Second Order Sliding Mode Control for Second Order Process with Delay Time Using Different Control Algorithms

B. Amarendra Reddy, N. Beauty, P. Sneha, and G. Neelima Sai

Abstract Regulation of second-order plus time-delay systems (SOPDT) plays a vital role because many of the industrial process (such as electrical, mechanical, and electromechanical systems) exhibits delayed response at outputs in response to the inputs. These time-delay systems are responsible for control complexity and degrade the system performance under disturbance conditions. To regulate the performance of these systems, robust controllers are needed. Sliding mode control is a robust control strategy. To regulate these systems and also to improve the performance under disturbance conditions, sliding mode control strategy is adapted. Here, second-order sliding algorithms such as twisting, super-twisting, and adaptive algorithms are incorporated for regulation of these systems. Pade approximations (0/1, 1/1, 1/0) are used for representation of the constant time delay and here (0/1) approximation is adapted. Simulation studies have been performed for these systems in the MATLAB environment.

Keywords Siding mode control (SMC) · Second-order sliding mode control (SOSMC) · Robust control · Second-order process with delay time (SOPDT) · Pade approximation

1 Introduction

Three important parameters are needed for representing an industrial process control system. These are (i) gain (ii) time delay and (ii) time-constant. The exact representation of such system using these parameters is very difficult due to model uncertainty, i.e., error between actual plant and the model. Modeling point of view this is one possible uncertainty and assume that the model error is negligible. In real time, there

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may be disturbances at the plant outputs, inputs parameter variations, measurement inaccuracies, and assume these are known to be within 15–20% from their respective values. Considering these into account, designing a controller is a challenging task. A controller is called as a robust controller which can reject these disturbances in effective way in regulating the process control variables [1]. There are a wide variety of robust controller design methods. Sliding mode control strategy is a robust control design methodology which is completely insensitive to the uncertainties. These are classified into first-order and higher-order control strategies.

The sliding mode control strategy design comprises of two stages. One is designing the switching surface; once the system states are sliding on the switching surface, the system characteristics are presided by the characteristics of sliding surface and the other is to design a control action, such that the system should reach the sliding surface in finite time. Usually, to obtain ideal sliding motion, fast switching of control action is required but in actual plants, switching occurs at finite frequency causing the trajectories to oscillate within the region of sliding surface. This phenomenon is called chattering effect. Generally, conventional first-order sliding mode control (FOSMC) exhibits chattering phenomena. Chattering phenomena leads to large undesired oscillations causes instability of the system. To overcome the above effects, second-order sliding mode control had been developed. The main advantage of SOSMC over FOSMC is it completely avoids chattering effect and higher accuracy can be obtained with respect to robustness and easiness of implementation [2].

This paper is organized as: Significance of system parameters of SOPDT model is explained Sect. 2. The SOSMC design procedure and control laws are discussed in Sect. 3. Simulated results for different algorithms in SOSMC are presented in Sect. 4 and conclusions from results are drawn in Sect. 5.

2 Significance of Time Constants, Gain, and Time Delay of SOPDT Model

Consider an industrial process control system which is described using SOPDT model and its transfer function is given in Eq. (1).

$$G(s) = \frac{Ke^{-\alpha_d s}}{(1+\alpha_1 s)(1+\alpha_2 s)} \tag{1}$$

Usually, study of system gain K, time constants α_1, α_2, and delay time α_d helps to understand the model behavior. For a certain change in controller output, the response of process variable behaves according to the change in gain. Simply, larger gain means the response of the system will be larger [3]. Time-constant illustrates how quickly the process responds to a certain change in controller output [4]. The process described using SOPDT model is having two time-constants, and by using this, the plant behavior can be modeled. The dead time illustrates how much delay is expressed in the output for a change in controller output but the dead time does not affect the nature of the response. The frequency response of the FOPDT model does not match with the actual plant at specific corner frequencies [3] as it contains less number of parameters. The mismatch in the frequency response can be minimized using SOPDT representations. Therefore, it is better to prefer SOPDT model rather than FOPDT model. The time delay of the SOPDT model is represented using (0/1) Pade approximation and it is described using Eq. (2).

$$e^{-\alpha s} = \frac{1}{1+\alpha s} \tag{2}$$

The transfer function of SOPDT model using Pade approximation is given using Eq. (3).

$$G(s) = \frac{K}{(1+\alpha_1 s)(1+\alpha_2 s)(1+\alpha_d s)} \tag{3}$$

Using phase variable canonical state space model, Eq. (3) is expressed as given in Eq. (4).

$$\dot{X} = AX + BU \tag{4}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -p_1 & -p_2 & -p_3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}$$

where

$$p_1 = \frac{1}{\alpha_1\alpha_2\alpha_d}, \; p_2 = \frac{\alpha_1+\alpha_2+\alpha_d}{\alpha_1\alpha_2\alpha_d}$$
$$p_3 = \frac{\alpha_1\alpha_2+\alpha_2\alpha_d+\alpha_1\alpha_d}{\alpha_1\alpha_2\alpha_d}$$

where $X_1(t)$ is the output, $u(t)$ is the control input and p_1, p_2, p_3 are functions of plant parameters.

3 Second-Order Sliding Mode Control (SOSMC) of a SOPDT Model

The second-order sliding mode controller is design involves design of (i) switching surface and also (ii) design of controller. Switching surface design involves designing a surface/plane 's' which is a the linear combination of the state variables [5]. The

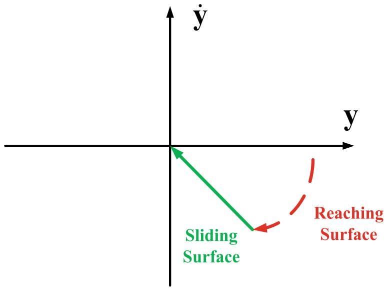


Fig. 1 Sliding mode control

role of the SOSMC controller is to keep the variables on this surface and also to make the system insensitive to the uncertainties. If the state variables are deviated from the pre-defined switching surface, then the controller has to bring these variables onto the surface. The switching surface is defined by equation $s = \dot{s} = 0$. [6]

(a) **Defining a Sliding Surface**

In SOSM control approach, the history of all state variables will converge to an equilibrium point when they obey $s = \dot{s} = 0$ of the state plane. The switching surface of the SOSMC strategy is stated using Eq. (5) (Fig. 1).

$$s = \left\{X \in R^K; s = \dot{s} = 0\right\} \tag{5}$$

The mathematical representation of switching surface, 's' is stated as $s(x) = Sx$, and here, x is a state vector and $s \in R^n$ is switching surface vector. The switching function, 's' and its first-time derivative with respect to time is continuous and its second-time derivative w.r.t time is discontinuous [7].

$$\dot{s} = \frac{\partial s}{\partial x}\dot{x} = \frac{\partial s}{\partial x}(f(x) + b(x)u)$$

Differentiating twice the switching variable "s" produces a relation as given in Eq. ().

$$\ddot{s} = \frac{\partial \dot{s}}{\partial x}(f(x) + b(x)u) + \frac{\partial \dot{s}}{\partial u}\dot{u}$$
$$\ddot{s} = \alpha(x) + \beta(x)\dot{u}$$

If $|s(x)| < s_0$, the following inequalities are assumed.

$$|\alpha(x)| < \phi, 0 < \Gamma_m \leq \beta(x) \leq \Gamma_M, \beta(x) = \frac{\partial \dot{s}}{\partial u}$$

The SOPDT system is represented in the regular form [8] and the respective matrices are obtained as given

$$A_{11} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} A_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} A_{21} = \begin{bmatrix} -a_1 & -a_2 \end{bmatrix} A_{22} = [-a_3]$$

The switching surface design involves the design of state feedback matrix M such that the eigen values of $(A_{11} - A_{12}M)$ are in the left half of s-plane and to achieve this, the desired characteristic equation is needed. It is the second-order quadratic equation as given in Eq. (6)) where parameters ξ and ω_n represent the desired damping ratio and natural frequency, respectively. The M matrix is obtained by comparing Eq. (6) with $(A_{11} - A_{12}M)$ and it is given in Eq. (7).

$$\lambda^2 + 2\varepsilon\omega_n\lambda + \omega_n^2 = 0 \tag{6}$$

$$M = \begin{bmatrix} m_1 & m_2 \end{bmatrix} = \begin{bmatrix} \omega_n^2 & 2\varepsilon\omega_n \end{bmatrix} \tag{7}$$

The sliding surface is given by

$$s = \begin{bmatrix} m_1 & m_2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$s = m_1 x_1(t) + m_2 x_2(t) + x_3(t) \tag{8}$$

where $m_1 = \omega_n^2, m_2 = 2\xi\omega_n$.

The time derivative of the switching surface (first order) is given using Eq. (9).

$$\dot{s}(t) = -p_1 x_1(t) + (m_1 - p_2)x_2(t) + (m_2 - p_3)x_3(t) + bu(t) \tag{9}$$

(b) **Control Law**

The control laws for SOPDT system using twisting super-twisting and adaptive algorithms are described in the following paragraphs.

Twisting Algorithm

Using twisting algorithm, the trajectories are plotted using s and $s$dot variables are twisting around the origin and its characteristic is as shown in Fig. 2. In the twisting algorithm of the second-order sliding mode control assumes that after a finite time interval the point $\mathbf{s} = \dot{s} = 0$ will be reached [9]. The control algorithm is defined by the control law as given in Eq. (10).

$$\dot{u}_{\mathrm{TW}} = \begin{cases} -u_{\mathrm{TW}} & for\ |u| > u_{\max} \\ -k_m \operatorname{sign}(s) & for\ s\dot{s} \le 0 \text{ and } |u| \le u_{\max} \\ -k_M \operatorname{sign}(s) & for\ s\dot{s} > 0 \text{ and } |u| \le u_{\max} \end{cases}$$

$$k_M > k_m > 0, k_m > \frac{4\Gamma_M}{s_0}, k_m > \frac{\phi}{\Gamma_m}, \Gamma_m k_M - \phi > \Gamma_M k_m + \phi \tag{10}$$

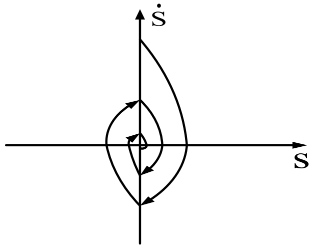


Fig. 2 Trajectory of s and $s$dot (twisting)

Super-Twisting

Using super-twisting algorithm, the trajectories plotted using s and $s$dot variables are twisted around the origin along with travel on $s$dot axis (super-twisting) and its characteristic is as shown in Fig. 3. The super-twisting sliding mode control algorithm relies on inserting an integrator into the control loop such that control becomes continuous time function (u_{st}). The control law u_{ST} is defined by two terms: the first is defined in terms of an integral of a discontinuous function of sliding variable, while the second is a continuos function of the sliding variable [10] . The control law is defined mathematically using Eq. (11).

$$
\begin{aligned}
u_{\mathrm{ST}} &= u_1 + u_2 \\
\dot{u}_1 &= \begin{cases} -u_{\mathrm{ST}}, & \text{for } |u_{ST}| > u_{\max} \\ -w\,\mathrm{sign}(s), & \text{for } |u_{ST}| \le u_{\max} \end{cases} \\
u_2 &= \begin{cases} -\lambda |s_o|^{\rho}\mathrm{sign}(s), & \text{for } |s| > s_o \\ -\lambda |s|^{\rho}\mathrm{sign}(s), & \text{for } |s| \le s_o \end{cases}
\end{aligned}
\tag{11}
$$

where

$$
w > \frac{\phi}{\Gamma_m}, \quad 0 < \rho \le 0.5
$$

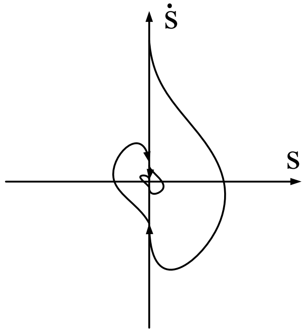


Fig. 3 Trajectory of s and $s$dot (super-twisting)

$$w > \frac{4\Gamma_M}{s_o}, \quad \rho(\lambda\Gamma_m)^{\frac{1}{\rho}} > (\Gamma_M w + \phi)(2\Gamma_M)^{\frac{1}{\rho}-2}$$

4 Adaptive Algorithm

Adaptive sliding mode is an effective control technique approach that has better disturbance rejection and chattering avoidance properties than classical SMC [11]. The control law for adaptive control is defined mathematically as

$$\dot{u} = -\theta \,\mathrm{sign}(p)$$

where $p = \dot{S} + |S|^{\frac{1}{2}}\mathrm{sign}(S)$

$$\dot{\theta} = \begin{cases} \overline{\theta}\|p\| \,\mathrm{sign}(\|p\| - \varepsilon) & \text{if } \theta > \mu \\ \mu & \text{if } \theta < \mu \end{cases} \tag{12}$$

where $\overline{\theta}$, μ and ε are positive controller parameters to be chosen. The value of μ has been set to restrict the gain to be positive value. The parameter ε defines the region around the switching surface, i.e., indicates closeness to the switching surface. It will effect the variation in switching gain and responsible to achieve ideal sliding mode [11].

5 Simulation Results

Second-Order Sliding Mode Control for Twisting Algorithm
Simulation studies has been carried out by considering the SOPDT system using second-order SMC strategy using a twisting algorithm. Here, the parameters ξ = 0.7 and w_n = 3.8 are considered for design of switching surface. Different initial conditions considered for simulation studies are X_0 = [0.0125, 0.0125, 0, 0], [0.05, 0.05, 0, 0], [0.1, 0.1, 0, 0], and [0.075, 0.075, 0, 0] with design constants as V_m = 0.7 and V_M = 1. The history of the state variables x_1, x_2, x_3 and the switching surface with time is shown, respectively, using Figs. 4, 5, 6 and 7 considering these initial conditions. It is observed that these state variables are returning to the stable equilibrium state using SMC with twisting control algorithm.

The trajectory of switching surface and its derivatives is shown in Fig. 8. The finite time convergence of state trajectories to the origin in the state plane is due to switching of the actual control signal between two different magnitudes. The time taken for the state variables to reach the stable equilibrium point using second-order twisting SMC is less compared to first-order SMC.

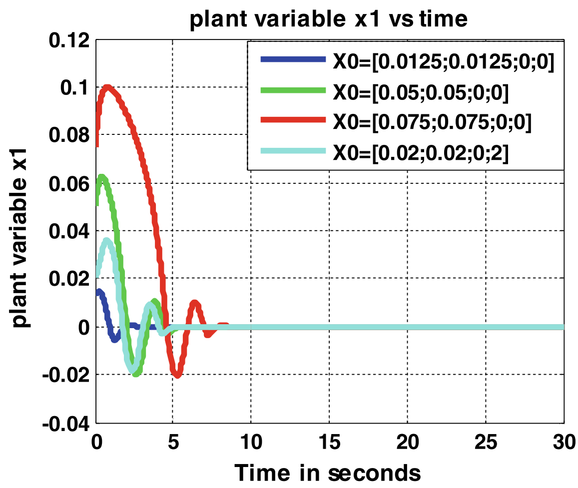


Fig. 4 Plant variable $x1$ versus time without disturbance condition

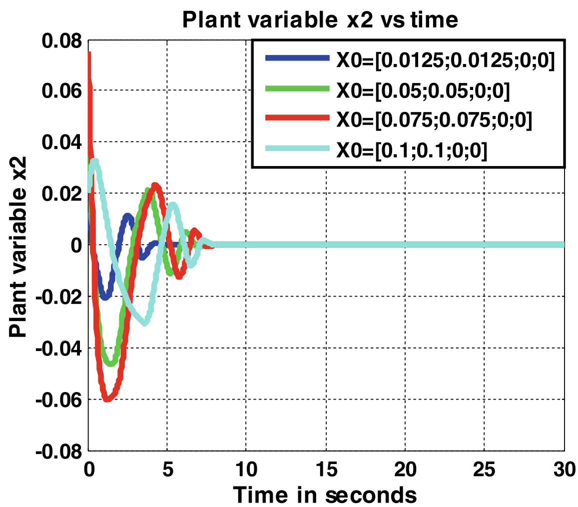


Fig. 5 Plant variable $x2$ versus time without disturbance

Second-Order Sliding Mode Control for Super-Twisting Algorithm

For the simulation studies of this system using this algorithm needs important parameters like W, ρ and λ. These are considered as follows $W = 3$, $\rho = 0.5$ and $\lambda = 4.75$. The trajectory of S and Sdot using super-twisting algorithm without disturbance for different initial conditions are graphically shown in Fig. 9. From this figure, it is observed that under different conditions the shape of this trajectory is following desired shape.

For the simulation studies of this system is also performed using this algorithm under disturbance conditions considering the parameters like W, ρ and λ. The controller parameters are chosen such that, they assure the convergence of the

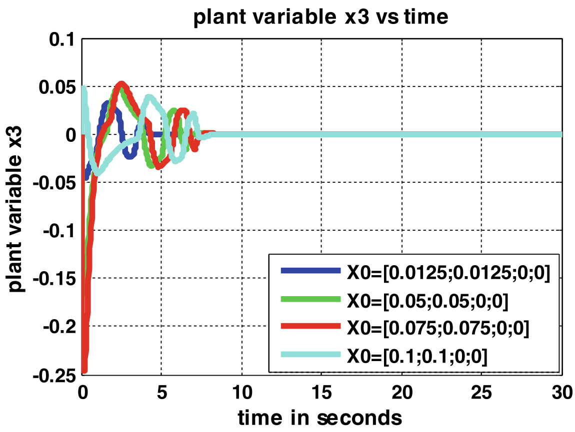


Fig. 6 Plant variable $x3$ versus time without disturbance

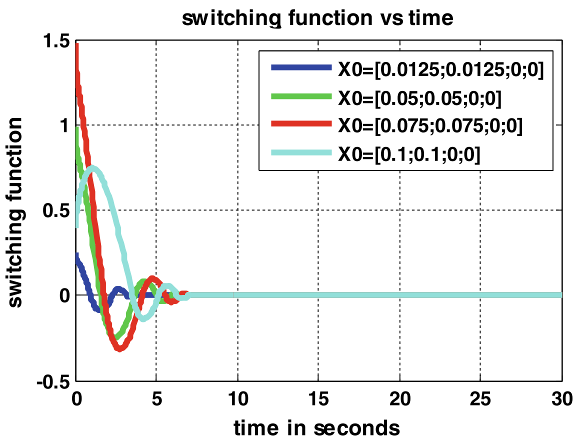


Fig. 7 Switching surface versus time without disturbance

state variables to an equilibrium point in a finite time, when this system is subjected to disturbance under different initial conditions. The constants are chosen as $W = 3$, $\rho = 0.5$, $\lambda = 4.75$, $\xi = 0.7$ and $w_n = 3.8$. The variation of plant variables x_1, x_2, x_3, S and Sdot with time are shown, respectively, in Figs. 10, 11, 12 and 13 under disturbance condition. These are plotted by considering four different initial conditions $x_0 = [0.0125, 0.0125, 0, 0]$, $[0.05, 0.05, 0, 0]$, $[0.075, 0.075, 0, 0]$ and $[0.1, 0.1, 0, 0]$. Here, the system is also subjected to step disturbance $d = 0.01 * (\text{Heaviside}(t - 16) - \text{Heaviside}(t - 16.5))$ with a magnitude 0.01 during the period 16–16.5 s. It is observed that when this system is subjected to the perturbation, these variables are returning to the original equilibrium state. It is observed that the control algorithm is effectively rejecting the disturbances and the regulation characteristics are also good.

Adaptive Second-Order Sliding Mode Control (ASOSMC)

In ASOSMC, $\overline{\theta}$, μ and ε parameters chosen such that, they assure a finite time convergence of the state variables when this system is subjected to parameter variation. The

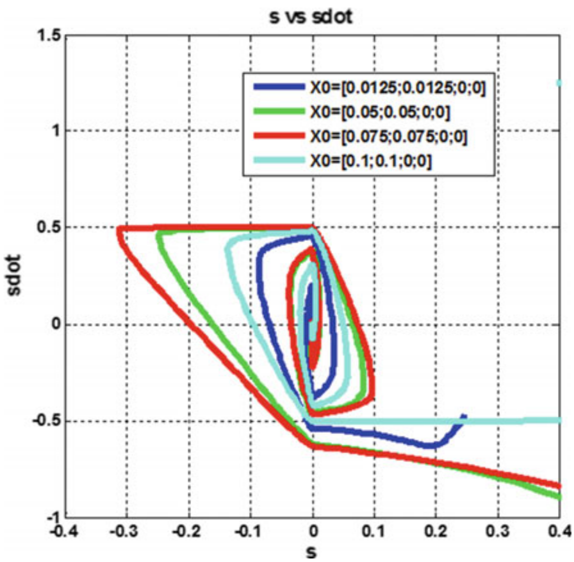


Fig. 8 Derivative of switching surface versus switching surface using twisting algorithm without disturbance

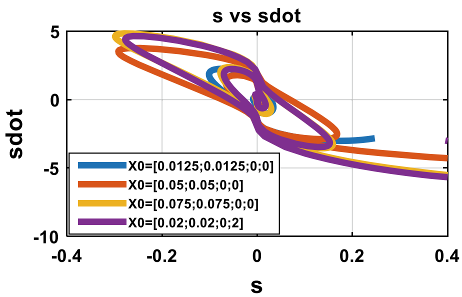


Fig. 9 *S* versus *S*dot trajectory without disturbance

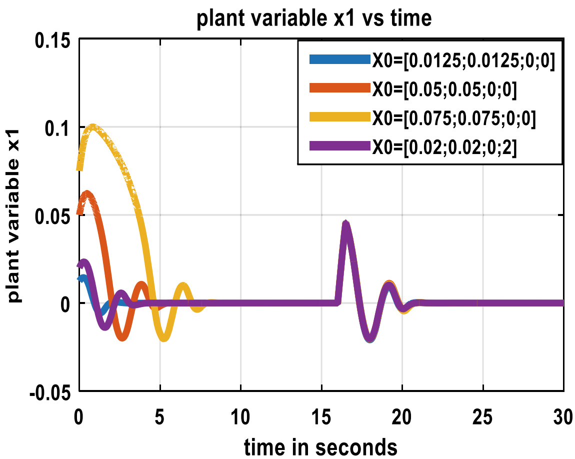


Fig. 10 Plant variable $x1$ versus time under disturbance condition

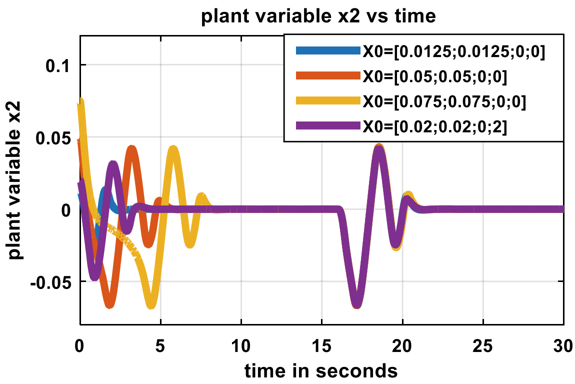


Fig. 11 Plant variable $x2$ versus time under disturbance condition

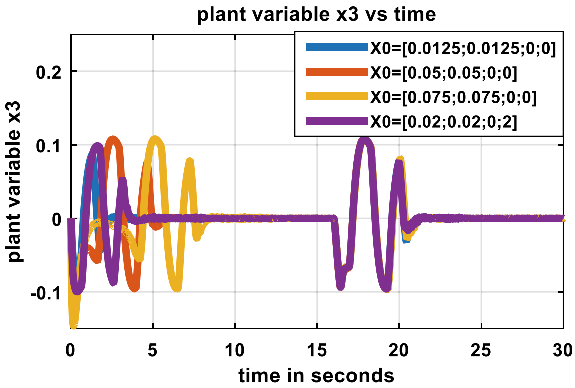


Fig. 12 Plant variable $x3$ versus time under disturbance condition

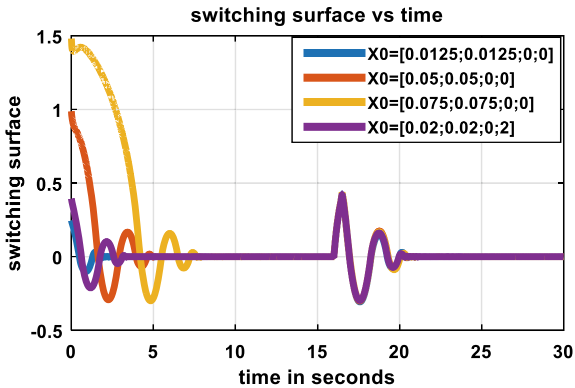


Fig. 13 Switching surface versus time under disturbance condition

Fig. 14 Plant variable $x1$ versus time under parameter variation

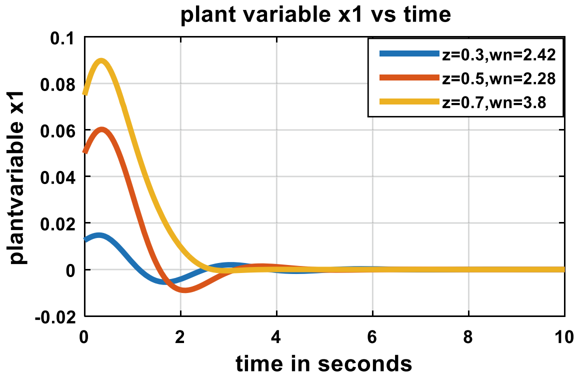
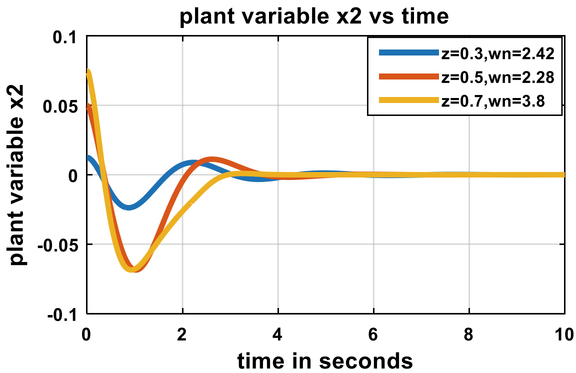


Fig. 15 Plant variable $x2$ versus time under parameter variation



constants are chosen as, $\overline{\theta} = 540$, $\mu = 95$ and $\varepsilon = 0.0001$. The system is subjected to parameter variations in damping ratio and natural frequency. The response is plotted with respect to time. The variations of the state variables, sliding surface, and derivative of sliding surface with respect to time under the parameter variation conditions are obtained graphically in the interval [0 10]. For three different damping ratios 0.3, 0.5, 0.7 and natural frequencies w_n as 2.42, 2.28, 3.8. The parameter variation of plant variables x_1, x_2, x_3, s and $s$dot with time is shown respectively in Figs. 14, 15, 16 and 17. It is observed that this SMC is effective under these conditions (Fig. 18).

ASOSMC provides smooth control to force the system states on sliding surface. There is no chattering effect and also insensitive to the parameter variation.

6 Conclusions

The design of the different sliding surfaces for twisting, super-twisting, and adaptive algorithms has been used for design of SOSMC to improve performance of second-order process with delay time. In SOSMC, existence of sliding mode has

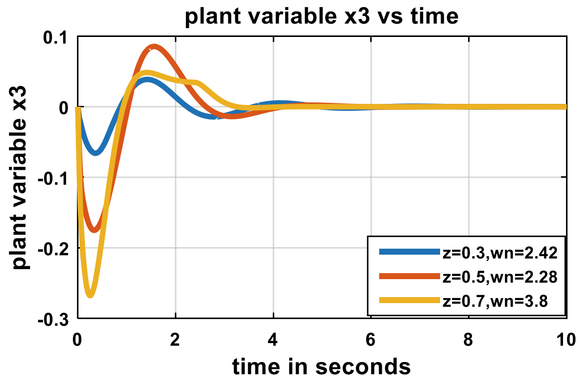


Fig. 16 Plant variable $x3$ versus time under parameter variation

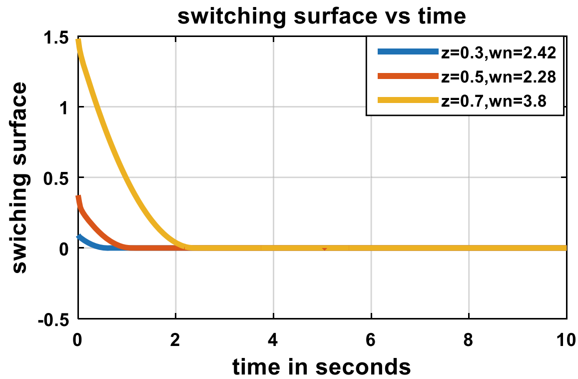


Fig. 17 s versus time under parameter variation

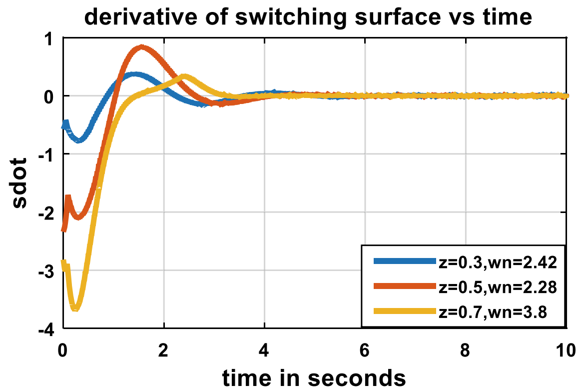


Fig. 18 $s$dot versus time under parameter variation

been proved for SOPDT systems with significant improvement of performances. The proposed methods are able to achieve low overshoot and small settling time simultaneously. Also robust and high performance control can be achieved with the above-explained algorithms of SOSMC. From simulation results, we can observe that SOSMC gives control avoiding chattering phenomena. Thus, the used SOSMC algorithms provide an improvement in transient performance along with steady-state accuracy with smooth control efforts and settling time are improved effectively.

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