

Multi-body Analysis for a Four-Bar Mechanism Using RecurDyn and MATLAB



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Abstract A planar four-bar mechanism is the simplest and the smallest movable closed chain linkage. It consists of four revolute pairs and is therefore often referred to as 4R mechanism. This paper presents the kinematic and inverse dynamic force analysis of a planar four-bar mechanism (in crank rocker configuration), assuming the motion of one crank as input. The equations for the forces and moments of respective links are obtained in the matrix form by combining the dynamic equilibrium equations and the kinematic relationships of each element by using Newton's second law of motion and Euler's equation. The numerical computation was performed using MATLAB. Alternatively, RecurDyn was used to perform multibody dynamic analysis, and the results of two are compared and found in good agreement. The planar four-bar mechanism can be seen in many everyday appliances, like in car wipers, bicycles, door frames (opening and closing), etc., and therefore, has several real-life applications.

Keywords Four-bar mechanism · Inverse dynamic analysis · RecurDyn · Crank-rocker

1 Introduction

In the spectrum of planar mechanisms, the simplest category of lower pair mechanisms are four-bar linkages. The four-bar mechanism is a single degree of freedom device and can also be used to form various other useful mechanisms through marginal variations, such as altering the character of the pairs or the proportions of the links, etc. [1].

A four-bar mechanism consists of four rigid members: the *frame* or fixed member, to which pivoted the *crank* and follower, connected through the *coupler*. These members are connected using four *revolute* pairs [2].

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There are two major categories of four-bar mechanisms depending upon the lengths of the links, namely Grashof and non-Grashof mechanisms.

A Grashof mechanism in which the drive crank is the shortest acts as a crank-rocker mechanism and the criterion to be met by the system for this configuration is:

$$L_{\max} + L_{\min} < L_a + L_b \quad (1)$$

where link lengths are measured between bearing centers, L_{\max} is the length of the longest link, L_{\min} of the shortest link, L_a and L_b are lengths of the remaining links [2].

2 Notations

The following set of notations was used while deriving the equations for analysis:

l_1 : Length of link 1 (Frame)

l_2 : Length of link 2 (Crank)

m_2 : Mass of link 2

θ_2 : Angular position of link 2 w.r.t horizontal

w_2 : Angular velocity of link 2

α_2 : Angular acceleration of link 2

I_2 : Moment of inertia of link 2

G_2 : Center of gravity of link 2

F_2 : External force on link 2

M_2 : Driving torque for link 2

l_3 : Length of link 3 (Coupler)

m_3 : Mass of link 3

θ_3 : Angular position of link 3 w.r.t horizontal

w_3 : Angular velocity of link 3

α_3 : Angular acceleration of link 3

I_3 : Moment of inertia of link 3

G_3 : Center of gravity of link 3

F_3 : External force on link 3

M_3 : Driving torque for link 3

l_4 : Length of link 4 (Rocker)

m_4 : Mass of link 4

θ_4 : Angular position of link 4 w.r.t horizontal

w_4 : Angular velocity of link 4

α_4 : Angular acceleration of link 4

I_4 : Moment of inertia of link 4

G_4 : Center of gravity of link 4

F_4 : External force on link 4

M_4 : Driving torque for link 4

Reaction Forces:

P_{O_2} : force acting on Revolute joint 1 (between link 2 and ground)

P_A : force acting on Revolute joint 2 (between link 2 and link 3)

P_B : force acting on Revolute joint 3 (between link 3 and link 4)

P_{O_4} : force acting on Revolute joint 4 (between link 4 and ground)

3 Theoretical Formulation

In the analysis, the laws of vector addition were used to derive the equations for the position, velocity, and acceleration analysis of the bodies.

From Fig. 1.

$$l_1 = l_2 + l_3 - l_4 \quad (2)$$

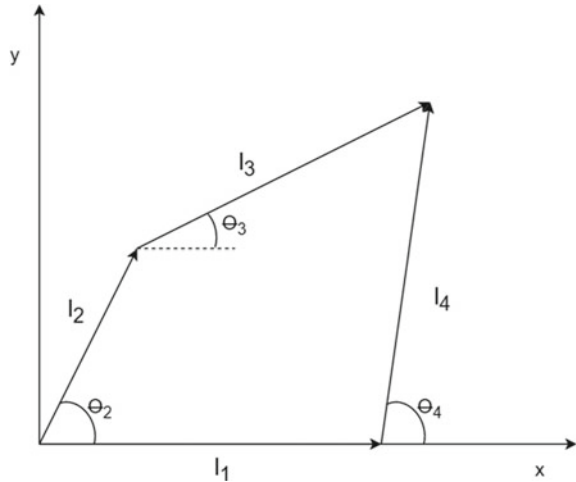
where

$$l_2 = \hat{i}.l_2\cos\theta_2 + \hat{j}.l_2\sin\theta_2$$

$$l_3 = \hat{i}.l_3\cos\theta_3 - \hat{j}.l_3\sin\theta_3$$

$$l_4 = \hat{i}.l_4\cos\theta_4 + \hat{j}.l_4\sin\theta_4$$

Fig. 1 Schematic diagram of the four-bar mechanism [3]



3.1 Kinematic Analysis

Since l_1 is a fixed vector; therefore, differentiating Eq. (2) w.r.t. time gives:

$$l_2 \sin \theta_2 \cdot w_2 = -l_3 \sin \theta_3 \cdot w_3 + l_4 \sin \theta_4 \cdot w_4 \quad (3)$$

$$l_2 \cos \theta_2 \cdot w_2 = -l_3 \cos \theta_3 \cdot w_3 + l_4 \cos \theta_4 \cdot w_4 \quad (4)$$

Again differentiating Eq. (2) gives:

$$\begin{aligned} -l_3 \sin \theta_3 \cdot \alpha_3 + l_4 \sin \theta_4 \cdot \alpha_4 &= l_2 \sin \theta_2 \cdot \alpha_2 + l_2 \cos \theta_2 \cdot (w_2)^2 \\ &\quad + l_3 \cos \theta_3 \cdot (w_3)^2 - l_4 \cos \theta_4 \cdot (w_4)^2 \end{aligned} \quad (5)$$

$$\begin{aligned} l_3 \cos \theta_3 \cdot \alpha_3 - l_4 \cos \theta_4 \cdot \alpha_4 &= -l_2 \cos \theta_2 \cdot \alpha_2 + l_2 \sin \theta_2 \cdot (w_2)^2 \\ &\quad + l_3 \sin \theta_3 \cdot (w_3)^2 - l_4 \sin \theta_4 \cdot (w_4)^2 \end{aligned} \quad (6)$$

While θ_4 and θ_3 are calculated using Freudenstein's equation and Eq. (2) respectively, taking θ_2 and lengths of links as inputs.

Taking the values of θ_2 , w_2 , and α_2 of crank(driver) as inputs, the corresponding values of w_3 , w_4 and α_3 , α_4 can be obtained by solving Eqs. (3), (4) and (5), (6) respectively.

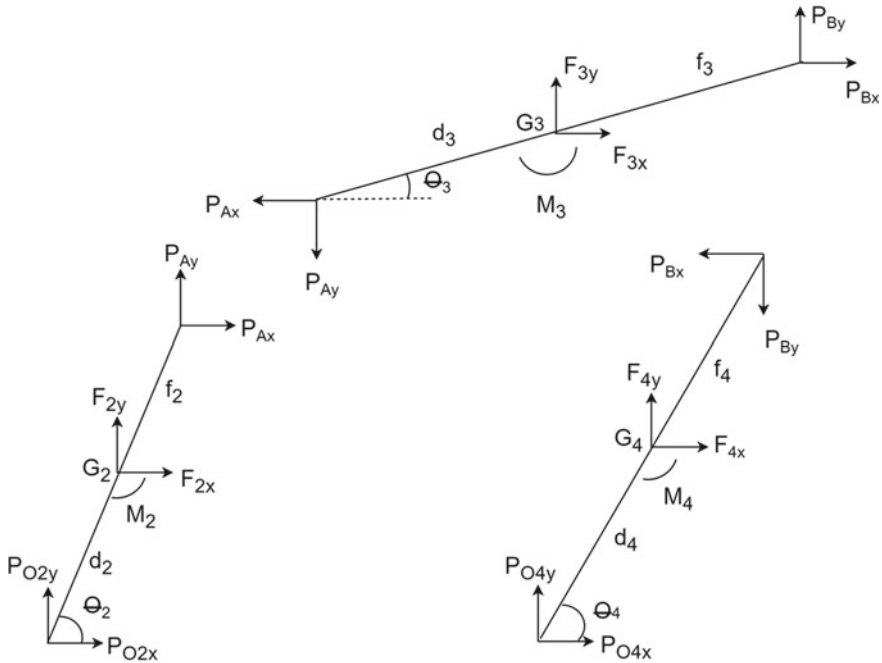


Fig. 2 Free body diagram (FBD) of the four-bar mechanism [3]

3.2 Dynamic Analysis

The reaction forces on links and the driving torque are calculated by applying Newton’s second law of motion and balancing the moments about the center of mass of links as shown in Fig. 2.

Since link 2 rotates about ground, thus the acceleration of its COM can be written as:

$$a_{G2x} = -d_2 \cos \theta_2 \cdot (\omega_2)^2 - d_2 \sin \theta_2 \cdot \alpha_2$$

$$a_{G2y} = d_2 \cos \theta_2 \cdot \alpha_2 - d_2 \sin \theta_2 \cdot (\omega_2)^2$$

Now, both links 2 and 3 have point A in common; therefore,

$$a_{G3} = a_A + a_{G3/A} \tag{7}$$

Using Eq. (7), the acceleration of COM of link 3 can be expressed as:

$$a_{G3x} = -[l_2 \cos \theta_2 \cdot (\omega_2)^2 + l_2 \sin \theta_2 \cdot \alpha_2] + [-d_3 \cos \theta_3 \cdot (\omega_3)^2 + d_3 \sin \theta_3 \cdot \alpha_3]$$

$$a_{G3y} = [l_2 \cos \theta_2 \cdot \alpha_2 - l_2 \sin \theta_2 \cdot (w_2)^2] + [d_3 \sin \theta_3 \cdot (w_3)^2 + d_3 \cos \theta_3 \cdot \alpha_3]$$

Similarly, link 4 also rotates about the ground; therefore,

$$a_{G4x} = -d_4 \cos \theta_4 \cdot (w_4)^2 - d_4 \sin \theta_4 \cdot \alpha_4$$

$$a_{G4y} = d_4 \cos \theta_4 \cdot \alpha_4 - d_4 \sin \theta_4 \cdot (w_4)^2$$

Applying Newton's second law of motion, for link 2,

$$F_{2x} + P_{O2x} + P_{Ax} = m_2 \cdot a_{G2x} \quad (8)$$

$$F_{2y} + P_{O2y} + P_{Ay} = m_2 \cdot a_{G2y} \quad (9)$$

$$\begin{aligned} M_2 + d_2 \sin \theta_2 \times P_{O2x} - d_2 \cos \theta_2 \times P_{O2y} \\ - f_2 \sin \theta_2 \times P_{Ax} + f_2 \cos \theta_2 \times P_{Ay} = I_2 \cdot \alpha_2 \end{aligned} \quad (10)$$

For link 3,

$$F_{3x} - P_{Ax} + P_{Bx} = m_3 \cdot a_{G3x} \quad (11)$$

$$F_{3y} - P_{Ay} + P_{By} = m_3 \cdot a_{G3y} \quad (12)$$

$$\begin{aligned} M_3 - d_3 \sin \theta_3 \times P_{Ax} + d_3 \cos \theta_3 \times P_{Ay} \\ - f_3 \sin \theta_3 \times P_{Bx} + f_3 \cos \theta_3 \times P_{By} = I_3 \cdot \alpha_3 \end{aligned} \quad (13)$$

For link 4,

$$F_{4x} - P_{Bx} + P_{O4x} = m_4 \cdot a_{G4x} \quad (14)$$

$$F_{4y} - P_{By} + P_{O4y} = m_4 \cdot a_{G4y} \quad (15)$$

$$\begin{aligned} M_4 + d_4 \sin \theta_4 \times P_{Bx} - d_4 \cos \theta_4 \times P_{By} \\ + f_4 \sin \theta_4 \times P_{O4x} - f_4 \cos \theta_4 \times P_{O4y} = I_4 \cdot \alpha_4 \end{aligned} \quad (16)$$

In the above equations, external forces on all links in the x-direction are assumed to be zero.

Therefore, $F_{2x} = F_{3x} = F_{4x} = 0$.

And external forces in the y-direction are due to link's own masses; thus,

$$F_{2y} = -m_2 \times g, F_{3y} = -m_3 \times g \text{ and } F_{4y} = -m_4 \times g.$$

Writing Eqs. (8)–(16) in matrix form gives us:

$$[A]\{X\} = \{B\} \tag{17}$$

where matrices $[A]$, $\{X\}$, and $\{B\}$ are as follows:

$$[A] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ d_2 \sin \theta_2 & -d_2 \cos \theta_2 & -d_2 \sin \theta_2 & d_2 \cos \theta_2 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -d_3 \sin \theta_3 & d_3 \cos \theta_3 & -f_3 \sin \theta_3 & f_3 \cos \theta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -d_4 \sin \theta_4 & d_4 \cos \theta_4 & f_4 \sin \theta_4 & -f_4 \cos \theta_4 & 0 \end{bmatrix}$$

$$\{X\} = \begin{bmatrix} P_{O2x} \\ P_{O2y} \\ P_{Ax} \\ P_{Ay} \\ P_{Bx} \\ P_{By} \\ P_{O4x} \\ P_{O4y} \\ M_2 \end{bmatrix} \qquad \{B\} = \begin{bmatrix} m_2 \cdot a_{G2x} - F_{2x} \\ m_2 \cdot a_{G2y} - F_{2y} \\ I_2 \cdot \alpha_2 \\ m_3 \cdot a_{G3x} - F_{3x} \\ m_3 \cdot a_{G3y} - F_{3y} \\ I_3 \cdot \alpha_3 - M_3 \\ m_4 \cdot a_{G4x} - F_{4x} \\ m_4 \cdot a_{G4y} - F_{4y} \\ I_4 \cdot \alpha_4 - M_4 \end{bmatrix}$$

The above set of equations (in matrix form) depicts nine equations which were solved using MATLAB to determine the values of nine unknowns in the $\{X\}$ matrix that represent the reaction forces on all four revolute joints and the driving torque to be applied on the crank.

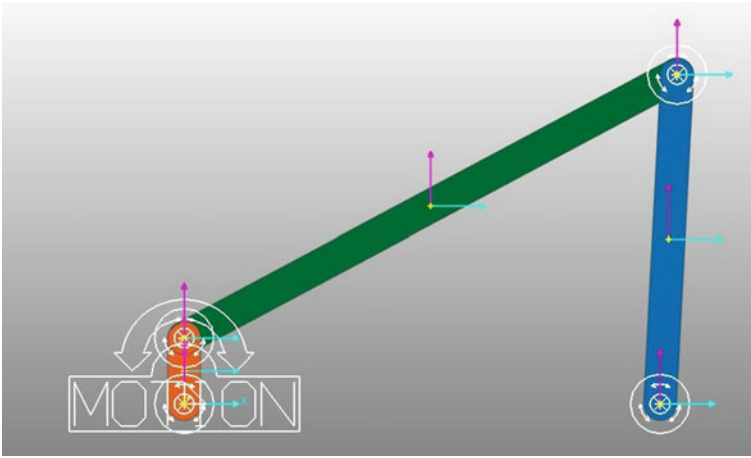


Fig. 3 Four-bar model using RecurDyn

4 Analysis Using RecurDyn

A similar model of the four-bar mechanism was built using RecurDyn (simulation software). The links were taken as cylindrical rods, and the values of the input variables were taken as follows (Fig. 3):

$$l_1 = 0.867 \text{ m}$$

$$l_2 = 0.120 \text{ m}, m_2 = 0.991 \text{ kg}, I_2 = 0.0012 \text{ kg m}^2$$

$$l_3 = 1.018 \text{ m}, m_3 = 9.028 \text{ kg}, I_3 = 0.7797 \text{ kg m}^2$$

$$l_4 = 0.600 \text{ m}, m_4 = 5.357 \text{ kg}, I_4 = 0.1607 \text{ kg m}^2$$

The kinematic and dynamic analysis of the proposed model was then performed on RecurDyn for one time period (T) of the crank, keeping the angular velocity and acceleration of the crank fixed at $w_2 = 2\pi$, $\alpha_2 = 0$.

$$\text{For } w_2 = 2\pi,$$

$$T = 2\pi/w_2 = 1 \text{ s}$$

5 Results and Discussion

The computed values of all the variables obtained after the analysis of the mechanism using both MATLAB and RecurDyn were plotted against each other to compare the results (Figs. 4, 5, 6, 7, 8, 9, and 10).

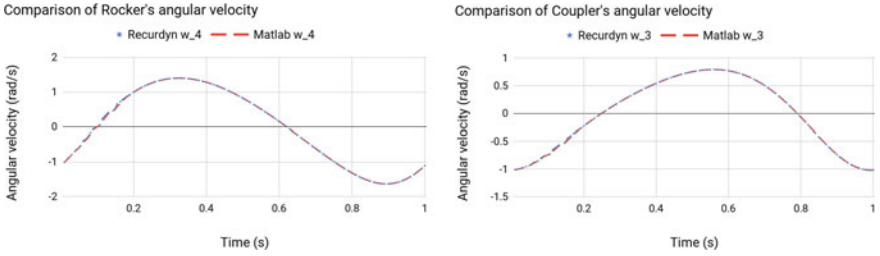


Fig. 4 Comparison of angular velocities of rocker and coupler

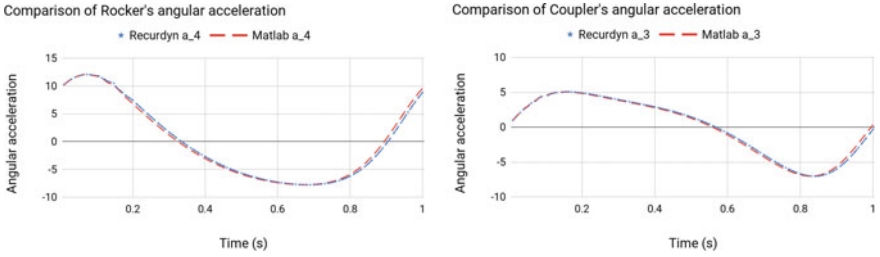


Fig. 5 Comparison of angular accelerations of rocker and coupler

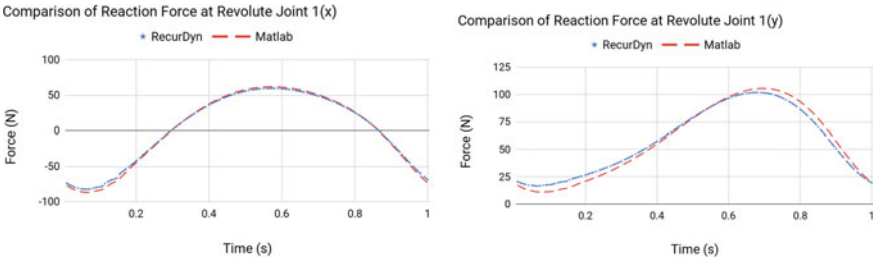


Fig. 6 Comparison of reaction forces on joint 1 (in x and y directions, respectively)

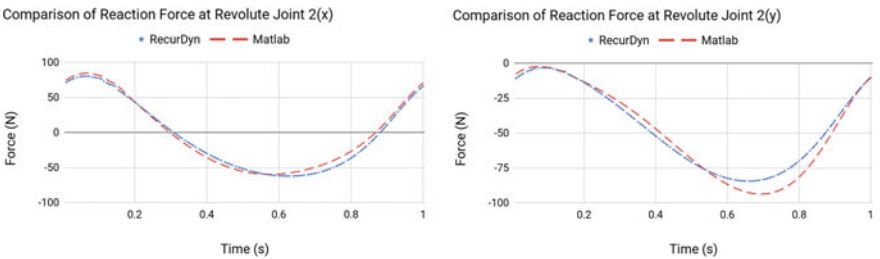


Fig. 7 Comparison of reaction forces on joint 2 (in x and y directions, respectively)

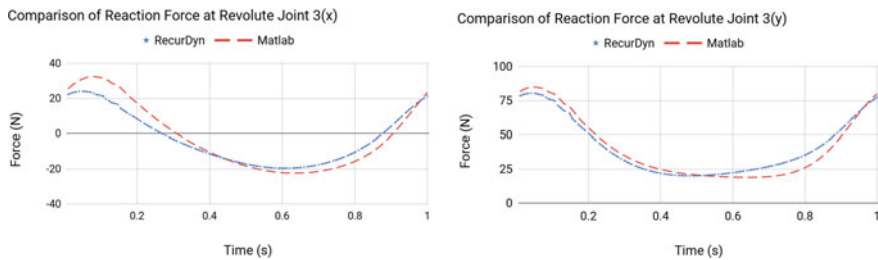


Fig. 8 Comparison of reaction forces on joint 3 (in x and y directions, respectively)

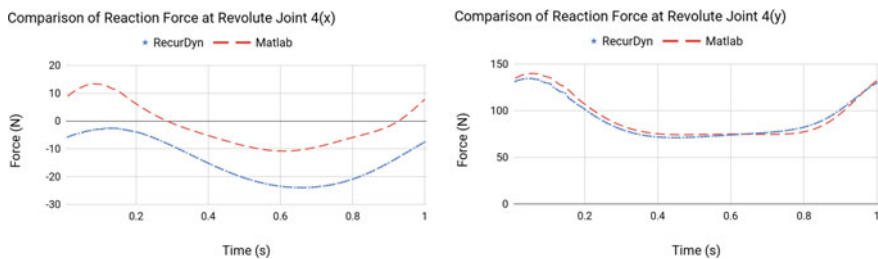


Fig. 9 Comparison of reaction forces on joint 4 (in x and y directions, respectively)

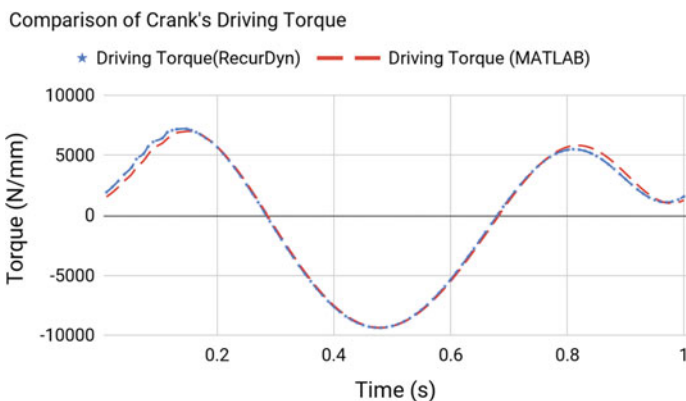


Fig. 10 Comparison of Crank's driving torque (turning moment)

From the above graphs, we can infer that the values of angular velocities and accelerations obtained by analysis through the two, MATLAB and RecurDyn, are found to be in good agreement with each other for both the Rocker and the Coupler.

On the other hand, it can also be observed that there is a little discord in the values of the reaction forces and the turning moment from the two sources. These difference in the values from MATLAB and RecurDyn can be ascribed to the unlike models in both the software, as in MATLAB, the four-bar mechanism is a 2-D model

comprising of links that are defined as 2-D lines. While in RecurDyn, the simulation is more exhaustive because of a 3-D model which comprises of links in the form of cylinders having considerable width as well unlike straight lines.

Thus, these differences can be attributed to dissimilarity in the type of simulation model; one being two dimensional and the other being three dimensional.

6 Conclusion

The study underlines the reactions forces acting on individual joints and the links; thus, it can help one choose the suitable materials and the dimensions of the respective links. Also, it draws special attention to the amount of driving torque required to sustain the mechanism at a given velocity and acceleration of the crank, and thus, it provides clear insights about the type and the capacity of actuator required to power the system.

Moreover, the study shows the dependence of kinematics and dynamics of four-bar mechanism on the type of model (2-D or 3-D); thus, it can assist a designer while working with the four-bar mechanisms to estimate the design parameters as per his/her design criteria.

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