Modelling and Optimization of Asymmetric Vehicle Routing Problem Using Particle Swarm Optimization Algorithm

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Abstract Various problems related to vehicle routing problem attract interest of researchers and industry. Specific optimization model and algorithm were developed to solve the problem. This intensive effort aims to reduce logistics costs and number of vehicle usage. In this context, most articles focus on different optimization method approaches. Asymmetric vehicle routing problem (AVRP) appeared when the const of delivering and returning route using the same path were difference. It was used in practical applications to solve AVRP problems identified for specific application. This paper used a well-known metaheuristic optimization method, Particle Swarm Optimization (PSO) for solving AVRP models. To optimize AVRP, three optimization objectives were recommended; the total travelling time, efficiency of the route and the number of vehicles. This is to optimize the number of targets visited. The performance of PSO is evaluated by comparing its results with other popular metaheuristics. The computational experiment was conducted using five test problems with different sizes. The optimization results indicated that this algorithm able to offer good solutions with the best answer for the practical problem. Finally, this study shows that the algorithm can significantly reduce travel costs via number of bus needed to serve all the stop points.

Keywords Asymmetrical vehicle routing problem · Particle swarm optimization · Vehicle routing problem

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1 Introduction

Public transportation sector is in high demand from time to time as it covers many aspects of transport, such as buses, trains between city trains, subways and high-speed trains. It is very active in regional and local passenger transport systems with regular routes and regular schedules. Public transport has the potential to provide people with mobility and low safety risks, promote a well environment and strong urban areas by reducing overcrowding and pollution. As per the population and the urban economy grow, the demand for public transport is growing rapidly.

Various types of studies have been conducted to ensure the efficiency of public transport. One of the studies focused on public transport is optimizing routing problems. Research area of this common problem is known as Vehicle Routing Problems (VRP). This problem dates to the 1950s when Dantzig and Ramser [1] defined the formulation of mathematical programming and algorithmic approaches to solve the problem of gasoline distribution to service stations.

VRP is one of the problems in managing distribution and logistics systems. This problem arises in many practical situations, especially when they include commercial vehicles such as trucks and buses. VRP mainly focuses on finding ways to visit a specific set of customers. The simplest model of the problem is that every vehicle has to visit a customer's place exactly and for every vehicle routing, starting and ending at the depot. Usually there are different VRP classes or editions such as Capacitive VRP (CVRP), VRP with Windows Time (VRPTW) [2]. In general, CVRP is a VRP where a fleet of a certain capacity must meet customer requirements for a single product from the same depot with minimal transportation costs. In other words, CVRP is a VRP with an additional limit for each vehicle that must have the same capacity as a product. The interval or relief on the depot is called the planning horizon.

Therefore, determining the optimal route used by vehicles to the users is a major problem in VRP. The goal is to minimize total transportation costs while increasing travel and service improvements. Transportation costs might be reduced by increasing travel distances and reducing the number of vehicles needed, or vice-versa.

Most real-world transportation issues are often more complex and require better optimization methods. That is why in practice the classic VRP problem is combined to achieve a better goal. In principle, every research starts from studying the VRP method to understanding the characteristics of the vehicle path problems through different solutions with more detailed methodologies.

The practical importance of VRP has increased the number of studies, to address the concerns of various parties. However, VRP is a complex and comprehensive problem, so when there is an increasing number of problems, it becomes more difficult and complex to find the right solution in a limited time. Although there were advanced solutions introduced, certain constraints on problem solving that often violated when dealing with real-world VRP, have left experts dissatisfied with the performance and usability of the algorithm. Vehicle routing problem and scheduling problem is combined problem that often appear in many real-word application as reviewed by Ellabib et al. [3]. VRP became an intensive research as a subject for both heuristic and exact optimization approach. Campbell and Savelsbergh [4] reported that different types of constraints including time windows and multiple uses of vehicles can efficiently handle with insertion heuristic. There are various approaches to VRP through case study including implementing the concept of VRP to complete distribution of home meal delivery by Bräysy et al. [5]. Another example of VRP application according to [6] is on redistribution of public bicycle system. Kara [7, 8] presents an integral programming formulation with the aim of setting for each route where the path capacity limitation is changed, considering the maximum route length along the time limit.

The use of optimization methods by researchers and industry increasingly since the introduction of Dantzig and Ramser in 1959, different types of algorithmic techniques have been used to solve VRP problems. An example such as Genetic Algorithm (GA) that inspired by Darwin's concept was introduced by Holland [9] and Ant Colony Optimization (ACO) [10] is also an attracted algorithm used by researchers to solve VRP problems. There are other optimization techniques and are used to study transport and VRP routing problems, including Simulated Annealing [11], a highly specialized metaheuristic, to optimize global optimization in a large search space for optimization problems. There are also optimization techniques based on the mathematical method of the tabu search, inspired [12]. In the tabu search, it contains methods that use the local search method used for mathematical optimization to improve the performance of local search.

Metaheuristic methods have been adapted and used extensively to solve different VRP variations. The main types of metaheuristics used to solve VRP are Tabu Search performed by Bolduc et al. [13], Ant Colonization Optimization was initiated by Donati et al. [14], Memetic Algorithm [15], Genetic algorithms proposed by [16–18], Variable Neighborhood studies were performed by Hemmelmayr et al. [19] and Adaptive Large Neighborhood Search proposed by Azi et al. [20].

In this paper, VRP problem will be modelled and optimized using Particle Swarm Optimization (PSO). PSO, inspired by Kennedy and Eberhart [22], is a population-based measurement method and draws inspiration from the behavior of organisms such as bees, birds and fish.

2 Asymmetrical Vehicle Routing Problem

Numerous research methods have been investigated in recent years and this contributes to important theoretical developments. These issues relate to different distribution management situations and affect their research. Laporte et al. [21] explain that vehicle routing problem (VRP) issues consist of optimal route planning for one or more vehicles through a range of locations (node, city), subject to different types of restrictions. They look for solutions using heuristic methods. While it provides accurate results, there is no way to determine whether an investigation is valid. As a result, AVRP should be used so that any result in a study is clearly visible. This is because every data set is optimized using different algorithmic techniques to see the differences in each of these techniques. This is a study that applies existing techniques but with different approaches.

The aim of this paper is to optimize one particular type of asymmetrical VRP using metaheuristic algorithm. It is generally stated that C is not symmetrical and A, a set of arcs, is directed. For each node I, except for the depot (e.g. city n), we associate a non-negative weight, d (d, = 0). The problem of capacitated vehicle routing (CVRP) is to determine a path for the same vehicle, each with a capacity of D and a fixed cost of f, such that.

(i) each vehicle begins and ends its journey in the depot;
(ii) each city (except the depot) is visited exactly by one vehicle;
(iii) the total weight of a given route must not exceed D;
(iv) the total fixed costs and routing costs are minimized.

In this study, three optimization objectives were considered, (i) minimize total travelling time, T_{sum} (ii), minimize route efficiency, RE (iii) minimize number of bus, B used. In addition, a constraint of travelling time limit is applied, to ensure that each bus must return to the depot within a maximum time.

$$T_{sum} = \sum_{i=1}^{M} \sum_{j=1}^{M} t_{ij} y_{ij} \tag{1}$$

$$y_{ij} \begin{cases} 1 \text{ if vehicle travels from } i \text{ to } j \\ 0 \text{ otherwise} \end{cases} \tag{2}$$

$$RE = \frac{T_{sum}}{B \cdot T_{\max}} \tag{3}$$

$$B = \sum_{k=1}^{V} b_k \tag{4}$$

$$b_k \begin{cases} 1 \text{ if vehicle } k \text{ used in the route} \\ 0 \text{ otherwise} \end{cases} \tag{5}$$

In Eq. (1), t_{ij} is the travelling time from i to j. Route efficiency refer to the travelling load balance between one route with another route. In the case that RE is low, it means that there is a driver whose completed the route faster, while another driver needs longer time to complete his/her job. When this situation happens, it will make the workload imbalance. It should be noted that $T_{\max}$ refers to the maximum time taken by any driver to complete his/her route.

Finally, a weighted sum approach is used for multiple objectives optimization;

$$\min f = w_1 \widehat{T}_{sum} + w_2 \widehat{RE} + w_3 \widehat{B} \tag{6}$$

Noted that the $\widehat{T}_{sum}$, $\widehat{R}E$ and $\widehat{B}$ are normalized T_{sum}, RE and B respectively. These optimization objectives were normalized between 0 and 10 to avoid bias in the fitness calculation since the original range of optimization objectives were different.

3 Particle Swarm Optimization

Particle Swarm Optimization (PSO) has been introduced by Kennedy and Eberhart [22]. It is a suitable metaheuristic algorithm for optimizing nonlinear continuous functions [22]. PSO is inspired by behavior of bird flocking and fish schooling. To explain how PSOs solve complex mathematical problems, the discussion about herds of animals is that birds are made. The movement of the herd only if each member of the herd shares information with each other. In this way, each member of the group has a means of experience and knowledge, also known as social knowledge. The problem of finding the best landing point described has optimization issues. The herd must identify the best points, such as latitude and longitude, to maximize the living conditions of the members. To do this, each bird searches and evaluates different points using multiple survival criteria at the same time. Each of them has the advantage of knowing where the best location is, until the herd is known.

Consequently, PSO has been one of the most effective optimization methods in solving difficult optimization problems [23]. Formally, the algorithm of PSO has a step to considered before taking to account.by general this is the step of PSO; Initialize the swarm form the solution space, then evaluate the fitness of each particle. After that update individual and global bests, update velocity and position of each particle using Eqs. (7) and (8) and repeat until termination condition. Flowchart of PSO is presented in Fig. 1.

$$V_i(k+1) = V_i(k) + Y_{1i}(Pi - X_i(k)) + Y_{2i}(G - x_i(k)) \tag{7}$$

$$X_i(k+1) = X_i(k) + V_i(k+1) \tag{8}$$

The PSO is randomly initialized with a group of particles and then optimized by updating the generation. With each iteration, each particle is updated by following two “best” values. The first is the best solution achieved if fitness values are also maintained. This is called Pbest. There are other “best” values that are maintained by particle group optimizers, which are the best values are derived by each particle in the population. The value is the best of any particle and is called the Gbest. The PSO at each step consists of changing the speed of each particle (accelerating) to the Pbest and Gbest location.

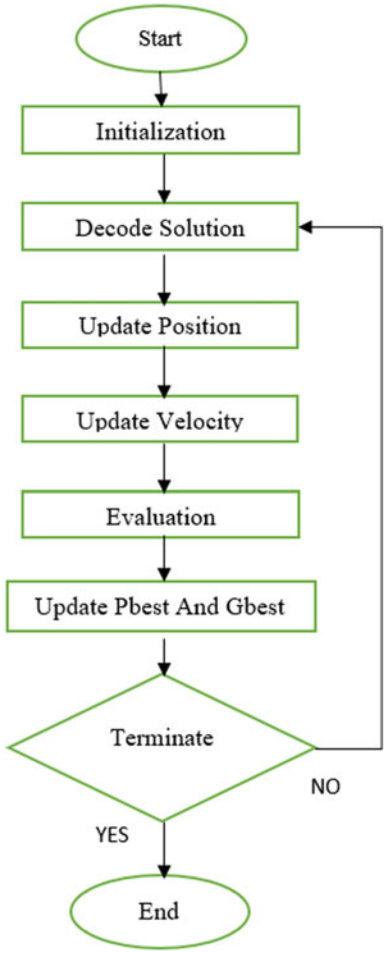

Fig. 1 Flowchart of particle swarm optimization

4 Computational Experiment

To evaluate the effectiveness of the PSO to optimize AVRP, an extensive computational experiment was performed. Matlab 2018 software was used to model the problem and optimize using PSO algorithm. The aim of this experiment was to look at the effectiveness of the PSO compared to the other two types of algorithms which were ACO and GA. These two algorithms were chosen as comparison algorithm because of popularity in the VRP optimization publication.

Each algorithm tested has its own advantage in optimization methods. The original goal was to use VRP optimization to find algorithms that could provide more efficient assessment or output. Each algorithm involved in this analysis is defined with the same parameters, the same repetition rate and the same data set. This is intended to enable any algorithm to show great success in the context of routing efficiency.

For the experiment, a set of test problems consists of different stop point has been used. These problems have been randomly generated to represent the location of the bus stop in a planning area. The generated problem consists of the stop point between 20 and 100, with increment of 20. Therefore, the total number of test problems used was five.

For each test problem and algorithm, 30 repetitions of run will be conducted to reduce the pseudo-random effect. Then, the minimum and average fitness will be recorded as comparison parameter. At the same time the best fitness will be decoded into three optimization objectives; T_{sum}, RE and B.

The optimization results for AVRP using GA, ACO and PSO is presented in Table 1. To verify the performance of the proposed algorithm, the results of each algorithm were compared to find the optimum solution. Each test problem from Table 1 is divided into three indicators based on experimental results that indicate mean fitness, standard deviation, and computational time.

For test problem size with 20 stop points, ACO performed the best in term of mean fitness. This was followed by PSO in the second rank and GA in the third rank. However, when the problem size increased to 40 and above, it was found that PSO outperformed GA and ACO in all test problems. This result was associated with the search space of the test problem. For smaller problem size, the algorithm with constructive cooperation like ACO tend to perform better because the gathered information from the search agent covered most of the search space. However, when the problem size increased, the information delivered to the ants were incomprehensive, since the search space were excessively increased, but the number of search agent (i.e. ants) remain the same.

Table 1 Computational experiment results

Problem	Indicator	GA	ACO	PSO
20	Mean fitness	19.8753	9.2861	18.3665
	Std. Dev.	6.9623	6.29	8.1382
	CPU time	1.2387	3.2025	1.6028
40	Mean fitness	19.5776	12.6926	12.5308
	Std. Dev.	4.3911	4.6163	5.5880
	CPU time	1.5463	7.5551	2.0437
60	Mean fitness	23.5468	18.4547	12.7830
	Std. Dev.	3.5378	2.8918	3.0099
	CPU time	1.9323	15.0358	2.5152
80	Mean fitness	29.0321	23.6022	18.1958
	Std. Dev.	2.2684	1.1745	3.2664
	CPU time	2.4894	24.1462	3.3218
100	Mean fitness	32.2843	26.7297	19.2644
	Std. Dev.	3.3782	1.5390	4.4114
	CPU time	3.0879	38.5702	4.2925

Table 2 Optimum results for test problems

Problem	Indicator	GA	ACO	PSO
20	Min fitness	6.2900	6.2900	5.9830
	T_{sum}	226	220	245
	RE	0.9040	0.88	0.98
	B	5	5	5
40	Min fitness	9.7878	8.9958	7.8077
	T_{sum}	497	487	490
	RE	0.8583	0.8855	0.9655
	B	11	11	10
60	Min fitness	15.5690	15.6290	7.9641
	T_{sum}	755	756	772
	RE	0.8489	0.8894	0.9153
	B	17	17	16
80	Min fitness	24.1126	18.3294	12.0367
	T_{sum}	1065	982	1041
	RE	0.86	0.8539	0.9052
	B	24	23	22
100	Min fitness	22.6326	22.3963	7.2584
	T_{sum}	1321	1269	1246
	RE	0.8658	0.8752	0.9273
	B	29	29	26

On the other hand, PSO was well-known for simple mechanism. In related with large size of test problem, this algorithm has a balance exploration and exploitation capability through local and global best mechanism. This makes the algorithm able to find better quality solution even though using smaller search agent number. Furthermore, the PSO only requires small parameter adjustment to achieve good solution.

Besides that, it was found that standard deviation in PSO was higher than other algorithms in majority of the test problems. Its indicated high variation in the final solution obtained by PSO. This finding is undesirable in optimization because of inconsistent in the final answer. This result also related with exploration of PSO that able to discover different region in search space.

Finally, the CPU time indicated that ACO consumed the longest time to complete the iteration. For this criterion, PSO was in the second rank behind GA. In ACO, a solution is constructed one by one dimension to complete a visiting route. This was clearly observed that the larger size of test problem, the CPU time excessively increased in ACO. In comparison with PSO, the CPU time is linearly increased with the problem size.

Table 2 presents the best solution obtained by each algorithm. For the minimum fitness, PSO clearly obtained the best solutions in all test problems. In contrast to the minimum fitness, PSO indicated mixed result in total travelling time (T_{sum}) for all test problem. For test problem with 100 stop point, it shown that PSO outperformed GA and ACO by percentage of 6%. This shows that PSO has higher probability and efficiency in problem solving. Comparison test results for algorithm selection are given in Table 2. This table presents the results for benchmark problems at 5 different sizes.

On the other hand, PSO consistently provided better *RE* for all test problems. In all problems, the *RE* by PSO exceeds 90%. It means that the load between drivers were almost equivalent for the solution provided by PSO. This result indirectly will maintain the drivers' motivation, since their colleague also having roughly similar workload.

In the meantime, the number of bus, *B* suggested by PSO solution was the best in four out of five problems. In problem with 20 stop point, PSO obtained similar result for number of bus as found by GA and PSO. Meanwhile for the remaining of the problem, PSO came out with better number of bus compared with comparison algorithms. It means that, the operational cost for the solution by PSO will be lower since they utilize less driver number, fuel and maintenance cost. It is expected that the saving will be between 6 and 11%, depending on the number of stops.

The results of minimum fitness, T_{sum}, *RE* and number of bus shows that PSO solution were better compared with GA and ACO. Although the $\mathrm{T_{sum}}$ for PSO were not the smallest, but the difference was less than 5%. This result was because of normalizing effect of the optimization objectives into similar range [0, 10]. For optimization objective with smaller range such as nob, small changes in the nob give higher impact on the normalized fitness. While the larger range optimization objective as T_{sum}, small changes on the value only give small effect on the fitness.

5 Conclusion

This paper optimized asymmetric vehicle routing problem using Particle Swarm Optimization. The AVRP has been modelled by considering three optimization objectives; total travelling time (T_{sum}), route efficiency (*RE*) and number of bus (*B*). A weighted sum approach was used to optimize all these objectives.

Computational experiment results indicated that PSO algorithm performed the best for the mean fitness with acceptable CPU time. In addition, the best solution provided by PSO consistently having the best *RE* and *B*. While the T_{sum} was behind another algorithm with less than 5% difference.

It can be concluded that PSO is superior compared with GA and ACO to optimize AVRP. In future, a further study to identify the best weightage for all optimization objectives is suggested to reduce the normalizing effect in problem modelling.

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