Fracture Behavior and Toughness of Fiber Reinforced Thermoset Composites

Alak Kumar Patra and Indrajit Ray

Abstract The fiber reinforced polymer composites (FRPC) are being widely used in several advanced engineering structures ranging from civil infrastructure to aircraft, spacecraft, ships, cars and in many other outdoor and household applications. The major advantage is its high specific strength, stiffness, and durability leading to sustainable applications. Thermoset resins have advantages of retaining shape and strength at a higher temperature and harsh environment and lower life-cycle cost compared to thermoplastic resins. However, the knowledge of their behaviors during fracture failures are essential to properly evaluate the performance of thermoset composites. This chapter is divided into following five sections: Sect. 1 provides the introduction; Sect. 2 discusses the fracture mechanism of FRP's from micromechanics and global response of components or structure as a whole. The detailed information on general fracture mechanics approach including different modes of fracture, fracture failure procedures, fracture mechanics approach in FRC problems, fracture under compression, and failures at different scales will be provided. Section 3 elaborates on various failure theories such as micromechanical failure of FRP with UD lamina, anisotropic failure theory including theory of maximum stress, maximum strains, deviatoric strain energy theory, theory of tensor polynomial, failure for damage mechanism, failures under creep, fatigue and rupture, high strain rate failures. Section 4 covers the various modes of experimental investigations on FRPC including mode I, mode II, mode III, and mixed mode fracture toughness.

Keywords Fiber reinforced composites · Fracture behavior · Fracture failure theory · Thermoset composites · Toughness

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1 Introduction

Fracture behavior of FRPCs includes the behavior of FRPCs in the course of their fractures. The behavior can be described in different ways. Fracture failure is the focal point of all these exercises. Inability of the element of structure to withstand against load or loss of material integrity can be defined as failure [29]. Failure of FRPC is a complicated phenomenon and a continuous growing field of research for its importance. There are obvious reasons behind it. The FRPCs are stronger in the direction of fiber in comparison to other directions. It is evident that the failures of FRPCs depend on the direction of stresses. FRPCs fail in much lower stresses in the direction normal to the fiber than that required to cause failure in the direction of fiber. Failure under tensile load is governed by the strength of the fiber and controlled by the bond strength between the fiber and the polymer matrix and the matrix-strength in the direction perpendicular to the fiber direction. But the failure in an angular direction other than the 0° or 90° depends on the direct stresses in 0° and 90° directions with respect to fibers and the shear stress also. It becomes an important point of investigation that under which stress (direct/shear) or stress combination the FRPC will fail. Matrix materials in FRPCs may be ductile or brittle. In case of ductile polymer matrix, the material may fail due to disruption in load transfer mechanism from matrix to fiber under large strain of matrix. Polymer composites with brittle matrices may exhibit numerous cracks about the fibers or in between the fibers causing disturbances in the transfer mechanism of the load to the fibers from the matrices leading to failures. Buckling or enormous deformation of fibers may cause the failure of FRPCs under compressive loads. Most of the times, failure is an ultimate result of initiation and maturity of a combination of more than one of these mechanisms which is a complex and intricate event. FRPCs are not only multi-phase materials but also multi-layered and may be composed of fibers in multiple directions which may be subjected to a variety of loads. Even if the failure of unidirectional (UD) lamina of FRPC is considered, it becomes difficult to understand for the dissimilarities in stress distribution in different phases and interfaces. This is because in UD lamina, matrix and fibers are of different strengths, interface behaves differently from the matrix and the manufacturing defects or flaws. It is now obviously understandable why failure of FRPCs is a complicated topic and why it is studied by so many researchers and groups of researchers till today. Instead of the importance of understanding the failure mechanisms in FRPCs, it is not possible to realize the details of every state of failure. From practical point of view, it is important to know for the safety of the structure whether a stress level or combination of stresses or that of strains are below some critical limit or not. Establishing a fracture criterion for the improvement in precision of predictability of a fracture behavior is essential [46]. Another important point is that the criteria of failure should not be much conservative without compromising its safety against failure, must be understandable and verifiable through experiments. At base, all these criteria are either maximum stress or maximum strain criteria or interaction between them, sometimes are modified by some specific observations obtained from experimental investigations. The objective of all these straight or intricate approaches is to predict the failure through some failure criteria. The significance of so many criteria is that instead of immense efforts, none of them is able to explain the failure behaviors of all the FRPCs subjected to any kind of loading system. The issue is same for isotropic materials also; some fail due to yielding while some others' failure is of brittle nature. Many criteria may be acceptable when the criteria are considered for indication, not for prediction of failures under all conditions.

Fracture behaviors of FRPCs can be assessed from the criteria for fracture [51]. The mechanisms from crack initiation to propagation is not clear for complicated relation between the material's microstructure and the stress field at macroscopic level [56]. One approach followed in fracture mechanics calculates critical rate of release of strain energy from macroscopic displacements and applied forces. A look into the cracks at microscopic level reveals that the strain energy induced in the location of crack supplies the energy required for the newly generated surfaces from the formation of cracks, heat, some additional release of elastic energy as well as energy required for plastic deformation. Instead of significant advancement in fracture mechanics, much insights are yet to be gained on the extension in front region of the crack tip as well as on the kinetic and kinematic instabilities in the displacement field around the crack tip [17, 76]. Reason is straightforward, it is very difficult to investigate directly on propagation of crack starting from the nucleation for the speed of propagation of cracks in solids.

Fracture behaviors of brittle thermoset polymers (mostly epoxy resins) were started to be addressed through kinetic approach to explain the fracture in thermoset polymers as ruptures of bonds [21, 80, 81]. In a latter approach [67, 68], the micro-level stresses have been taken into account to address the kinetics of polymer fracture. The kinetic theory is much applicable to the initiation of fracture in brittle polymer and cannot be efficiently used for the crack propagation especially for non-brittle polymers with deformations consisting of inelastic and/or plastic components [37]. There are some other approaches also for explanation of fractures in polymers like molecular fracture etc. [37]. Thermoset polymers are well known brittle materials. These thermosets are reinforced with toughners and/or fibers to improve its characteristics and toughness against fracture. Consequently, the fracture behavior of thermoset FRPCs are often studied by application of fracture mechanics. Though detailed scientific criteria to address the failures of all FRPCs under all conditions are not available, phenomenological procedures are there to address the failures of such materials due to formation of cracks. Mechanics of cracks or fracture mechanics is based on Griffith's and Irwin's approaches [19, 20, 30]. These approaches can be followed to describe the fracture in ceramics, polymers or metals. But for FRPCs, heterogenous ingredients of different length scales are infused into homogeneous and isotropic matrices with some other consequences for arresting, branching or deflection of cracks along with mixed mode features. Consequently, some conditions assumed in classical theories or hypotheses become insufficient for these composites. Moreover, with the advent of new varieties of matrices and fibrous materials, additional theories are being employed to explain the fracture behaviors of FRPCs. Diverse field of development of different theories are becoming important day by day to address the fracture problems of fiber reinforced polymeric composites. These FRPCs are used in many applications for their appreciable performances in resisting fractures or fracture toughness.

2 Fracture of FRPCs

Fracture of FRPCs is a type of failure that can be explained by several approaches. One of the approaches deals with micromechanics of composites. Another approach is to estimate the global response of components or structure as a whole which is very much important to the scientists, engineers and policy makers of the world. There is one intermediate approach between the two. In this approach, the deformation or the load carrying capacity of each layer or that of the number of layers of a laminate with the interactions between the different layers of laminated composite is addressed. The topics are discussed in a little more detail in the following section.

Based on reinforcing materials, polymeric composites can be divided in to two broad classes (as described in details in first chapter): Particulate and fiber reinforced composites. Based on matrices, they are primarily divided into two types: thermoplastic and thermoset polymeric composites. The target of discussion in this chapter is the fracture behavior of the fiber reinforced thermoset polymeric composites. It is clear from the fore going discussions that fracture in FRPCs is a complicated phenomenon. Nevertheless, fibers play key roles in FRPCs. Legitimate approach is to start with the interaction between fibers and matrices during fractures. Fibers and matrices are treated as elements of separate constituents in numerous studies. The failure problem may be addressed from different points of views. One of such views of addressing this problem is studying the fiber-stresses, stresses in matrices holding the fibers in positions, the interface stresses, fiber breakage, matrix cracks, fiber interactions, varying distances between the fibers, influence of other fibers on the broken one, or localized yields of matrix or fibers. This view of addressing localized effects of interaction between matrices and fibers is micromechanics [32, 41]. On the other hand, different intellectuals of the world are interested in the global response of composite structures or structural components made of FRPCs. The global responses like deformation, buckling and fatigue loads, thermal behaviors, damping, energy absorbing capacity, effect of holes or similar discontinuities etc. in the components or the structure as a whole are more important to them from application point of view. Response of a layer or a number of layers are intermediate in nature between these two approaches. In this approach, the deflection or load carrying capacity of individual layer, or that of the laminates of a groups of layers along with the interactions between the different layers of laminates are studied to predict their failures. Role of fiber orientation or constituent materials in a single layer or the layers of a laminate is an active field of study due to invention and development of new constituent materials for both fibers and matrices (as mentioned in the previous section). In this technique, the responses like deformations of some element of a layer with many fibers are addressed without going to the micromechanical responses of individual fiber, matrix and their interactions.

The next section will be dedicated to failures accompanied with fractures only as the topic of this section is fracture of FRPCs followed by much more insights from micro, meso and macro level study of failures.

2.1 General Fracture Mechanics Approach

Fracture mechanics deals with the propensity of inherent cracks of materials to grow under applied loads. The strength of component or structures is reduced due to existing flaws or cracks. If the cracks are long enough, the structure or structural component will fail much below the design loads. The defect criticality assessment for the performance of a structure is the prime objective of fracture mechanics. Moreover, fracture mechanics is applied to calculate the maximum allowable size of cracks.

The material science and applied mechanics are generally integrated in fracture mechanics to study the behaviors of materials with defects. Applied mechanics is applied to find the relationship between the stress field and deformation at the tip of a crack in a cracked material body. The resistance of the materials against fracture (due to crack) is estimated in material science applying fracture mechanics for developing more robust materials through better processing and design of materials.

The defect or flaws plays the key role in the strength exhibited by any material. These flaws are very much important in fracture failure of FRPCs [32]. The micro or macro-cracks governs the strength of the FRPCs. Thus, fracture mechanics is an essential field of study in the fracture failures of the FRPCs, be it at micro or macro levels for strength of materials approach is essential but not sufficient to explain the behaviors of FRPCs.

Three major stages are to be considered in fractures due to cracks: nucleation or initiation of microcracks, growth and coalescence and crack propagation. In the growth phase, microcracks grow stably to join with other micro cracks to form a macro crack. In the final stage, these macro cracks propagate fast leading to fracture. This happens at a stress level which is critical for unstable crack growth. While these stages are prominent in thermoplastic FRPCs, due to matrix ductility and crack arresting capability of fibers at the interface between the fibers and matrix, stage two is not prominent to be realized in thermoset FRPCs for brittle character of the thermosets. It is already mentioned that polymer composites with brittle matrices like thermoset resins are prone to exhibit numerous cracks about the fibers or in between the fibers causing disturbances in the transfer mechanism of the load to the fibers from the matrices leading to failures.

Failures may occur under one or combination of more than one of the modes of fractures. Based on applied loads, following modes of fractures are encountered in fracture mechanics.

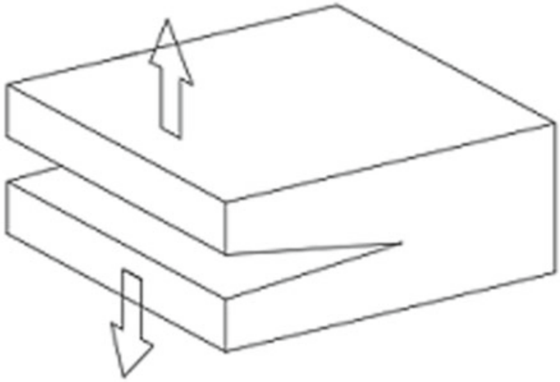

Fig. 1 Fracture under mode I loading

1. Mode I fracture
2. Mode II fracture
3. Mode III fracture failures and
4. Mixed mode fractures.

2.1.1 Mode I Fracture

In mode I, tensile load is applied perpendicular to the facture plane leading to opening mode of fracture. It is termed as opening mode for the joints between the fracture surfaces at the crack tips open like opening some page in writing pad as shown in Fig. 1.

2.1.2 Mode II Fracture

In mode II fracture, the load applied is parallel to the fracture plane and results in sliding the two newly developed fracture surfaces parallel to the crack surface and in the same direction to the crack front due to shear stress perpendicular to the crack front. The loading direction, cracked surfaces and the crack front are presented in Fig. 2. This is sometimes termed as sliding mode for the newly developed fractured surfaces which are parallel to the crack-plane, slide in directions opposite to each other as shown in figure. Definitely, in-plane shear forces play important role in this type of fracture.

2.1.3 Mode III Fracture

Mode III fracture occurs when load is applied in such a way that the shear stress is parallel to the crack front and crack plane both. The load is applied out of plane to

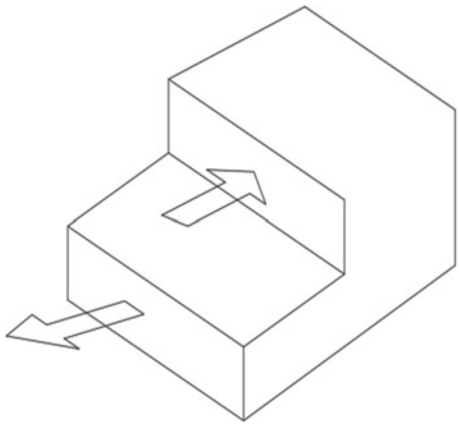

Fig. 2 Fracture under mode II loading

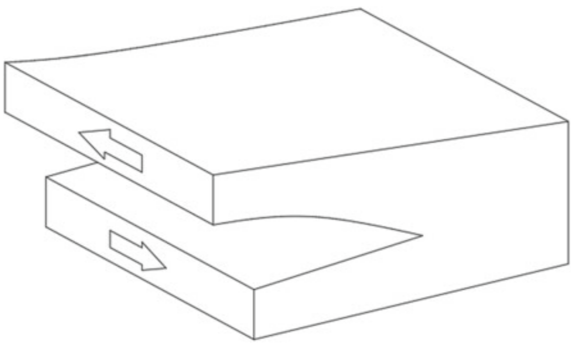

Fig. 3 Fracture under mode III loading

the shearing plane. This is called as tearing mode of fracture for this mode of fracture is often realized in tearing some component. As for example, this mode of fracture observed when a paper is torn by applying forces by two hands at one edge in reverse directions. Mode III fracture is schematically presented in Fig. 3.

2.1.4 Mixed Mode of Fractures

A combination of the different pure modes of fractures (mode I, mode II and mode III) are frequently experienced in practical cases of complex situations. This mode of fracture can be characterized by the presence of two or more modes (Mode I and mode II, Mode I and Mode III or all modes) mixed at the crack front. This mixed mode fracture is common in case of sandwich composites due to asymmetry in both material and geometry of layers and in several loading cases.

It is already mentioned that different mechanisms ultimately result in failures. Several mechanisms which finally lead to failures in FRPCs are presented in the following section.

2.2 Fracture Failure Procedures

Two well accepted hypotheses of fracture mechanics are:

1. There are inherent flaws in every real material, and
2. Higher stresses are induced at the location of flaws than the surrounding region of the materials which finally lead to fracture.

Several anomalies or flaws in microscales exist in FRPCs due to defects in manufacturing or other reasons [4]. These defects are the breeding points of crack-initiations leading to propagation due to the higher stress concentration at flaws than the surrounding materials. Failure in macroscopic scales within FRPCs can include the following four mechanisms.

1. Transverse cracking,
2. longitudinal cracking
3. Delamination and
4. Fiber breakage.

Based on observations, the resultant event of failure in laminated composites starts with transverse cracks which leads to series of failure events such as: cracking in longitudinal direction, delamination and breakage of fibers. The failure mechanisms in macroscopic scales mentioned above are briefly presented below.

2.2.1 Transverse Cracking

Crack in the matrix in a direction transverse to the direction of applied load is one of the mostly observed mode of damage in laminated composites [77]. The transverse cracking in laminates has been shown in aqua color in Fig. 4. Transverse crack

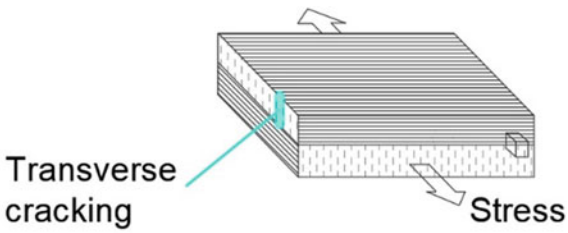

Fig. 4 Transverse cracking in FRP laminates

initiation and propagation is an enduring issue over some decades. The microcracks at 90° plies due to loads applied along plies at 0° direction are commonest form of damage [43]. Stiffness of laminates is reduced due to it followed by stress singularity at the location of crack tip along with initiation of delamination between the laminae. Delamination leads to disband between the matrix and the fibers and breakage in fibers. Finally, the integrity in the structures is lost inviting failures in structures. It is now understandable that failure of laminated composites starts with transverse cracks which can be followed by cracking in longitudinal direction, delamination and breakage of fibers. Analytical and experimental investigations on the failures of laminates have been concentrated over decades on the initiation of cracks transverse to the loading direction and their propagation in laminated composites. Finite element method was employed in a three-dimensional analysis of specimen [78] to calculate the rate of energy release for initiation and propagation of crack across the width. Behavior of neighboring plies due to transverse cracking in matrix and generalized plane strain induced delamination was studied by Yokozeki et al. [79]. Propagation of crack in the matrix of continuous carbon fiber reinforced epoxy composite was predicted [62] by another research group applying finite element method in non-linear domain.

2.2.2 Longitudinal Cracking

Investigation based on numerical analysis reveals that matrix-crack in the direction transverse to the loading direction is first damage mode and longitudinal cracking [48] is the second mode of damage. Longitudinal cracks are developed when the direction of primary loading is parallel to the direction of fibers. While longitudinal cracks are not developed in the life time of some laminates, they are observed to occur before the failure in some of the laminated composites. Sometimes, for this reason, longitudinal cracks are not taken into account in modelling laminates. But they can develop in cross-ply laminates [48]. The longitudinal cracking in laminates has been shown in aqua color in Fig. 5.

2.2.3 Delamination

In case of laminated or layered composites, delamination is separation of different laminae or layers of composites. It is a critical mechanism of failure [73] in composites made of polymeric matrix reinforced with different fibers. This is a basic difference between the behaviors of the FRPCs and metals. Delamination is resulted from higher stresses in the interfacial region accompanied with lower strength through the thickness. This is evident as the fibers provided along the plane of laminae do not render any reinforcing effect in the through-thickness direction leaving weaker matrix only to carry the loads in that direction. In addition, the tendency of delamination is increased by the matrices like brittle resins. It may occur due to failure in adhesives or glues joining the layers. Material fibers of higher strengths like carbon

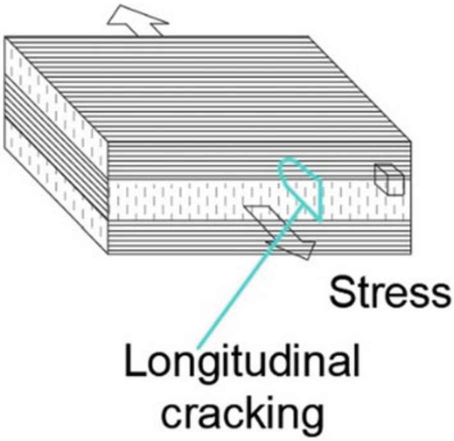

Fig. 5 Longitudinal cracking in cross-ply FRP laminates

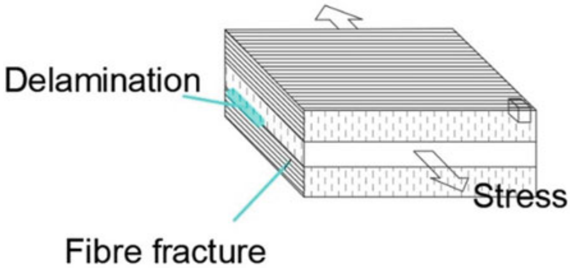

Fig. 6 Delamination and fibre fracture in FRPCs

or glass fibers are bonded together by lower strength matrices. Load applied in the direction perpendicular to the layer of higher strength or some shear force may cause fracture failure of matrix or debonding of fibers from the matrix. The separation of two layers of layered composite is schematically shown by aqua color in Fig. 6. Delamination may be there as manufacturing defect or may originate within the body of the composite component under service loads which are not visible on the exposed surfaces of the component. It may grow under applied loads or due to environmental effects finally leading to catastrophic failures and huge loss of wealth or life. This is one of the major concerns of the material scientists and engineers working with composite structures.

2.2.4 Breakage of Fibers

This mechanism finally leads to failures in fiber reinforced polymer composites. The name of the mechanism is self-explanatory. Typical breakage in fiber is schematically shown in Fig. 6. As per fiber bundle approach, a number of fibers must break before ultimate failure under tensile loads [55]. Mechanical performance of laminated and other fiber reinforced composites can be degraded due to breakage in fibers [65]. Pullout and breakage of fibers appear to be common mechanisms of failures in the impact tests under low velocity. Due to shear force or high flexural stresses at the stress field induced in the sides opposite to the face indented or hit by the projectiles, fibers break or fail. Breakage of fibers may be caused by the high stress or instantaneous rise in temperature during indentation or experiment. High level fiber breakage may be experienced by a fibrous composite specimen at the location of impact [6]. This breakage of fibers obviously depends on many factors like fiber types, fiber volume ratio, original aspect ratio of fibers at initial stage and the stress–strain condition during manufacturing of fibers. Modelling on fiber breakage is exercised by research-group on ruptures in carbon composite [31, 42, 63] to address both the theory and applications.

2.3 Fracture Mechanics Approach in FRPC Problems

Some concepts of isotropic material approach in linear elastic fracture mechanics is difficult to implement for FRPCs for anisotropy and inhomogeneity. Anisotropy and inhomogeneity are there both in materials, and stacking sequence of laminae (i.e. several laminas in different directions) within the laminates invite complex problems.

The anisotropic materials with homogeneity can be used with fracture mechanics approach to deal with the FRPCs. Here, composite materials are assumed to be homogeneous but anisotropic. But in most of the cases FRPCs are not homogeneous revealing that this assumption is also insufficient to explain the behaviors of the FRPCs in true sense. Stress distribution around the crack tip is reported by Wu [74]. He finds that the stress intensities around the crack in affected by the properties of anisotropic materials, crack orientation with respect to the principal material axis and crack parameters. The researchers worked on the advancements in application of fracture mechanics to the problems of composite materials [10, 59, 69]. The problem is more critical in the case of fatigue. It is a complex phenomenon. Crack tends to grow parallel to the fibers in a self-similar pattern if the crack is cut in parallel to the fiber direction, whereas the crack growth is parallel to the fibers if cut at an angle to the fibers not parallel to the crack itself. In a laminate of various layers, the crack growth is much more complicated. Therefore, in reality, the growth prediction of different cracks in FRPCs is a complex problem.

2.4 Fracture in FRPCs Under Compression

Fracture and delamination in laminates may be caused by compression beyond a certain limit.

Fig. 7 Delamination of carbon fiber reinforced polymer under compression load [Courtesy: Wikimedia Commons (under free reuse license)]

This type of delamination followed by fracture due to compressive load beyond limit is presented in Fig. 7.

The crack-interactions among the cracks in orthotropic layered composites under compression were analyzed in a non-classical approach of fractures mechanics [72]. The problem was treated as transversely isotropic one with parallel fibers embedded in matrix and analyzed by finite element method. The investigators reported that the critical strain in layered materials under compressive loading not only depends on the interaction of cracks but also depends on the crack size and their mutual positions. There are several other works also on the fracture of FRPCs under compressive loads.

2.5 Fracture Failures of FRPCs in Different Scales

It is clear from the foregoing discussions that the fracture failure analysis of FRPCs is a complicated problem. Different approaches have been proposed to explain the behaviors of FRPCs subjected to fractures. Some of them are competent to explain certain behavior of particular or a group of FRPCs, but insufficient for others. This is

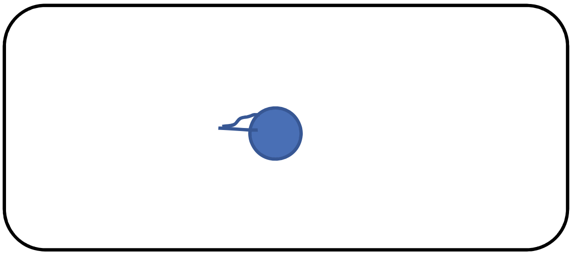

Fig. 8 Micro level flaw near a fiber in FRP

still an open field of research. To handle this complex problem of fracture failure, one such approach may be acceptable. The problem can be examined in different length scales to make it convenient for study [58]. In this context, it is logical to discuss on the failure mechanisms those occur in different scales, i.e. micro, meso and macro scales [36]. The study of fracture behaviors of fiber reinforced composites will be briefly addressed in the following section on the next page.

2.5.1 Fracture Study in Micro-scale

It is evident that there are inherent flaws within FRPCs due to manufacturing defects or some other reasons. These are the hotspots for the fracture initiation and further growths. Fracture starts at molecular or atomic levels at these hotspots. One of such flaws in the matrix at the location of a fiber is shown in Fig. 8 hiding all neighboring fibers. Crack may grow under service loads. The problem is much more complex when treated with fracture mechanics. Each of these mechanisms at micro level depends on the types of loading at macroscopic levels. How the effects of loads are distributed among these micro cracks, how they are affected is not clear till date. The bonds are damaged due to force exceeding some limits to initiate crack growth. For the uncertainties associated with these mechanisms, Monte-Carlo simulation was used by the investigators for prediction of failure [23, 35]. But there are limitations in capturing the complicacy as a whole. The diameter of the fiber (in microns) and their lengths are taken as input parameters which are much and much larger than atomistic scales which shows the incapability of this simulation in constituting microstructure explicitly. This is so much important for it permits to realize different failure mechanisms besides failure of individual ingredients. It is started with the flaws in atomistic scales at fiber location.

2.5.2 Fracture Study in Meso-scale

Meso scale is characterized by several hundred microns. Therefore, it addresses flaws extending through several fibers. One of such fractures extending through multiple

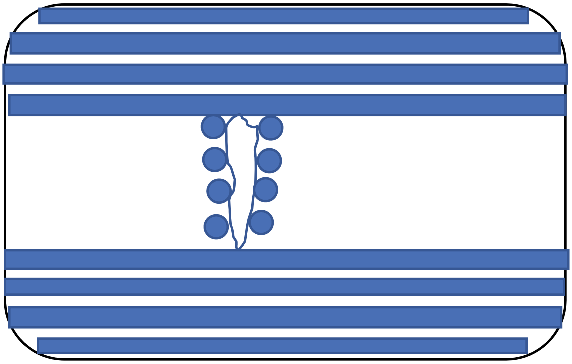

Fig. 9 Meso scale crack in FRP

fibers in a cross-ply laminate is shown in Fig. 9. In this type of fracture, the micro level mechanisms can grow and conglomerate through complicated mechanisms under external loading. Meso scale fracture study is useful for FRPCs made of fibers arranged with textile fashion for laminates with UD fibers are very weak in transvers direction with respect to the fiber direction. Laminates are often manufactured by stacking UD laminae in different directions, woven fabrics etc. Mesoscale is useful when the interactions of constituent plies of laminates in fracture is considered. Meso-scale fracture study can well address the inter-ply delamination as well as splitting. The important input parameter for the inter-ply delamination is the adhesion between the fiber and the matrix. The surfaces of fracture exhibit noticeable mutilated or hackle structures through branching and deflection of cracks [18] for increased bond strength consuming more energy for the same propagation length of relevant macroscopic crack.

If the individual ply is weak, then interlaminar crack can grow and propagate through it. But the bonding between the matrix and fiber is the key factor for delamination between the plies. Delamination may occur within a layer also if it is subjected to high stress in out-of-plane direction with rough surfaces of fracture and noticeable splitting of the layers of fibers as shown in Fig. 10.

Several plies of these laminates are subjected to different levels of stresses in axial direction. Strain coupling [57] is possible in this case if there is no interfacial damage between the laminae. The strength and elastic behaviors really depend on the orientation of the fibers at different plies with different states of stresses. Cracks between the fibers of the off-axis plies are readily observed under tension. Fracture depends on the configuration of loads and the sequence of the stack. These cracks

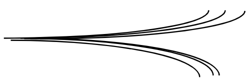

Fig. 10 Interlaminar delamination from inter-fiber cracks

may develop within the fibers in both the direction of loads or in a transverse direction to it. Consecutive plies with fibers in different directions may stop or halt the growth of cracks for the fibers in loading direction can avoid the matrix failure under local coupling of strains. But such cracks among the fibers may cause to more damage due to crack tip-stress concentration and lead to propagation of delamination between the layers of laminates.

Fiber bridging is another type of failures which may occur under tensile loading at mesoscopic scales due to low bonding strength between the matrix and fiber. Even after some pull out, the fibers in transverse direction to the crack propagation can resist the propagation of load to some extent.

In addition to the above some pores at mesoscopic level may exist within the laminated composites which can act as the local spot of stress concentration leading to delamination or buckling or may reduce the effective area of cross section to increase the stress level.

2.5.3 Fracture Study in Macro-scale

The scale ranges from some millimeters to meters. In this study, the laminate thickness or a whole structure can be covered. This macro-scale fracture is defined by the types of applied loads for a large range of mechanisms are covered within micro and meso scales. Fracture type inter-fiber may not result into fracture in macroscopic scales. But it may impair the stiffness. Macro level fracture may be due to an integral effect of several microscopic and mesoscopic failures. It can be observed from a test of T-pull, pattern of damage or fracture in a woven fabric composite. In fracture of woven fabric components, it is observed that through-thickness macroscopic fracture surfaces are results of branching of cracks into several mesoscopic cracks causing many inter laminar debonding, micro buckling of several layers and other micro or mesoscopic damages. On the microscale, cracks between fibers with breakage of fibers are observed. One of such macroscopic failure resulting from several micro and meso level failures may be damage due to impact. In the case of impact of low velocity, the damage may not be directly visible on the exposed surfaces but may cause serious damages in micro and meso scales causing delamination within the laminates finally leading to fractures.

3 FRPC Failure Theories

Many theories have been proposed to predict the failures of FRPCs over the decades to interlink the different microscopic and mesoscopic failure mechanisms leading to failures at macro-scales. Additionally, these vary for different type of loading and materials. The theories can be grouped under different categories based on their field of applications as follows:

1. Failure theories for static or quasi-static loads
2. Failure theories for damage mechanism, growth and degradation
3. Theories of failures under creep, fatigue and rupture due to stress
4. Theories for high strain rate failures.

Laminae are the building blocks of laminated composites. Before going to a brief review on most significant developments in this theoretical area, it will be legitimate to start with the micromechanics of UD FRPC lamina. Primarily following three modes of failures are observed at micro level [41, 66] in UD laminae made of FRPCs.

1. Failure dominated by fibers
2. Failures dominated by matrix
3. Interfacial failures or failures dominated by flaws.

Instead of failures are induced in FRPCs by the above-mentioned causes, global failures are not usually treated with them (which are considered at microlevels).

FRPCs fail gradually, stresses are redistributed among the other laminae when a particular lamina is affected by failure. Basically, in the analysis of failures. Strength is considered to be the important criterion. This strength of laminae is affected by directionality and there is significant difference between tensile and compressive strengths of UD lamina. Additionally, the direction of shear stress in comparison to the direction of fibers plays important role on the strength of UD lamina.

3.1 Micromechanical Failure of Fiber Reinforced Polymeric UD Lamina

The failure of UD FRPC lamina with their strengths with respect to the three phases (fiber, matrix and interfaces) will be briefly presented below.

3.1.1 Tension in Longitudinal Direction

The ingredient with lower ultimate strain will fail before the other for fiber and matrix exhibit different values of ultimate strains under tension. The stress in longitudinal direction of lamina is given by

$$\sigma_{1t} = \sigma_{1tf} V_f + \sigma_{1tm} V_m \quad (1)$$

where

σ_{1tf} is the average stress in fibers under uniaxial tensile load.
σ_{1tm} is the average stress in matrix under uniaxial tensile load.
V_f volume fraction of fiber present in the FRPC.
V_m volume fraction of matrix (resins or other polymers) in the FRPC.

There is a possibility of two cases.

Firstly, it is possible that the ultimate strain in matrix is greater than that of the fiber. The UD lamina will fail when strain in longitudinal direction exceeds ultimate strain in fiber. In that case, the strength of lamina in longitudinal direction is approximately given by

$$\mathrm{S_{1t}} = \mathrm{S_{1tf}}\mathrm{V_f} + \sigma_{avm} V_m \tag{2}$$

where

S_{1t} is tensile strength of the lamina in longitudinal direction.
S_{1tf} is tensile strength of the fiber in longitudinal direction.
σ_{avm} is the average stress in the matrix in longitudinal direction at the time of ultimate strain in the fiber.

Secondly, the ultimate strain in fiber may be greater than that of the matrix.

The UD lamina will fail when strain in longitudinal direction exceeds ultimate strain in matrix. In that case, the strength of lamina in longitudinal direction is approximately given by

$$\mathrm{S_{1t}} = \mathrm{S_{mt}}\mathrm{V_m} + \sigma_{avf} V_f \tag{3}$$

where

S_{mt} is tensile strength of the matrix.
σ_{avf} is the average stress in the fiber in longitudinal direction at the time of ultimate strain in the matrix.

This may be written in another form as

$$\mathrm{S_{1t}} = \mathrm{S_{mt}}\left(V_f \frac{E_{f1}}{E_m} + \mathrm{Vm}\right) \tag{4}$$

E_{f1} is the elastic modulus of fiber in longitudinal direction.
E_m is the Modulus of elasticity of matrix material.

But the statistical distribution of the strength of fibers and matrix is not considered in above discussion [52, 53]. Strength in longitudinal direction in this case is governed by the strength of fibers.

Three different types of failures can be observed in FRPCs.

1. Transverse matrix crack for stronger interface and brittle matrix.
2. Matrix-fiber debonding for weaker interface and higher ultimate strain of fibers.
3. Failure through conical shear for stronger interface and matrix-ductility which does not occur in thermoset resins for thermosets are well known for their brittleness.

The stresses are redistributed among the other fibers when a particular fiber break. This is a localized phenomenon. A greater number of fibers break with increased loads. These localized failures result in eventual failures when they join together and interacts among them. The pattern of failure depends on the volume fractions of the constituents and their properties.

3.1.2 Compression in Longitudinal Direction

It is very difficult to verify the results of numerical analysis of UD FRPC lamina through experiment for two reasons.

Firstly, it is very difficult to arrange experimental setup for UD composite lamina through which it will be subjected to true compression only. Secondly, the experts may have the opinions that these structural components or the structures are rarely subjected to true compression. Except some special cases, instability influences the failure in these cases before failure under pure compression. Significant investigations have been exercised to address the compressive strength of FRPC [47].

Compression failure in longitudinal direction occurs due to micro-buckling or kinking of fibers. In out-of-phase micro-buckling, the compressive strength at low V_f is expressed as

$$S_{1c} \cong 2V_f\sqrt{\left[\frac{E_m E_{f1} V_f}{3(1 - V_f)}\right]} \tag{5}$$

compressive strength at higher V_f is expressed as

$$S_{1c} \cong \frac{G_m}{1 - V_f} \tag{6}$$

Expression (6) gives the compressive strength for the failure for higher V_f occurs due to in-phase or shear mode and G_m is the modulus of rigidity or shear modulus of the matrix. The compressive strength may be governed by the strength of fiber in shear also in another failure mode. In that case, compressive strength is expressed as

$$S_1 \cong 2S_{fs}\left[V_f + (1 - V_f)\frac{E_m}{E_{f1}}\right] \tag{7}$$

where

S_{fs} is the strength of the fiber in shear.

3.1.3 Tension in Transverse Direction

In case of UD lamina of FRPC, transverse tension is most critical for failure. Matrix and interphase experience very high strain or stress concentration under this loading. Distribution of stress about the fiber can be determined experimentally or theoretically. Experimentally it can be determined by photo elastic method. The methods of complex variable, finite element, boundary element or finite difference can be applied to determine the theoretical stress distribution. Primarily, the critical stress and the critical strains are developed in the interface between the fiber and the matrix.

Concept of stress concentration factor is used to determine the strength of the lamina under tension in transverse direction and that is related to the strain concentration factor as follows [14].

$$\mathrm{K_e} = \frac{e_{2\,\max}}{e_2} \cong K_s\left(\frac{E_{2t}}{E_m}\right)\frac{(1-\nu_m)(1-2\nu_m)}{1-V_m} \tag{8}$$

where

K_e is the factor of strain concentration.
K_s factor of stress concentration.
e_{2max} Maximum value of strain in transverse direction.
e_2 Average value of transverse strain.
ν_m Poisson's ratio for the matrix material.

Composite laminas are subjected to thermal stress in many cases. Curing of matrix induces residual stress and residual strains. The induced stresses cause difference in values of thermal coefficients in the ingredients. Failures of composite laminae subjected to thermal stresses are predicted through the linear relation between the stress and strains. If the thermal stress is considered with criterion for the maximum stress, then the tensile strength of FRPC in transverse direction can be expressed as

$$\mathrm{S_{2t}} = \frac{1}{K_s}(S_{tm} - \sigma_{mr}) \tag{9a}$$

where

σ_{mr} Maximum value of residual stress.

If the thermal stress is considered with criterion for the maximum strain, then the tensile strength of FRPC in transverse direction can be expressed as

$$\mathrm{S_{2t}} = \frac{(1-\nu_m)}{K_s(1+V_m)(1-2\nu_m)}(S_{tm} - e_{mr}E_m) \tag{9b}$$

where

e_{mr} Maximum value of residual strain.

3.1.4 Compression in Transverse Direction

Several mechanisms in UD FRPCs may result in compression failures. Stress concentration at of high magnitude at the interface under compression may lead to matrix failure or fiber failure through crushing. The transverse compressive stress of UD composite lamina is

$$\mathrm{S}_{2\mathrm{C}} = \frac{1}{K_s}(S_{cm} - \sigma_{mr}) \tag{10}$$

where

S_{cm} matrix-compressive-strength.

Overall shear failure may be observed at higher stress under compression for higher stress at the interface resulting in delamination or shear failure in matrix.

3.1.5 In Plane Shear

The interface between the matrix and fiber of UD lamina is the worst affected region under inplane shear. It can be fairly assumed that the matrix of UD lamina fails under inplane shear, and the predicted shear strength is given by

$$\mathrm{S}_6 = \frac{S_{sm}}{K_\tau} \tag{11}$$

where

K_τ concentration factor for the shear strength.
S_{sm} Shear strength of the matrix.

The values of K_τ varies with the materials and $\mathrm{V_f}$ which can be determined by the method of finite difference [1].

3.2 Anisotropic Failure Theories

Prediction on failure initiation from micromechanical analysis is acceptable at critical points. In case of global failures, that is an approximation only. Micromechanically, laminae are anisotropic and the strength is a function of the direction of fibers. Lamina may have five parameters for in plane micromechanical strengths—$\mathrm{S_{1t}}$, $\mathrm{S_{c1}}$, $\mathrm{S_{2t}}$, $\mathrm{S_{2C}}$ and $\mathrm{S_6}$.

Theories of micromechanical failures of composites are adoption and extensions of isotropic theories. There is a good review on the anisotropic composite failure theories [60].

Four failure criteria are basically employed to explain failures of FRPCs.

1. Theory of maximum stress
2. Theory of maximum strain
3. Theory of deviatoric strain energy or Tsai-Hill theory
4. Theory of tensor polynomial or Tsai-Wu theory.

3.2.1 Theory of Maximum Stress

This theory states that the UD lamina fails when a stress component reaches a value of the strength in the direction of corresponding principal axis. The stresses in a complex state can be transformed into stresses in principal axes of the materials. Failure takes place when

$$\sigma_1 = \left.\begin{array}{c} S_{1t} \text{ when } \sigma_1 \text{ is positive} \\ -S_{c1} \text{ when } \sigma_1 \text{ is negative} \end{array}\right\} \tag{12a}$$

$$\sigma_2 = \left.\begin{array}{c} S_{2t} \text{ when } \sigma_2 \text{ is positive} \\ -S_{2c} \text{ when } \sigma_2 \text{ is negative} \end{array}\right\} \tag{12b}$$

$$|\sigma_6| = S_6 \tag{12c}$$

$$\begin{aligned} \sigma_1 &= \sigma_x \cos^2\theta \\ \sigma_2 &= \sigma_x \sin^2\theta \\ \sigma_6 &= -\sigma_x \cos\theta \sin\theta \end{aligned} \tag{13}$$

Equating strengths to corresponding stresses, strengths in off-axis direction (S_x) are determined as follows (Fig. 11).

For σ_x is tensile, i.e., $\sigma_{x,}$ is positive

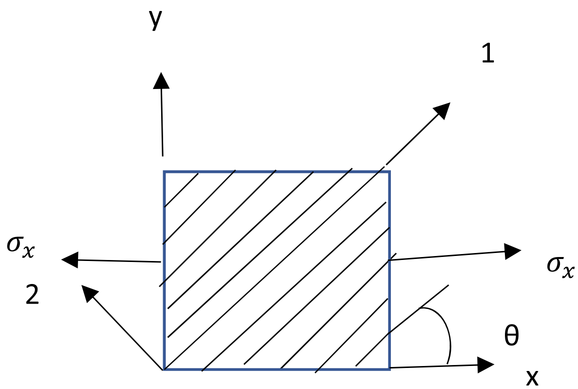

Fig. 11 Unidirectional lamina under off-axis load

$$\mathrm{S_{xt}} = \frac{S_{1t}}{\cos^2\theta}$$
$$\mathrm{S_{xt}} = \frac{S_{2t}}{\sin^2\theta}$$
$$\mathrm{S_{xt}} = \frac{S_6}{\cos\theta\sin\theta} \tag{14}$$

For σ_x is compresive, i.e., $\sigma_{x,}$ is negative

$$\mathrm{S_{xc}} = \frac{S_{1c}}{\cos^2\theta}$$
$$\mathrm{S_{xc}} = \frac{S_{2c}}{\sin^2\theta}$$
$$\mathrm{S_{xc}} = \frac{S_6}{\cos\theta\sin\theta} \tag{15}$$

According to theory of maximum stress, only three out of five subcriteria are applicable depending on whether σ_x is compressive or tensile. As per this theory, failure in any direction is not dependent on the stresses in its perpendicular directions. The interaction of stresses as in biaxial stresses is not considered in this theory.

3.2.2 Theory of Maximum Strains

This theory states that the UD lamina fails when a strain component reaches a value of the ultimate strain in the direction of corresponding principal axis. The stresses in a complex state can be transformed into stresses in principal axes of the materials. Failure takes place when

$$e_1 = \left.\begin{matrix} \mathrm{e_{1tu}\ if\ e_1\ is\ positive} \\ \mathrm{e_{1cu}\ if\ e_1\ is\ negative} \end{matrix}\right\} \tag{16a}$$

$$\mathrm{e_2} = \left.\begin{matrix} \mathrm{e_{2tu}\ if\ e_2\ is\ positive} \\ \mathrm{e_{2cu}\ if\ e_2\ is\ negative} \end{matrix}\right\} \tag{16b}$$

$$\mathrm{e_6} = \mathrm{e_{6u}} \tag{16c}$$

where

e_{1tu} is ultimate tensile strain in longitudinal direction.
e_{1cu} is ultimate compressive strain in longitudinal direction.
e_{2tu} is ultimate tensile strain in transverse direction.
e_{2cu} is ultimate compressive strain in transverse direction.
e_{6u} is in plane ultimate shear strain.

In case of biaxial stresses, stresses in xy-coordinate axes are expressed in 1–2 axes (principal axes system of material) through transformation. The components of strains in 1–2 axes system can be obtained from the relation between stress and strains.

$$e_1 = \frac{1}{E_1}(\sigma_1 - \nu_{12}\sigma_2) \tag{17a}$$

$$e_2 = \frac{1}{E_2}(\sigma_2 - \nu_{21}\sigma_1) \tag{17b}$$

$$e_6 = \frac{\sigma_6}{E_6} \tag{17c}$$

The signs in principal direction of the material for the shear stresses are shown in Figs. 12 and 13.

In case of UD lamina, ultimate strains can be determined from basic parameters of the strengths.

$$e_{1tu} = \frac{S_{1t}}{E_1}$$

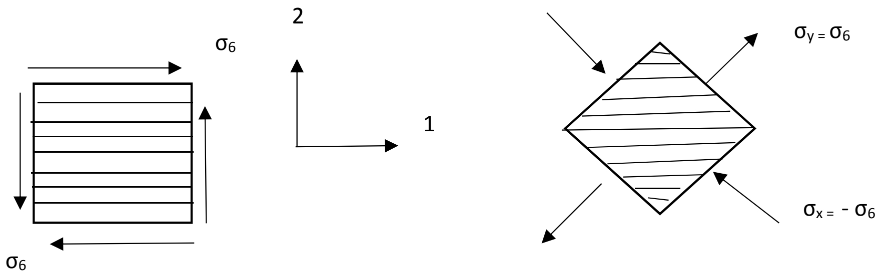

Fig. 12 Positive shear stress

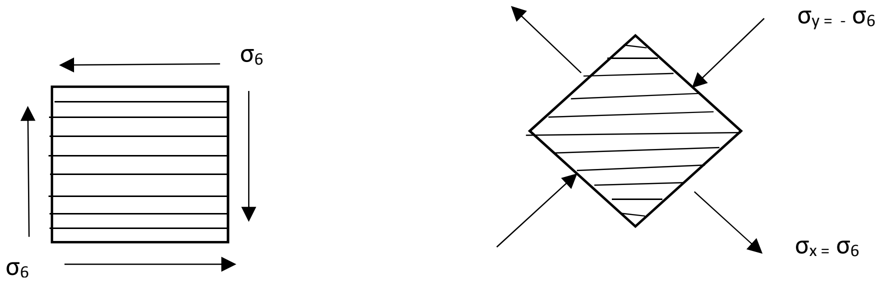

Fig. 13 Negative shear stress

$$e_{1cu} = -\frac{S_{1c}}{E_1}$$
$$e_{2tu} = \frac{S_{2t}}{E_2}$$
$$e_{2cu} = -\frac{S_{2c}}{E_2}$$
$$e_{6u} = \frac{S_6}{E_6} \tag{18}$$

The criteria for failure in general state of biaxial stress can be written as

$$\sigma_1 - \nu_{12}\sigma_2 = \left.\begin{array}{c} S_{1t} \text{ if } e_1 \text{ is positive} \\ -S_{1c} \text{ if } e_1 \text{ is negative} \end{array}\right\} \tag{19a}$$

$$\sigma_2 - \nu_{21}\sigma_1 = \left.\begin{array}{c} S_{2t} \text{ if } e_2 \text{ is positive} \\ -S_{2c} \text{ if } e_2 \text{ is negative} \end{array}\right\} \tag{19b}$$

$$|\sigma_6| = S_6 \tag{19c}$$

According to theory of maximum strains, number of subcriteria is five. Material can experience strain in any direction without any applied stress in that direction due to Poisson's or residual thermal effects. The interaction of strains is not considered in this theory also.

3.2.3 Theory of Deviatoric Strain Energy or Tsai-Hill Theory

In three-dimensional stress field, the yield criterion proposed by von Mises for isotropic materials is expressed as

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1 = \sigma_{yp}^2 \tag{20}$$

where

σ_{yp} is material's yield stress.

This criterion for isotropic material was modified by Hill [24] to describe anisotropic behavior of isotropic metals for large deformation in plastic region.

$$A\sigma_1^2 + B\sigma_2^2 + C\sigma_1\sigma_2 + D\sigma_6^2 + E\sigma_3^2 + F\sigma_2\sigma_3 + G\sigma_3\sigma_1 + H\sigma_4^2 + I\sigma_5^2 = 1 \tag{21}$$

Equation (21) can be expressed as following for a two-dimensional problem

$$A\sigma_1^2 + B\sigma_2^2 + C\sigma_1\sigma_2 + D\sigma_6^2 = 1 \tag{22}$$

The constants A, B, C, etc. depend on the anisotropy of the material.

Assuming FRPCs as transversely isotropic, and 1-direction as the direction of fibers Tsai and Azzi [5] expressed the following relations for FRP composite in two-dimensional stress field from the relation (22) provided by Hill.

In case of failure under in plane shear.

$\sigma_{6u} = S_6$ and $\sigma_2 = \sigma_1 = 0$. The relation (22) is modified as

$$D = \frac{1}{S_6^2} \tag{23}$$

In case of failure under uniaxial loading in transverse direction.

$\sigma_{2u} = S_2$ and $\sigma_6 = \sigma_1 = 0$. The relation (22) is modified as

$$B = \frac{1}{S_2^2} \tag{24}$$

In case of failure under uniaxial loading in longitudinal direction.

$\sigma_{1u} = S_1$ and $\sigma_6 = \sigma_2 = 0$. The relation (22) is modified as

$$A = \frac{1}{S_1^2} \tag{25}$$

Only C is the remaining parameter which can be determined from the interaction between σ_2 and σ_1 through biaxial test. Alternately, for equal loading along both the axes, though $\sigma_6 = 0$; $\sigma_2 = \sigma_1 \neq 0$. Failure of the material can be defined by criteria of maximum stress. That is the material will fail when σ_2 reaches composite strength in transverse direction for $S_2 \ll S_1$. The relation (22) can be expressed as

$$C = -\frac{1}{S_1^2} \tag{26}$$

Then Eq. (22) can be expressed with these values of A, B, C, D as

$$\frac{1}{S_1^2}\sigma_1^2 + \frac{1}{S_2^2}\sigma_2^2 - \frac{1}{S_1^2}\sigma_1\sigma_2 + \frac{1}{S_6^2}\sigma_6^2 = 1 \tag{27}$$

where

$$S_1 = S_{1t} \text{ or } S_{1c}$$

$$S_2 = S_{2t} \text{ or } S_{2c}$$

The tensile strength is to be used for S_1 and S_2 values when the composite is subjected to tension and the compressive strength is to be used for S_1 and S_2 values when compressive load is applied on the composite.

Equation (27) can be written as

$$\frac{1}{S_1^2}\sigma_1^2 + \frac{1}{S_2^2}\sigma_2^2 - \left(\frac{\sigma_1}{S_1}\right)\left(\frac{\sigma 2}{S_2}\right) + \frac{1}{S_6^2}\sigma_6^2 = 1 \tag{28}$$

The relation can be expressed as the following one for uniaxial off-axis load (Figs. 10 and 11) using Eqs. (13) and (28).

$$\frac{1}{S_x^2} = \frac{m^4}{S_1^2} + \frac{n^4}{S_2^2} + \left(\frac{1}{S_6^2} - \frac{1}{S_1 S_2}\right)\mathrm{m}^2\mathrm{n}^2 \tag{29}$$

where $\mathrm{m} = \cos\theta$, $\mathrm{n} = \sin\theta$ and $S_x = \sigma_{xu}$.

For advanced composites

$S_1 \gg S_6$, Eq. (29) is transformed to

$$\frac{1}{S_x^2} = \frac{m^4}{S_1^2} + \frac{n^4}{S_2^2} + \frac{m^2 n^2}{S_6^2} \tag{30}$$

Tsai-Hill criterion is a single criterion. This is the primary advantage of it over theories of maximum stress and maximum strain (with many criteria). Though the interaction between S_1, S_2 and S_6 is considered but the tensile and compressive strengths are not separately considered in Tsai-Hill criterion.

3.2.4 Theory of Tensor Polynomial or Tsai-Wu Theory

A theory of tensor polynomial has been proposed and modified by Tsai and Wu [64] with the assumption of existence of a failure surface within stress space which can be expressed with contracted notations as follows:

$$S_i\sigma_i + S_{ij}\sigma_{ij} = 1 \tag{31}$$

S_i is second rank strength tensor.
S_{ij} is forth rank strength tensor.

Equation (31) is a very complicated one. The expanded form of Eq. (31) for an orthotropic lamina in plane stress is given by

$$S_1\sigma_1 + S_2\sigma_2 + S_6\sigma_6 + S_{11}\sigma_1^2 + S_{22}\sigma_2^2 + S_{66}\sigma_6^2 + 2S_{12}\sigma_1\sigma_2 + 2S_{16}\sigma_1\sigma_6 + 2S_{26}\sigma_2\sigma_6 = 1 \tag{32}$$

S_{12} incorporates the interaction between σ_1 and σ_2.

Tsai-Wu theory of failure is most general among all the theories discussed in Sect. 3.2.

Maximum stress criterion, maximum strain criterion, and Tsai-Wu theory have been discussed as they are the most popular criteria for strength of composites for these are based on simple specimens subjected to tension, compression or shear. Further, these can be used to predict the failure loads of composite structures subjected to combined stress.

Important developments in the failure theories will be summarized below for the FRPCs without going to detailed discussion on complicated Tsai-Wu theory. The current developments and usage of experimentally determined input parameters of the theories will be emphasized in the following sections.

It is already mentioned in Sect. 3 that the failure theories can be grouped under four major classes:

1. Failure theories for static or quasi-static loads
2. Failure theories for damage mechanism, growth and degradation
3. Theories of failures under creep, fatigue and rupture due to stress
4. Theories for high strain rate failures.

3.2.5 Failure Theories for Static or Quasi-static Loads

Tsai-Wu-Criteria is one of the famous such failure theories [64, 75]. Hashin [22] showed that this criterion is not consistent with all the stress states of FRPCs inviting new development in the theories. Hashin adopted Mohr's criterion [39] for fracture failure of brittle materials. Hinton with the team performed a notable task to work out predictive capabilities of the failure theories [26, 27, 33]. In this series of attempts, the theories provided by Pinho et al. [45], Cuntze [12] and other two researcher groups were capable of holding best position for predicting the behaviors under 3-D state of stresses.

3.2.6 Failure Theories for Damage Mechanism, Growth and Degradation

Failure at macroscopic level through the summation of microscopic failure mechanism is experimentally challenging. In addition to it, the computational exercise in microscopic level in fact is very much intensive from computational point of view. Stress–strain relations in macro-scales, homogeneity, orthotropic symmetry with layers free from defects are considered to be acceptable for analytical analyses. In this approach, classical laminate theory (CLT) is valid. Layer-wise theory is used to predict the failures. Progress of damage within laminates with fracture mechanics concepts using strain energy of laminates subjected to triaxial loading is studied [16]. In another notable study, the capability of different criteria to predict delamination and initiation of cracks within the matrix has been compared [34] to conclude that experimental verification will be necessary to select which theory will be best in this approach to predict the behavior.

3.2.7 Theories of Failures Under Creep, Fatigue and Rupture Due to Stress

The investigations on prediction of failures due to creep, fatigue and ruptures caused by stresses are limited. As per definition [47] a strain rate less than 10^{-6} s^{-1} is considered as creep. In case of polymers reinforced with fibers creep generally cause unacceptable degradation but not always. Besides some studies, there are no much theories proposed for this type of failures of the FRPCs. In case of stress rupture, different theories produced similar results in prediction of life time. Some accelerated tests discussed [44] in literature needs further validation regarding stress-rupture. The investigation on viscoelastic materials in this field is extremely limited. Study of FRPCs under fatigue load is being continued for a long time. Behavior of FRPCs under fatigue load is extremely complicated and challenging field both from experimental as well as computational point of view. Studies are still being continued in several composite research centres. Prediction of actual point of time for initiation of cracks is really challenging. Statistical approach is essential for large number of scattered data. Nevertheless, the study and prediction of fatigue lives are essential for important structural designs. Different criteria have been proposed with their competencies and limitations.

3.2.8 Theories for High Strain Rate Failures

Currently, use of FRPCs in crash-worthy structures has been increased. This ensures the importance of studies under high strain rates. Unlike the other failures discussed in foregoing sections, the failure is sudden. A The rate of strain beyond 10^2 per second is considered as high strain rate. A strain rate of 10^6 per second can also be realized from special cases. The failure is of different nature for there is no scope of stress redistribution or relaxation etc. Instead of failure theories, several investigations have been carried out in this field with continuous refinements and developments along with the use of digital image correlation (DIC) techniques [38]. The location and the impact energy are being monitored by using sensors.

There is no criterion which is accepted universally for the failure of composites [25].

Instead of an appreciable number of failure theories has been developed, no criterion alone can accurately predict the failure of all FRPCs under all classes of loadings [29].

Large scale applicability of Tsai-Wu criteria in industrial composites is due to its capability in predicting composites' strengths reasonably. But the failure mode is not predicted through this criterion [25]. On the other hand, it is to be noted that World-Wide -Failure-Exercise concluded that no failure model predicted the failure accurately [25].

In early 1960s, the competitors of metals among FRPCs were boron or carbon fiber reinforced polymeric composites. Polymeric matrices like epoxy resins with low toughness were not considered for determining the toughness of the composites. Improved matrices with considerable toughness were developed as per civil and military demands of applications in 1980s. Moreover, new fibers are also developed in the composite world. This resulted in improvements and development of experimental techniques for investigation on the toughness and tolerance of FRPCs against fracture/damage. New test methods were proposed and developed for testing FRPCs including different modes of fracture tests under experimental fracture mechanics. These experimental investigations will be addressed in the following section.

4 Experimental Investigations on FRPCs

Experimental mechanics, especially experimental fracture mechanics are frequently used to measure the parameters and to realize the behaviors of FRPCs for the different limitations of the mathematical prediction theories and criteria. Experimental investigation itself may suffer from errors due to error in experimental setup and different other factors. Experimental setup for advanced fiber reinforced composites is generally expensive but experimental approach is the only way where reliable input data are not available for numerical simulations or analytical studies. Recently, both the industrial and political sectors of some countries have realized the importance of research on FRPCs. Experimental investigations play a great role towards fulfilment of this demand goals of research in this field.

Numerous experiments were performed by Wu [74] on FRPC material for realizing the extent of applicability of fracture mechanics of linearly elastic materials. He investigated on UD FRPCs with central cracks for critical loads and lengths (of cracks) at the initiation of rapid extension of cracks with different crack orientation and several loadings with respect to direction of fibers. Cracks were recorded to propagate colinearly with the original cracks. In addition, the opening mode was led by the symmetric loading while sliding mode was observed under skew symmetric loadings. For tensile load $\sigma\infty$ perpendicular to the central crack parallel to fibers, the intensity factors for stress are

$$\begin{aligned} k_1 &= \sigma\infty(a)^{1/2} \\ k_2 &= 0 \end{aligned} \tag{33}$$

Critical intensity factors for stress are

$$\begin{aligned} k_{1c} &= \sigma_c\infty(a_c)^{1/2} \\ k_{2c} &= 0 \end{aligned} \tag{34}$$

where

$\sigma_c\infty$ is defined as critical stress and the half crack length at the point of initiation of fast crack extension is denoted by a_c. Equation (34) can be written as

$$\log k_{1c} = \log \sigma_c\infty + \frac{1}{2}\log a_c \tag{35a}$$

$$\text{i.e., } \log \sigma_c\infty = -\frac{1}{2}\log a_c + \log k_{1c} \tag{35b}$$

Equation (35b) indicates that a plot of log $\sigma_c\infty$ versus log a_c will be a straight line with a slope of −(1/2) if k_{1c} is considered as a constant expressing material property. In fact, the slope is determined to be −0.49 [18] which shows that the theory is applicable to orthotropic thin lamina with cracks in principal direction of the material, i.e., in the direction parallel to the fibers. Numerous works of researchers through decades have advanced the level of knowledge on application of fracture mechanics to the fracture problems of FRPCs.

4.1 Fracture Toughness of FRPCs

Selection of a materials for an application is based on its toughness.

Fracture toughness (FT) indicates the resistance of a material against the growth of a crack which is measured in terms of intensity factor for stress or rate of release of the strain energy G_c at critical stage of crack growth. In anisotropic FRPCs, the highest fracture toughness is measured during in-plane fracture. This in-plane fracture is associated with pull-out or breakage of fibers. Lowest fracture toughness is recorded in interlaminar delamination. Fracture toughness of wood or fiber reinforced such other natural composites also reflect the anisotropic characters. This parameter largely depends on the properties of constituent materials of the FRPCs. The toughness of FRPCs against fracture can be improved through materials modifications like toughening matrices or by structural modifications like stitching, pinning etc. Additionally, many factors (Wu proposes seven factors) influence the resistance of the materials against fracture. Dr. Hyer demonstrated that none of the prediction models are capable of predicting the fracture behaviors of the each FRP composites under all conditions. In addition, the most general one, i.e., Tsai-Wu criteria is a very involved one with its limitations. Experimental investigations on mode I, mode II, mode III and mixed mode fractures are frequently used for determination of fracture toughness of FRPCs under quasi static loads for understanding the fracture behaviors. Numerous investigations on fracture behaviors of FRPCs under fatigue loads have been undertaken through decades. The uncertainties associated with fracture of FRPCs needs rigorous statistical approaches for determining a most probable fracture and fracture toughness under fatigue loadings. To introduce the readers first with the fracture events under quasi static loads will be briefly introduced in the following sections before addressing complicated fatigue behaviors of FRPCs.

4.2 Experimental Investigations on Fracture Toughness of FRPCs

A majority of the research works on FRPCs from early 1970s has focused on application of LEFM or linear elastic fracture mechanics in determining the fracture toughness. In case of thermoset polymeric composites, with randomly oriented short fibers more or less 0.5 mm long, the inelastic zone in front of crack tip is small enough to apply LEFM on the crack initiation and propagation. But the inelastic region ahead of the crack tip for long fibers (>25 mm) is sufficiently extended and the assumption of small yielding required for application of LEFM is no longer valid. In that case, some alternative procedure like that based on tension-softening plot or fracture mechanics applicable to non-linear characters are more acceptable to determine the fracture toughness of FRPCs [40].

As already stated in Sect. 2.1, fracture mechanics is the mechanics of cracks which includes the mechanisms from initiation to propagation of cracks and is applied in analysis and design of composites materials and structures. One of the primary objectives of analysis through fracture mechanics is prediction of initiation of crack and its growth in a material body with a cracks-size. LEFM is applicable to cracks in some composites [71] and in UD composites of brittle matrices [70]. The state of crack can be understood from the elastic stress intensity in the field about the crack tip. It is realized from the investigations that stress singularity is a function of—$(r_e)^{1/2}$ at a distance r_e from crack tip in homogeneous and isotropic or orthotropic materials [2]. If the plane of crack follows any plane of symmetry of orthotropic materials, the intensity factors for stress can be calculated [61]. Loadings at crack tip can be partitioned into three modes of fractures as shown in Figs. 1, 2 and 3. Mode I, Mode II and mode III [8].

It is becoming a common tendency to investigate interfacial cracks applying the concept of rate of energy release G. It is derived from energy considerations, defined mathematically and can be measured experimentally. It is based on original Griffith's criterion of fracture which considers the energy required for extension of crack from the available energy.

Potential of elastic energy in a body with crack can be expressed as

$$V = W_e - U_i \tag{36}$$

where

W_e is the work done by external forces,
U_i is the strain energy due to strain of the body.

The energy required for creation of unit area of crack (G_c) due to crack growth is given by

$$\delta V \geq Gc\delta A \tag{37}$$

In which, increase in the area of crack is given by δA.

At critical condition, the net energy is used to just balance the energy required for creation of unit area of crack (G_c). The condition can be expressed as

$$\delta V = Gc\delta A \tag{38}$$

The crack grows when the equilibrium is disturbed. i.e.,

$$\delta V > G_c\delta A \tag{39}$$

The rate of release of energy G is given by

$$\mathrm{G} = \frac{\delta V}{\delta A} \tag{40}$$

Then, the fracture criteria can be expressed as

$$\mathrm{G} > G_c \tag{41}$$

This can be demonstrated through Fig. 14 for a body which is linearly elastic with initial crack size "a". It is assumed that the growth of crack occurs at fixed load or constant displacement (grip is fixed).

In case of fixed load,

$$\delta U_i = {}^1\!/_2(\mathrm{F}\delta \mathrm{x}) \tag{42}$$

$$\delta W_e = \mathrm{F}\delta \mathrm{x} \tag{43}$$

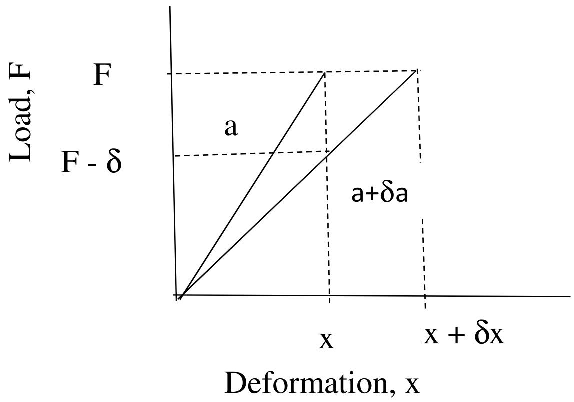

Fig. 14 Load deformation plot for crack size a and a + δa

From Eq. (36),

$$\delta V = F\delta x - {}^{1}\!/_{2}(F\delta x) = {}^{1}\!/_{2}(F\delta x) \tag{44}$$

Then, Eq. (40) can be written as

$$G = {}^{1}\!/_{2}(F\delta x/\delta A) \tag{45}$$

In case of *constant displacement*, i.e., when the grip is fixed, the W_e is zero and

$$\delta V = {}^{1}\!/_{2}(x\delta F) \tag{46}$$

Stiffness is reduced due to crack extension, for that, the δF is negative.

$$G = -{}^{1}\!/_{2}(x\delta F/\delta A) \tag{47}$$

As the material is linearly elastic, therefore,

$$x = CF \tag{48}$$

In Eq. (48), compliance is denoted by C.
Using the Eqs. (48) and (45) for fixed load,

$$G = {}^{1}\!/_{2}F^{2}(\delta C/\delta A) \tag{49}$$

In case of *fixed displacement*, i.e., when the grip is fixed, substituting F = x/C in Eq. (47)

$$G = \frac{x^2}{2C^2}\frac{\partial C}{\partial A} = \frac{F^2}{2}\frac{\partial C}{\partial A} \tag{50}$$

It is clear from Eqs. (49) and (50) that both for fixed load and fixed displacement cases, the rate of energy release G is given by same expression.

If a crack in principal plane of material is considered, it can be decomposed into three modes of cracks shown in Figs. 1, 2 and 3. That is

$$G = G_I + G_{II} + G_{III} \tag{51}$$

where

G_I rate of energy release due to mode I component of crack,
G_{II} rate of energy release due to mode II component of crack and
G_{III} rate of energy release due to mode III component of crack.

Theoretically, the above components can be determined from Irwin's concept which states that the energy release for a small crack is equal to the energy required to close the crack for the same length. To maintain the direction of this section, the experimental methods for determining these components will be discussed in the following section.

4.2.1 Experimental Investigations on Mode I Fracture Toughness of FRPCs

Method of mode I interlaminar fracture toughness of unidirectional fiber-reinforced polymer matrix composites is specified in ASTM D 5528-13 [3]. The schematic of the specimen is shown in Fig. 15. The initial delamination in DCB testing of composites is placed symmetrically at mid-plane as shown in Fig. 15. The two cantilever limbs on both top and bottom of pre-crack is assumed to be firmly supported at the remaining portion of the specimen. Load is applied through the hinge tab fixed at top of the upper cantilever limb of the sample while the advancement of crack is recorded with the load displacement data of the experiment. Determination of global compliance (C) as a function of crack length (a) is typically obtained in the analysis of fracture test specimens. The energy release rate *G* is obtained by differentiating C with respect to *a* and is given by the following equation,

$$G = \frac{P^2}{2b}\frac{dC}{da} \tag{52}$$

where *b* is the specimen width and *P* is the applied load. C is the compliance given by

$$\mathrm{C} = \delta/\mathrm{P} \tag{53}$$

and δ is the displacement of the loading point.

At critical condition,

$$G_c = \frac{P_c^2}{2b}\frac{dC}{da} \tag{54}$$

and

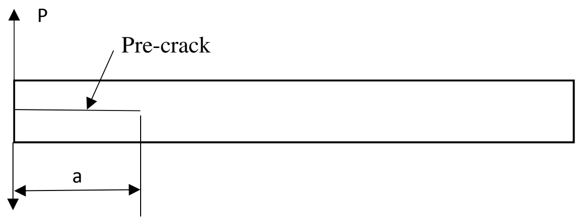

Fig. 15 Double cantilever beam specimen

$$C_c = \frac{\delta_c}{P_c} \tag{55}$$

Analytical solution of compliance (C) as a function of crack length, C(a) is possible in some specimen with simpler geometry, loading and support conditions. The energy release rate (G) is obtained by differentiation of C(a) with respect to crack length (a) as per Eq. (52). Experimentally, compliance versus crack length data of the DCB specimen is fitted to a power function of crack length as

$$C = C_0 a^n \tag{56}$$

From Eqs. (54) and (56),

$$G = \frac{nP\delta}{2ab} \tag{57}$$

Under critical condition like crack initiation point in DCB test

$$G_c = \frac{nP_c\delta_c}{2ab} \tag{58}$$

The value of parameter n can be determined by curve fitting to the experimental compliance (C) versus crack length (a) plot. Once the value of n is experimentally determined, the energy release rate G can be calculated from Eq. (58) for each crack length (a).

4.2.2 Experimental Investigations on Mode II Fracture Toughness of FRPCs

End notch flexure (ENF) test specimen used for mode II fracture characterization originally developed for wooden beam and applied to laminated composite by Russell [54] as shown in Fig. 16 who used this specimen as an end-delaminated three-point bend specimen to measure the Mode II interlaminar fracture energy of graphite/epoxy laminates. This specimen is used for mode II test in which the element of monolithic composites at crack tip is subjected to shear stress due to this experimental setup.

4.2.3 Experimental Investigations on Mode III Fracture Toughness of FRPCs

In spite of numerous works in mode I, mode II or mixed mode (mode I and mode II) testing, the study on mode III testing of sandwich is remarkably less due to presence of unwanted mode II component at the crack tip caused by in-plane bending moment higher near the edge. Among the test methods for mode III, Split Cantilever Beam (SCB) test set up shown in Fig. 17is most commonly used which is first used by

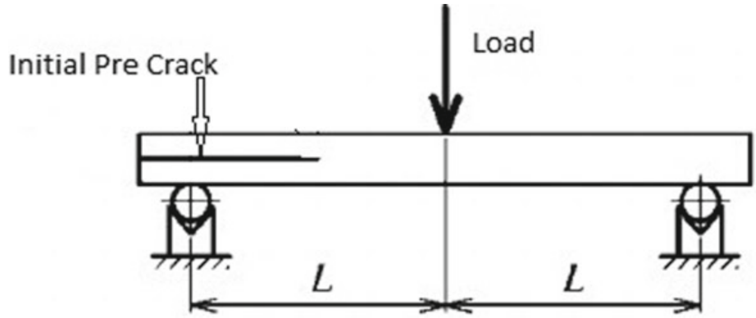

Fig. 16 End notch flexure test (ENF) specimen for mode II test

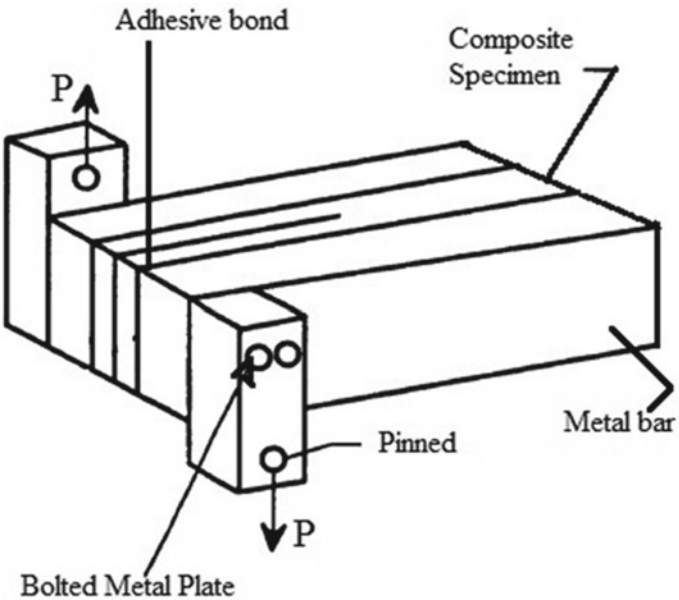

Fig. 17 Split cantilever beam test configuration

Donaldson [15]. Robinson and Song [50] modified the split cantilever beam setup by introducing additional loading points to reduce the unwanted mode II component.

4.2.4 Experimental Investigations on Mixed Mode Fracture Toughness of FRPCs

In a number of cases, failure due to delamination is associated with mixed mode loading (i.e. mixed mode I, mode II and /or mode III loading). In laminated composites, initiation of crack or its propagation is caused by influence of both mode I (causing normal stress) and shear stresses (mode II). The procedure of test for delamination under mixed-mode loading was executed by combining setup of loading DCB specimens for mode I test and the setup of loading ENF specimens for mode II test of UD laminates. A single load (F) applied through a lever can induce both mode I and mode II components of loading on the specimen as shown in Fig. 18. Delamination under combined normal and shear stresses was experimentally investigated by Crews [11] and Reeder [49]. Experimentally, laminated composite specimens are tested with mixed mode bending (MMB) apparatus. Loading in MMB apparatus can be considered as the combined mode I and mode II loadings [3, 4].

The test can be executed for different mode ratios (G_I/G_{II}) by changing the positions of load applications on the lever. The standard testing procedure for MMB

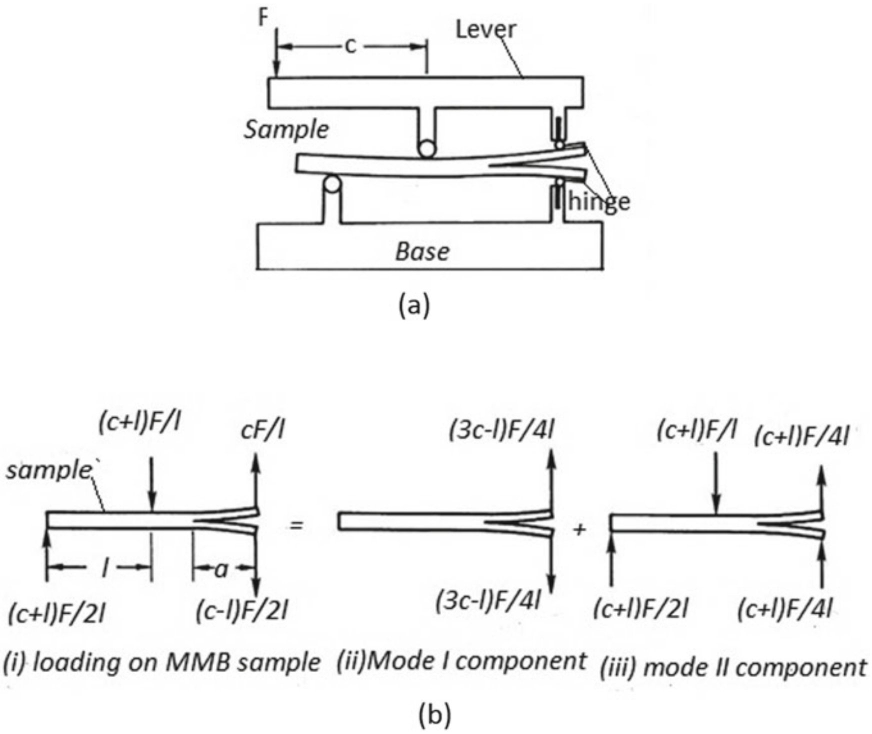

Fig. 18 MMB apparatus: **a** Mixed mode apparatus with laminate specimen, **b** components of mixed mode

fracture of UD fiber reinforced laminated samples is demonstrated in ASTM standard [28]. In the interest of conciseness, the first author requests to accept apologies for restricting the discussions on laminated composites only without going through the analysis of sandwich composites. Advanced learners are requested to go through the references [4, 7–9, 11, 13, 28, 49] for the detailed theoretical analysis and experimental investigations.

5 Conclusions and Future Perspective

The chapter started with an introduction to fracture behaviors and toughness of fiber reinforced polymer matrix composites (FRPCs). Fracture failure mechanisms in different length scales (micro, meso and macro) are addressed. The primary failure theories for fiber reinforced thermoset composites have been discussed. Failures of UD lamina under different loading conditions have been addressed from micromechanical approaches. Anisotropic failure theories applicable to FRPCs have also been presented in this chapter. Damage mechanisms, and theories of failures under creep, fatigue and high strain rates are briefly presented. The fracture behaviors of thermoset composites under different modes of fractures are theoretically discussed. Finally, the fracture behaviors and toughness of fiber reinforced thermoset polymer reinforced composites are addressed through the procedures of experimental fracture mechanics. Existing research gaps have been indicated in different sections.

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