t\right]$, where $\underline{h}_t = \underline{\theta} c(z_{t+1}) \overline{w}_t$ and $\overline{h}_t = \overline{\theta} c(z_{t+1}) \overline{w}_t$.

9.2.5 The Dynamic Equation of the Model

In the economy, changes in the number of the educated workers E_t over time are explained by two types of intergenerational mobility: one is upward mobility and the other is downward mobility. The upward mobility implies that individuals born to the uneducated acquire education and hence become belonging to the educated at the next period, while the downward mobility implies that individuals born to the educated do not acquire education and hence become belonging to the uneducated at the next period. Therefore, if the number of upward-moving individuals, $(1 - E_t)\left[\left(\widehat{h}_t^u - \underline{h}_t\right) / \left(\overline{h}_t - \underline{h}_t\right)\right]$, exceeds that of downward-moving individuals, $E_t\left[\left(\overline{h} - \widehat{h}_t^u\right) / \left(\overline{h}_t - \underline{h}_t\right)\right]$, then E_t will increase, and vice versa. The dynamics of E_t can be expressed as:

$$
\begin{aligned}
E_{t+1} - E_t &= (1 - E_t)\frac{\widehat{h}_t^u - \underline{h}_t}{\overline{h}_t - \underline{h}_t} - E_t\frac{\overline{h}_t - \widehat{h}_t^e}{\overline{h}_t - \underline{h}_t} \quad or \quad E_{t+1} \\
&= (1 - E_t)\frac{\widehat{h}_t^u - \underline{h}_t}{\overline{h}_t - \underline{h}_t} + E_t\frac{\widehat{h}_t^e - \underline{h}_t}{\overline{h}_t - \underline{h}_t},
\end{aligned} \tag{9.21}
$$

Taking $\overline{h}_t = \overline{\theta} c(z_{t+1}) \overline{w}_t$, $\underline{h}_t = \underline{\theta} c(z_{t+1}) \overline{w}_t$, $\overline{w}_t = E_t w_t^e + (1 - E_t) w_t^u = Y_t$, Eqs. (9.19) and (9.20) into account, Eq. (9.21) can be rewritten as follows:

$$
E_{t+1} = \frac{1 - \omega(E_t, E_{t+1}, a)^{-2}}{2\left(\overline{\theta} - \underline{\theta}\right) c(a + \varphi(E_t))} - \frac{\underline{\theta}}{\overline{\theta} - \underline{\theta}},
$$

or, the following implicit function:

$$
G(E_t, E_{t+1}, a) = E_{t+1} - \frac{1 - \omega(E_t, E_{t+1}, a)^{-2}}{2\left(\overline{\theta} - \underline{\theta}\right) c(a + \varphi(E_t))} + \frac{\underline{\theta}}{\overline{\theta} - \underline{\theta}} = 0, \tag{9.22}
$$

where $\omega(E_t, E_{t+1}, a) = (a + \varphi(E_t))^{(\sigma - 1)/\sigma}((1 - E_{t+1})/E_{t+1})^{1/\sigma}$. The behavior of the economy is summarized by the above first-order nonlinear difference equation.

9.3 The Analysis

We can analyze the dynamic behavior of intergenerational mobility and inequality as well as the effects of technological change on them by investigating (9.22). Totally differentiating (9.22), we have the following:

$$G_2 dE_{t+1} = -G_1 dE_t - G_3 da, \tag{9.23}$$

where $G_1 = \frac{\varphi'\left[\left(\omega^2-1\right)\varepsilon_c - 2\varepsilon_\omega\right]}{2\left(\overline{\theta}-\underline{\theta}\right)c\omega^2 z}$, $G_2 = 1 - \frac{\omega_2}{\left(\overline{\theta}-\underline{\theta}\right)c\omega^3} > 1$, $G_3 = \frac{\left(\omega^2-1\right)\varepsilon_c - 2\varepsilon_\omega}{2\left(\overline{\theta}-\underline{\theta}\right)c\omega^2 z} = \frac{G_1}{\varphi'}$, $\varepsilon_c \equiv \frac{gc'}{c} = \frac{\partial c/c}{\partial z/z}$ and $\varepsilon_\omega \equiv \frac{g\omega_3}{\omega} = \frac{\partial \omega/\omega}{\partial z/z} = \frac{\sigma-1}{\sigma}$. As these definitions show, ε_c is the elasticity of skill-acquisition cost with respect to technological progress, while ε_ω is the elasticity of wage premium with respect to technological progress. Although ε_ω is positive by the assumption that $\sigma > 1$, ε_c can be either zero or positive. Thus the nature of the economy depends crucially on the sign and the value of ε_c, namely, the sign and the value of $c^{'}(z_{t+1})$.

9.3.1 Dynamics When $\varepsilon_c = 0$, i.e., $c^{'}(z_{t+1}) = 0$

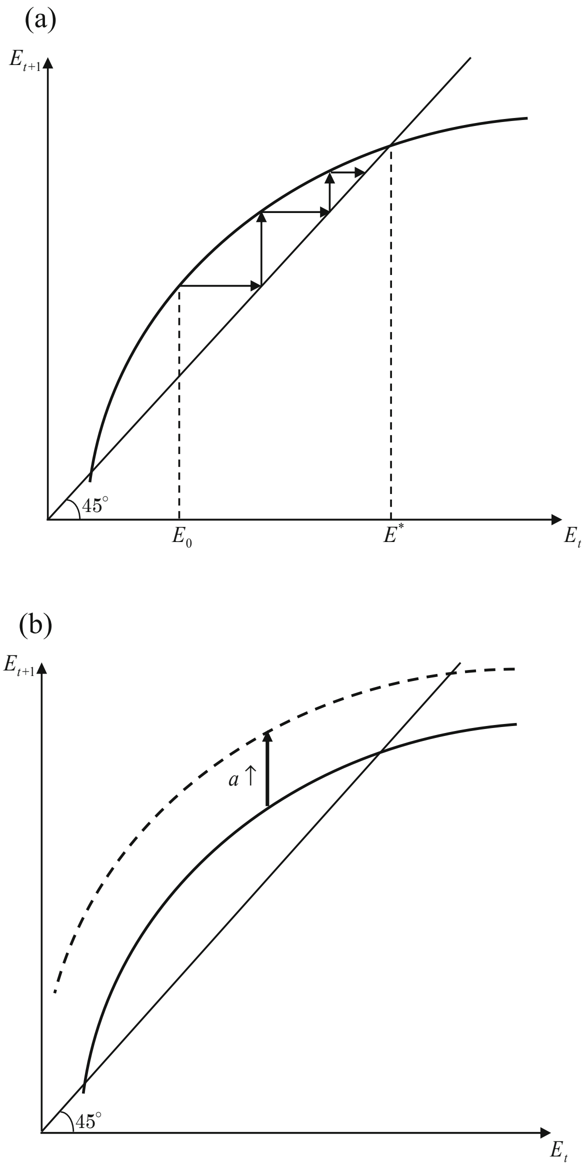
When $\varepsilon_c = 0$, namely, $c^{'}(\cdot) = 0$, $G_1 = -\varepsilon_\omega \varphi' / \left(\overline{\theta} - \underline{\theta}\right) c\omega^2 z < 0$. From Eq. (9.18), we have

$$\frac{dE_{t+1}}{dE_t} = -\frac{G_1}{G_2} > 0. \tag{9.24}$$

If the skill-acquisition cost is constant regardless of the rate of technological progress, namely, $c^{'}(z_{t+1}) = 0$, Eq. (9.22) is upward-sloping in the (E_t, E_{t+1}) plane as Fig. 9.1a shows. Hence, E_t increases monotonically toward the steady state. During the transition, the growth rate of income and intergenerational mobility monotonically increase, while income inequality monotonically decreases over time.

Proposition 1 *Suppose that skill-acquisition cost is constant regardless of the rate of technological progress. Then, the growth rate of income, intergenerational mobility and income inequality monotonically approach the steady state levels. If the initial number of educated workers is lower (higher) than that at the steady state, the growth rate of income and intergenerational mobility increase (decrease), while income inequality decreases (increase) over time.*

Fig. 9.1 When $c^{'}(z_{t+1}) = 0$ or $\varepsilon_c = 0$. (**a**) Transitional Dynamics of E_t. (**b**) Effects of Exogenous Technological Progress



9.3.2 *The Effects of Exogenous Technological Progress*

In this subsection, let us examine the effect of a change in the rate of exogenous technological progress a when $c^{'}(z_{t+1})$ is zero or small. From Eq. (9.23) we have:

$$\frac{dE_{t+1}}{da} = -\frac{G_3}{G_2} > 0. \tag{9.25}$$

Namely, an increase in the exogenous rate of technological progress shifts up the locus of Eq. (9.22) as Fig. 9.1b shows. Hence, technological progress increases the number of educated workers, intergenerational mobility and income at the stationary state.

The stationary state wage premium ω^* is determined by

$$\omega^* = (a + \varphi(E^*))^{(\sigma-1)/\sigma}((1 - E^*)/E^*)^{1/\sigma},$$

or

$$\ln\omega^* = ((\sigma - 1)/\sigma)\ln(a + \varphi(E^*)) + (1/\sigma)[\ln(1 - E^*) - \ln E^*].$$

Hence, the effect of a on ω^* is

$$\frac{d\ln\omega^*}{da} = \frac{\sigma - 1}{\sigma(a + \varphi(E^*))} + \left[\frac{(\sigma - 1)\varphi'(E^*)}{\sigma(a + \varphi(E^*))} - \frac{1}{\sigma(1 - E^*)E^*}\right]\frac{dE^*}{da}. \tag{9.26}$$

Since the first term and dE^*/da in the right hand side are positive, the above is positive as long as the terms inside the square brackets are non-negative. Even when they are negative, the above can be positive if the value is small in absolute value. Hence, a sufficient condition for the sign of $d\omega^*/da$ to be positive is as follows:

$$\sigma \geq 1 + \frac{a + \varphi(E^*)}{\varphi'(E^*)(1 - E^*)E^*}. \tag{9.27}$$

When the above is satisfied, then the skill premium at the stationary state ω^* increases with the number (share) of educated (skilled) workers. This is the phenomena known as the skill-premium puzzle.

Although the second term in the right hand side of inequality (9.27) is positive, it can be less than unity if $\varphi'(E^*)$ is large, namely, the skilled workers' contribution to technological progress is large. In other words, the right hand side can be smaller than 2. In Acemoglu (2002, 2009), $\sigma > 2$ is required for the puzzle. In the model presented in this chapter, the puzzle can occur even under $\sigma \leq 2$.

Proposition 2 *If the educated workers' contribution to technological progress is large, then the skill premium puzzle can occur even when the elasticity of substitution between the educated and the uneducated is smaller than 2.*

The above proposition refers just to the comparison of the steady states. After an exogenous increase in technological progress, the technology gap z_{t+1} first increases, which expands the wag premium, and then the number of the educated E_{t+1} gradually increases through intergenerational mobility, which shrinks the premium. Thus, the effect to increase the premium is larger than that to decrease it at the early stage of the transition after the shock. Hence, the puzzle is expected to almost always occur right after an exogenous increase in technological progress.

9.3.3 Dynamics When $c'(z_{t+1}) > 0$

As long as $c'(z_{t+1})$ is sufficiently small, i.e., $\varepsilon_c \approx 0$, the dynamics of the economy and the effects of technological progress are the same as in the previous subsections. In contrast, if $c'(z_{t+1})$ is large enough to satisfy the following:

$$(\omega^2 - 1)\varepsilon_c > 2\varepsilon_\omega,$$

then G_2 and G_3 become positive. Therefore, the following holds:

$$\frac{dE_{t+1}}{dE_t} = -\frac{G_1}{G_2} < 0. \tag{9.28}$$

In other words, if the skill-acquisition cost increases rapidly with the rate of technical progress, then Eq. (9.22) is down-sloping in the (E_t, E_{t+1}) plane as Fig. 9.2a shows. In this case, aggregate growth, intergenerational mobility and inequality exhibit cyclical behavior.

The intuition behind the above findings is straightforward. Whether to acquire education or not depends on the net benefit from education, namely, the gross benefit that comes from the wage gap net of the cost to acquire education. Only the individuals with the positive net benefit acquire education. An increase in E_t enhances technological progress at period t, z_{t+1}. This increases both the education cost at period t and the wage gap at period $t + 1$. If the cost increase is small, then people choose to acquire education. As a result, E_{t+1} increases and the increase will continue during the transition, as Fig. 9.1a shows. However, if the cost increment is sufficiently large, then people choose not to acquire the education. This decreases E_{t+1}. The decreased E_{t+1} lowers technological progress at period $t + 1$, z_{t+2}. Although both the gross benefit from the wage gap and the cost to acquire education decrease, the cost drops more sharply than the benefit when $c'(z_{t+1})$ is large. As a result, E_{t+2} increases. Under a large marginal education cost $c'(z_{t+1})$, the net benefit fluctuates throughout the transition. This explains the cyclical behavior of income, intergenerational mobility, and income inequality, as Fig. 9.2a shows.

Proposition 3 *Suppose that skill-acquisition cost increases rapidly with the rate of technological progress. Then, the growth rate of income, intergenerational mobility and income inequality exhibit cyclical behavior.*

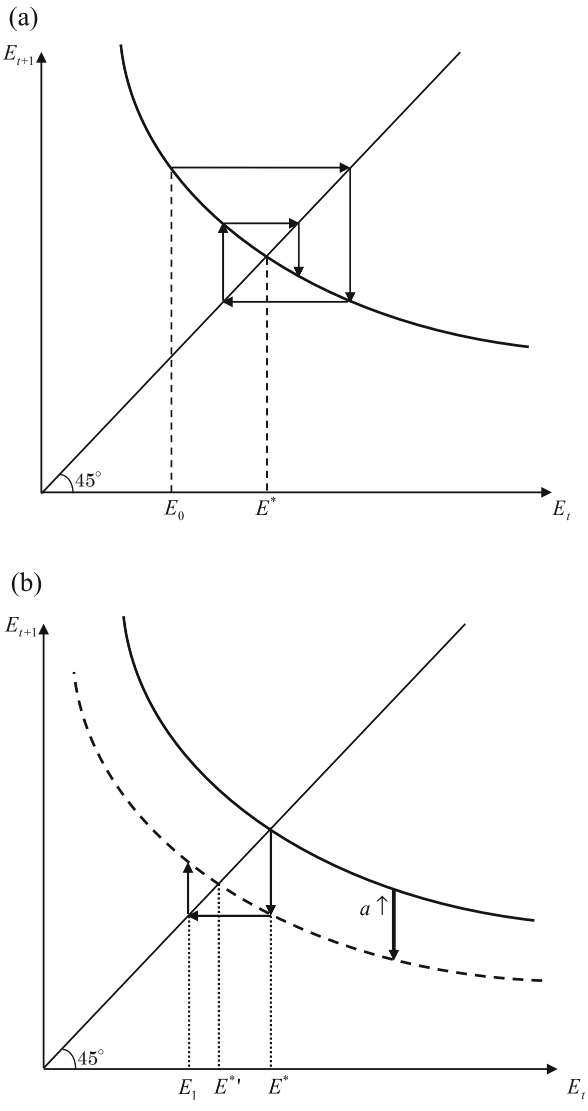


Fig. 9.2 When $c^{'}(z_{t+1}) > 0$ or $\varepsilon_c > 0$. (**a**) Transitional Dynamics of E_t. (**b**) Effects of Exogenous Technological Progress

9.3.4 The Effects of Exogenous Technological Progress

In this subsection, let us examine the effect of a change in the rate of exogenous technological progress a when $c^{'}(z_{t+1})$ is sufficiently large. From Eq. (9.23) we have:

$$\frac{dE_{t+1}}{da} = -\frac{G_3}{G_2} < 0. \tag{9.29}$$

As the above shows, an increase in the exogenous rate of technological progress shifts down the locus of Eq. (9.22) as Fig. 9.2b shows. Hence, technological progress decreases the number of educated workers and intergenerational mobility at the steady state.

The steady state gross growth rate z^* is

$$z^* = a + \varphi(E^*).$$

The effect of a on z^* therefore becomes

$$\frac{dz^*}{da} = 1 + \varphi'(E^*)\frac{dE^*}{da}. \tag{9.30}$$

Since dE^*/da is negative in this case, the above can be negative when $\varphi'(E^*)$ is large.

In addition, as Fig. 9.2b shows, z_t undershoots the new steady state value, $E^{*'}$, during the transition. Suppose, for instance, that the economy is at the steady state at the beginning of period 0, and the exogenous rate of technical progress increases by Δa once-and-for-all. Then the gross growth rate at period 1 is $z_1 = a + \Delta a + \varphi(E_1)$, while the growth rate at period 0 is $z_0 = a + \Delta a + \varphi(E^*)$ because the number of the educated is E^*. Since $\varphi'(E_t)$ is positive and $E_1 < E^*$, the following inequality holds:

$$z_0 - z^* = \Delta a > 0 \quad \text{and} \quad z_1 - z_0 = \varphi(E_1) - \varphi(E^*) < 0. \tag{9.31}$$

The above shows that a permanent increase in the rate of technical progress surely decreases the growth rates at the early stage of the transition.

Proposition 4 *Suppose that education cost increases rapidly with the rate of technological progress, and the educated workers' contribution to technological progress is large. Then, an exogenous increase in technological progress may decrease the long-run growth rate. Also, the exogenous increase lowers the growth rates at the early stage of the transition even if it does not lower the long-run growth rate.*

9.4 Concluding Remarks

Incorporating skill-biased technological progress contributed by human capital (educated labor) into the seminal model of Maoz and Moav (1999), this chapter has investigated the dynamics of income inequality and intergenerational mobility in an endogenously growing economy. Also, the chapter has analyzed the effects of technological progress on inequality and intergenerational mobility. As a result, it is shown that the shape of the education cost function plays a crucial role in determining both the dynamic behavior of the model and the effects of technological progress.

The cost to acquire education is assumed to increase with technological progress. If the cost does not increase rapidly with technological progress, then the economy monotonically approaches toward the steady state. If, in contrast, it increases rapidly, then the growth rate of income, intergenerational mobility and income inequality exhibit cyclical behavior.

In the case of monotonic convergence, an increase in exogenous technological progress may increase both wage premium and the number of the educated workers when the skilled workers' contribution to technological progress is large. Putting it differently, the phenomenon known as the skill-premium puzzle can be explained by the model under weaker conditions than in Acemoglu (2002, 2009). In contrast, if the economy converges to the stationary state with cyclical behavior, then a permanent increase in the rate of technical progress may decrease the long-run growth rate. It surely lowers the growth rates at the early stage of the transition even if it does not lower the long-run growth rate.

Since thus the dynamic behavior and the effects of technological progress depend crucially on the nature of the education cost, the empirical investigations on the education cost should be carefully carried out. It is hoped that this chapter stimulates the future research of income inequality and intergenerational mobility.

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