

Chapter 15

Use of Technology by Experienced and Competent Mathematics Teachers in Singapore Secondary Schools



Joseph B. W. Yeo

Abstract This chapter reports how 30 experienced and competent Singapore mathematics teachers used technology inside and outside their classrooms. The first ICT (Information and Communication Technology) Masterplan in Education for Singapore was launched in 1997 to provide some comprehensive strategies to harness ICT for teaching and learning. Since then, there were quite a number of local research studies on the use of ICT in mathematics teaching. However, in 2019, it was noted that research interest in this area had dwindled after 2004. Therefore, this chapter provides a timely update on whether Singapore mathematics teachers still use technology in their teaching and if yes, how. The video recording of 209 lessons of the 30 teachers were analysed and it was found that 23 of them made use of ICT in various ways. The most common modes were the use of technology as a tool for students to explore and discover mathematics and as teacher aids to project various resources onto the screen. Fewer teachers made use of computer-assisted instructions (CAI) on the Internet. Teachers could emulate the practices of the 30 teachers to harness the affordances of technology as a tool for students to construct their own knowledge through interactive investigative activities with the help of graphing or dynamic geometry software.

Keyword Technology · ICT · Tool mode · Tutor mode · Investigation

15.1 Introduction

The first ICT (Information and Communication Technology) Masterplan in Education for Singapore was launched by the Ministry of Education (MOE) in 1997 to build a strong foundation to harness ICT for teaching and learning (MOE, 2020a). It called for core ICT training for all teachers, development of ICT infrastructure and support for all schools, and production of educational software and resources

J. B. W. Yeo (✉)

National Institute of Education, Nanyang Technological University, Singapore, Singapore
e-mail: josephbw.yeo@nie.edu.sg

© Springer Nature Singapore Pte Ltd. 2021

B. Kaur et al. (eds.), *Mathematics Instructional Practices in Singapore Secondary Schools*,
Mathematics Education – An Asian Perspective,
https://doi.org/10.1007/978-981-15-8956-0_15

303

for relevant subjects. It was during this period that MOE bought the dynamic geometry software, the Geometer's Sketchpad (GSP), for every school to use for their mathematics lessons.

The second ICT Masterplan in Education was unveiled in 2002 (for the period 2003–2008) and it built on the first ICT Masterplan to seed innovation (MOE, 2020b). It called for a more pervasive use of ICT as a tool to customise education to meet the needs and abilities of students so as to support and develop lifelong learners. Some key priorities of the masterplan were to set baseline standards for students' learning experiences and teachers' ICT integration practises, give greater autonomy to schools to take full ownership of their ICT implementation through devolved ICT funds, and generate innovative practices through more recognition schemes.

The third ICT Masterplan in Education was launched in 2008 (for the period 2009–2014) to strengthen and scale up the use of ICT in the first two masterplans to harness ICT to transform learners (MOE, 2020c). The enabler goals were for school leaders to provide direction and create conditions to harness ICT for teaching and learning, for teachers to have the capacity to plan and deliver ICT-enriched learning experiences, and for the ICT infrastructure to support learning anytime and anywhere. The outcome goals were for students to develop competencies for self-directed and collaborative learning through the effective use of ICT as well as become discerning and responsible ICT users.

The fourth ICT Masterplan in Education (for the period 2015-present) built on the first three masterplans to deepen learning and sharpen practices in order to prepare students to be future-ready and responsible digital learners in the twenty-first century (MOE, 2020d). This was also in line with MOE's direction towards a values-based and student-centric education in 2012 (Kaur, 2019; Natarajan, Lim, & Cheah, 2018). The implementation of the masterplan focused on four areas: deeper integration of ICT in curriculum, assessment and pedagogy; sustained professional learning among school teams and learning communities; translational research, innovation and scaling in ICT-enabled pedagogies and practices; and a connected ICT learning ecosystem in terms of both physical and socio-cultural infrastructure.

A common theme among all the four ICT Masterplans is still the encouragement and strengthening of the effective use of technology in teaching and learning for the development of students. Since the first ICT Masterplan in Education was introduced in 1997, there was a proliferation of local research on the use of ICT in mathematics teaching, peaking in 2002 and started to dwindle after 2004 (Ng, Teo, Yeo, Ho, & Teo, 2019). Therefore, we only had a sense of what Singapore mathematics teachers were doing in the infusion of ICT in their classrooms before 2004, and after that, there was a dearth of local research. Hence, this research project on how 30 experienced and competent Singapore mathematics teachers enacted the school curriculum was timely because it provided an insight into how these teachers harnessed technology inside and outside their classrooms for teaching and learning the subject. Unlike other chapters which reported the findings from the survey of 677 mathematics teachers, it was beyond the scope of the same survey instrument to ask these teachers how they use ICT in their teaching.

Thus the research questions addressed in this chapter are:

- (1) How many of the 30 experienced and competent teachers leveraged on technology in their lessons?
- (2) How do the experienced and competent teachers infuse technology in their teaching?

15.2 Literature Review

Taylor (1980) classified the use of the computer in the school during the 1960s and 1970s into three modes: tutor, tool and tutee. In the tutor mode, the computer is the tutor and so students learn *from* the computer. This involves using computer-assisted instruction (CAI) to tutor and drill pupils in procedural skills. In the tool mode, the computer is the tool and so students learn *with* the computer. In those days, this involved using application software such as *LOGO* to explore mathematical ideas. Designed by Seymour Papert and others in 1967, the *LOGO* software consists of a turtle and the user explores geometrical concepts by moving the turtle around. In the tutee mode, the computer is the tutee and so students learn *through* teaching (e.g. programming) the computer. There are generally two approaches to the tutee mode. The first approach is to learn a programming language. But Jensen and Williams (1993) believed it was unwise to spend so much curriculum time studying a programming language that is constantly changing. The second approach is to learn some simple programming, like *LOGO*, and then programme the turtle to move around to explore mathematical ideas. Therefore, *LOGO* programming to explore concepts involves both the tutee and the tool modes.

In the 1990s, Jensen and Williams (1993) and Manoucherhri (1999) observed that the use of CAI to teach students mathematics was the most frequently used mode since the 1970s. With the advance of ICT when technology is no longer confined to just a desktop computer but there are handheld instruments (such as graphing calculators, tablets and smart phones), Internet and social media, are Taylor's three modes still relevant today? If one were to search the Internet for mathematics resources, one would find that most of the websites just contain non-interactive reading materials or videos of a person teaching. Lagrange, Artigue, Laborde, and Trouche (2003) called these resources "CAI on the Internet" (p. 255), which they "assumed to be the actual technological engagement of an average student, at least in developed countries" (ibid.). Even for flipped classrooms where students learn the materials before going to class for discussion, those materials are usually in the form of notes, PowerPoint slides or videos of the teacher teaching (Ng et al., 2019). All of these uses of technology are just Taylor's tutor mode. This suggests that the tutor mode is still very much prevalent even today.

However, many mathematics researchers seem to advocate the use of the tool mode. For example, Kaput (1992), in the *Handbook of Research on Mathematics Teaching and Learning*, talked about using the computer as a tool for exploration, e.g. virtual manipulative, dynamic geometry environments such as CABRI Geometry

and the Geometer's Sketchpad (GSP), and probability, statistics and data modelling. Zbiek, Heid, Blume, and Dick (2007), in the *Second Handbook of Research on Mathematics Teaching and Learning*, wrote about the use of cognitive technological tools, such as microworlds, simulations and representational toolkits (which include graphing calculators, spreadsheets and computer algebra system), for exploring and constructing mathematical ideas.

In the same vein, Balacheff and Kaput (1996), in the *International Handbook of Mathematics Education*, wrote about computer-based learning environments in mathematics, such as microworlds, computer algebra systems (CAS), dynamic geometry software and mathematical and statistical modelling. Lagrange et al. (2003), in the *Second International Handbook of Mathematics Education*, found that most of the research on the use of ICT in mathematics education from 1994 to 1998 tended to focus on dynamic computer software or symbolic calculators, rather than on CAI on the Internet or on CD-ROM. In the *Third International Handbook of Mathematics Education*, there are four whole chapters dedicated to the use of ICT as a tool for modelling (Williams & Goos, 2013) and exploration in geometry (Sinclair & Robutti, 2013), algebra (Heid, Thomas, & Zbiek, 2013) and statistics (Biehler, Ben-Zvi, Bakker, & Makar, 2013). Some of the computer tools used are TI-Nspire (a CAS), GSP and ThinkerPlots (an interactive statistical software). Even in the chapter in the same handbook on learning with the use of the Internet, Borba, Clarkson, and Gadanidis (2013) stated that they "have chosen not to report on studies that are predominantly text based and/or use rapid response modes aimed mainly at testing students' abilities" (p. 700). Instead, they would rather "report on studies that seem to push the boundaries of how the Internet can be used creatively and with worth in mathematics education" (ibid.): all the examples given use some kinds of tools to explore, and nothing on CAI.

Based on the review of literature, it seems that there exists a dichotomy of how researchers would use technology as a tool to explore mathematics and how teachers and students actually use technology as the tutor via CAI. This is understandable because researchers usually prefer students to construct their own knowledge through investigative activities, according to the theory of constructivism (Ernest, 1994; von Glasersfeld, 1990) while direct instruction via CAI may not be so helpful to do so. It will be interesting to find out which mode is favoured by experienced and competent mathematics teachers in Singapore.

15.3 Research Design

The research design for the collection of data reported in this chapter has been outlined in Chapter 2. In this section, I will briefly describe how the data were analysed to answer the research questions for this chapter. The 209 lessons of the 30 experienced and competent teachers were examined to pick up episodes of the teacher teaching mathematics with the help of technology in the classroom. Because some teachers

told their students to view their pre-recorded lessons at home or made reference to answering queries from their students through WhatsApp after school, the use of technology by the teachers also included those done outside the classroom. I will now present the findings.

15.4 Findings and Discussion

It was discovered that most of the 30 experienced and competent mathematics teachers infused technology inside and outside the classrooms using the modes described in Table 15.1. Out of the 30 teachers, 4 taught the Integrated Programme (IP), 10 taught the Express course, 8 taught the Normal (Academic) (N(A)) course and the remaining 8 taught the Normal (Technical) (N(T)) course. The reader can refer to Chapter 2 for a more detailed description of these four courses of study, but the abilities of the students are generally higher for the IP course than students in the

Table 15.1 Use of technology by the 30 experienced and competent teachers

Instructional Approach	Number (and Percentage) of Teachers				
	IP (<i>n</i> = 4)	Express (<i>n</i> = 10)	N(A) (<i>n</i> = 8)	N(T) (<i>n</i> = 8)	Total (<i>N</i> = 30)
Tutor mode: direct-instruction videos in YouTube or e-learning portals; or self-recorded direct-instruction videos uploaded onto YouTube	1 (25%)	2 (20%)	2 (25%)	0 (0%)	5 (16.7%)
Tool mode: exploration using interactive software or online templates such as Geometer's Sketchpad, GeoGebra, Desmos, TI-Nspire, Excel and algebra discs; or apps such as a sound meter	4 (100%)	3 (30%)	2 (25%)	2 (25%)	11 (36.7%)
Engagement mode: amusing videos to engage the hearts of students	0 (0%)	1 (10%)	0 (0%)	1 (12.5%)	2 (6.7%)
Assessment mode: practice and quizzes using software such as Kahoot, e-learning portals or school-created websites	0 (0%)	1 (10%)	0 (0%)	2 (25%)	3 (10%)
Teacher aids: projection of textbooks, e-books, PowerPoint slides, teacher's notes, questions and/or student work onto a screen with the help of laptop, iPads/Apple TV or visualiser (i.e. document camera)	1 (25%)	4 (40%)	2 (25%)	4 (50%)	11 (36.7%)
Student aids: to seek help for homework outside curriculum time, e.g. through WhatsApp	0 (0%)	1 (10%)	1 (12.5%)	0 (0%)	2 (6.7%)
No evident use of technology	0 (0%)	3 (30%)	1 (12.5%)	3 (37.5%)	7 (23.3%)

Express course, which in turn are higher than those in the N(A) course; while the abilities of the students in the N(T) course are generally the lowest.

From Table 15.1, we observed that 23 of the 30 teachers (76.7%) made use of technology in one way or another while 7 teachers (23.3%) did not use any form of technology at all. Of the 23 teachers who made use of technology, 10 of them used more than one mode (not shown in Table 15.1). If we exclude the 3 teachers who used only the visualiser and/or PowerPoint slides, then only 20 of the 30 teachers (66.7%) made use of more modern technology in or outside the classrooms. As far as teaching and learning with the help of technology is concerned, 5 teachers (16.7%) used the tutor mode and 11 teachers (36.7%) used the tool mode, out of which 2 of them (6.7%) utilised both tutor and tool modes. In other words, a total of 14 teachers (46.7%) used either the tutor or tool mode or both. None of the teachers used the tutee mode for teaching and learning.

As for the 7 teachers who did not use any form of technology, the topics taught by them at the time of the video recording were: differentiation (Teacher 10 and Teacher 28), vectors (Teacher 27), volume and surface area of prisms and cylinders (Teacher 14), trigonometric ratios of acute angles (Teacher 25), bearings and 3-dimensional problems using trigonometry (Teacher 15), and simultaneous linear equations (Teacher 4). Some of these topics did not lend themselves to the use of ICT or there may be other equally effective pedagogy (such as the use of concrete manipulative) to teach them, which may explain why these teachers did not engage in technology just for the sake of using ICT when it does not enhance student learning.

On closer analysis, all the 4 IP teachers (100%) used the tool mode (with one of them using the tutor mode to show some YouTube videos as well) while 25–30% of the teachers in each of the other 3 streams used the tool mode (with one Express teacher using the tutor mode as well). It seems that the tool mode is more popular with IP teachers, who teach high-progress learners. But it does not mean that teachers teaching the other three courses of study does not use the tool mode.

In addition, 11 of the 30 teachers (36.7%) used technology as an aid for themselves, mainly to project various resources and/or student work onto the screen via visualisers or laptops (all classrooms in Singapore have visualisers, or what some countries call document cameras; and all teachers are issued with a laptop). Only 3 teachers (10%) used technology for assessment, 2 teachers (6.7%) used it outside curriculum time for students to seek help for homework, and 2 teachers (6.7%) used it to engage the hearts of the students (by showing interesting mathematics videos). Let us now examine in more details what the teachers did for each of the various modes.

15.4.1 Tutor Mode

With the advance of the Internet and social media, the tutor mode does not change. Instead of installing a program or using a CD-ROM with CAI, teachers in Singapore can now use direct-instruction videos in YouTube or in an e-learning portal which their school has to subscribe, for their students to view and learn the contents. Among

Table 15.2 Use of tutor mode by 5 experienced and competent teachers

Types	Topics	No. of teachers (Course of Study)
Existing YouTube videos	<ul style="list-style-type: none"> • Parts of a circle • Laws of logarithm, mathematical constant e and natural logarithm 	2 (1 N(A) and 1 IP)
Existing videos in an e-learning portal	<ul style="list-style-type: none"> • Hypotenuse of a triangle • Applications of trigonometry 	2 (1 Express and 1 N(A))
Self-recorded videos posted on YouTube	<ul style="list-style-type: none"> • Proofs in plane geometry 	1 (1 Express)

the 5 teachers who utilised the tutor mode, 2 of them used a video from YouTube to teach their students topics like parts of a circle (Teacher 11), the laws of logarithm, the mathematical constant e and natural logarithm (Teacher 12) (see Table 15.2). Another two teachers made use of a video in an e-learning portal to teach their students topics such as the hypotenuse of a triangle (Teacher 6) and applications of trigonometry (Teacher 19). Teacher 3 recorded himself teaching how to prove some geometrical properties and posted the series of videos on YouTube.

Most of the videos were viewed in the classrooms or computer labs, with the exception of the YouTube video on the mathematical constant e and natural logarithm, and some of the self-recorded videos on proofs in plane geometry, which the teachers expected their students to view at home. According to the annual survey on infocomm usage in household and by individuals for 2019, nearly 100% of resident households with school-going children have broadband (i.e. high-speed) Internet access at home, so it is not an issue for students to view some of these videos at home (Infocomm Media Development Authority, 2019, p. 8).

In other words, the Internet and social media did not change the tutor mode but they only make it easier for the tutor mode to be implemented by making the videos more readily available, and they also provide a platform for teachers to post their self-recorded direct-instruction videos for easy access by the students.

15.4.2 Tool Mode

The Internet also makes the tool mode more easily accessible. Before the World Wide Web was able to host interactive templates, mathematics software such as Geometer's Sketchpad (GSP) or Graphmatica had to be installed in the teacher's and students' desktop computers, and pre-designed templates or files had to be uploaded onto the computers. With the advance of technology, teachers and students can now use interactive online software or templates such as GeoGebra and Desmos without any pre-installation and they can use them on handheld devices such as iPad or mobile phones without having to go to a computer lab in the school. Secondary school students are not allowed to use graphing calculators in examinations, so they usually

do not have access to graphing calculators for such purpose. Most of them will use their own mobile phones or the teacher will loan school iPads for them to use. There was also a plan by MOE for every student to get a laptop or tablet by 2028 (Ang, 2020), but this plan has been brought forward to 2021 (Ong, 2020) due to the need for home-based learning during the circuit breaker (or lockdown) to stem the spread of the coronavirus Covid-19.

Among the 11 teachers who used the Tool mode, almost half of them (i.e. 5 teachers) used a graphing software, such as Desmos and TI-Nspire, for students to investigate the properties of graphs of linear functions (Teacher 26), quadratic functions (Teacher 13 and Teacher 21) and logarithmic functions (Teacher 12), as well as the relationships between the sine and cosine ratios of acute and obtuse angles (Teacher 17) (see Table 15.3). An almost equal number of teachers (i.e. 4 teachers) made use of a dynamic geometry software, such as the Geometer's Sketchpad (GSP) and GeoGebra, for students to explore and discover angle properties of circles (Teacher 5), Pythagoras' Theorem (Teacher 6 and Teacher 24) and Cosine Rule (Teacher 17). One of the teachers, Teacher 20, utilised a statistical software, namely Excel, as a tool to compile students' experiment results of tossing a coin and compute the experimental probability of obtaining a Head. Two teachers tapped on an online interactive applet for different uses: one of them used an applet as a visualising tool to help students observe how many surfaces a triangular prism has (Teacher 23); the other, Teacher 18, used the AlgeDisc™ application in AlgeTools™ to explore the balancing of an equation using algebra discs (AlgeTools™ was created by the Ministry of Education of Singapore and has since been decommissioned). One of the teachers, Teacher 12, also used a sound meter as a tool for students to measure the sound intensity of certain activities, such as normal breathing, soft whisper and classroom noise. Out of the 11 teachers, two of them used more than one tool: Desmos and GSP, or Desmos and sound meter.

Figure 15.1 shows a Desmos template used by Teacher 17 in the IP for students to explore the relationship between the sine and cosine ratios of acute and obtuse angles (the tangent ratio of obtuse angles is not in the syllabus because students only need sine and cosine ratios of obtuse angles when applying sine rule and cosine rule respectively). Because the circle is a unit circle, the coordinates of the point on the circle are $(\cos a, \sin a)$. The slider for the angle a allows the user to drag and change the value of a .

Figure 15.2 shows a GSP template used by the same teacher, Teacher 17, for her class to explore and discover cosine rule. The students would click and move each of the vertices of the triangle, and the measures of the angles and lengths of the triangle would change automatically and instantaneously. The values in the table would also change accordingly. Students would then observe that certain values in the table would always be equal regardless of how the triangle was changed. This would lead them to discover the cosine rule.

To summarise, most of the 11 teachers used either a graphing software or a dynamic geometry software. Three of the 11 teachers (Teacher 18, Teacher 23 and Teacher 26) used the tool purely for teacher demonstration while 7 of them (Teacher 5, Teacher 6, Teacher 12, Teacher 13, Teacher 20, Teacher 21 and Teacher 24) let their

Table 15.3 Use of tool mode by 11 experienced and competent teachers

Types	Topics	No. of teachers (Course of Study)
Graphing Software (e.g. Desmos, TI-Nspire)	<ul style="list-style-type: none"> Investigate properties of graphs of linear functions Investigate properties of graphs of quadratic functions Investigate characteristics of graphs of logarithmic functions Investigate relationships between the sine and cosine ratios of acute and obtuse angles 	5 (4 IP and 1 N(A))
Dynamic Geometry Software (e.g. Geometer's Sketchpad, GeoGebra)	<ul style="list-style-type: none"> Investigate angle properties of circles Investigate to discover Pythagoras' Theorem Investigate to discover Cosine Rule 	4 (1 IP, 2 Express and 1 N(T))
Statistical Software (e.g. Excel)	<ul style="list-style-type: none"> Teacher used Excel as a tool to compile students' experiment results of tossing a coin and compute experimental probability of obtaining a Head 	1 (1 Express)
Online Interactive Applets	<ul style="list-style-type: none"> IAs a visualising tool for students to observe how many surfaces a triangular prism has 	1 (1 N(T))
AlgeDisc™ application in AlgeTools™	<ul style="list-style-type: none"> This is an online interactive tool using algebra discs to balance an equation (it has since been decommissioned) 	1 (1 N(A))
Sound Metre	<ul style="list-style-type: none"> Measure sound intensity of certain activities such as normal breathing, soft whisper and classroom noise 	1 (1 IP)

students use the software to investigate the mathematics. The last teacher, Teacher 17, did both: on two occasions, it was purely teacher demonstration; on another occasion, she let the students used GSP to explore cosine rule. Student-centred investigation was done either in the computer lab (by Teacher 5 and Teacher 6), or in the classroom with laptops or iPads provided by the school. Sometimes, the students had to use their own mobile devices for such activities.

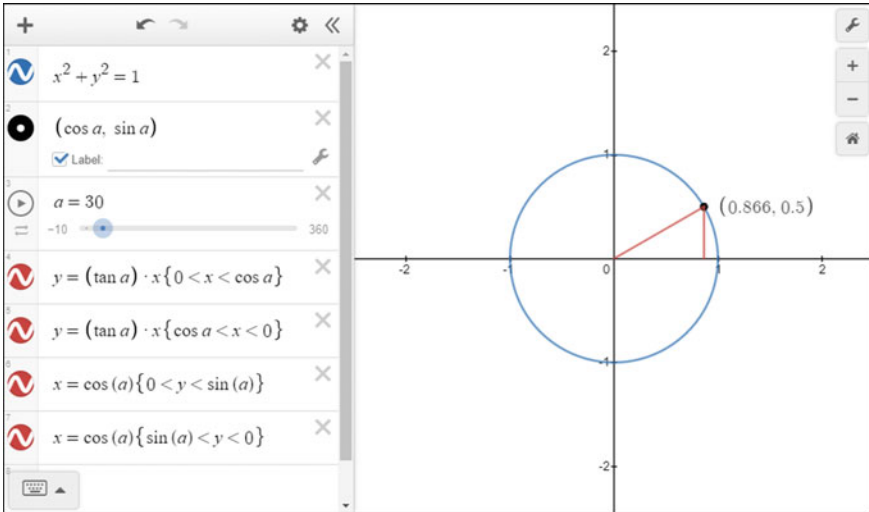


Fig. 15.1 Teacher 17’s Desmos template for exploring the sine and cosine ratios of acute and obtuse angles

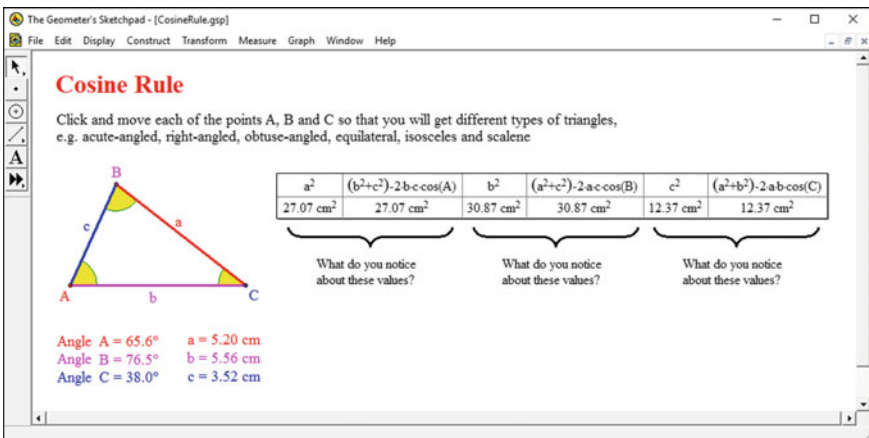


Fig. 15.2 Teacher 17’s GSP template for students to explore and discover cosine rule

15.4.3 Engagement Mode

Two of the teachers, Teacher 9 and Teacher 6, did not just use videos to teach students mathematics but they use them to engage their hearts and to arouse their interest. Teacher 9 screened a video from YouTube in class on the Great Pyramid of Giza: the video did not teach students any mathematics but just provided some information about the pyramid such as its history and its dimensions. The teacher hoped to use

this real-life example of a pyramid to motivate her Secondary 4 N(T) students to learn more about finding the volume and surface area of a pyramid.

Teacher 6 showed her Secondary 2 Express class a 10-minute video containing snippets of a Korean drama (with English subtitles) about a girl who managed to travel back in time to ancient Korea and helped the king solve a mathematics problem using Pythagoras' theorem. The whole class found the drama funny because of the slapstick humour and situational jokes, but there was really nothing much about the theorem. However, the teacher designed three problems with contexts that continued the storyline in the drama for her students to solve in class using Pythagoras' theorem. In other words, the teacher used the Korean drama to arouse the interest of her students to solve word problems involving the application of Pythagoras' theorem. For more details on the three problems and the outcome, please refer to Chapter 7 in this book.

15.4.4 Assessment Mode

Three teachers, Teacher 7, Teacher 20 and Teacher 30, used ICT for assessment. Teacher 7 used Kahoot, an app that allows students to answer multiple choice questions, to assess his Secondary 4 N(T) students' learning of the three types of averages, namely mean, median and mode. Teacher 20 used an e-learning platform which the school has subscribed, for his Secondary 2 Express students to take an online quiz on probability over the weekend. Teacher 30 asked her Secondary 1 N(T) class to take some quizzes on angles from an online portal in class using laptops loaned from the school.

15.4.5 Teachers' Aids

A significant proportion of the 30 teachers (11 teachers or 36.7%) also used ICT as aids for themselves. They were Teacher 1, Teacher 2, Teacher 3, Teacher 7, Teacher 8, Teacher 9, Teacher 16, Teacher 21, Teacher 22, Teacher 23 and Teacher 24. The main usage was to project various resources (e.g. textbooks, e-books, PowerPoint slides, teacher's notes, questions and/or student work) onto the screen with the help of their laptop/iPad or visualiser (i.e. document camera) so that their students could view them.

15.4.6 Students' Aids

Two of the teachers, Teacher 8 and Teacher 29, also used ICT as a means for students to seek their help outside curriculum time through the use of WhatsApp. This leverage of technology was not possible before the invention and proliferation of social media.

15.5 Conclusion

Slightly more than 75% of the 30 experienced and competent Singapore mathematics teachers harnessed the use of technology in their teaching in numerous ways. The most common modes were the use of ICT as a tool for students to learn mathematics through investigation (36.7%), and as teacher aids to project various resources onto the screen for students to see (36.7%). One sixth (or 16.7%) of the teachers also used ICT in the tutor mode for direct instruction. Across the four courses of study, IP teachers seem to favour the tool mode over the tutor mode more than the other teachers, although there were teachers in the other three courses who also used the tool mode. Technology may advance but for mathematics, it seems that the tool mode and the tutor mode do not change much. Also, these experienced and competent local teachers are using the tool mode as advocated by many researchers (as discussed in Sect. 15.2) more than the use of the tutor mode. Therefore, Singapore teachers could follow the examples of the 30 teachers to harness the affordances of technology as a tool for students to construct their own knowledge through interactive investigative activities with the help of a suitable graphing or dynamic geometry software.

References

- Ang, J. (2020, March). Parliament: All secondary school students to have personal digital devices by 2028, \$200 Edusave top-up to support purchase. *The Straits Times*. <https://www.straitstimes.com/politics/parliament-all-secondary-school-students-to-have-personal-digital-devices-by-2028-200>.
- Balacheff, N., & Kaput, J. J. (1996). Computer-based learning environments in mathematics. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 469–501). Dordrecht, The Netherlands: Kluwer Academic Press.
- Biehler, R., Ben-Zvi, D., Bakker, A., & Makar, K. (2013). Technology for enhancing statistical reasoning at the school level. In M. A. Clements, A. J. Bishop, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *Third international handbook of mathematics education* (pp. 643–689). New York: Springer.
- Borba, M. C., Clarkson, P., & Gadanidis, G. (2013). Learning with the use of the Internet. In M. A. Clements, A. J. Bishop, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *Third international handbook of mathematics education* (pp. 691–720). New York: Springer.
- Ernest, P. (1994). Social constructivism and the psychology of mathematics education. In P. Ernest (Ed.), *Constructing mathematical knowledge: Epistemology and mathematical education* (pp. 62–72). London: The Falmer Press.
- Heid, M. K., Thomas, M. O. J., & Zbiek, R. M. (2013). How might computer algebra systems change the role of algebra in the school curriculum? In M. A. Clements, A. J. Bishop, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *Third international handbook of mathematics education* (pp. 597–641). New York: Springer.
- Infocomm Media Development Authority. (2019). *Annual survey on infocomm usage in household and by individuals for 2019*. https://www.imda.gov.sg/-/media/Imda/Files/Infocomm-Media-Landscape/Research-and-Statistics/Survey-Report/2019-HH-Public-Report_09032020.pdf?la=en.
- Jensen, R. J., & Williams, B. S. (1993). Technology: Implications for middle grades mathematics. In D. T. Owens (Ed.), *Research ideas for the classroom: Middle grades mathematics* (pp. 225–243). New York: National Council of Teachers of Mathematics & Macmillan.

- Kaput, J. J. (1992). Technology and mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 515–556). Reston, VA: National Council of Teachers of Mathematics & MacMillan.
- Kaur, B. (2019). Overview of Singapore's education system and milestones in the development of the system and school mathematics curriculum. In T. L. Toh, B. Kaur, & E. G. Tay (Eds.), *Mathematics education in Singapore* (pp. 13–33). Singapore: Springer.
- Lagrange, J. B., Artigue, M., Laborde, C., & Trouche, L. (2003). Technology and mathematics education: A multidimensional study of the evolution of research and innovation. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second international handbook of mathematics education* (pp. 237–269). Dordrecht: The Netherlands: Kluwer Academic Publishers.
- Manoucherhri, A. (1999). Computers and school mathematics reform: Implications for mathematics teacher education. *Journal of Computers in Mathematics and Science Teaching*, 18(1), 31–48.
- Ministry of Education (MOE). (2020a). *Masterplan 1*. <https://ictconnection.moe.edu.sg/masterplan-4/our-ict-journey/masterplan-1>.
- Ministry of Education (MOE). (2020b). *Masterplan 2*. <https://ictconnection.moe.edu.sg/masterplan-4/our-ict-journey/masterplan-2>.
- Ministry of Education (MOE). (2020c). *Masterplan 3*. <https://ictconnection.moe.edu.sg/masterplan-4/our-ict-journey/masterplan-3>.
- Ministry of Education (MOE). (2020d). *Masterplan 4*. <https://ictconnection.moe.edu.sg/masterplan-4>.
- Natarajan, U., Lim, K., & Cheah, H. M. (2018). *Twenty years of Thinking Schools, Learning Nation (TSLN) vision: Reflections on Singapore's ICT Masterplans* (The HEAD Foundation Working Papers Series No. 2 / 2018).
- Ng, W. L., Teo, B. C., Yeo, J. B. W., Ho, W. K., & Teo, K. M. (2019). Use of technology in mathematics education. In T. L. Toh, B. Kaur, & E. G. Tay (Eds.), *Mathematics education in Singapore* (pp. 313–348). Singapore: Springer.
- Ong, Y. K. (2020). *Opening address by Mr Ong Ye Kung, Minister for Education at the 2020 Schools and Institutes of Higher Learning Combined Workplan Seminar*. <https://www.moe.gov.sg/news/speeches/opening-address-by-mr-ong-ye-kung-minister-for-education-at-the-2020-schools-and-institutes-of-higher-learning-combined-workplan-seminar>.
- Sinclair, N., & Robutti, O. (2013). Technology and the role of proof: The case of dynamic geometry. In M. A. Clements, A. J. Bishop, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *Third international handbook of mathematics education* (pp. 571–596). New York: Springer.
- Taylor, R. (Ed.). (1980). *The computer in the school: Tutor, tool, tutee*. New York: Teachers College Press.
- von Glasersfeld, E. (1990). An exposition of constructivism: Why some like it radical. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics: Journal for Research in Mathematics Education Monograph Number 4* (pp. 19–29). Reston, VA: National Council of Teachers of Mathematics.
- Williams, J., & Goos, M. (2013). Modelling with mathematics and technologies. In M. A. Clements, A. J. Bishop, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *Third international handbook of mathematics education* (pp. 549–569). New York: Springer.
- Zbiek, R. M., Heid, M. K., Blume, G. W., & Dick, T. P. (2007). Research on technology in mathematics education. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1169–1207). Charlotte, NC: Information Age Publishing & National Council of Teachers of Mathematics.

Joseph B. W. Yeo Yeo is a Lecturer in the Mathematics and Mathematics Education Academic Group at the National Institute of Education, Nanyang Technological University, Singapore. He is the first author of the think! Mathematics textbooks (previously called New Syllabus Mathematics textbooks) used in many secondary schools in Singapore. His research interests are on innovative

pedagogies that engage the minds and hearts of mathematics learners. These include an inquiry approach to learning mathematics, ICT and motivation strategies to arouse students' interest in mathematics (e.g. catchy maths songs, amusing maths videos, witty comics and intriguing puzzles and games). He is also the Chairman of Singapore and Asian Schools Math Olympiad (SASMO) Advisory Council, and the creator of Cheryl's birthday puzzle that went viral in 2015.