Chapter 9
Meaningful Mathematics Talk That Supports Mathematics Learning in Singapore Secondary Schools

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Abstract A wide variety of talk may occur within a mathematics lesson, but the mere presence of talk does not ensure that understanding follows—only meaningful mathematics talk can enhance learning. Talk may be used to convey meaning or to generate meaning. There is evidence to suggest that conceptual understanding is more likely to be associated with dialogic talk than with univocal discourse. We can examine mathematics talk from the perspectives of the teacher (teaching talk) and the students (learning talk) according to Alexander's dialogic teaching framework. Teaching episodes illustrate the kinds of mathematics talk (univocal and dialogic) enacted in the interactions between an experienced and competent teacher and his students. They show how the teacher uses the students' talk to generate meaning for both himself and the students, creates the learning moment by using students' responses as thinking devices, and thus provides opportunities for students to construct their own knowledge. The implications for mathematics teachers in Singapore secondary schools are discussed, as we acknowledge the reality of a teacher's classroom, which includes the competing demands of depth versus breadth in content coverage, students' differing abilities and interests, and time constraints.

Keywords Mathematics talk · Univocal · Dialogic · Thinking device · Generate meaning

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9.1 Introduction

The role of talk as central to knowledge building in mathematics classrooms has been recognised for many decades, and there have been a number of research that study the role of talk in supporting learning (e.g. Weaver, Dick, & Rigelman, 2005). Analyses of classroom talk have identified the provision of opportunity for students to voice and share ideas as an important component of learning that yields higher level of conceptual exchanges and leads to more robust learning (Alexander, 2004). However, the mere presence of talk does not ensure that understanding follows – only meaningful mathematics talk can enhance learning. The quality and type of talk are crucial to helping students think conceptually about mathematics (Kazemi & Stipek, 2009; Lampert, Blunk, & Pea, 1998; Nathan & Knuth, 2003; Van Zoest & Enyart, 1998).

The teacher's role is critical in how mathematics talk plays out in a mathematics classroom, and research reveals that teachers' instructional practices often give students little opportunity to talk, discuss, conjecture, reason, and justify. The Kassel project in 1995 on general features of mathematics instruction in Singapore classrooms reported that teachers "presented knowledge to the pupils as a class by telling and explaining" (Kaur, 1999, p. 195). The Learner's Perspective Study (LPS) in 2005 also revealed that "teachers played the most active role in expounding mathematical concepts and problem-solving skills" (Kaur, 2009, p. 340) and the most common interaction pattern was the initiation-response-feedback (IRF) discourse format (Sinclair & Coulthard, 1992) where the teacher asked a question, students responded and teacher gave feedback. In their study of nature of teacher questions (performative, procedural, and conceptual), Hogan, Rahim, Chan, Kwek, and Towndrow (2012) also noted that the prevalence of mundane IRF talk structure and that a substantial proportion of performative questions eventually lead on to procedural and explanatory talks, thus suggesting that Singapore mathematics classrooms provide limited opportunities for students to engage in rich classroom conversations.

9.2 Meaningful Mathematics Talk

Most research on mathematics talk anchor on two perspectives on teaching and learning: Vygotskian, and constructivism and socio-constructivism. A Vygotskian viewpoint suggests that teaching is beneficial when it "awakens and rouses to life those functions which are in a stage of maturing, which lie in the zone of proximal development" (Gallimore & Tharp, 1990, p. 177), and learning occurs when assistance is provided at opportune points in the learner's zone of proximal development. Thus, in a mathematics-talk learning classroom, both the teacher and students move through their own learning zones of proximal development as they assist one another in a recursive process of talking.

Constructivism suggests that students make sense of their learning by relating new information or ways of understanding to existing ideas or ways of thinking, and hence, actively constructs new understanding. Piaget and Inhelder (1969) pointed that new knowledge and experience can be *assimilated* when they fit comfortably into our existing schema; but when new ideas do not fit, we are forced to *accommodate* them by changing our schema, and that we sometimes resist. As cited in Atwood, Turnbull, and Carpendale (2010), "Piaget considered cooperative interaction especially conducive to learning because within conditions of cooperation individuals are more likely to share their perspectives with others, perspectives that can be questioned, affirmed, or revised" (p. 359), and Chapman's (1991) reconstruction of Piagetian theory supported that "the experience of interpersonal argumentation provides children with the need and the occasion to justify their assertions, ideally with arguments that have force even for persons who do not share the same perspectives" (p. 220). In other words, learning in schools is a social activity and the discussion of learning moves from the individual to the group.

This implies the need to set up a learning environment that encourages students to relate new ideas to existing ones in order to modify them, and together develop knowledge as a co-constructed activity of all classroom members, constituted in and through talk. Douglas Barnes (1992) advocated the idea that coming to terms with new knowledge requires working on understanding, which can most readily be achieved through talk because "the flexibility of speech makes it easy for us to try out new ways of arranging what we know, and easy also to change them if they seem inadequate" (Barnes, 2008, p. 5). According to him, two kinds of talk, exploratory and presentational, contribute to learning but each has a different place in the sequence of lessons.

There are many types of mathematics talk. In a research by Oregon Mathematics Leadership Institute (OMLI) that addressed the research question: *Can student achievement in mathematics be significantly improved by increasing the quantity and quality of meaningful mathematical discourse in mathematics classrooms?*, the team developed a Classroom Observation Protocol, specific to student talk. In this protocol, they define 9 types of discourse (see Fig. 9.1).

These types represent a continuum of the mathematics discourse desired in mathematics classrooms where students are thinking and talking about mathematics. The order of the discourse types represents the continuum of discourse in terms of increasing levels of cognitive demand. That is, giving a short right or wrong answer to a direct question represents the lowest level of cognitive demand and justifying mathematical ideas and procedures and making generalisations represent the highest levels.

According to Lotman (1988), talk may be used to convey meaning or to generate meaning. Wertsch (1991) used the term univocal and dialogic, respectively to represent these two functions. In a univocal talk, the listener receives the "exact" message that the speaker intends for the listener to receive, and once the speaker's intention has been conveyed, the talk ends. In contrast, in a dialogic talk, there is a give-and-take communication that extends beyond the conveyance of an exact message

Level	Definition	Explanation
1	Answering	A student gives a short answer to a direct question from the teacher or another student.
2	Making a Statement or Sharing	A student makes a simple statement or assertion, or shares his or her work with others and the statement or sharing does not involve an explanation of how or why. For example, a student reads what she wrote in her journal to the class.
3	Explaining	A student explains a mathematical idea or procedure by stating a description of what he or she did, or how he or she solved a problem, but the explanation does not provide any justification of the validity of the idea or procedure.
4	Questioning	A student asks a question to clarify his or her understanding of a mathematical idea or procedure.
5	Challenging	A student makes a statement or asks a question in a way that challenges the validity of a mathematical idea or procedure. The statement may include a counter example. A challenge requires someone else to reevaluate his or her thinking.
6	Relating	A student makes a statement indicating that he or she has made a connection or sees a relationship to some prior knowledge or experience.
7	Predicting or Conjecturing	A student makes a prediction or a conjecture based on their understanding of the mathematics behind the problem. For example, a student may recognize a pattern in a sequence of numbers or make a prediction about what might come next in the sequence or state a hypothesis a mathematical property they observe in the problem.
8	Justifying	A student provides a justification for the validity of a mathematical idea or procedure by providing an explanation of the thinking that led him or her to the idea or procedure. The justification may be in defense of the idea challenged by the teacher or another student.
9	Generalizing	A student makes a statement that is evidence of a shift from a specific example to the general case.

Fig. 9.1 OMLI Classroom Observation Protocol for student talk (Weaver et al., 2005)

leading to generation of meaning through dialogue as a "thinking device" (Lotman, 1988). There is evidence to suggest that conceptual understanding is more likely to be associated with dialogic talk than with univocal discourse (Knuth & Peressini, 2001; Wertsch & Toma, 1995).

Robin Alexander (2004) proposed that a different type of 'talk' is required within the classroom to stimulate students' thinking and learning, and he developed a pedagogical approach to classroom teaching known as 'dialogic teaching'. Dialogic teaching is teaching based on more equal dialogue between teachers and students and among students themselves. The principles of dialogic teaching provide a framework to develop purposeful and authentic learning activities. According to Alexander (2004), dialogic teaching harnesses the power of talk to stimulate and extend students' thinking, and advance their learning and understanding as students' talk is used as a thinking device. It helps the teacher more precisely to diagnose students' needs, frame their learning tasks, and assess their progress.

Dialogic teaching is not just any talk. It is as distinct from the question-answer and listen-tell routines of traditional teaching as it is from the casual conversation of informal discussion. Dialogic teaching draws on a broad repertoire of strategies and techniques—talk for everyday life, learning talk, teaching talk, and classroom organisation.

Students in dialogic classrooms do not just provide brief factual answers to 'test' or 'recall' type of questions, or answers that they think the teacher wants to hear. Instead they are engaged in a spectrum of strategies specific to learning (known as learning talk)—narrate, explain, analyse, speculate, imagine, explore, evaluate, discuss, argue, justify, and even ask questions of their own (Alexander, 2010). While Alexander did not provide further descriptions or explanations of these talk strategies in the literature, the following descriptors are used in our identification of learning talks occurred during the teaching episodes:

- Narrate: mere telling
- Explain: making an idea clear by providing more details
- Analyse: examine information in detail so as to explain and interpret it
- Speculate: predicting an outcome based on information provided
- Imagine: forming a supposition (of some idea not actually present)
- Explore: developing a concept through an investigation or finding alternatives
- Evaluate: forming an assessment or a judgement
- Discuss: talking about a topic in detail, taking into account different ideas
- Argue: exchanging or providing different views, with reasons in support
- Justify: showing or proving to be right or reasonable
- Ask questions of their own: (self-explained).

In Alexander's dialogic teaching framework (2010), the spectrum of talk strategies specific to teaching (known as teaching talk) are:

- Rote: the drilling of facts, ideas, and routines through constant repetition;
- Recitation: the accumulation of knowledge and understanding through questions designed to test or stimulate recall of what has been previously encountered, or to cue pupils to work out the answer from clues provided in the question;
- Instruction/Exposition: telling the pupil what to do, and/or imparting information, and/or explaining facts, principles or procedures;
- Discussion: the exchange of ideas with a view to sharing information or solving problems; and
- Dialogue: achieving common understanding through structure, cumulative questioning and discussion which guide and prompt, reduce choices, minimise risk and error, and expedite 'handover' of concepts and principles.

According to Alexander (2010), rote, recitation, instruction, and exposition are frequently used, and they are probably the default modes of teaching talk. While there is always a place for these talk strategies, discussion and dialogue, which are less common, are what students need to experience much more frequently. By using discussion and dialogue, students do not merely listen and answer, but are empowered both cognitively and socially to think, engage, and take decisions about their learning.

Dialogic teaching requires interactions that encourage students to think, and to think in different ways; questions which invite much more than simple recall; answers which are justified, followed up and built upon rather than merely received; feedback which informs and leads thinking forward as well as encourages; contributions which are extended rather than fragmented; exchanges which chain together into coherent and deepening lines of enquiry; discussion and argumentation which probe and challenge rather than unquestioningly accept; professional engagement with subject matter which liberates classroom discourse from the safe and conventional; and classroom organisation, climate, and relationships which make all this possible (Alexander, 2010). Using Alexander's dialogic teaching framework, we can examine mathematics talk from the perspectives of the teacher (teaching talk) and the students (learning talk).

Teaching episodes in the next section will illustrate the kinds of mathematics talk (univocal and dialogic) enacted in the interactions between an experienced and competent teacher and his students. The various talk strategies specific to teaching and learning talks in Alexander's dialogic teaching framework are identified in these teaching episodes to illustrate how the teacher uses the students' talk to generate meaning for both himself and the students, creates the learning moment by using students' responses as thinking devices, and thus provides opportunities for students to construct their own knowledge.

9.3 Mathematics Talk Enacted by an Experienced and Competent Teacher

The teacher in focus is Teacher 27. An experienced and competent mathematics teacher, he is the Head of Mathematics Department in his school. He is in the age range of 40–49 years with 20–25 years of mathematics teaching experience. The lessons of Teacher 27 were selected for study of mathematics talk as they represented a comprehensive range of mathematics talk that was present in the lessons of the 30 experienced and competent teachers who participated in Phase 1 of the project. Teacher 27 taught his secondary 4 class, of 17 students in the Express course of study, the topic of Vectors that spanned 495 min of instruction time over a period of 8 lessons. In this section, the three teaching episodes illustrate the kinds of mathematics talk (teaching and learning talks) enacted in the interactions between Teacher 27 (T) and his students (S). The goal of the lesson was to develop student understanding of vectors and representations.

Episode 9.1

Line	Teaching episode	Teaching/learning talks
(1)	T: You may have heard of vectors when you're studying physics. Now, can you give me an example of what you already studied in physics which you understand as vectors? What did you already know about vectors?	Recitation

(continued)

(continued)

Line	Teaching episode	Teaching/learning talks
(2)	S: Gravity	Narrate
(3)	T: Gravity, what else? I'm not going to correct you now. I'm just letting you tell me what you understand about vectors. Tell me what else you know about vectors	
(4)	S: Got direction	Narrate
(5)	T: So vectors have direction. Is that correct? [Students nod.] Give me an example	Rote Exposition
(6)	S: Velocity	Narrate
(7)	T: Velocity. Does velocity have direction?	Exposition
(8)	S: Yes	
(9)	T: If I run towards Sean [pointing at Sean] at a speed of 4 km/h from here. Then I ask Hadi to run towards Sean also at a speed of 4 km/h from there [pointing at Hadi], are the two of us travelling at the same velocity? Hadi, let us run towards Sean now [Both T and Hadi move towards Sean.] We are both running towards Sean but are we running in the same direction?	Exposition
(10)	S: No. Towards the same direction, yes. Ay? So same direction?	Narrate
(11)	T: We are both running TOWARDS Sean but are we running in the same direction? I don't know. Yes or no, I'm not sure. You discuss	
	[Students discuss among themselves.]	
(12)	T: So, are Hadi and I running in the same direction?	
(13)	S: No	
(14)	T: We are not running in the same direction. Why?	Exposition
(15)	S: Because one person is pointed this way and the other person is pointed the other way	Explain
(16)	T: So we are running in different directions although we are both running at the same speed	Rote

Up to line 16 in Episode 9.1, the mathematics talk enacted is primarily univocal because the teacher's intention is to convey the message that vectors have directions. Teacher 27 ensures that his intended message for this lesson is adequately conveyed by using a live demonstration of two persons running at the same speed but in different directions. His focus thus far is on how well everyone understands his perspective rather than on making sense of the students'. The teaching talks invoked are Rote, Recitation, and Instruction/Exposition; while the learning talks invoked are Narrate and Explain.

However, the following Episode 9.2 reveals how the teacher carries on to leverage students' responses (Line 10 in Episode 9.1) as generators of meaning, illustrating the essence of a dialogic mathematics talk. The teaching talks invoked are still Recitation and Exposition; while the learning talks have included Justify.

Episode 9.2

Line	Teaching episode	Teaching/learning talks
(17)	T: But some of you have this idea that we're running in the same direction because we are running towards the same person? How to disprove that? How can we show that both of us are not running in the same direction?	Recitation
(18)	S: Bearings	Narrate
(19)	T: Bearings? Can you show me how?	Exposition
(20)	S: This is Sean. [S draws a point.] Teacher is running towards Sean in this direction. [S draws an arrow to the point.] Hadi is also running towards Sean in this direction. [S draws another arrow to the point.] North is in this direction. [S draws another arrow to denote North.] We measure the bearing of the two of you from Sean. So we can see that the two bearings are not the same	Justify

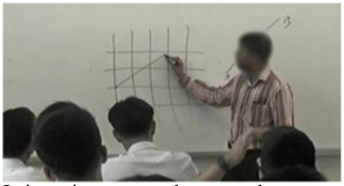
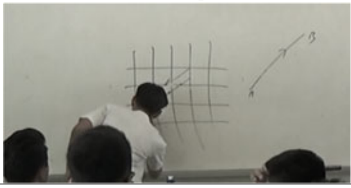
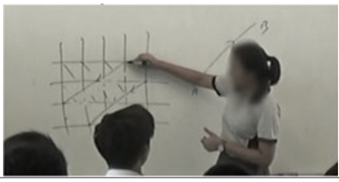
The following Episode 9.3 further illustrates a mathematics talk that embodies the dialogic characteristics.

Episode 9.3

Line	Teaching episode	Teaching/learning talks
(21)	T: Let's work in pairs. I want Partner A to draw any vector and label it $\vec{AB}$. Now Partner B, how are you going to draw a vector $\vec{PQ}$ such that $\vec{AB} = \vec{PQ}$, that is, to replicate exactly the same vector your partner has drawn? [Students discuss in pairs.]	Instruction
(22)	S: Use a protractor	Narrate
(23)	T: How to use a protractor to draw another vector that is equal to this vector?	Exposition
(24)	S: Draw another vector of same length and is parallel.	Narrate
(25)	T: So how do we make sure the two vectors are parallel?	Exposition
(26)	S: Use bearing. I use the protractor to measure the bearing like this	Explain
(27)	T: If you do not have a protractor, then how? Is there another way?	Dialogue
(28)	S: Use tracing paper	Imagine
(29)	T: That's a good idea. What if I make it difficult for you and say cannot use tracing paper? What other paper will you use?	Dialogue

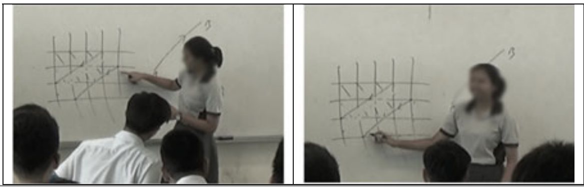
(continued)

(continued)

Line	Teaching episode	Teaching/learning talks
(30)	S: I draw horizontal lines like lines in the exercise book. I also draw vertical lines. Then I count how many lines and then draw my vector like this	Explore
(31)	T: Oh, that’s smart! So if the vector is drawn on grid like this [T shows a vector drawn on grid]  Is it easier now to draw another vector that is equal to this? Shane, show us how. [S draws a vector on the board.] 	Discussion
(32)	T: How do the others ensure that the lengths of the two vectors are the same?	Dialogue
(33)	S: Use the boxes	Analyse
(34)	T: How to use the boxes?	Dialogue
(35)	S: The vector you draw is between 6 boxes, so you find another 6 boxes and draw the vector	Explain
(36)	T: I don’t quite understand what you’re saying. Can anyone help to explain?	Dialogue
(37)	S: The vector cuts across these 6 boxes. [S points the 6 boxes.] So I copy and draw my vector that cuts same 6 boxes like this. [S points the other 6 boxes.] 	Explain
(38)	T: Is everyone convinced that the two vectors are equal? How are you so sure that they are equal?	Dialogue

(continued)

(continued)

Line	Teaching episode	Teaching/learning talks
(39)	S: The vector is from here to here. [S points at initial and terminal points of the vector.] I start from here, it goes down by 2 boxes and then goes left by 3 boxes 	Analyse
(40)	T: So how are you counting?	Dialogue
(41)	S: Vertically and horizontally	Analyse
(42)	T: So if each box is a unit, the vector represents a movement of 3 units to the left and 2 units down	Rote
(43)	T: How can we express this vector in a form that represent 3 units to the left and 2 units down?	Discussion

Again, Teacher 27 first attempts to see/hear what the students understand of equal vectors and uses the students' talk to generate meaning for both himself and the students. He creates the learning moment by using student's response (line 30) to incept the idea of representing a vector horizontally and vertically. Rather than telling the class directly how a vector can be represented in a column vector, Teacher 27 turns to the whole class for inquiry and discussion. He prompts students to use peers' responses as thinking devices and provides opportunities for students to construct their own knowledge.

Teaching Episode 9.3 is primarily dialogic. A significant mark of Teacher 27's classroom is the degree to which the students took ownership of the learning situation. The student-generated responses that emerged during the lesson encouraged dialogues and discussions in a productive manner. The teacher encourages students to build ideas on the basis of one another's insights. Student collaboration is evident as students attempt to refine one another's ideas, help one another explain, and verify one another's claims. Teacher 27 does not attempt to convey a particular message by engaging his students in a specific approach. Instead, he is open to his students' ideas and allows his students to pursue approaches, that may be quite unexpected to him, to generate new mathematical understanding, and this is the essence of a dialogic mathematics talk.

9.4 Mathematics Talk in the Classrooms of Mathematics Teachers in General

As part of the survey, 677 teachers reflected on their lessons for a specific course of study—Integrated Programme (IP), Express, Normal (Academic) (N(A)), or Normal

(Technical) (N(T)), and indicated the frequency of their use of the kinds of teaching talk. Chapter 2 provides details about the different courses of study in Singapore secondary schools and also details of the survey. The aggregated data is shown in Table 9.1.

From the data in Table 9.1, we see that about 60% or less of the teachers for the IP course but 80% or more of the teachers for the Express/N(A)/N(T) courses frequently or mostly/always draw on Rote and Recitation; about 55% of the teachers for the IP course and more than 65% of the teachers for the other courses frequently or mostly/always draw on Instruction/Exposition; 80% or more of the teachers for the IP course and approximately 60–75% of the teachers in the other courses frequently or mostly/always draw on discussion and dialogue. It is apparent that teachers for the IP course draw less on the basic repertoire of teaching talk (rote, recitation, and instruction/exposition) but more on the larger oral repertoire (discussion and dialogue) as their students are of higher learning ability. Nonetheless, it is encouraging to see that more than 50% of the teachers for the other courses are also harnessing the power of dialogic teaching talk to engage students, stimulate and extend their thinking, and advance their learning and understanding.

In the survey, teachers were also asked to reflect on the kinds of learning talk they engaged their students in and indicate the frequencies. Table 9.2 shows the aggregated data.

The data in Table 9.2 informs the use of the basic repertoire of learning talk (Narrate and Explain). Less than 60% of teachers for all the courses frequently or mostly/always engage their students in Narrate; and about 70% of the teachers for all the courses, except N(T), frequently or mostly/always engage their students in Explain. However, on the use of the larger oral repertoire of learning talks, the teachers for the IP course have provided more opportunities for their students to develop their repertoire of learning talk (Speculate, Explore, Analyse, Evaluate, Discuss, Argue, Justify, and Question). In fact, 55% or less of the teachers for the Express/N(A)/N(T) courses frequently or mostly/always engage their students in learning talks, such as Explore, Evaluate, Discuss (except for Express), and Justify.

9.5 Conclusion

The distinction between univocal and dialogic mathematics talks is at times difficult to discern. A mathematics talk can be a continuum between univocal and dialogic. Both univocal and dialogic can be appropriate forms of mathematics talk, depending on the instructional goals. However, instances of meaningful mathematics talk in which students are actively engaged in and are transforming one other's thinking are rare. Some challenges teachers faced when orchestrating meaningful mathematics talk include supporting students to make contributions that are productive to further the dialogue (Heaton, 2000; Staples, 2007); managing the mathematical direction that the mathematics talk takes (Jaworski, 1994; Sherin, 2002a; Silver & Smith, 1996);

Table 9.1 Use of teaching talk

Kind of talk/sample survey item	Course of study	% of respondents			
		Never/rarely	Sometimes	Frequently	Mostly/always
Rote • I use "I do, We do, You do" strategy • I emphasise basic facts/steps for students to memorise them • I provide students with sufficient questions to practise so as to develop procedural fluency • I ask students to recall past knowledge • I get students to automatise steps leading to a solution through repetitive exercises	IP	9.7	27.9	37.2	25.2
	Express	2.2	17.2	46.7	33.9
	N(A)	1.6	11.5	52.6	34.3
	N(T)	1.1	6.6	49.3	43.0
	All	2.5	15.5	47.6	34.4
Recitation • I ask direct questions to stimulate students' recall of past knowledge/check for understanding of concepts being developed in the lesson • I provide students with directed guidance (close-ended questions) when they face difficulty with a mathematical task they are doing, focussing them on the concept/skill necessary to do the task	IP	4.3	35.3	44.0	16.4
	Express	1.3	18.7	54.5	25.5
	N(A)	0.0	15.2	56.6	28.1
	N(T)	0.6	14.2	54.5	30.7
	All	1.2	18.8	54.1	26.0
Instruction/Exposition • I use the textbooks to introduce concepts/skills • I use exposition (teacher at the front talking to whole class) to explain mathematical ideas • I tell students how to do it when they face difficulty with a mathematical task they are doing • I explain what exemplary solutions of mathematics problems must contain • I ask questions to encourage reasoning, not just to elicit right answers	IP	15.9	27.6	41.4	15.2
	Express	6.1	27.1	45.5	21.4
	N(A)	4.5	29.4	50.2	15.9
	N(T)	4.1	29.1	43.6	23.2
	All	6.3	27.9	45.9	19.9

(continued)

Table 9.1 (continued)

Kind of talk/sample survey item	Course of study	% of respondents			
		Never/rarely	Sometimes	Frequently	Mostly/always
Discussion • I use examples and non-examples to engage students in discussion to make sense of a concept • I focus on mathematical processes (such as compare and contrast, logical reasoning) to facilitate the development of concepts or student understanding • I lead whole class discussion (with guided questions) to facilitate the development of concepts	IP	0.0	15.5	56.9	27.6
	Express	1.4	21.1	55.1	22.4
	N(A)	2.2	24.5	55.2	18.1
	N(T)	2.3	26.9	47.0	23.9
	All	1.6	22.2	54.2	22.1
Dialogue • I exchange ideas with students on how to solve a problem • I ask students open-ended questions and allow them to build on one another's responses to develop concepts or clarify their understanding • I build on students' responses rather than merely receiving them	IP	2.3	18.4	58.6	20.7
	Express	2.5	31.1	49.6	16.8
	N(A)	3.1	37.1	47.9	11.9
	N(T)	3.4	31.4	50.4	14.8
	All	2.7	31.4	50.1	15.8

Note Due to rounding errors the percentages do not always add up to 100

Table 9.2 Use of learning talk

Kind of talk/sample survey item	Course of study	% of respondents			
		Never/rarely	Sometimes	Frequently	Mostly/always
Narrate • I get my students to provide answers or solutions (without any explanation) to my questions • I get my students to practise a similar problem after I have shown them how to do it on the board • I get my students to state/list what they have learnt at the beginning/end of the lesson	IP	18.4	35.6	36.2	9.8
	Express	16.0	30.3	34.8	18.9
	N(A)	17.4	28.0	33.8	20.8
	N(T)	17.0	24.2	31.4	27.3
	All	16.6	29.4	34.3	19.6
Explain • I get my students to explain how their solutions or how their answers are obtained • I get my students to teach/explain to another classmate while doing individual assigned seatwork • I get my students to explain how they would correct an error or a misconception that I have put on the board	IP	2.3	27.6	54.6	15.5
	Express	2.1	26.7	55.4	15.8
	N(A)	3.8	29.4	54.1	12.8
	N(T)	23.1	50.4	24.2	2.3
	All	2.5	27.0	54.4	16.1
Speculate • I get my students to predict outcomes	IP	1.7	12.1	69.0	17.2
	Express	2.6	32.4	47.9	17.1
	N(A)	5.3	32.5	51.7	10.6
	N(T)	11.4	30.7	47.7	10.2
	All	4.3	30.4	50.5	14.8
Explore • I get my students to develop concepts through exploratory/investigate activities • I get my students to work collaboratively as a group on mathematical tasks and discuss the solution method before presenting to the whole class • I get my students to explore alternative solution methods for a problem besides the one I have shown on the board	IP	2.9	40.8	48.9	7.5
	Express	6.3	49.4	36.3	8.0
	N(A)	8.6	52.1	34.2	5.1
	N(T)	5.7	43.9	40.5	9.8
	All	6.5	48.5	37.5	7.5

(continued)

Table 9.2 (continued)

Kind of talk/sample survey item	Course of study	% of respondents			
		Never/rarely	Sometimes	Frequently	Mostly/always
Analyse • I get my students to analyse why a procedure (that I have shown on the board) works or why a solution method makes sense • I get my students to review their mistakes and identify possible causes by themselves	IP	0.9	19.8	62.1	17.2
	Express	2.1	25.4	56.6	15.9
	N(A)	4.6	29.8	52.0	13.6
	N(T)	2.8	23.9	52.3	21.0
	All	2.7	25.7	55.5	16.2
Evaluate • I get my students to compare multiple procedures/solution methods I have shown on the board and evaluate the efficiency, appropriateness, ease of use, or other advantages/disadvantages • I get my students to critique one another's work presented on board so as to improve their understanding of concepts or elegance in their presentation/solution	IP	2.6	29.3	50.9	17.2
	Express	3.8	40.4	43.3	12.5
	N(A)	6.3	43.7	41.7	8.3
	N(T)	4.5	42.0	41.5	11.9
	All	4.4	40.4	43.4	11.9
Discuss • I get my students to develop concepts together with me through class discussion • I get my students to discuss features of a problem that I have shown on the board	IP	1.7	25.9	62.9	9.5
	Express	4.5	36.6	47.2	11.7
	N(A)	5.3	45.0	45.0	4.6
	N(T)	8.5	41.5	36.4	13.6
	All	4.9	38.2	46.7	10.2
Argue • I get my students to offer alternative solution method(s) to a problem I have shown on the board	IP	0.0	24.1	56.9	19.0
	Express	0.3	33.7	53.2	12.9
	N(A)	3.3	47.7	41.7	7.3
	N(T)	1.1	37.5	45.8	15.9
	All	1.0	36.5	49.9	12.6

(continued)

Table 9.2 (continued)

Kind of talk/sample survey item	Course of study	% of respondents			
		Never/rarely	Sometimes	Frequently	Mostly/always
Justify • I get my students to justify why their solution(s) to a problem is different from the one I have put on the board • I get my students to defend and explain to classmate(s) why their approach/method to solve a problem is better	IP	4.3	41.4	49.1	5.2
	Express	8.0	39.1	42.9	10.0
	N(A)	11.6	40.7	43.4	4.3
	N(T)	12.5	33.5	41.5	12.5
	All	9.1	38.9	43.4	8.6
Question • I get my students to ask questions when they do not understand • I get my students to ask self/classmate questions to check their understanding • I get my students to ask questions (such as "what if") to probe further or for deeper understanding	IP	4.6	23.6	52.3	19.5
	Express	5.1	29.1	46.8	19.0
	N(A)	7.3	30.9	45.9	15.9
	N(T)	5.7	25.0	45.8	23.5
	All	5.6	28.5	46.9	19.0

Note Due to rounding errors the percentages do not always add up to 100 maintaining a 'common ground' which enables all students to follow the mathematical direction and to contribute appropriately (Staples, 2007); respecting the students' claims that are mathematically incorrect while trying to transform them and support the development of appropriate mathematical ideas (Chazan & Ball, 1999; Staples, 2007); seeing beyond one's own long-held and taken-for-granted mathematical ideas in order to hear and work with students' ideas (Heaton, 2000); creating appropriate norms for talking and interacting in the classroom (Cobb, 2000; Lampert, 2001); and most crucially, the teachers' sense of efficacy in anticipating and preparing for their role in instruction (Sherin, 2002b; Smith, 1996, 2000).

We also have to acknowledge the reality of a teacher's classroom, which includes the competing demands of depth versus breadth in content coverage, the students' differing abilities and interests, and time constraints. These factors often influence the learning goals, which in turn influence the kinds of mathematics talk. Thus, extensive mathematics talk may not be included in everyday lessons. Nonetheless, we encourage teachers to refrain from telling too much but to probe for students' ideas. Our data reveals that the mathematics talks enacted in our Singapore classrooms are often straddling between univocal and dialogic such that no one kind of talk is dominant over a significant period of time in a lesson. The two groups of univocal and dialogic talks are not mutually exclusive, and all kinds of talks have their place. We encourage teachers to continue to strive to engage the students in more dialogic mathematics talks so that they can acquire a deeper understanding of mathematics when they use their own responses, as well as those of their peers and teacher, as thinking devices.

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