Time–Frequency Characteristics of Seismic Signal Using Stockwell Transform

Gayatri Devi, P. Sumathi, and Ashvini Kumar

Abstract Ground motions always create great interest for seismologists and engineers worldwide. For these signal, an accurate and precise analysis of non-stationary spectral variation are a longstanding problem aiming at some characteristics of signal like any underlying periodicity. Fourier transform is a conventional tool, used to study the seismic signals. In the last few years, researchers have become attentive to the limitations of the Fourier transform. It decomposes the signal into its constituent frequency components, but does not reveal, where changes in the frequency contents occur. To overcome, it joint time–frequency representations have been introduced which is a representation of both time and frequency. Some conventional method to obtain the desired time–frequency information contained in these signals are short-time Fourier transform (STFT) and Wavelet Transform. These methods show limitation in terms of resolution. The *S*-transform (ST) proposed by Stockwell et al. [10] is fusion of short-time Fourier transform (STFT), and Wavelet Transform. *S*-transform is based on a moving and scalable localizing Gaussian window. It provides frequency-dependent resolution while maintaining a direct relationship with the Fourier spectrum. In this paper, Stockwell transform (ST), STFT, and CWT-based technique for joint time–frequency representation of seismic signal has been used. Effectiveness of ST is evaluated by comparing the result of developed non-stationary synthetic signal and real-time ground motion signal of Uttarkashi earthquake ($M_w = 6.8$, 20 October 1991). Stockwell transform is capable for improving the resolution of non-stationary signals as well as clearly identified the spots of concentration in energy.

Keywords Ground motion · Fourier transform · Time–frequency representation · Stockwell transform

G. Devi (✉)
CSIR-Central Building Research Institute, Roorkee, Uttarakhand 247667, India
e-mail: gayatrikapil@gmail.com

P. Sumathi
Indian Institute of Technology Roorkee, Roorkee, Uttarakhand 247667, India

A. Kumar
H.V.M. (PG) College, Raisi, Uttarakhand, 247671, India

T. G. Sitharam et al. (eds.), *Seismic Hazards and Risk*, Lecture Notes in Civil Engineering 116, https://doi.org/10.1007/978-981-15-9976-7_17

1 Introduction

Earthquake ground motions are inherently non-stationary in nature. For better understanding about the characterization of local site effects subject to earthquake, reliable earthquake signal processing is essential. A time history is a most widely used method for explaining the ground motion. The strong motion duration of an earthquake is the time interval during which most of the energy is contained. Because seismic waves are scattered during propagation, it shows a time evolving frequency composition. The records of these seismic waves exhibit non-stationary characteristics. Seismic signal shows non-stationary characteristics due to attenuation and absorption of seismic energy [1].

Damage of structures depends on earthquake's time duration, their amplitude and frequency content of waves. Fourier-based analysis neglects the time duration information of dominant frequency of strong ground motion signal. Fourier transform can identify the frequencies which are present in signal but it does not reveal, where changes occur in the frequency contents [2].

To overcome this type of problem, joint time–frequency representations have been introduced which is a representation of both time and frequency. For the proper interpretation of seismic data in terms of varying frequency content, there is a need of time–frequency representation techniques jointly [2–4]. A proper analysis of ground motions is very necessary, if building design is to be constructed in seismically active areas, predicting earthquakes, quantification of damage from the recorded motions. To overcome this type of problem, joint time–frequency representations have been introduced which is a representation of both time and frequency. For the proper interpretation of seismic data in terms of varying frequency content, there is a need of time–frequency representation techniques jointly [2].

2 Time–Frequency Distributions: Fundamental Ideas

Time–frequency representation unfolds temporal information and maps a time series into 2D quantity of time and frequency with effective characterization of the time–frequency image. It describes how the spectral content of the signal changes with time. Through the view of mathematical point, this joint distribution will provide fractional energy of signal's total energy at frequency (ω) and time (t) [5].

If the ground motion signal is represented by $x(t)$, then the energy density of signal is represented by $|x(t)|^2$.

The total energy E_x is

$$Ex = \int |x(t)|^2 \mathrm{d}t$$

The frequency domain representation $x(f)$ for the signal $x(t)$ is

$$x(f) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-j2\pi f t}\mathrm{d}t$$

$x(t)$ and $x(f)$ are uniquely related.

The energy or intensity per unit frequency at frequency f is $|x(f)|^2$.

The total energy

$$E_x = \int |x(f)|^2 \mathrm{d}f$$

This should be equal the total energy of the signal calculated directly from the time waveform

$$E_x = \int |x(t)|^2 \mathrm{d}t = \int |x(f)|^2 \mathrm{d}f$$

Jointly distribution of energy of $x(t)$ over both the time and frequency variables amounts to looking for an energy distribution $P_x(t, f)$ such that

$$E_x = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_x(t, f)\mathrm{d}t\mathrm{d}f$$

For analysis of non-stationary signal, TFD is an appropriate tool. Various time–frequency representation method has been developed in last two decades for analysis of non-stationary signals. Some early form of JTF representation is STFT and CWT. The *S* transform is also one of JTF representation method [6]. It has some unique advantage as it gives frequency-dependent resolution and also maintain a direct relationship with Fourier spectrum [7].

3 Method

3.1 Stockwell Transform

It is an extension of the ideas of the short-time Fourier transform and is based on a moving and scalable localizing Gaussian window [7] so before to define the Stockwell transform, a general idea about STFT and CWT has been described here.

3.2 Short-Time Fourier Transform

Fourier transform, when made the function of time gives this STFT, i.e. this technique involves the division of signal into narrow time slots using the window template such that the segmented signal is considered to be stationary [8]. The FT of the segmented signal is then taken to get the frequency spectrum of that particular segment. So it can be defined as Fourier transform of the product of the signal and shifted version of window function [9, 10].

The mathematical relation that gives the STFT of a signal $p(t)$ is

$$S(\tau, \omega) = \int_{-\infty}^{+\infty} p(t)w(t-\tau)\mathrm{e}^{-j\omega t}\mathrm{d}t$$

where

$p(t)$ is the signal to be transformed.

$w(t)$ is the analysis window.

$S(\tau, \omega)$ is the STFT of the signal.

τ is the centre position of the window.

The width of the window length is fixed, which leads to the disadvantage of fixed time–frequency resolution. Because of fixed window length, there is a difference in number of cycles in that window along frequencies and that makes it inconvenient of having good time resolution comparatively frequency resolution at higher frequency [11].

One another problem through which STFT suffer is leakage due to window effect. Fourier transform of rectangular window used in this transform is a 'Sinc' function that has narrow main lope width and larger side lopes, which result spectral leakage.

Continuous wavelet transform: The basic and main principles of wavelet are review here briefly to understand its application in seismic signal. The continuous wavelet transform is the cross-correlation function, which is calculated using the signal correlated with wavelets. These wavelets are generated through the original mother wavelet by its scaled and translated versions [12–14]. Mother wavelet is termed as the main function, and the modified functions are called wavelets.

Mathematical formulation of the CWT is given by [15]

$$W_{\mathrm{s}}(a, \tau) = \int_{-\infty}^{+\infty} p(t)\frac{1}{\sqrt{|a|}}\psi * \left(\frac{t-\tau}{a}\right)\mathrm{d}t$$

where $p(t)$ is the signal.

$W_{\mathrm{s}}(a, \tau)$ are the CWT coefficients.

$\psi(t)$ is the mother wavelet, dilated by scale a and shifted in time τ.

$\frac{1}{\sqrt{|a|}}$ is known as the multiplication factor, and it ensures the normalization of energy, means the wavelet which has unit energy at all scales.

Like STFT, the wavelet transforms also suffer from uncertainty principle [16]. Both a good time and frequency resolution cannot be achieved simultaneously.

3.3 Stockwell Transform

The Stockwell transform is a mid way between STFT and CWT. This time–frequency representation method has quite similarity to STFT while because of the use of multiresolution tactics, it makes somewhere closer to wavelet transform [11]. A big advantage of *S*-transform is that because of its simple concept, it gives a simple understanding of multiresolution approach as have been introduced in wavelets and hardly require any additional knowledge excepting STFT [11]. In Stockwell transform for window function, Gaussian window is used, which can be time shifted by τ and is inversely proportional to the linear frequency f [10]. The *S*-transform may comparable to the CWT as the Gaussian template is comparable to the mother wavelet with a phase shift.

The mathematical expression for the Gaussian taper is given as

$$w(t) = \mathrm{e}^{\frac{-t^2}{2\sigma^2}},$$

where σ is taper width and inversely proportional to the frequency $\sigma = \frac{k}{|f|}$. The parameter k can be tuned to obtain better frequency localization at the cost of reduced time localization by controlling the width of Gaussian taper [15] here the parameter k is considered 1. Gaussian taper function is normalized to achieve the Gaussian template.

The S transform can be derived from STFT by replacing the window function $w(t)$ with the Gaussian function shown as

$$w(t) = \frac{|f|}{\sqrt{2\pi}} \mathrm{e}^{\frac{-t^2 f^2}{2}}$$

As STFT defined as

$$S(\tau, f) = \int_{-\infty}^{+\infty} p(t) w(t - \tau) \mathrm{e}^{-j2\pi f t}.$$

Then, the S transform is defined as

$$S_{\mathrm{T}}(\tau, f) = \frac{|f|}{\sqrt{2\pi}} p(t) \mathrm{e}^{\frac{-(t-\tau)^2 f^2}{2}} \mathrm{e}^{-j2\pi f t}$$

where $S_{\mathrm{T}}(\tau, f)$ represents the *S*-transform of signal.

$p(t)$ is continuous time signal.

τ parameter controls the position of the Gaussian window on t-axis.

The advantage of S-transform is that it may give multiresolution analysis beside keeping the absolute phase of each frequency [17]. The Gaussian window which localization is inversely proportional to the frequency is an improvement over STFT as fixed width window used in this.

S-transform's phase referenced to time origin which gives useful and accompanying information about the spectra, and this is not obtainable from locally referenced phase information in continuous wavelet transform [10].

4 Result

To demonstrate the performance of ST, one non-stationary synthetic signal and one real-time earthquake signal record are considered.

A. **Synthetic Signal**
A synthetic signal quite similar to synthetic example [18] is generated with three sinusoidal components to illustrate the features of ST. The synthetic signal is a sum of three components which consists of sinusoidal waves. The sampling frequency of the signal is 500 Hz, and the signal-to-noise ratio (SNR) is 10 db. The length of signal is 10 s. Here, η(t) is the Gaussian noise. The details of synthetic signal are given by the following equation and are shown in Fig. 1 with its three components.

$$S(t) = S_1(t) + S_2(t) + S_3(t) + \eta(t)$$

$$S_1(t) = [2 + 0.2\sin(t)] \cdot \sin[2\pi(3t + 0.6\sin(\mathrm{t}))]$$

$$S_2(t) = 0.6\left[1 + 0.3\sin(2t) \cdot \exp\left(-\frac{t}{20}\right) \cdot \cos\left[2\pi\left(5t + 0.6t^{1.8} + 0.3\cos(t)\right)\right]\right]$$

$$S_3(t) = 0.4\sin[2\pi(9t)]$$

The frequency of $S_1(t)$ is the lowest among all the three components but the amplitude of which is highest. The frequency of $S_2(t)$ increases with time, and amplitude becomes smaller when frequency becomes higher. The frequency of $S_3(t)$ is constant, and its amplitude is smallest. The specially designed signal components are so that the higher is the amplitude of a component, the lower the frequency of the component.

Figure 1a shows the synthetic signal and its components considered for the analysis. The TF representations of synthetic signal using STFT, CWT, ST are shown in Fig. 2a–c, respectively. Hanning window with window length (196 in sample) used for STFT and Morse wavelet has been used for CWT; the width factor (K) is 1 for

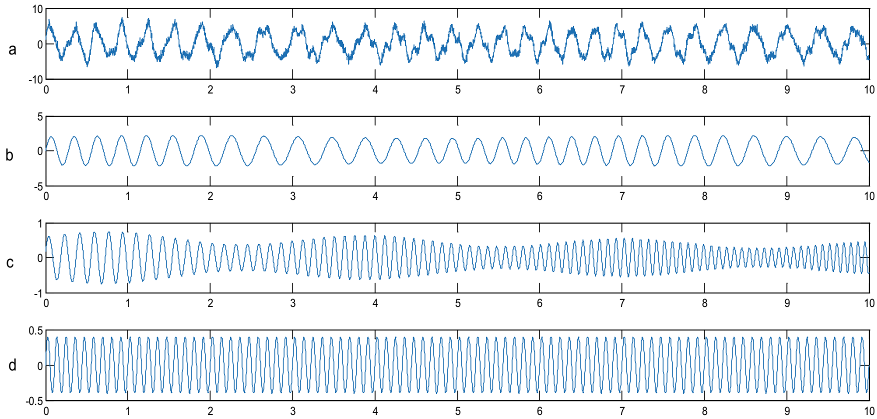


Fig. 1 Developed synthetic signal in (**a**). **b**–**d** represents the three components $S_1(t)$, $S_2(t)$ and $S_3(t)$, respectively,

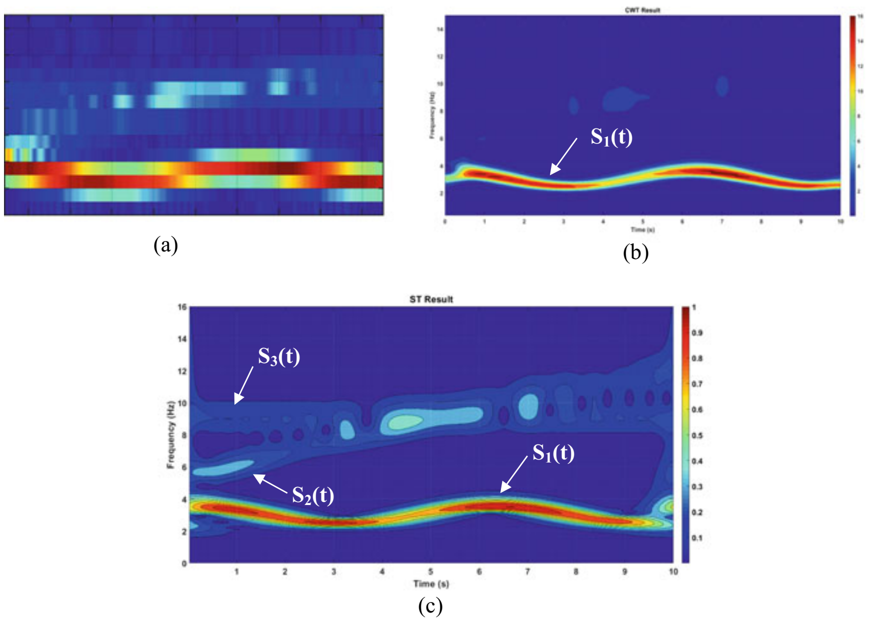
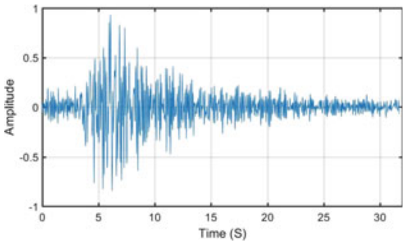
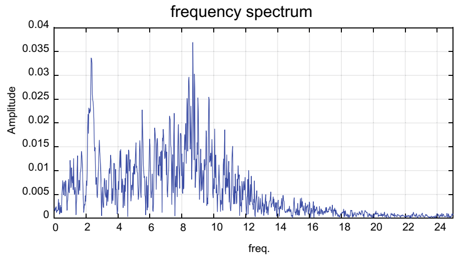


Fig. 2 Time–frequency representations of synthetic signal **a** STFT **b** CWT **c** ST

ST. All three components of $S(t)$, namely $S_1(t)$, $S_2(t)$, $S_3(t)$ are visible in the result of TF spectra of ST while in the result of CWT only one frequency component $S_1(t)$ is clearly visible.



(a) Acceleration record of N-S component of Uttarkashiearthquake recorded at Badkot station



(b) Frequency Spectrum of Acceleration record

Fig. 3 **a** Acceleration record of *N*-*S* component of Uttarkashi earthquake recorded at Badkot station, **b** frequency spectrum of acceleration record

B. **Real-Time Earthquake Record**

The real-time earthquake record is of Uttarkashi earthquake occurred in October 1991. This earthquake was recorded at 13 stations of a strong motion network installed by Department of Earthquake Engineering, Indian Institute of Technology, Roorkee in Garhwal Himalaya under the research scheme funded by Department of Science and Technology, Govt. of India.

The acceleration record of Uttarkashi earthquake recorded at Barkot station at 20 October 1991 at 02:53 IST with a moment magnitude of 6.8. Signal was recorded through a three channel (North–South, Vertical, East–West) accelerograph. The frequency sampling rate of accelerogram's recording is 50 sps. *N*-*S* component of this accelerogram, which duration is 31.74 s used for analysis.

The acceleration time history of the signal and corresponding FT of the signal are shown in Fig. 3a, b. *T*-*F* representation of seismic signal using STFT, CWT and ST is shown in Fig. 4a–c, respectively. The parameter used for analysis through different methods is given in Table 1.

Figure 4a–c shows at least 3–4 frequency components. These components are difficult to find only frequency domain based analysis. These are marked as $f1$, $f2$, $f3$ and $f4$. The time localization of $f1$, $f2$, $f3$, $f4$ is very good in the result of ST comparatively STFT and CWT. In the result of ST, the frequencies are annotated at different time as $f1 = 7.5$ Hz at 5.6 s, $f2 = 2.5$ Hz at 6.5 s, $f3 = 9$ Hz at 7.2 s and $f4 = 8.5$ Hz at 8.5 s. It may visualized from the results that resolution of time–frequency representation obtained using ST is better as compared to the STFT and CWT representation.

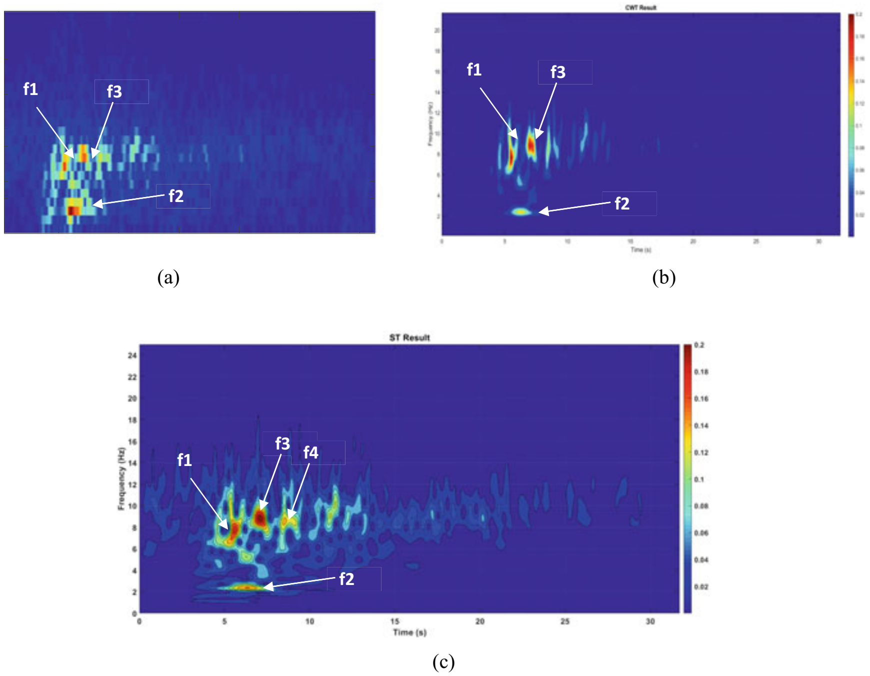


Fig. 4 Time–frequency representations of Uttarkashi station record **(a)** STFT **(b)** CWT **(c)** ST

Table 1 Paramaters used in various T-F Representations

Signal	STFT	CWT	ST
Ground motion acceleration record of Uttarkashi earthquake at Barkot station	Hanning window with window length (196 in sample)	Morse wavelet, cwt uses 10 voices per octave	The width factor $(k) = 1$

5 Conclusion

The Stockwell transform-based joint time–frequency technique is introduced for seismic signals and is validated for real-time ground motion signal. The time–frequency representations of synthetic signal using STFT, CWT, ST are compared. All three components of $S(t)$, namely $S_1(t)$, $S_2(t)$, $S_3(t)$ are visible in the result of time frequency spectra of Stockwell transform while in the result of CWT only one frequency component $S_1(t)$ is clearly visible. The time–frequency representations of real-time signal using STFT, CWT, ST are also compared. These components are difficult to find only frequency domain-based analysis. These are marked as $f1$, $f2$, $f3$ and $f4$. The time localization of $f1, f2, f3, f4$ is very good in the result of

ST comparatively STFT and CWT. It may visualized from the results that resolution of time–frequency representation obtained using ST is better as compared to the STFT and CWT representation. Stockwell transform is capable for improving the resolution of non-stationary signals as well as clearly identified the spots of concentration in energy. For seismic signals, the *S*-transform provides good time localization. Frequency localization is not so good; it shows frequency smearing. Frequency smearing problem may be minimized through synchrosqueezing of Stockwell transform.

References

1. Bormann P (2009) Seismic signals and noise. New manual of seismological observatory pratice (NMSOP), pp 1–34
2. Devi V, Sharma M (2016) Recent spectral decomposition techniques and its applications in analysis of seismological data: a review. Int J Innov Res Sci Eng Technol 5(1):213–220
3. Kuyuk HS, Kuyuk RT (2010) Investigation on displacement, velocity and acceleration waveforms of 1999 Kocaeli earthquake using wavelet and Stockwell. In: 19th international geophysical congress and exhibition. Ankara, Turkey
4. Nagarajaiah S, Basu B (2009) Output only modal identification and structural damage detection using time frequency and wavelet techniques. Earthq Eng Eng Vib 8(4):583–605
5. Huerta CL, Shin Y, Powers EJ, Roesset JM (2000) Time-frequency analysis of earthquake records. In: Proceedings of 12th WCEE
6. Kuyuk HS (2015) On the use of Stockwell transform in structural dynamic analysis. Sadhana 40(1):295–306
7. Han B (2013) The analysis of signal based on the S-Transform. In: 2nd international conference on measurement, information and control, Harbin, China
8. Allen JB, Rabiner LR (1977) A unified approach to short-time. Fourier analysis and synthesis. Proc IEEE 65(11):1558–1564
9. Black CJ (1998) Dynamic analysis of civil engineering structures using joint time-frequency method. M.A.Sc. thesis, The University of British Columbia
10. Stockwell RG, Mansinha L, Lowe RP (1996) Localization of the complex specturm: the S transform. IEEE Trans Sig Process 44(4):998–1001
11. Ventosa S, Simon C, Schimmel M, Danobeitia JJ, Manuel A (2008) The S-Transform from a wavelet point of view. IEEE Trans Sig Process 56(7):2771–2780
12. Rioul O, Flandrin P (1992) Time-scale energy distributions: a general class extending wavelet transforms. IEEE Trans Signal Process 40(7):1746–1757
13. Daubechies I (1992) Ten lectures on wavelets. CBMS-NSF regional conference series in applied mathematics, vol 61. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA
14. Flandrin P (1999) Time-frequency/time-scale analysis. Volume 10 of wavelet analysis and its applications. Academic Press Inc., San Diego, CA
15. Tary JB, Herrera RH, Han J, Baan MVD (2014) Spectral estimation-what is new? What is next? Rev Geophys 52(4):723–749
16. Sinha S, Partha SR, Phil DA, Castagna JP (2005) Spectral decomposition of seismic data with continuous-wavelet transform. Geophysics 70(6):19–25
17. Pinnegar CR, Mansinha L (2003) The S-transform with windows of arbitrary and varying shape. Geophysics 68(1):381–385
18. Huang ZL, Zhang J, Zhao T, Sun Y (2016) Synchrosqueezing S-transform and its application in seismic spectral decomposition. IEEE Trans Geo-sci Remote Sens 54(2):817–825