




# Rethinking the Limits of Optimization Economic Order Quantity (EOQ) Using Self- Generating Training Model by Adaptive- Neuro Fuzzy Inference System

A. Stanley Raj<sup>1</sup> , H. Mary Henrietta<sup>2</sup>, K. Kalaiarasi<sup>3</sup>,  
and M. Sumathi<sup>4</sup>

<sup>1</sup> Loyola College, Chennai 600034, Tamilnadu, India  
stanleyraj\_84@yahoo.co.in

<sup>2</sup> Saveetha Engineering College, Chennai 602105, Tamilnadu, India

<sup>3</sup> Cauvery College for Women (Affiliated to Bharathidasan University),  
Trichy 620018, Tamilnadu, India

<sup>4</sup> Khadir Mohideen College (Affiliated to Bharathidasan University),  
Adirampattinam 614701, Tamilnadu, India

**Abstract.** This paper deals gives an alternate approach to the traditional way of solving an optimization of Economic order quantity (EOQ) by applying self-generating training model in ANFIS. The inventory control for a successful organization has to be sustained with varying parameters such as demand, setup cost and ordering cost. This research work combines the fuzzy inference system and Adaptive neuro-fuzzy inference system to acquire the optimal order quantity with fuzzy logic. The proposed algorithm of self-generating training dataset proclaims the efficient model for predicting EOQ with variable demand. This algorithm is tested with various numbers of datasets and the results are compared with the fuzzified and crisp model. The performance evaluation is done and the results are satisfactory to apply any nonlinear problems.

**Keywords:** Economic order quantity (EOQ) · ANFIS · Fuzzy logic to EOQ · Optimization of total cost

## 1 Introduction

It was Zadeh [20] in 1965 introduced the fuzzy sets that refers to vagueness and uncertainty in real life. In the year 1983, Zimmerman [21] observed fuzziness in the study of operational research. Park [12] fuzzified the ordering cost and holding(storage) cost as trapezoidal fuzzy numbers in the traditional EOQ to solve the non-linear programming that resulted in fuzzy EOQ models. Inventory is a part of the major segments in economy which is ambiguous in nature that concentrates on a proper management, by increasing the customers and minimizing the inventory costs. This resulted in an EOQ model introduced by Harris [9] in 1913. Chen [5] was the first person to propose a probabilistic approach to an inventory with imperfect items. In 2007, Wang [18] studied the randomness in fuzzy EOQ model for items with imperfection. Dutta [7] in

the year 2004, fuzzified the inventory parameters demand, holding cost and ordering cost. In inventory management the economic order quantity is the strategy used as a replenishment model that is used to determine the total inventory costs and also manipulates to minimize it. Earlier time demand was assumed to be a constant that resulted as a pitfall in the EOQ model. So, there were models brought with fluctuating demands to face the consistent seasonality in business. To put up with the complications, EOQ can be customized using inventory management software suggesting a well-organized ordering solution. To deal with these limitations it is essential to bring in the artificial intelligence techniques in solving the real problems.

Although many researchers had simulated the artificial intelligence in inventory control for production systems. The study of Artificial intelligence in inventory management was first started off by Jang [10] by combining the fuzzy inference system with adaptive networks. Gupta and Maransa [8] proposed a two-stage stochastic programming uncertainty demand model where in the first, production decisions was considered and supply chain decisions in the second stage.

Fuzzy based study could be convenient for such uncertain situations and works best if combined with artificial neural networks (ANN) in the real business world. When Fuzzy merges with ANN, it results in neuro fuzzy system and fuzzy neural system. The renowned work of Aliev [2] proposed two different design, namely neuro fuzzy systems whose key task is to operate mathematical relations, whereas the fuzzy neural systems is applied to discover the numerical information and knowledge-based data which are represented by fuzzy numbers. In 1991, Pedrycz [14] generated models by its conduct towards unpredictability and characterized the connection between the fuzzy theory and neural networks. Also, Pedrycz [15] extended his study of neurons in pattern classifiers in the year 1992. Further, NFN was defined by a fuzzy structure instructed by an algorithm by a smart model by Jang [11]. Fuzzy neural networks are many distinguished by the connection of their neurons. In 1943, McCulloch [19] developed a mathematical model using a single neuron which established the authenticity of a neuron activation in the brain that was widely accepted in theoretical possibilities. The start of examining the connected nodes to modify the connection weights was carried out by Hebb [6] in the year 1949. It was Rosenblatt [16] brought the perceptron neural model that satisfied the neuron's behavior.

An adaptive neuro-fuzzy logic inventory control system was introduced in the year 2012 by BalazsLenart [4]. Later, Aksoy et al. [1] applied ANFIS in a garment trade with demand prediction. In 2015, Aengchuan and Phruksaphanrat [3] compared the three methods namely fuzzy inference system (FIS), Adaptive Neuro-Fuzzy inference system (ANFIS) and Artificial neural networks with varying membership functions in solving inventory problems. Out of these, the ANFIS along with gaussian membership function resulted with the minimum total cost. In 2015, yet another result that forecasted the advantage of ANFIS over ANN was carried out by Paul, Azeem and Ghosh [13] maintaining the optimum inventory level in the inventory management problem. The economic order quantity equation Kalaiarasi et al. [22] was taken and the adaptive neuro fuzzy inference system is applied to the obtain the desired model.

Many researchers had formulated the hybrid methods of artificial intelligence which had resulted in better results by applying ANFIS in solving complex EOQ problems. ANFIS is a learning tool that is extensively used to get the desired output by applying fuzzy logic with highly interconnected neural network. In this model an inventory management with the advantage of Adaptive neuro-fuzzy inference system (ANFIS) has been developed with modelling done in fuzzy using a given set of data.

## 2 Inventory Model for EOQ

The input parameters for the corresponding model are.

- R - Ordering cost
- d - constant demand rate coefficient
- $\alpha$  - price-dependent demand rate coefficient
- S - selling price
- Q - Order size
- p - unit purchasing cost
- g - constant holding cost coefficient

The total cost [17] per cycle is given by

$$T(Q) = \frac{R(d - \alpha S)}{Q} + p(d - \alpha P) + \frac{gpQ}{2} \quad (1)$$

Partially differentiating w.r.t Q,

$$\frac{\partial T}{\partial Q} = \frac{R(d - \alpha S)}{Q^2} + \frac{gp}{2} \quad (2)$$

Equating  $\frac{\partial T}{\partial Q} = 0$  we obtain the economic order quantity in crisp values [22]

$$Q = \sqrt{\frac{2R(d - \alpha S)}{gp}} \quad (3)$$

### 2.1 ANFIS Model Development

Initially, the data have been subjected to certain degree of membership grade so that at each iterations the firing strength will decide the consequent parameters (Fig. 1).

ANFIS system consists of 5 layers; Output of each layer is symbolized by  $O_i$ ,  $i$  with  $i$  is a sequence of nodes and  $l$  is the sequence showing the lining. Here is an explanation for each layer (Jang 1993), namely:

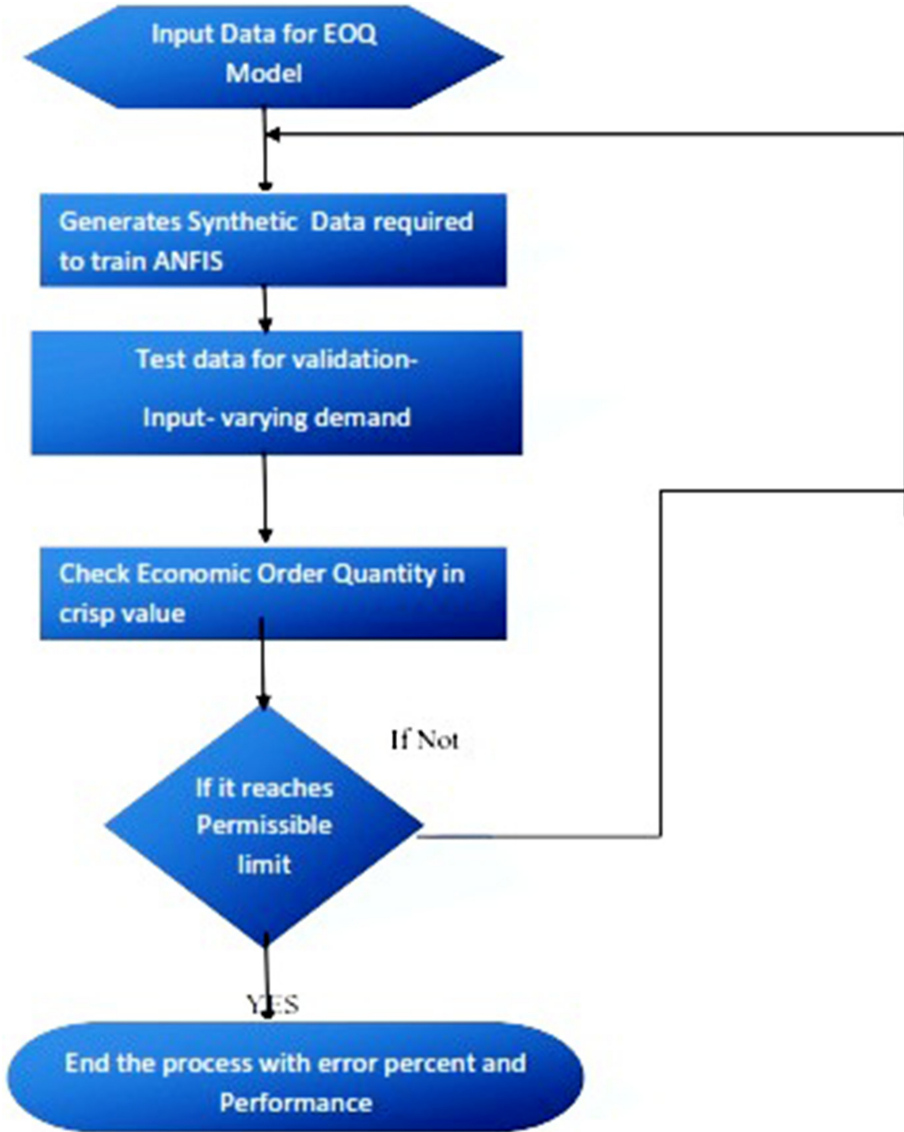


Fig. 1. Flow chart for ANFIS methodology to predict EOQ for varying Demand

**Layer 1**

Serves to raise the degree of membership and the membership used here is Gaussian membership function.

$$O_{1,i} = \mu_A(x), i = 1, 2 \tag{4}$$

and

$$O_{1,i} = \mu_B(y), i = 1, 2 \tag{5}$$

$$f(x, \sigma; c) = e^{\frac{-(x-c)^2}{2\sigma^2}}$$

by  $\{\sigma$  and  $c\}$  are the parameters of membership function or called as a parameter premise.  $\sigma$  signifies the cluster bandwidth and,  $c$  represents the cluster center.

**Layer 2**

Serves to evoke *firing-strength* by multiplying each input signal.

$$O_{2,i} = w_i = \mu_A(x) \times \mu_B(y), i = 1, 2. \tag{6}$$

**Layer 3**

Normalizes the *firing strength*

$$O_{3,i} = \bar{w}_i = \frac{w_i}{w_1 + w_2}, \quad i = 1, 2. \tag{7}$$

**Layer 4**

Calculates the output based on the parameters of the rule *consequent*  $\{p_i, q_i$  and  $r_i\}$

$$O_{4,i} = \bar{w}_i f_i = \bar{w}_i(p_i x + q_i y + r_i) \tag{8}$$

**Layer 5**

Counts the ANFIS output signal by summing all incoming signals will produce

$$\sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \tag{9}$$

ANFIS uses the input data scaling by  $x$ bounds = [min max] command used in MATLAB software which represents the scaling parameter of the input function varies between minimum to maximum value of the data point. Each data point is scaled for pre-processing of training initially by normalizing it.

**2.2 Model Development and Application**

**Step 1:**

The user can import the data for EOQ model. The imported data must be of any form that includes text files, excel spreadsheets, CSV etc.

**Step 2:**

For ANFIS training is more important. Thus, based on the number of iterations and random permutations of data, ANFIS generates synthetic data. This self-generation of synthetic data will be helpful to evaluate noisy and missed data. Thus, this step is a very important step in this algorithm.

**Step 3:**

This the key step where the neuro fuzzy algorithm generates synthetic dataset for each data with connection to the adjacent data that controls the noises and errors present in data.

**Step 4:**

In this step the algorithm calculates the error percentage of the data based on the coefficient of variation.

**Step 5:**

Every iteration the system generates one synthetic data and a model is generated the based on the number of iterations decided by the user. More the number of synthetic data better performance of the algorithm.

**Step 6:**

Finally, the EOQ model based on variable demand rate is predicted by the neuro fuzzy algorithm.

### 2.3 Algorithm Description and Application

This algorithm works on the basis of self-generating training dataset. In self-generating model, the system considers the mean, standard deviation, upper (Maximum value) and lower (minimum value) bounds. After considering all the statistical values from the given input data, the algorithm generates the synthetic data using random permutations. For each iteration, it generates a single synthetic data. For getting a greater number of synthetic data, user has to fix the number of iterations. The consistency may be lost once the user fixes a greater number of iterations than the memory allocated by the system to train the data. The system became unstable after a particular number of epochs.

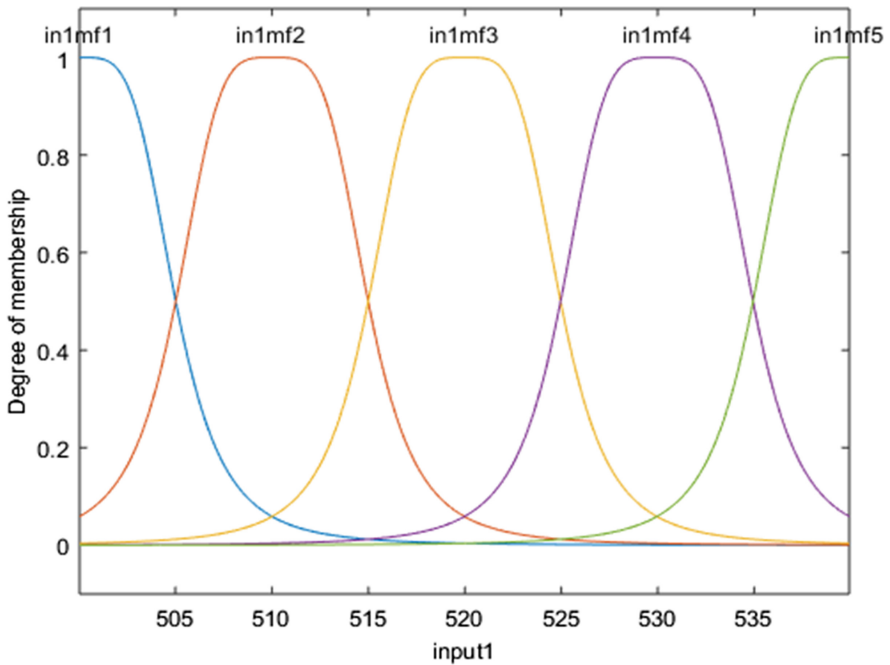
## 3 Results and Discussion

ANFIS will test the data using the synthetic training dataset generated by this algorithm. Figure 2 shows the membership function used to train the data (Figs. 3, 4 and 5).

It is very necessary for a company to know the strategy of variable demand with Economic Order Quantity. Thus, using this algorithm, we can easily predict the EOQ for variable demand. This algorithm is proved successful while comparing with crisp and fuzzified model (Fig. 6). Figure 7 represents the three-dimensional model of EOQ for variable demand. In ANFIS training there is a virtual connection between synthetic dataset and output. ANFIS is one of the potential soft computing algorithms integrates neural networks and fuzzy logic. In this EOQ model, Gaussian membership function is applied to guesstimate the output.

Self-generation of data have certain advantages.

1. Errors or noises in the data can be eliminated.



**Fig. 2.** Represents the Gaussian membership function used for training the data

2. If there is any missing data between two data points, based on the standard deviation and the trend, it can be interpolated. The trend will be maintained.
3. Synthetic training datasets are elastic in nature because the datasets can run between maximum and minimum of the original data and predicts the exact results though the data is nonlinear.
4. Developing synthetic data using this algorithm helps the ANFIS system to decide the output very easily. Fixing the membership functions for a bunch of data, though it consumes time, it will predict the exact result after defuzzification.

### **Error Estimation [23]**

L1-norm error estimation was used to minimize the errors during iteration and this method can be applied in numerous fields due to its robustness compared to L2-norm. L2-norm squares the errors that makes the model more efficient when noisy data are applied.

In this model the over fitting problems which normally occurs in ANFIS were avoided by fixing the allowable error percentage to minimum (10% in this study). To choose the appropriate model parameters during the iteration the permissible error was fixed by the user.

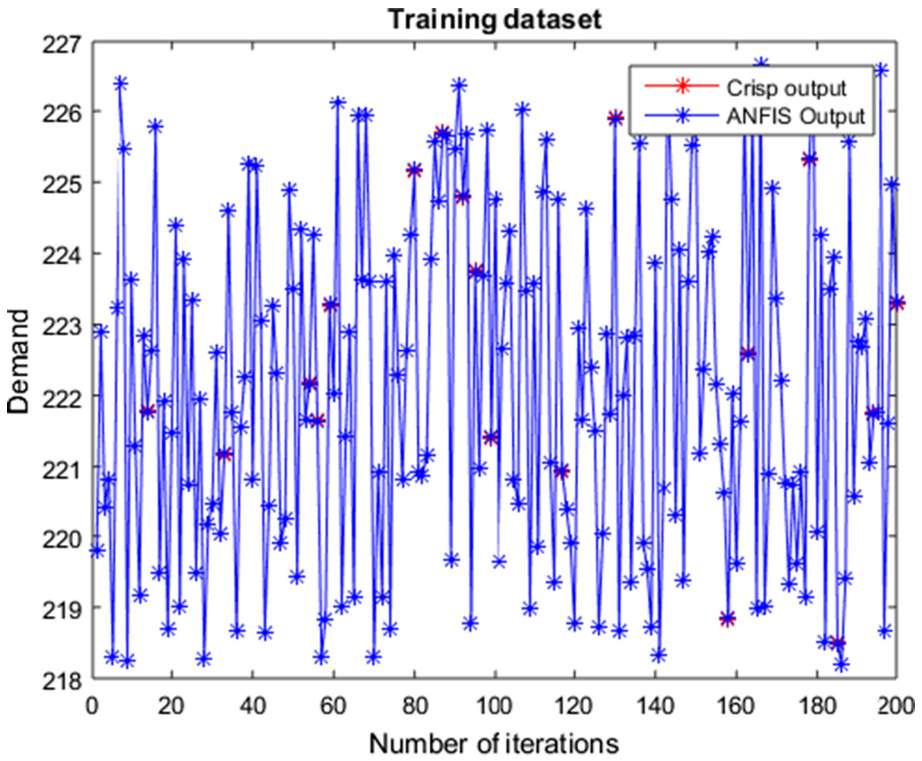
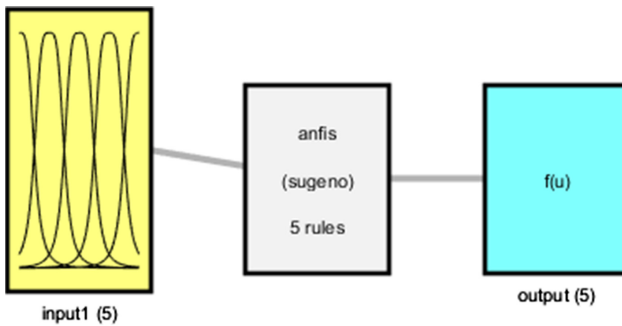


Fig. 3. Shows the self-generated training dataset compared with the original output



System anfis: 1 inputs, 1 outputs, 5 rules

Fig. 4. ANFIS architecture for EOQ model prediction



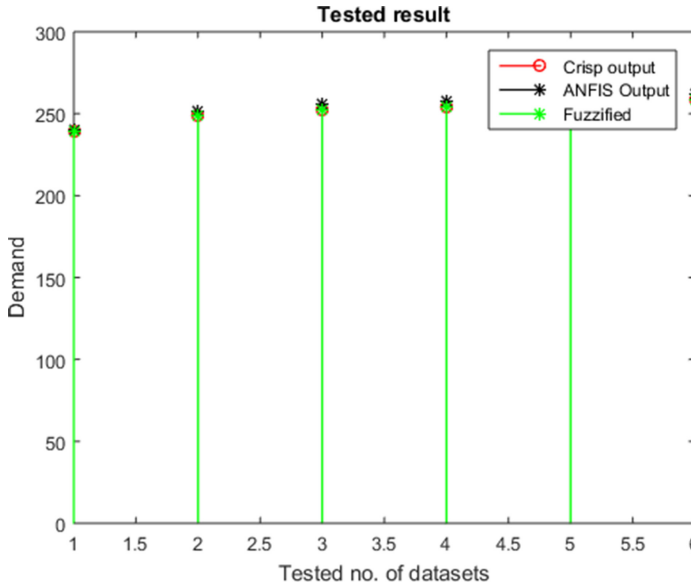


Fig. 5. Represents the tested data using synthetic training dataset by self-generating algorithm

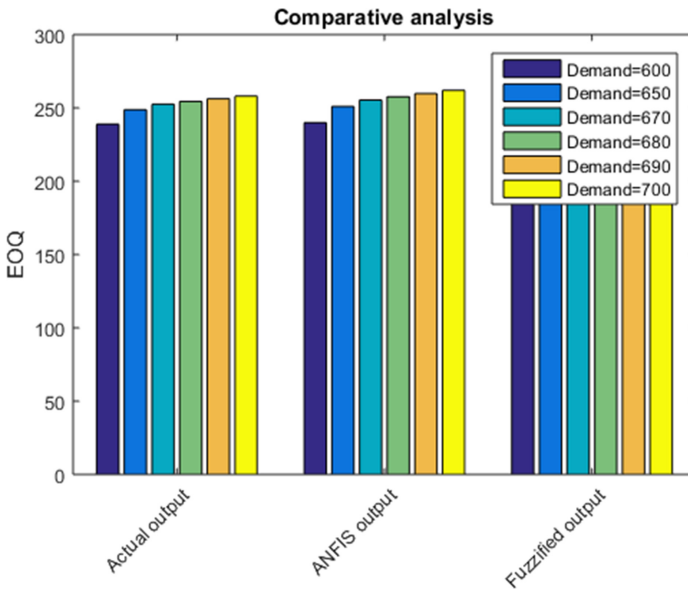
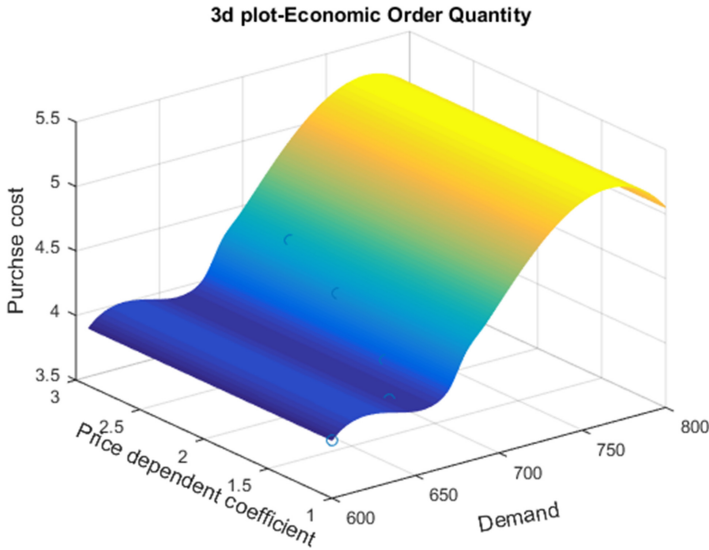


Fig. 6. Comparative analysis of ANFIS output with crisp and fuzzified model.



**Fig. 7.** Three-dimensional model for price dependent coefficient, purchase cost and demand

## 4 Conclusion

This method of intelligent technique is applied for efficient management for acquiring economic order quantity. The comparative analysis in conventional methods of linear solving methods with artificial intelligent techniques has proven to be a valuable and logically expert tool is assessing EOQ. On combining the neural networks and fuzzy logic, ANFIS has been applied by training with large number of synthetic datasets generated by the random permutations of software algorithm. The linguistic knowledge present in the fuzzy logic approach is used to model complex nonlinear process of EOQ model. In this paper EOQ model has been solved with different techniques and tested with the actual crisp model. It reveals that the following method of ANFIS works well and it can be applied in any logistic expert to analyze the EOQ. Neurofuzzy training time is quite short and the expert training dataset will provide exact results. The flow of goods and the procurement of other goods can be easily modeled using ANFIS. The sensitivity analysis is done and the results revealed that the ANFIS tool has been universally applied for any type of logistic problem.

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